Contents

Foreword 5
User Guide 7

Chapter 1: The Four Operations

Introduction 11
Warm Up: Mental Maths 13
The Order of Operations 15
Equations 17
Revision: Addition and Subtraction 19
Revision: Multiplication and Division 22
Partial Products, Part 1 25
Partial Products, Part 2 28
The Multiplication Algorithm 31
More Multiplication 36
Revision of Long Division 41
A Two-Digit Divisor 44
More Long Division 47
Division with Mental Maths 51
Long Division and Repeated Subtraction 53
Divisibility and Factors 58
More on Divisibility 61
Primes and Finding Factors 63
Prime Factorisation 66
Chapter 1 Revision 71

Chapter 2: Large Numbers and the Calculator

Introduction 75
A Little Bit of Millions 77
Exponents and Powers 80
The Place Value System 83
Multiplying Numbers by Powers of Ten 86
Adding and Subtracting Large Numbers 91
Rounding 94
The Calculator 97
When to Use the Calculator 100
Mixed Revision Chapter 2 102
Chapter 2 Revision 104

Chapter 3: Problem Solving

Introduction 107
Balance Problems and Equations 1 109
Balance Problems and Equations 2 112
Problem Solving with Bar Models 1 115
Problem Solving with Bar Models 2 118
Problem Solving with Bar Models 3 120
Problem Solving with Bar Models 4 122
Miscellaneous Problems 124
Mixed Revision Chapter 3 126
Chapter 3 Revision 128

Chapter 4: Decimals, Part 1

Introduction 131
Revision: Tenths and Hundredths 133
More Decimals: Thousandths 136
Decimals on a Number Line 141
Add and Subtract Decimals—Mental Maths 143
Add and Subtract Decimals in Columns 147
Comparing Decimals 149
Rounding Decimals 151
Multiply a Decimal by a Whole Number 153
More on Multiplying Decimals 157
More Practice and Revision 159
Divide Decimals by Whole Numbers 1 161
Divide Decimals by Whole Numbers 2 165
Mixed Revision Chapter 4 167
Chapter 4 Revision 169

Chapter 5: Graphing

Introduction 173
The Coordinate Grid 175
The Coordinate Grid, Part 2 178
Number Patterns in the Coordinate Grid 180
More Number Patterns in the Coordinate Grid 183
Line Graphs 186
Double and Triple Line Graphs 190
Average (Mean) 193
Mixed Revision Chapter 5 196
Chapter 5 Revision 199

Foreword

Math Mammoth Grade 5, South African Version, comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 5, only customised for South African use in a few aspects (listed below). Please note that the curriculum is not following the South African official syllabus for grade 5 maths. For the most part, Math Mammoth exceeds South African standards. Some standards may not be covered.

Math Mammoth South African version has been customised to South Africa in the following manners:

- Some names used are South African names (instead of Jack and Jill, there are Sipho and Sibongile).
- The currency used in word problems is rand.
- The material is all metric. In other words, the US customary measuring units are not used.
- Spelling follows British English instead of American English.
- Large numbers are formatted with a space as a thousands separator.
- The decimal separator is a comma.
- Paper size is A4.

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorising two-dimensional figures (especially triangles);

The year starts out with a study of the basic operations, some algebraic concepts, and primes and divisibility. In chapter 2, we go on to study place value, large numbers, and the usage of the calculator.

In chapter 3, students solve simple equations with the help of a pan balance. Then they learn to solve a variety of word problems using the bar model as a visual aid.

Chapter 4 is all about decimals and decimal arithmetic. Several lessons here focus on mental maths strategies based on place value.

The last chapter in this part A is on graphing. Students encounter the coordinate plane and simple number patterns that are plotted as points on the grid. They also plot and read line graphs.

In part 5-B, students study more decimal arithmetic, all fraction operations, and geometry.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don’t automatically assign all the exercises. Use your judgement, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Worktext 5-A	
Chapter 1	21 days
Chapter 2	12 days
Chapter 3	9 days
Chapter 4	18 days
Chapter 5	11 days
TOTAL	**71 days**

Worktext 5-B	
Chapter 6	20 days
Chapter 7	15 days
Chapter 8	20 days
Chapter 9	12 days
TOTAL	**67 days**

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A	88	10	178	78	2,28	11,4
5-B	92	10	188	82	2,29	11,5
Grade 5 total	180	20	366	160	2,29	11,4

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A			178			
5-B			188			
Grade 5 total			366			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on maths. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

Working space, the usage of additional paper, and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1000). Typically, I have intended that such exercises should be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue to enjoy learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: The Four Operations
Introduction

We start fifth grade by studying the four basic operations. The topics include the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought in the beginning portion of the chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students determine which expression matches the given word problem, and write simple expressions for word problems, using the correct order of operations. Thus, they are learning how to represent a situation symbolically, which is a very important step in using mathematics to solve problems.

We also briefly study the concept of an equation, and students solve simple equations in several lessons.

Next, we revise multi-digit multiplication, starting with partial products (including a geometric visualisation), and then going on to the standard multiplication algorithm with more digits than in 4th grade.

Then it is time for long division, especially practising long division with two-digit divisors. We also study why long division works, in the optional lesson *Long Division and Repeated Subtraction*. You can use the lesson as time allows and according to student interest. Throughout the lessons there are also word problems to solve.

The lessons for long multiplication often ask the student to estimate the answer before calculating. The lessons for long division ask for the student to check the answer by multiplying. Both of these methods serve the same purpose: to help them gauge whether the calculation is correct. Too often, students simply calculate something and hurry on by, without paying attention to their own work. We need to foster in them a sense of carefulness with calculations, and the habit of checking one's own work for accuracy. If necessary, assign less problems (especially similar calculations) so that students have time to think about and check their answers.

Lastly, we study the topics of divisibility, primes and factoring. Students revise or learn the common divisibility rules for 2, 3, 4, 5, 6, 9, and 10. In prime factorisation, we use factor trees. The topic of finding factors is revision from 4th grade. Prime factorisation is a new topic; it is also studied in 6th grade.

Although the chapter is named "The Four Operations," the idea is not to practise each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are developing the students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, and "undo" operations.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Warm Up: Mental Maths	13	*2 pages*	1 day	
The Order of Operations	15	*2 pages*	1 day	
Equations	17	*2 pages*	1 day	
Revision: Addition and Subtraction	19	*3 pages*	1 day	
Revision: Multiplication and Division	22	*3 pages*	1 day	
Partial Products, Part 1	25	*3 pages*	1 day	
Partial Products, Part 2	28	*3 pages*	1 day	
The Multiplication Algorithm	31	*5 pages*	2 days	
More Multiplication	36	*5 pages*	2 days	
Revision of Long Division	41	*3 pages*	1 day	

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
A Two-Digit Divisor	44	*3 pages*	1 day	
More Long Division	47	*4 pages*	1 day	
Division with Mental Maths	51	*2 pages*	1 day	
Long Division and Repeated Subtraction (optional)	53	*(5 pages)*	(2 days)	
Divisibility and Factors	58	*3 pages*	1 day	
More on Divisibility	61	*2 pages*	1 day	
Primes and Finding Factors	63	*3 pages*	1 day	
Prime Factorisation	66	*5 pages*	2 days	
Chapter 1 Revision	71	*3 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*56 pages*	21 days	
with optional content		*(61 pages)*	(23 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch1

SCAN ME

Warm-up: Mental Maths

Add in parts.	**Use rounded numbers, then correct the error.**
57 + 34 = ? Add the tens: 50 + 30 = 80. Add the ones: 7 + 4 = 11. Lastly, add the two sums: 80 + 11 = 91.	29 + 18 = ? 29 is close to 30, and 18 is close to 20. 30 + 20 = 50. But that is 3 too many, so the correct answer is 47.
Subtract in parts. 81 − 34 = ? Subtract 30 first: 81 − 30 = 51. Then subtract four: 51 − 4 = 47.	**Use rounded numbers, then correct the error.** 75 − 39 = ? 39 is close to 40, so subtract 75 − 40 = 35. You subtracted one too many, so add one to get the correct answer 36.

1. Add and subtract using the tricks explained above.

a.	b.	c.
19 + 19 = ______ 28 + 47 = ______	19 + 19 + 57 = _______ 44 + 12 + 29 = _______	100 + 200 + 2000 + 5500 = _________ 400 + 12 000 + 5000 + 320 = _________
d. 33 − 17 = ______ 81 − 47 = ______	**e.** 34 − 19 + 12 = _____ 85 − 12 + 55 = _____	**f.** 1500 − 250 − 250 = __________ 400 − 7 − 40 − 100 = __________

2. A track has four legs of different lengths: (a) 1 km 200 m, (b) 700 m, (c) 1 km 500 m, and (d) 900 m. What is the total length of the track?

Hint: "kilo" in kilometre refers to one thousand.

3. A cold front just arrived, and the temperature dropped 17 degrees. It is now 11°C. What was it before?

4. Four crates of apples weigh a total of 56 kg. The first one weighs 12 kg, the second one 15 kg, and the third one 22 kg. Find the weight of the fourth crate of apples.

5. Solve in your head.

a. 127 + _______ = 200	**b.** 250 + ________ + 300 = 760	**c.** ______ − 34 = 56

6. Multiply.

a. 20 × 6 = ________	**b.** 10 × 35 = ________	**c.** 400 × 500 = ________
200 × 6 = ________	100 × 35 = ________	60 × 80 = ________
200 × 600 = ________	20 × 100 = ________	100 × 430 = ________

7. Continue the patterns for the next five numbers.

a. 60, 120, 180, 240, ...

b. 1080, 960, 840, 720, ...

c. 130, 170, 210, 250, ...

8. Estimate the cost of buying two skirts for R269,50 and three pairs of socks for R32,90 each. (Use rounded numbers.)

Multiply part-by-part Multiply ones, tens, and hundreds separately. Add. **3 × 62 = <u>3 × 60</u> + <u>3 × 2</u>** = 186	**5 times a number** Find 10 times half of the number. 5 × 28 = **<u>10 × 14</u>** = 140.
9 times a number Find 10 times a number and subtract that number once. 9 × 55 = **<u>10 × 55 − 55</u>** = 550 − 55 = 495	**11 times a number** Find 10 times the number, and then add that number. 11 × 38 = **<u>10 × 38 + 38</u>** = 380 + 38 = 418

9. Multiply using the “tricks” explained above.

a. 5 × 26 = ________ **b.** 5 × 43 = ________ **c.** 6 × 41 = ________

d. 5 × 107 = ________ **e.** 9 × 15 = ________ **f.** 9 × 32 = ________

g. 7 × 205 = ________ **h.** 3 × 211 = ________ **i.** 11 × 25 = ________

j. 11 × 18 = ________ **k.** 4 × 32 = ________ **l.** 9 × 109 = ________

The Order of Operations

Mathematicians have decided that if there are many operations, they are to be done in a certain order. This is to prevent confusion.

1. First solve whatever is inside brackets.

Brackets mark what operations are priorities to be done first.

2. Next, solve multiplications and divisions, from left to right.

This does not mean multiplications are to be done before divisions. Instead, they are all equally important, or "on the same level". For example, in 45 ÷ 5 + 2 × 8, do both the division and the multiplication first, before the addition. (It won't matter whether you divide or multiply first.)

If there are several multiplications and divisions in a row (without addition or subtraction in between), do them from left to right. For example, in 36 ÷ 9 × 5, solve 36 ÷ 9 first.

3. Last, solve additions and subtractions, from left to right.

Again, this doesn't mean additions are done before subtractions. Instead, they are to be done from left to right. For example, in 200 − 50 + 30 + 7, solve 200 − 50 first.

1. Solve what is in the parentheses first. You can enclose the operation to be done first in a "bubble."

Example 1. (36 + 4) ÷ (5 + 5) \ / \ / = 40 ÷ 10 = 4	**a.** (50 − 2) ÷ (3 + 5)	**b.** 20 × (1 + 7 + 5)
	c. 2 × (600 ÷ 60) + (19 − 8)	**d.** 180 ÷ (13 − 7 + 3)

2. Solve. When there are several multiplications and divisions in a row, do them from left to right.

Example 2. 24 ÷ 3 × 2 ÷ 4 \ / = 8 × 2 ÷ 4 \ / = 16 ÷ 4 = 4	**a.** 36 ÷ 4 ÷ 3	**b.** 1200 ÷ 4 × 5 ÷ 3
	c. 7 × 90 ÷ 2 × 2 ÷ 10	**d.** 5 × 6 ÷ 3 ÷ 2 × 20

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A Little Bit of Millions

If you count by whole thousands... *(read the numbers aloud)*

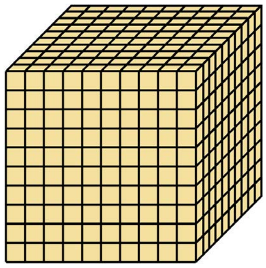
995 000
996 000
997 000
998 000
999 000

...what comes after 999 thousand?

1 000 000

A thousand thousands!
It is called
ONE MILLION.

How big is one million? You've seen a cube like this to illustrate one thousand. Now imagine that *each little cube* in it was a 1000-cube in itself.



It's a lot! It is 1000 × 1000 — a thousand copies of one thousand.

A comma separates the millions places (digits) from the rest. After the millions, the rest of the number is read just like you have learned before.

347 500 000	**19 020 000**	**5 040 326**
347 million 500 thousand	19 million 20 thousand	5 million 40 thousand 326

1. Continue the skip-counting patterns until you reach **one million**.

a.	**b.**	**c.**	**d.**
500 000	940 000	999 600	999 994
600 000	950 000	999 700	999 995

2. Write the numbers.

a. 18 million

b. 906 million

c. 2 million 400 thousand

d. 70 million 90 thousand

3. Draw lines in the numbers to separate the millions and thousands.
Fill in the missing parts. Read the numbers aloud.

a. 7 2 4 0 0 0 0 0 0 ________ million	**b.** 8 6 0 0 0 0 0 0 ________ million	**c.** 8 3 4 5 0 0 0 ________ million _______ thousand

d. 2 2 9 0 6 3 0 0 ________ million _______ thousand _______	**e.** 5 1 4 3 1 0 0 6 9 ________ million _______ thousand _______

In the following, there are NO thousands—so we don't even say the word "thousand."

f. 1 0 7 0 0 0 4 5 3 ______million ~~___thousand~~ _______	**g.** 7 2 0 0 0 0 9 0 ______million ~~___thousand~~ ________	**h.** 2 8 0 0 0 0 0 6 ______million ~~___thousand~~ ________

4. Write as numbers.

a. 41 million 400 thousand 20

b. 80 million 67

c. 5 million 6 thousand

d. 299 million 9

5. Continue the patterns.

a. 10 × 1 = ___________	**b.** 100 × 1 = ___________
10 × 10 = ___________	100 × 10 = ___________
10 × 100 = ___________	100 × 100 = ____________
10 × ________ = ___________	100 × ________ = ____________
10 × __________ = _____________	100 × __________ = ______________
10 × ____________ = _____________	100 × ____________ = ______________

6. How much is missing from one million?

a. 800 000 + _____________ = 1 million

b. 300 000 + _____________ = 1 million

c. 450 000 + _____________ = 1 million

d. 960 000 + _____________ = 1 million

e. 105 000 + _____________ = 1 million

f. 90 000 + ______________ = 1 million

7. Match.

a.

1/2 million	100 000
two hundred thousand	1 000 000
1/10 million	500 000
2 × 500 000	10 000 000
ten million	200 000

b.

1 million − 50 000	945 000
1 million − 500 000	500 000
1 million − 5000	950 000
1 million − 555 000	995 000
1 million − 55 000	445 000

8. Compare and write < or > between the numbers.

a. 6 111 050 ☐ 5 990 099	**b.** 2 223 020 ☐ 2 222 322	**c.** 192 130 659 ☐ 192 130 961
d. 18 000 000 ☐ 181 000	**e.** 13 395 090 ☐ 13 539 099	**f.** 2 367 496 ☐ 988 482
g. 6 009 056 ☐ 6 090 045	**h.** 1 000 999 ☐ 1 001 000	**i.** 17 199 066 ☐ 71 857 102

9. Find five large numbers in a newspaper or a news website with the help of an adult. Write the numbers here.

10. (*Optional*) A project with large numbers. Choose one of the options below, or one of your own. Use an encyclopaedia, the Internet, or some other source, and make a list *in descending order*—that is, from the largest number to the smallest in order:

a. of the South African provinces and their populations

b. of Asian countries and their populations

c. of the number of monthly visitors to a large amusement park

d. of the African countries and their land areas

Exponents and Powers

An exponent is used to signify repeated multiplication. For example, the expression 5^6 ("five to the sixth power") simply means we multiply number 5 by itself, repeatedly, six times:

$$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

The number 5 is called the **base**. It tells us what number we are multiplying repeatedly. The little raised number is the **exponent**, and it tells us how many times the number is repeatedly multiplied.

Example 1. 2^4 means $2 \times 2 \times 2 \times 2$. It is read as "two to the fourth power." Its value is 16.

Example 2. 9^2 means 9×9 and is commonly read as "nine squared" (think of the area of a square with side length 9). Similarly, 11^2 is read as "eleven squared". (What is its value?)

Example 3. 4^3 means $4 \times 4 \times 4$ and is commonly read as "four cubed" (because of the volume of a cube with edges 4 units). Similarly, 10^3 is read as "ten cubed". (What is its value?)

1. Write using exponents, and solve.

a. $4 \times 4 \times 4 = \square^{\square} =$ ________	**e.** five cubed = $\square^{\square} =$ ________
b. eight squared = $\square^{\square} =$ ________	**f.** two to the fifth power = $\square^{\square} =$ ________
c. $10 \times 10 \times 10 = \square^{\square} =$ ________	**g.** $3 \times 3 \times 3 \times 3 = \square^{\square} =$ ________
d. $1 \times 1 \times 1 \times 1 \times 1 = \square^{\square} =$ ________	**h.** zero to the tenth power = $\square^{\square} =$ ________

2. Multiplication is repeated addition, and a power is repeated multiplication. Compare.

a. $2 + 2 + 2 + 2 = 4 \times 2 =$ ______	**b.** $5 + 5 + 5 =$ ___ × ___ = _____
$2 \times 2 \times 2 \times 2 = \square^{\square} =$ ________	$5 \times 5 \times 5 = \square^{\square} =$ ______

3. Read the powers aloud. Then find their values.

a. $5^2 =$	**c.** $3^3 =$	**e.** $1^6 =$
b. $2^3 =$	**d.** $7^2 =$	**f.** $0^7 =$

Powers of ten are expressions where the number **10 is multiplied by itself.** For example, 100 is a power of ten because it is 10 × 10 or 10^2. Or, 100 000 is a power of ten because it is 10 multiplied by itself, five times (10^5).

4. Write these powers of ten as normal numbers. Notice there is a shortcut and a pattern!

a. 10^2 = ______________	**d.** 10^5 = ______________
b. 10^3 = ______________	**e.** 10^6 = ______________
c. 10^4 = ______________	**f.** 10^7 = ______________

SHORTCUT: In a power of ten, the exponent tells us how many ___________ the number has after the digit 1.

Example 4. Let's say a child asked you how much in total is five R100-banknotes. You would think that's easy— the total is five hundred rand! In symbols, $5 \times 10^2 = 500$.

Similarly, seven copies of (or seven times) one million equals seven million.
In symbols, 7 × 1 000 000 = 7 000 000 or 7×10^6 = 7 000 000.

5. Fill in.

a. nine copies of a hundred thousand ____ × __________ = ____________	**b.** eight copies of ten thousand ____ × __________ = ____________

c. 5×10^4 = __________	**d.** 7×10^6 = __________	**e.** 3×10^8 = __________

6. Study the patterns in these powers of ten, and fill in the missing parts.

a. 10×10^2 = *1000* $10 \times 10 \times 10^2$ = __________ $10 \times 10 \times 10 \times 10^2$ = __________	**b.** 10×10^3 = __________ = 10 100×10^3 = __________ = 10 1000×10^3 = __________ = 10
c. ______ $\times 10^3$ = 100 000 ______ $\times 10^4$ = 100 000 ______ $\times 10^4$ = 1 000 000	**d.** ______ $\times 10^5$ = 1 000 000 ______ $\times 10^5$ = 100 000 000 ______ $\times 10^3$ = 10 000 000

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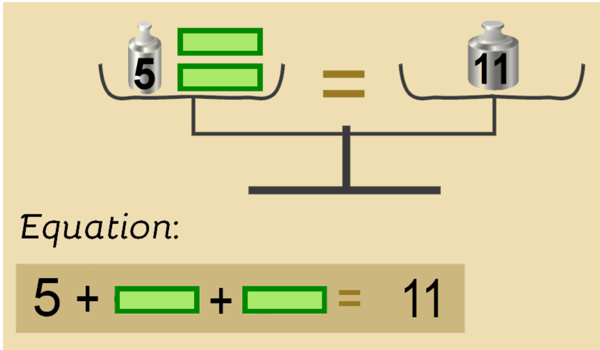
Balance Problems and Equations 1

Here you see a pan balance, or scales, and some things on both pans. Each rectangle represents an unknown (and "weighs" the same, or has the same value).

Since the balance is *balanced* (neither pan is going down—they are level with each other), the two sides (pans) of the scales weigh the same.

This portrays a mathematical equation: what is in the left pan *equals* what is in the right pan. (Things in the same pan are simply added.)



The equation is:

5 +  +  = 11

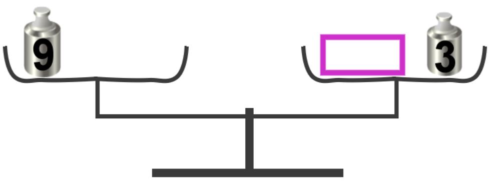
(If it helps you, you can think of kilograms.)

When we figure out how much the unknown shape weighs, we solve the equation.

The solution is:  = 3

1. Write an equation for each balance. Then use mental maths to solve how much each geometric shape "weighs." You can write a number inside each of the geometric shapes to help you.

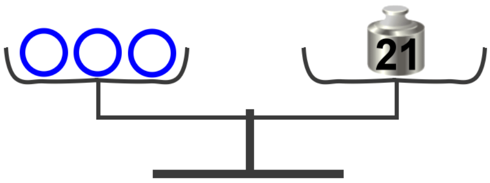
a.



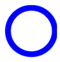
Equation: 9 =  + 3

Solution:  = 6

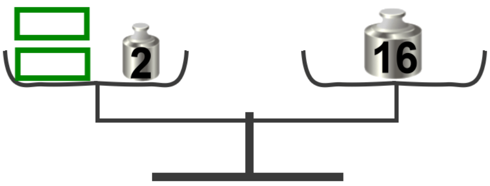
b.



Equation: ______________________

Solution:  = ________

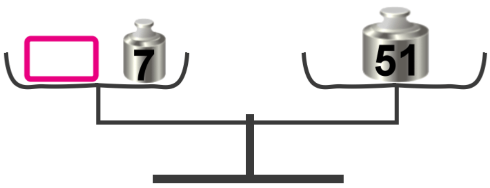
c.



Equation: ______________________

Solution:  = ________

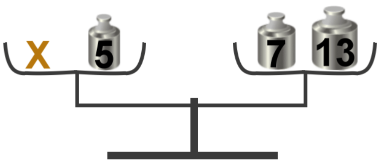
d.



Equation: ______________________

Solution:  = ________

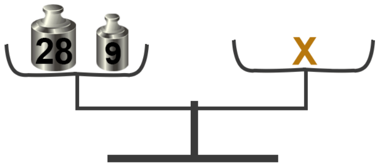
From now on we will use ***x*** for the unknown instead of a geometric shape.
It is the most commonly used letter of the alphabet to signify an unknown.



$$x + 5 = 7 + 13$$
$$x + 5 = 20$$
$$x = 15$$

Example 1. To solve this equation, first add 7 and 13 that are in the right "pan".

We get $x + 5 = 20$. The solution is easy to see now with mental maths: $x = 15$. You can also use subtraction: $x = 20 - 5 = 15$.

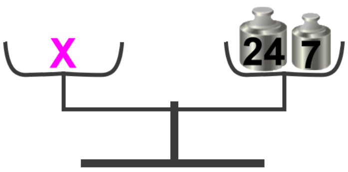


$$28 + 9 = x$$
$$37 = x$$
$$x = 37$$

Example 2. Sometimes x is on the right side of the equation. That is not a problem. In the last step you can flip the sides, so that your last line will be x = (something).

Notice that we *align the equal signs* when solving an equation. It keeps everything tidy and easy to read.

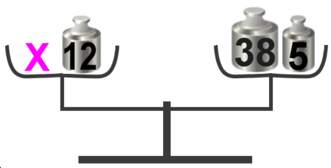
2. Write an equation. Write a second step if necessary. Lastly write what x stands for.



a.

________ = ________

x = ______



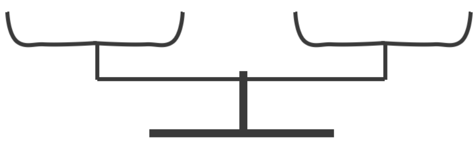
b.

________ = ________

________ = ________

x = ______

3. Draw x's and weights on the left and right sides on the two pans to match the given equation, then solve. You may not need all the empty lines provided.



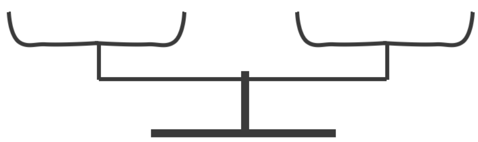
a.

$x + 18 = 5 + 31$

________ = ________

________ = ________

________ = ________



b.

$8 + 17 = 11 + x$

________ = ________

________ = ________

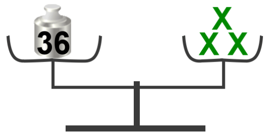
________ = ________

Whenever there are lots of x's in the same pan, use this shorthand notation:

- $x + x$ is written as $2x$. It means 2 times x.
- $x + x + x$ is written as $3x$. It means 3 times x.
- $x + x + x + x$ is written as $4x$, and so on.

We simply omit the multiplication sign between a number and a letter (such as 4 and x).

Example 3.



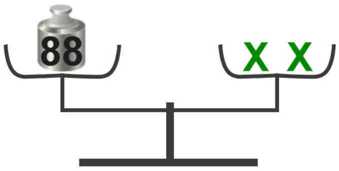
You can use *division* to solve this.

Lastly, flip the sides. →

$$36 = 3x$$
$$12 = x$$
$$x = 12$$

4. Write an equation to match the balance. Then solve what x stands for.

a.

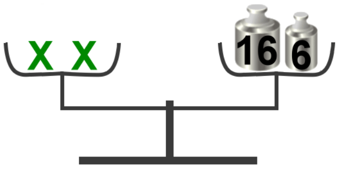


________ = ________

________ = ________

x = ________

b.

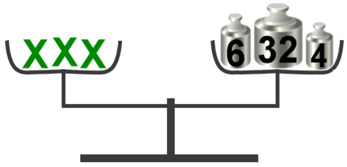


________ = ________

________ = ________

x = ________

c.



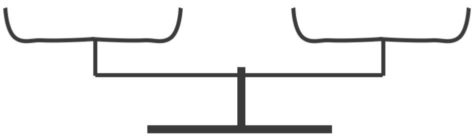
________ = ________

________ = ________

x = ________

5. Draw x's and weights on the left and right sides on the two pans to match the given equation, then solve. You may not need all the empty lines provided.

a.



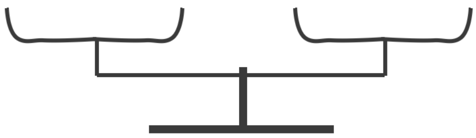
$3x = 16 + 35$

________ = ________

________ = ________

________ = ________

b.



$2 + 27 + 25 = 6x$

________ = ________

________ = ________

________ = ________

Puzzle Corner

Solve the equations.

a. $3928 + 3943 = 17x$

b. $10\,000 - 5493 - 834 - 3673 = 22x$

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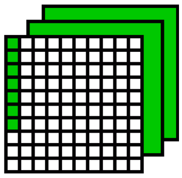
Revision: Tenths and Hundredths

Example 1. Read the decimal 2,07 as “two and seven hundredths”.

(The letters H, T, and O in the chart mark the hundreds, the tens, and the ones places. Then, “t” is for tenths and “h” is for hundredths.)

H	T	O		t	h
		2	**,**	**0**	**7**

$= 2\frac{7}{100} = \frac{207}{100} =$

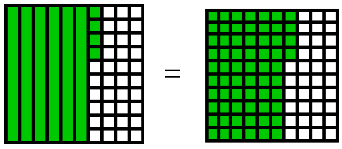


Example 2. The number 0,64 has six tenths and four hundredths. Yet, we read it as 64 hundredths. How can that be?

It is because its 6 tenths are equal to 60 hundredths. Therefore:

6 tenths + 4 hundredths
= 60 hundredths + 4 hundredths = 64 hundredths

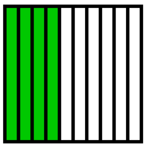
H	T	O		t	h
		0	**,**	**6**	**4**

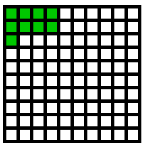


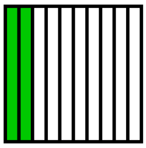
0,6 + 0,04 = 0,64

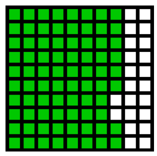
$\frac{6}{10} + \frac{4}{100} = \frac{64}{100}$

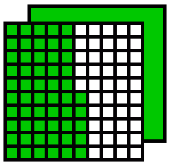
1. Write as fractions/mixed numbers and as decimals.

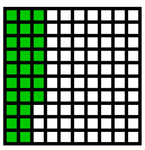
a.  = ________

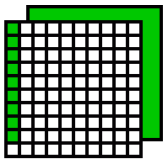
b.  = ________

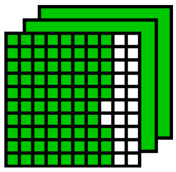
c.  = ________

d.  = ________

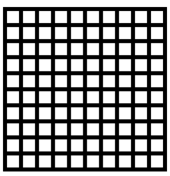
e.  = ________

f.  = ________

g.  = ________

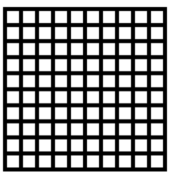
h.  = ________

2. Colour parts to show the decimals, and then write each as a single decimal number.



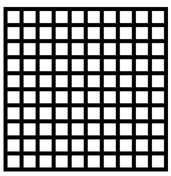
a. 32 hundredths

= __________



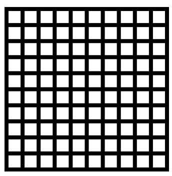
b. one tenth

= __________



c. 0,2 + 0,07

= __________



d. 0,04 + 0,6

= __________

Reading decimal numbers

- If there is one decimal digit, read it as tenths. For example, 0,9 is nine tenths.
- If there are two decimal digits, read the number formed by them as hundredths. For example, 0,38 is 38 hundredths.
- If the decimal is more than 1, read the whole-number part first, followed by the word "and." For example, 2,08 is read as "two and eight hundredths."

Also, people often just read the decimal using the word "comma" and spelling the digits one by one, saying the letter "oh" for zero. For example, 4,02 can also be read "four comma oh two."

Writing decimals as fractions

A fraction has: $\frac{\textit{numerator}}{\textit{denominator}}$

Use all the digits of the number (without the decimal comma) for the numerator. Then...

- If the decimal has ONE decimal digit, it signifies **tenths**, so use **10** as a denominator.
- If the decimal has TWO decimal digits, it signifies **hundredths**, so use **100** as a denominator.

If the decimal is more than 1, it can also be written as as a mixed number.

Examples: $9{,}1 = \frac{91}{10} = 9\frac{1}{10}$ $30{,}7 = \frac{307}{10} = 30\frac{7}{10}$ $145{,}08 = \frac{14\,508}{100} = 145\frac{8}{100}$

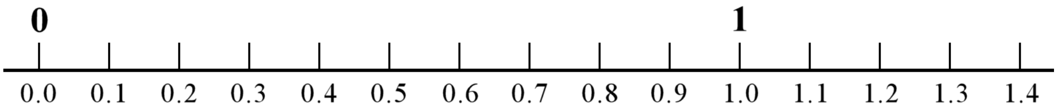
3. Write as a fraction and as a mixed number, and write (or say) how you would read each number.

	fraction	mixed number	read as ...
a. 0,45		(not applicable)	
b. 3,97			
c. 5,02			
d. 3,6			
e. 12,60			

4. Compare (write < , = , or >).

a. 0,70 ☐ 0,7	**b.** 0,03 ☐ 0,2	**c.** 1,3 ☐ 0,13	**d.** 0,50 ☐ $\frac{1}{2}$	**e.** 2,09 ☐ 2,2
f. 4,14 ☐ 4,4	**g.** 5,78 ☐ 5,7	**h.** 3,8 ☐ 3,28	**i.** 3,05 ☐ $3\frac{1}{2}$	**j.** 6,5 ☐ 6,32

5. Write an addition *or* subtraction sentence for each "number line jump," using decimals.



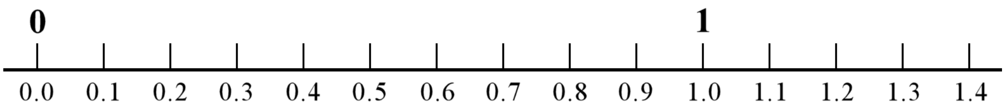
a. You are at 0,5, and you jump *six tenths* to the right. ______________________________

b. You are at 0,9, and you jump *five tenths* to the right. ______________________________

c. You are at 1,3, and you jump *eight tenths* to the left. ______________________________

d. You are at 1,3, and you jump *nine tenths* to the left. ______________________________

6. Draw number line jumps to show that 0,6 + 0,6 is *not* 0,12.



Example 3. Notice how we tag a zero to 0,2 to make it 0,20, and then it is very easy to add:

0,2 + 0,53 = 0,2<u>0</u> + 0,53 = 0,73

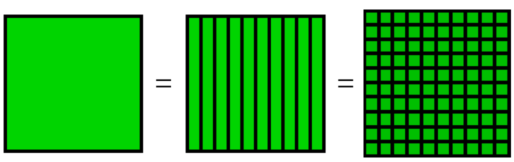
It is easy to add now because both numbers are now so many *hundredths*.

7. Add or subtract.

a. 0,5 + 0,07 = ________	**b.** 0,7 − 0,05 = ________	**c.** 0,1 + 0,71 = ________
d. 0,41 + 0,22 = ________	**e.** 0,7 + 0,5 = ________	**f.** 0,6 − 0,45 = ________

Recall that one whole is 10 tenths, and also 100 hundredths. You can use this fact to solve decimal calculations mentally.

Example 4. To solve 1 − 0,48, think of 1 as 100 hundredths. Then you have 100 hundredths − 48 hundredths, which equals 52 hundredths. So, **1 − 0,48 = 0,52**, or 1,00 − 0,48 = 0,52.



$1 = \frac{10}{10} = \frac{100}{100}$

1 = 1,0 = 1,00

8. Find the missing numbers.

a. 0,6 + ________ = 1	**b.** 0,2 + ________ = 1	**c.** 2,4 + ________ = 3	**d.** 5,8 + ________ = 6
e. 0,22 + ________ = 1	**f.** 0,52 + ________ = 1	**g.** 4,78 + ________ = 5	**h.** 3,21 + ________ = 4

More Decimals: Thousandths

The number 0,825 not only has 8 tenths and 2 hundredths, but it also has five *thousandths*. Read it as "825 thousandths."

$0{,}825 = 0{,}8 + 0{,}02 + 0{,}005$

$= \frac{8}{10} + \frac{2}{100} + \frac{5}{1000}$

O		t	h	th
0	,	8		
0	,	0	2	
0	,	0	0	5
0	**,**	**8**	**2**	**5**

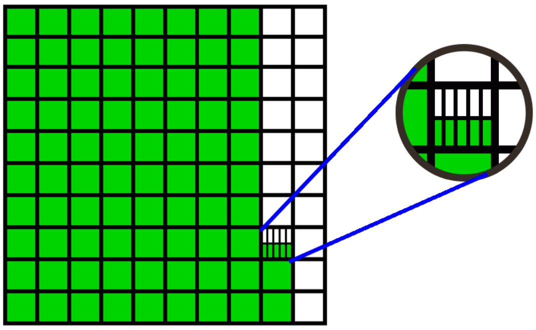
Notice the new column in the place value chart, with "th" for thousandths.

Here's a visual illustration of 0,825.

The entire square represents one whole. It is divided into a hundred parts (hundredths). Then, ONE of those hundredths is divided further into <u>ten new parts</u>, and five of those parts are shaded.

Those five tiny parts represent 0,005 or $\frac{5}{1000}$ (five thousandths).

Can you find the 8 tenths and the 2 hundredths in the illustration? (Remember, each entire column is one tenth.)



When a decimal has three decimal digits, read it as having so many *thousandths*.

$0{,}391 = \frac{391}{1000}$ = 391 thousandths	$0{,}008 = \frac{8}{1000}$ = 8 thousandths	$0{,}047 = \frac{47}{1000}$ = 47 thousandths

1. Fill in, following the model. Lastly, read the number aloud.

a. 0,06 + 0,9 + 0,001 =

O		t	h	th
0	**,**	**9**	**6**	**1**

$= \frac{961}{1000}$

b. 0,004 + 0,7 + 0,02 =

O		t	h	th
	,			

= ——

c. 0,08 + 0,2 + 0,007 =

O		t	h	th
	,			

= ——

d. 0,05 + 0,009 =

O		t	h	th
	,			

= ——

e. 0,001 + 0,06 =

O		t	h	th
	,			

= ——

f. 0,9 + 0,05 =

O		t	h	th
	,			

= ——

2. Write each number as a sum of its "parts". Some of them only have two parts.

a. 0,382 = *0,3* + *0,08* + *0,002*

b. 0,207 = ________ + ________

c. 0,639 = ______ + ________ + ________

d. 0,067 = ________ + ________

e. 0,199 = ______ + ________ + ________

f. 0,18 = ________ + ________

We can "tag" decimal zeros to the end of a decimal number, and its value will not change.
For example:

O		t	h	th
0	**.**	**7**		
0	**.**	**7**	**0**	
0	**,**	**7**	**0**	**0**

0,7 = 0,70 = 0,700

Seven tenths = 70 hundredths = 700 thousandths

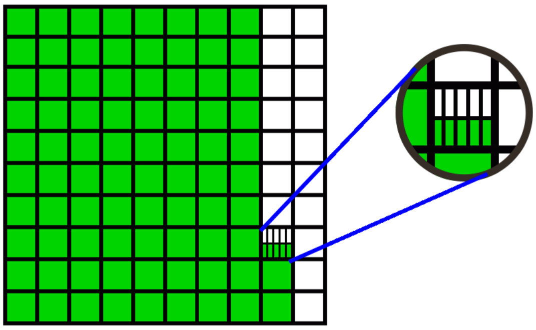
$\frac{7}{10} = \frac{70}{100} = \frac{700}{1000}$

Notice in the chart above that even though these numbers have an equal value, we will read them differently. We read them according to how many decimal digits they have.

To see this more clearly, look at the illustration. The big square represents 1 whole. One full column is **one tenth = 10 hundredths** (10 little squares).

If we divide one little square (1/100) into ten parts, those tiny parts are **<u>thousandths</u>**. Each hundredth is 10 thousandths: 1/100 = 10/1000.

Also, 1 tenth = 10 hundredths = 100 thousandths (0,1 = 0,10 = 0,100).



Can you see what number the shaded area represents?

This is why 0,825 $= \frac{8}{10} + \frac{2}{100} + \frac{5}{1000}$

$= \frac{800}{1000} + \frac{20}{1000} + \frac{5}{1000} = \frac{825}{1000}$

So in total, 0,825 is equal to 825/1000, and that is why we read it as "825 thousandths."

3. Fill in, following the model. Note: the phrase "copies of" corresponds to <u>multiplication</u>, and "makes" corresponds to the <u>equals sign</u>.

a. <u>100</u> *copies of* 1/100 *makes* one whole.

$100 \times \frac{1}{100} = 1$

b. ______ *copies of* 1/100 *makes* one tenth.

______ × ——— = ———

c. ______ *copies of* 1/1000 *makes* one hundredth.

______ × ——— = ———

d. ______ *copies of* 1/10 *makes* one whole.

______ × ——— = ______

e. ______ *copies of* 1/1000 *makes* one tenth.

______ × ——— = ———

f. ______ *copies of* 1/1000 *makes* one whole.

______ × ——— = ______

Remember, when reading a decimal number, separate the whole-number part with the word "and".

Decimal		Read as		Mixed number		Fraction
H T O , t h th / 5 6 2 , 7 8	=	562 and 78 hundredths	=	$562 \frac{78}{100}$	=	$\frac{56\,278}{100}$
H T O , t h th / 1 4 , 0 9 5	=	14 and 95 thousandths	=	$14 \frac{95}{1000}$	=	$\frac{14\,095}{1000}$

4. Write the numbers in the place value charts, and as fractions.

a. seven thousandths

O	,	t	h	th
	,			

= ——

b. 8 tenths, 2 thousandths

O	,	t	h	th
	,			

=

c. 3 and 371 thousandths

O	,	t	h	th
	,			

=

d. 26 and 39 thousandths

H	T	O	,	t	h	th
			,			

=

e. 101 and 41 hundredths

H	T	O	,	t	h	th
			,			

=

5. Write as decimals.

a. $\frac{3}{1000}$ =	**b.** $\frac{12}{1000}$ =	**c.** $\frac{319}{1000}$ =	**d.** $\frac{50}{1000}$ =
e. $42\frac{34}{1000}$ =	**f.** $2\frac{4}{1000}$ =	**g.** $9\frac{1}{100}$ =	**h.** $1\frac{80}{100}$ =

6. Write as fractions.

a. 0,42	**b.** 0,091	**c.** 3,009	**d.** 73,42
e. 0,08	**f.** 921,09	**g.** 42,5	**h.** 201,392

7. Write as mixed numbers.

a. 6,7	**b.** 10,06	**c.** 3,902	**d.** 3,005

You already know how to write a whole number in **expanded form**, such as:

$$9387 = 9 \times 1000 + 3 \times 100 + 8 \times 10 + 7 \times 1$$

We take each digit and multiply it by a power of ten (by 1, 10, 100, 1000, etc).

To write a decimal in expanded form, we do the same, writing the number as a sum, where each digit is multiplied by a power of ten. (Not only are 1000, 100, and 10 powers of ten, but 1/10, 1/100, and 1/1000 are also.)

T	O		t	h	th
6	3	,	9	2	6

$$= 6 \times 10 + 3 \times 1 + 9 \times \frac{1}{10} + 2 \times \frac{1}{100} + 6 \times \frac{1}{1000}$$

← *Think of 6 × 1/1000 as 6 copies of 1/1000. It is equal to 6/1000.*

8. Write in expanded form. Write the numbers also as fractions. Follow the example. Be careful! There are lots of details here.

a.

O		t	h	th
0	,	9	0	6

$= \frac{906}{1000}$

$= 9 \times \frac{1}{10} + 0 \times \frac{1}{100} + 6 \times \frac{1}{1000}$

b.

O		t	h	th
0	,	2	4	4

$= \frac{\square}{\square}$

$= \square \times \frac{1}{10} + \square \times \frac{1}{100} + \square \times \frac{1}{1000}$

c.

O		t	h	th
0	,	6	5	5

$= \frac{\square}{\square}$

$= \square \times \frac{1}{10} + \square \times \frac{1}{100} + \square \times \frac{1}{1000}$

d.

O		t	h	th
0	,	1	8	

$= \frac{\square}{\square}$

=

e.

O		t	h	th
0	,	8	0	2

$= \frac{\square}{\square}$

=

f.

O		t	h	th
0	,	7	1	1

$= \frac{\square}{\square}$

=

9. These numbers may have hundreds, tens, and ones, too. Write them in expanded form.

a.

T	O		t	h	th
6	3	,	9	2	6

$= \square \times 10 + \square \times 1 + \square \times \frac{1}{10} + \square \times \frac{1}{100} + \square \times \frac{1}{1000}$

b.

H	T	O		t	h	th
7	6	5	,	2	4	4

$= \square \times 100 +$

c.

T	O		t	h	th
	4	,	9	0	2

$= \square \times 1 +$

d.

H	T	O		t	h	th
1	5	1	,	9		

=

The different places in the place value chart are symmetrically around the ones place:

We have both thousands and thousand*th*s.
We have both hundreds and hundred*th*s.
We have both tens and ten*th*s.

Be careful so that you will not confuse them!

TH	H	T	O		t	h	th
1	**5**	**8**	**2**	**,**	**0**	**1**	**7**

$= 1 \times 1000 + 5 \times 100 + 8 \times 10 + 2 \times 1 + 1 \times \frac{1}{100} + 7 \times \frac{1}{1000}$

10. Write in expanded form.

a. 5400,95 =

b. 1,405 =

c. 244,781 =

d. 4860,4 =

e. 0,664 =

11. What number is formed from the "parts"? Give your answer as a decimal.

a. $\frac{3}{1000} + \frac{2}{10} + \frac{7}{100} =$	**b.** $\frac{8}{10} + 400 + 5 + \frac{9}{100} =$
c. $\frac{2}{10} + 7 + \frac{3}{1000} =$	**d.** $\frac{5}{10} + 90 + \frac{2}{100} + 3000 =$
e. $\frac{1}{10} + 7 + \frac{8}{1000} + 10 =$	**f.** $200 + \frac{8}{1000} + 5 =$

12. **a.** Use these "parts" to form TWO numbers in such a way that both numbers have at least one decimal digit. Use all the parts.

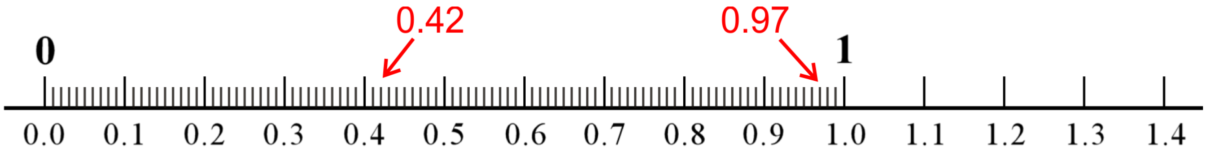
90 000 $\frac{2}{100}$ $\frac{1}{10}$ 70 400 3000 $\frac{8}{1000}$ 5

b. Now make two *different* numbers in a similar manner.

c. What is the *sum* of your two numbers in (a)?

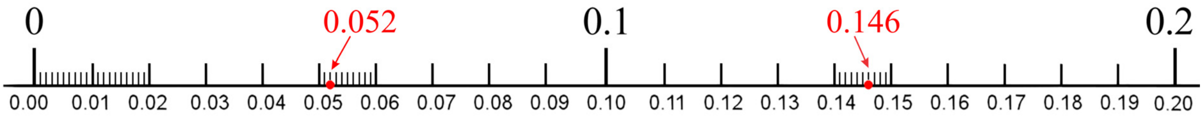
In (b)?

Decimals on a Number Line



This number line shows the interval from 0 to 1 divided into 100 parts—hundredths. The tenths are marked, and in between each two tenths are little tick marks marking the hundredths.

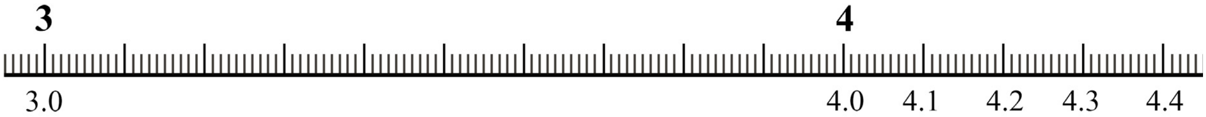
Now we will zoom in to just a part of the above number line.



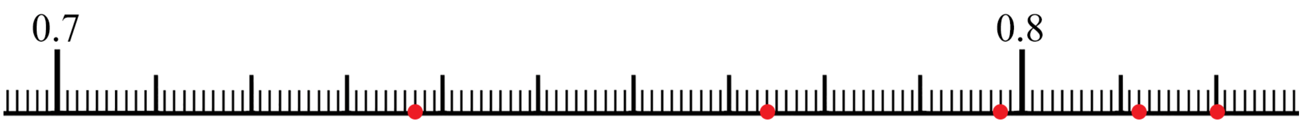
The labels 0,1 and 0,2 have been moved above the tick marks for clarity. The tick marks for hundredths have been labelled, all the way from 0,00 to 0,20.

In between 0,00 and 0,01 (and also in between and 0,01 and 0,02) are new tick marks, marking *thousandths*. They divide the interval from 0,00 to 0,01 (which is one hundredth) into ten parts. The marks are so tiny, you may not even see them clearly, but they are there!

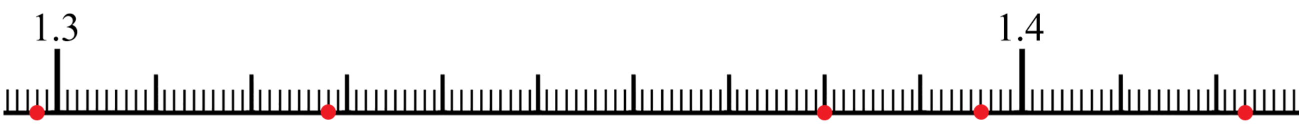
1. Mark these decimals on the number line: 3,6 3,43 3,89 4,11 2,98 4,05



2. Now we have zoomed in to see the *thousandths*. Write the decimals indicated by the dots.



3. Write the decimals indicated by the dots on the number line.



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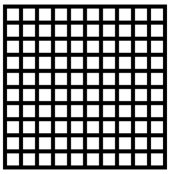
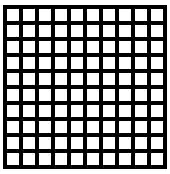
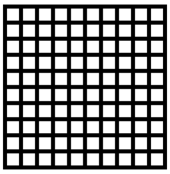
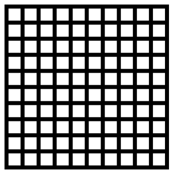
Chapter 4 Revision

1. Colour in parts to show the decimals.

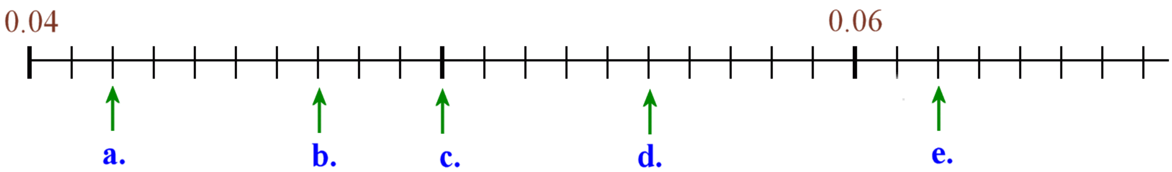
 **a.** 0,2 + 0,04	 **b.** 0,09 + 0,05	 **c.** seven hundredths	 **d.** 0,6

2. Write in expanded form.

a. 0,495

b. 2,67

3. Write the decimals indicated by the arrows.



a. ____________ **b.** ____________ **c.** ____________ **d.** ____________ **e.** ____________

4. Compare using < , = , and > .

a. 0,25 ☐ 0,215	**b.** 0,3 ☐ 0,19	**c.** 4,033 ☐ 4,33	**d.** 0,65 ☐ $\frac{1}{2}$	**e.** 0,065 ☐ 0,2

5. Write as decimals.

a. $\frac{3}{100}$ = ________ **b.** $\frac{48}{1000}$ = ________ **c.** $1\frac{209}{1000}$ = ________ **d.** $3\frac{39}{100}$ = ________

6. Write as fractions or mixed numbers.

a. 1,3

b. 2,15

c. 0,008

d. 0,038

7. Round the numbers to the nearest one, to the nearest tenth, and to the nearest hundredth.

rounded to...	nearest one	nearest tenth	nearest hundredth
4,608			
23,109			
2,299			
0,048			

8. Add or subtract.

a. 0,3 + 0,005 = ___________ 0,03 + 0,5 = ___________	**b.** 0,9 − 0,7 = ___________ 0,9 − 0,07 = ___________
c. 0,008 + 0,9 + 5 = _________ 0,9 + 0,8 + 0,17 = _________	**d.** 2,5 − 1,02 = _________ 7,8 − 0,9 − 0,04 = _________

9. Complete the addition sentences.

a. 0,21 + ________ = 1	**b.** 0,004 + _________ = 1	**c.** 4,391 + __________ = 5

10. **a.** Find the number that is 5 hundredths and 7 tenths *more* than 3,194.

b. Find the number that is 3 thousandths and 8 tenths *less* than 0,902.

11. Five children divided R250 equally, and then each one bought ice cream for R20,50.

a. Choose an expression that matches the problem.

b. Find the exact amount that each child has left now.

250 – R20,50 ÷ 5
250 ÷ 5 – R20,50
250 – 5 × R20,50

12. Solve.

a. 0,4 × 8 = _________	**c.** 20 × 0,5 = _________	**e.** 9 × 0,002 = _________
b. 6 × 0,009 = _________	**d.** 100 × 0,3 = _________	**f.** 6 × 0,03 = _________

13. Divide. Add decimal zeros to the dividend, as necessary.

a. Give your answer to 3 decimals.	**b.** Round your answer to 2 decimals.
$8\overline{)\ 5,4}$ Check:	$7\overline{)\ 4\ 4}$ Check:

14. Multiply and divide. Use the grid.

a. 5 × 0,614

b. 23 × 0,79

c. 2,485 ÷ 7

15. Six friends shared the cost of a dinner as equally as they could. The dinner cost R925,45 in total. Were they able to share the cost equally? How much did each person pay?

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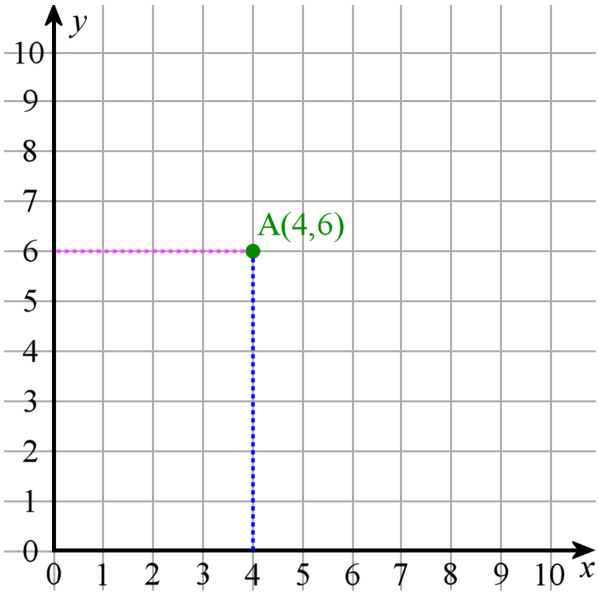
The Coordinate Grid

This is a **coordinate grid**. It consists of two number lines that are set perpendicular (at right angles) to each other.

The horizontal number line is called the ***x*-axis**. The vertical one is called the ***y*-axis**.

You can see one point, called "A," that is drawn or *plotted* on the grid.



Since we have two number lines, we use *two* numbers (4 and 6) to signify its location. Those numbers are the **coordinates** of the point A.

The first number, 4, is the ***x*-coordinate** of the point A. It is called the *x*-coordinate because point A is four units from zero in the horizontal direction (direction of the *x*-axis).

We can see that by drawing a straight line down from A. The line *intersects*, or "hits," the *x*-axis at 4.

The second number is the **y-coordinate** of the point A. In the vertical direction, point A is six units from zero. When we draw a line directly towards left from A, it intersects the *y*-axis at 6.

We write the two coordinates of a point inside brackets, separated by a comma: (4, 6).

Note: (4, 6) is an **ordered pair**: the order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is always the *y*-coordinate, not vice versa.

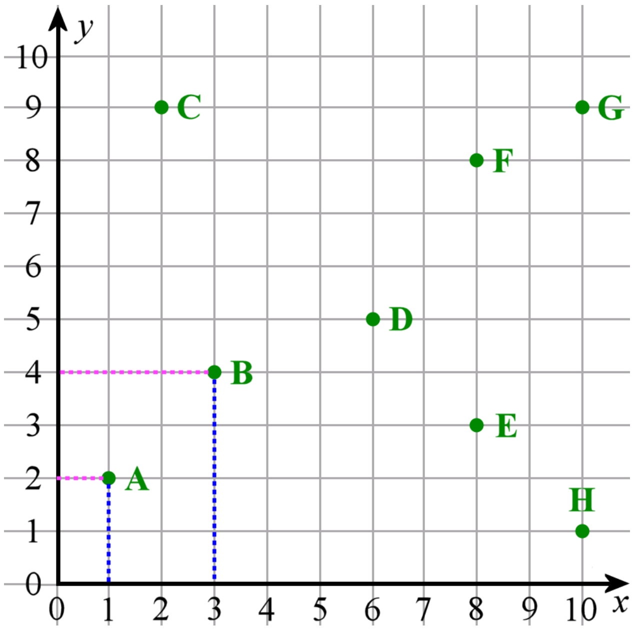
1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in. (The helping lines are not necessary to draw; they are just that — *helping* lines. You can draw them if they help you.)

A (___, ___) B (___, ___)

C (___, ___) D (___, ___)

E (___, ___) F (___, ___)

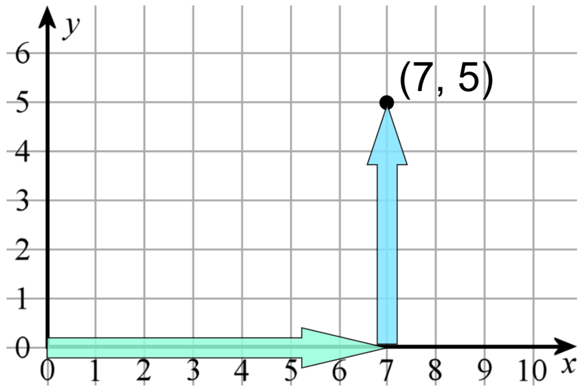
G (___, ___) H (___, ___)



To plot points, you can first “travel” on the *x*-axis from the point (0, 0) (the **origin**) the number of units indicated by the *x*-coordinate.

Then travel UP as many units as the *y*-coordinate indicates.

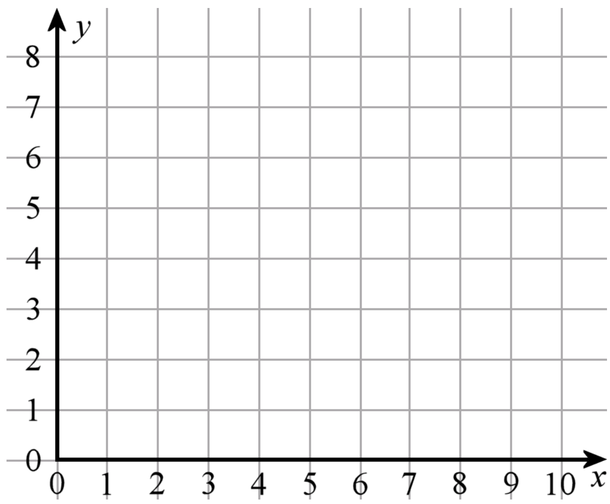
The image shows an example of how to plot (7, 5).



2. Plot the following points on the coordinate grid. Then join them with line segments in the alphabetical order. What do you get?

   A(1, 5) B(4, 3) C(4, 6)

   D(7, 5) E(6, 8)

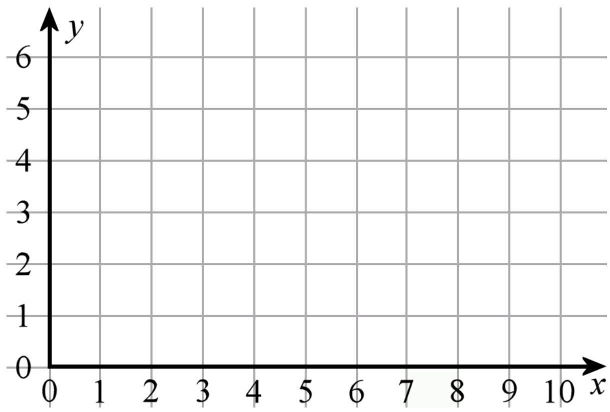


3. **Zero as a coordinate.** Plot the following points in the grid on the right.

   A(0, 6) B(0, 3) C (0, 0)

   D(5, 0) E(9, 0)

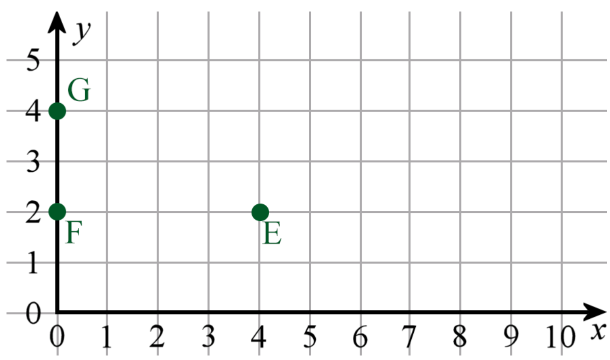
   What do you notice?



4. **a.** Write the coordinates of the points E, F, and G.

   b. Plot a fourth point, H, so that when you join E, F, G, and H with line segments, you will get a rectangle.

   c. What are the coordinates of H?



5. In this grid, the *y*-axis is scaled differently.

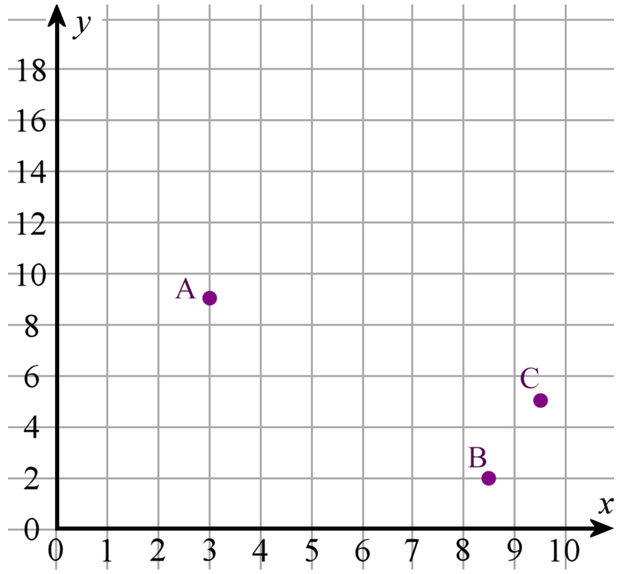
a. Write the coordinates of these points:

A(____, ____)   B(____, ____)

C(____, ____)

b. Plot these points. Note that the points don't necessarily fall on the gridlines.

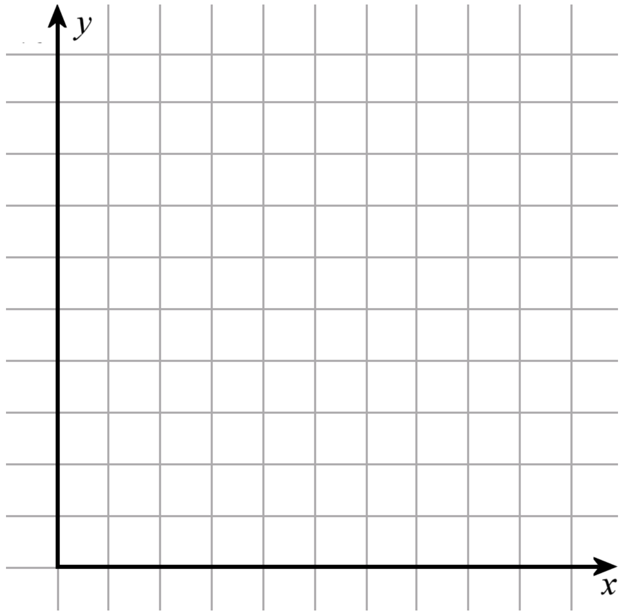
D(7, 11)   E(1 ½, 9)   F(9 ¼, 2)



6. **a.** Design a scaling for the axes so that the point P(36, 38) will fit on this grid.

b. Then plot these points also, and connect the points with line segments in order. What shape is formed?

Q(36, 28)   R(16, 18)   S(26, 38)



7. Here, "LINE (5,6) - (2,7)" means a line segment that is drawn from (5, 6) to (2, 7).

Draw the following line segments (joining the two given points). Use a ruler! The first one is already done for you.

What figure is formed?

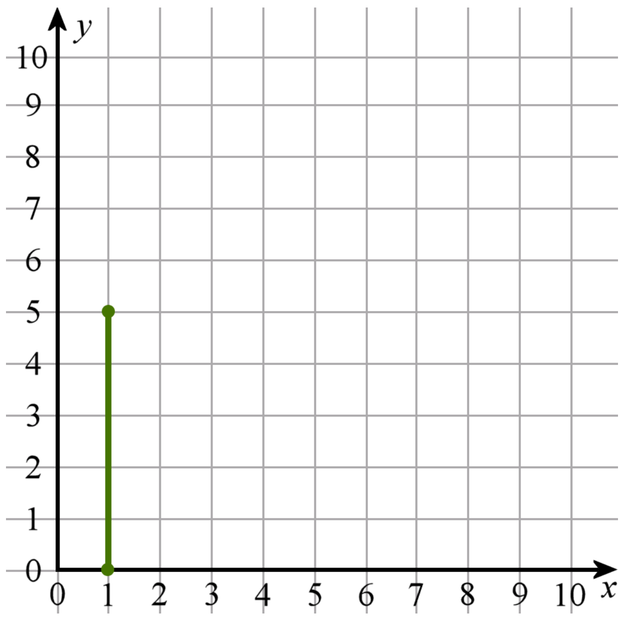
LINE (1, 0) - (1, 5)   LINE (1, 5) - (0, 5)

LINE (0, 5) - (4, 7)   LINE (4, 7) - (8, 5)

LINE (8, 5) - (7, 5)   LINE (3, 0) - (3, 3)

LINE (5, 0) - (5, 3)   LINE (3, 3) - (5, 3)

LINE (1, 0) - (7, 0)   LINE (7, 0) - (7, 5)



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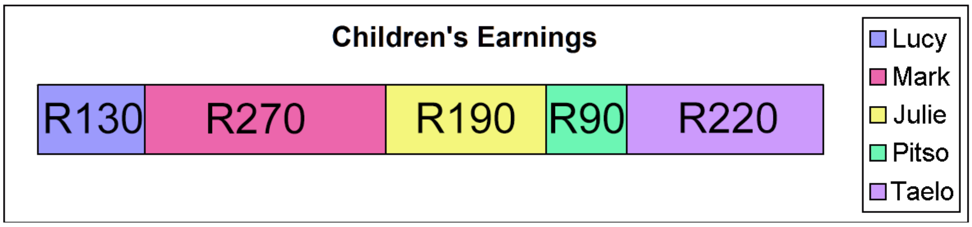
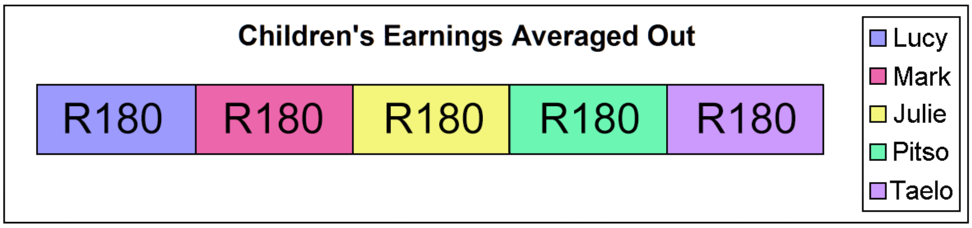
Average (Mean)

Example 1. Five children earned these amounts of money for a job: R130, R270, R190, R90, and R220. The graph below shows visually how much each child earned.



Together, they earned R900. If this R900 had been divided equally among the children, each child would have gotten R180. (It was not, because the children got paid according to how much they worked.) This R180 is the **average** pay.



The graph above shows the situation *if* each child had received the average earning (R180). Notice that R180 is sort of in the "middle" or in between the lowest and highest earnings.

- To calculate the average, first add all the numbers in the data set, and then divide the sum by the number of items in the data set. In other words,

$$\text{average} = \frac{\text{sum of all the items}}{\text{the number of items}}$$

- The average is always somewhere in the middle of a set of data: it is more than the smallest number and less than the largest number of the data.

- The average is also called the **mean**. We will use both terms in this lesson so you get used to both.

1. Calculate the average of the data sets. Do not use a calculator.

a. 2, 4, 5, 9, 0, 4, 1, 7

b. 13, 16, 20, 22, 16, 13, 17, 12, 15

2. Calculate the mean of the data sets to the nearest tenth. This time use a calculator.

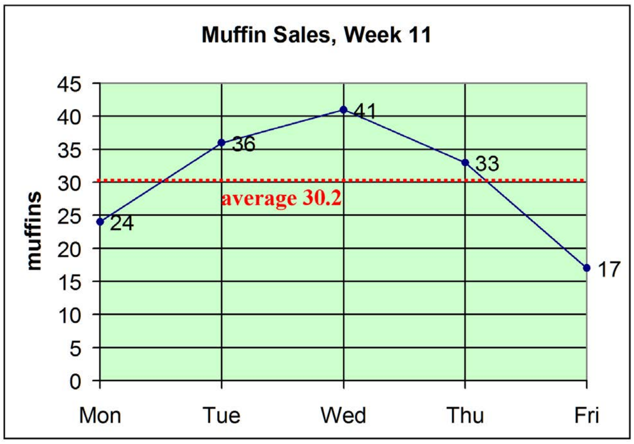
a. 2, 4,3, 5, 9, 4,7, 9,4, 3,7, 5,1

b. 312, 288, 284, 329, 293, 302

Average and Line Graphs

Let's look at the muffin sales again.

Muffin Sales, Week 11	
Day	**Muffins sold**
Mon	24
Tue	36
Wed	41
Thu	33
Fri	17



The average is $\dfrac{24 + 36 + 41 + 33 + 17}{5}$ = 30,2 muffins per day.

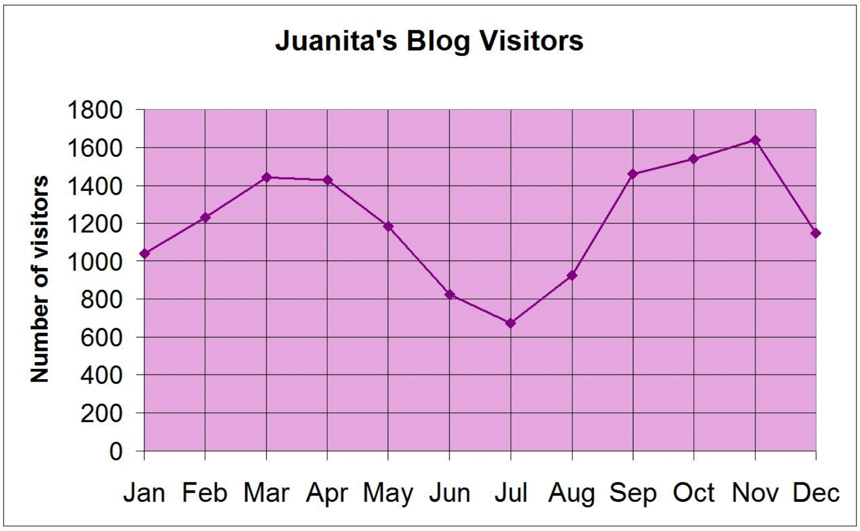
On average, Margaret sold 30,2 muffins or about 30 muffins per day. See how this 30,2 muffins is plotted on the line graph, using a dashed line. Notice that the average is somewhere in the middle of the data: some data points are above, some are below it.

If Margaret had sold 30,2 (or 30 1/5) muffins every day, she would have sold the same total amount in five days as she actually did: 151 muffins.

If Margaret had sold 30,2 muffins every day, what would the line graph look like?

It would be a horizontal line, with each day's data value at the same level of 30,2.

3. **a.** The monthly average visitor count to Juanita's blog over one year was 1211 visitors/month. Plot the average in the line graph with a dashed line, like in the example above.

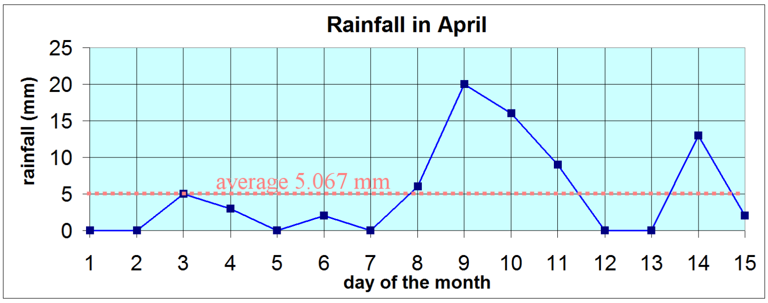


Month	Visitors
Jan	1039
Feb	1230
Mar	1442
Apr	1427
May	1183
Jun	823
Jul	674
Aug	924
Sep	1459
Oct	1540
Nov	1638
Dec	1149

b. In which months was the visitor count below average?

c. Find the average visitor count for September through December (to the nearest whole number).

4. The average rainfall in the first 15 days of April was 5,067 mm. What was the total rainfall in the first 15 days of April (in millimetres)?
Hint: you don't have to add 15 different numbers in order to find that out. There is a quicker way.



5. The birth weights of a certain litter of piglets were:
1400 g 1480 g 1250 g 1710 g 1630 g 1250 g 1700 g 1820 g 1500 g

a. Find the average weight to the nearest gram.

b. How many grams *lighter* than the average were the lightest (two) piglets?

c. How many grams *heavier* than the average was the heaviest piglet?

d. Remove the two lightest piglets' weights from the data.
Now calculate the average again.
Did the average change? If it did, by how much?

6. The data below gives the daily salaries of StarMop Inc. employees:
R1146 R1178 R1189 R1209 R1209 R1210 R1213 R1215 R3400

a. Calculate the mean.

b. Remove the person with the highest salary from the data set.
Calculate the mean again, rounding it to the nearest rand. How did it change?

Puzzle Corner

Sylvia checked the price of a certain plasma TV in four different stores. In three of the stores the price was R5490, R5890, and R5990. She calculated that the average price was R5670.

What was the price in the fourth store?
Choose the right answer: **a.** R6090 **b.** R5310 **c.** R4600 **d.** R5670

Mixed Revision Chapter 5

1. **a.** Write a number that is 5 thousandths, 2 tenths, and 8 hundredths more than 1,004. (Add and Subtract Decimals—Mental Maths/Ch.4)

   b. Write a number that is 3 thousandths and 3 tenths less than 3,411.

2. Figure out what was done in each step (either addition or subtraction)!
(Add and Subtract Decimals—Mental Maths/Ch.4)

2,2 → 2,25 → 2,35 → 2,09 → 2,1 → 2,172 → 2,2 → 2,01 → 2

3. Jan cut three 0,82-metre pieces from a 4-metre board.
How long was the piece that was left?

4. **a.** Estimate the total cost in dollars.

   b. Find the total.

   c. Find the error of estimation.

beans	R43,50
milk	R29,90
dog food	R113,80
broccoli	R21,40
chicken	R76,40

Estimate:

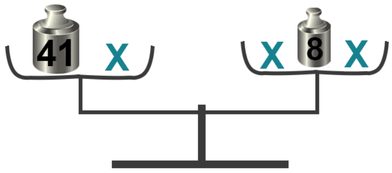
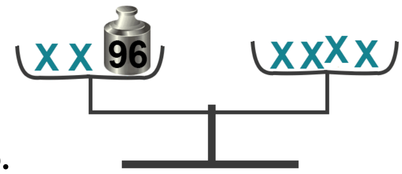
5. Factor the following numbers to their prime factors. (Prime Factorisation/Ch.1)

a. 48 / \	**b.** 71 / \	**c.** 93 / \

6. Multiply mentally. (Multiplying Numbers by Powers of Ten/Ch.2)

a. 500 × 200 =	**b.** 2000 × 400 =
c. 2 × 800 × 20 =	**d.** 30 × 40 × 50 =

7. First write the equation as the balance shows it. Then solve. (Balance Problems and Equations/Ch.3)

a. 	**b.** 
______ = ______ ______ = ______ ______ = ______	______ = ______ ______ = ______ ______ = ______

8. Write a division equation where the quotient is 210, the divisor is 52, and the dividend is unknown. Use a letter for the unknown.

Then find the value of the unknown.
(Revision: Multiplication and Division/Ch.1)

______ ÷ ______ = ______

9. Mark the numbers given in the problem in the bar model.
Mark what is asked with “?”. Then solve the problem.
(Problem Solving with Bar Models 2/Ch.3)

Mary and Luisa bought a gift together for R230.
Mary spent R30 more on it than Luisa.
How many rand did each girl spend?

10. With the money John earned from his summer job, he paid his phone service for two months (R480 per month), spent R1200 for a bike, and still had half of his money left. How much did he earn?

11. Calculate. (Multiplying Numbers by Powers of Ten/Ch.2)

a. $10\,000 \times 459 =$	**b.** $820 \times 10^6 =$
c. $43 \times 10^8 =$	**d.** $100\,000 \times 2098 =$

12. Divide. Check your answer by multiplying. (A Two-Digit Divisor/Ch.1)

a. $38\overline{)3\ 9\ 5\ 2}$	$\times\ 3\ 8$	**b.** $17\overline{)2\ 6\ 8.6}$	$\times$

13. Solve the problems. Choose if you should use <u>mental maths</u> or a <u>calculator</u>. (The Calculator/Ch.2) 

a. 140 − 70 − 30 calculator/mental maths	**b.** 529 − 71 × 6 calculator/mental maths	**c.** 7 × 80 + 1000 calculator/mental maths
d. Your change from R1000 when buying three packets of paper for R166,5 each calculator/mental maths	**e.** The total cost of five flash drives at R249,50 each calculator/mental maths	**f.** How many eggs are in five dozen eggs calculator/mental maths

14. Match the two problems (a) and (b) below to the correct expressions below them. Then solve each problem. What does your answer signify?

a. Johannes has R1700 to buy groceries for the week. First, Johannes sets aside R230 to buy treats; then he divides the remaining money evenly for each day of the week.

b. Johannes has R1700 to buy groceries for the week. Johannes decides to use R230 per day for food, and to use whatever is left for treats.

R1700 − 7 × R230 | **R1700 − R230 ÷ 7** | **(R1700 − R230) ÷ 7**

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Contents

Foreword 5
User Guide 7

Chapter 6: Decimals, Part 2

Introduction 11
Multiply and Divide by Powers of Ten 1 13
Multiply and Divide by Powers of Ten 2 16
Multiply and Divide by Powers of Ten 3 19
Multiply Decimals by Decimals 1 21
Multiply Decimals by Decimals 2 23
Multiplication as Scaling 26
Decimal Multiplication—More Practice 30
Dividing Decimals—Mental Maths 32
More Division with Decimals 35
The Metric System, Part 1 38
The Metric System, Part 2 42
Divide Decimals by Decimals 1 45
Divide Decimals by Decimals 2 48
Problem Solving 53
Mixed Revision Chapter 6 57
Chapter 6 Revision 59

Chapter 7: Fractions: Add and Subtract

Introduction 63
Fraction Terminology 65
Revision: Mixed Numbers 66
Adding Mixed Numbers 69
Subtracting Mixed Numbers 1 72
Subtracting Mixed Numbers 2 76
Equivalent Fractions 1 78
Equivalent Fractions 2 81
Adding and Subtracting Unlike Fractions 83
Finding the (Least) Common Denominator 86
Add and Subtract: More Practice 89
Adding and Subtracting Mixed Numbers 92
Comparing Fractions 95
Word Problems 100
Mixed Revision Chapter 7 102
Chapter 7 Revision 105

Chapter 8: Fractions: Multiply and Divide

Introduction 109
Simplifying Fractions 1 111
Simplifying Fractions 2 114
Multiply Fractions and Whole Numbers 1 117
Multiply Fractions and Whole Numbers 2 119
Multiply Fractions by Fractions 1 121
Multiply Fractions by Fractions 2 124
Fraction Multiplication and Area 126
Simplifying Before Multiplying 133
Multiplying Mixed Numbers 136
Multiplication as Scaling/Resizing 139
Fractions Are Divisions 142
Dividing Fractions: Sharing Divisions 146
Dividing Fractions: Fitting the Divisor 149
Dividing Fractions: Summary 152
Dividing Fractions: The Shortcut 154
Mixed Revision Chapter 8 157
Chapter 8 Revision 160

Chapter 9: Geometry

Introduction 165
Geometry Vocabulary Reference Sheet 167
Revision: Angles 168
Polygons 172
Classifying Quadrilaterals 1 175
Classifying Quadrilaterals 2 178
Classifying Quadrilaterals 3 181
Classifying Triangles 1 183
Classifying Triangles 2 186
Volume 188
Volume of Rectangular Prisms 193
Volume Is Additive 196
Area and Perimeter Problems 199
Star Polygons 203
Mixed Revision Chapter 9 205
Chapter 9 Revision 208

Foreword

Math Mammoth Grade 5, South African Version, comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 5, only customised for South African use in a few aspects (listed below). Please note that the curriculum is not following the South African official syllabus for grade 5 maths. For the most part, Math Mammoth exceeds South African standards. Some standards may not be covered.

Math Mammoth South African version has been customised to South Africa in the following manners:

- Some names used are South African names (instead of Jack and Jill, there are Sipho and Sibongile).
- The currency used in word problems is rand.
- The material is all metric. In other words, the US customary measuring units are not used.
- Spelling follows British English instead of American English.
- Large numbers are formatted with a space as a thousands separator.
- The decimal separator is a comma.
- Paper size is A4.

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study graphing. The main areas of study in Math Mammoth Grade 5 are:

- Multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors)
- Solving problems involving all four operations;
- The place value system, including decimal place value
- All four operations with decimals and conversions between measurements;
- The coordinate system and line graphs;
- Addition, subtraction, and multiplication of fractions; division of fractions in special cases;
- Geometry: volume and categorising two-dimensional figures (especially triangles);

This book, 5-B, covers more on decimal arithmetic, in chapter 6. The focus is on decimal multiplication and division, and on conversions between measurement units. Chapter 7 has to do with fraction addition and subtraction, and chapter 8 with fraction multiplication and division. The last chapter (chapter 9) is about geometry. Students classify quadrilaterals and triangles, and learn about volume.

The part 5-A covers the four operations, place value and large numbers, problem solving, decimals, and graphing.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In fifth grade, chapter 4 should be studied before chapter 6, and chapter 7 before chapter 8, but you can be flexible with the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 2 or 3 pages long, intended for one day. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Worktext 5-A	
Chapter 1	21 days
Chapter 2	12 days
Chapter 3	9 days
Chapter 4	18 days
Chapter 5	11 days
TOTAL	**71 days**

Worktext 5-B	
Chapter 6	20 days
Chapter 7	15 days
Chapter 8	20 days
Chapter 9	12 days
TOTAL	**67 days**

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A	88	10	178	78	2,28	11,4
5-B	92	10	188	82	2,29	11,5
Grade 5 total	180	20	366	160	2,29	11,4

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
5-A			178			
5-B			188			
Grade 5 total			366			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on maths. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

Working space, the usage of additional paper, and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1000). Typically, I have intended that such exercises to be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 6: Decimals, Part 2
Introduction

This chapter focuses on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying and dividing decimals by powers of ten, presented with the help of place value charts. This is familiar to students from chapter 2, where they multiplied and divided whole numbers by powers of ten. The number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten.

As a shortcut, we can move the decimal comma. However, the movement of the decimal comma is an "illusion"—that is what seems to happen—but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value chart.

Next, we study how to multiply decimals by decimals. The common rule (or shortcut) for it says to multiply the numbers without the decimal commas, and then add the decimal comma to the product (answer) so that it has as many decimal digits as the factors have in total. We justify this rule using the recently learned technique for dividing decimals by powers of ten. Students are also encouraged to use estimation in decimal multiplications, and they solve problems connected to real life.

Then students learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0,4 × 1,2, as repeated addition. Instead, they are viewed as scaling—shrinking or enlarging—the number or quantity by a scaling factor. So, 0,4 × 1,2 is thought of as scaling 1,2 by 0,4, or as four-tenths *of* 1,2. You may recognise this as the same as 40% of 1,2.

Next, we go on to decimal divisions that can be done with mental maths. Students divide decimals by whole numbers (such as 0,8 ÷ 4 or 0,45 ÷ 4) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. 0,9 ÷ 0,3 or 0,072 ÷ 0,008).

In the lesson *More Division with Decimals*, we revise long division with decimals, when the divisor is a whole number.

Then, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekalitres to hectolitres. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimetre is a hundredth part of a metre, since the prefix "centi" means 1/100. Knowing that, gives us a means of converting between centimetres and metres.

The second lesson deals with more metric units, even those not commonly used, such as dekalitres and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Next, we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor).

After learning that, students practise measurement conversions within the customary system and do some generic problem solving with decimals.

Recall that not all students need all the exercises; use your judgement. Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

Pacing Suggestion for Chapter 6

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Multiply and Divide by Powers of Ten, Part 1	13	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 2	16	*3 pages*	1 day	
Multiply and Divide by Powers of Ten, Part 3 (optional)	19	(*2 pages*)	(1 day)	
Multiply Decimals by Decimals 1	21	*2 pages*	1 day	
Multiply Decimals by Decimals 2	23	*3 pages*	1 day	
Multiplication as Scaling	26	*4 pages*	2 days	
Decimal Multiplication — More Practice	30	*2 pages*	1 day	
Dividing Decimals—Mental Maths	32	*3 pages*	1 day	
More Division with Decimals	35	*3 pages*	1 day	
The Metric System, Part 1	38	*4 pages*	2 days	
The Metric System, Part 2	42	*3 pages*	1 day	
Divide Decimals by Decimals 1	45	*3 pages*	1 day	
Divide Decimals by Decimals 2	48	*4-5 pages*	2 days	
Problem Solving	53	*4 pages*	2 days	
Mixed Revision Chapter 6	57	*2 pages*	1 day	
Chapter 6 Revision	59	*5 pages*	2 days	
Chapter 6 Test (optional)				
TOTALS		*48 pages*	20 days	
with optional content		(*51 pages*)	(21 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr5ch6

SCAN ME

Multiply and Divide by Powers of Ten 1

Remember? The number system we use is based on number 10. Therefore, each place value unit is always ten times the previous unit: 10 ones makes a ten, 10 tens makes a hundred, 10 hundreds makes a thousand. Because of this, when a number is multiplied by ten, the digits of the number essentially *move* in the place value chart!

Example 1. When 215 is multiplied by 10, each of its digits moves one slot to the left in the place value chart.

- The "2" in the hundreds place, signifying 200, becomes 2000.
- The "1" in the tens place, signifying 10, becomes 100.
- The "5" in the ones place (signifying 5) becomes 50.

Th	H	T	O		t	h	th
	2	**1**	**5**	**,**			

becomes

Th	H	T	O		t	h	th
2	**1**	**5**	**0**	**,**			

It works **the same way with decimals:** each place value unit is ten times the previous unit.

Example 2. 10 hundredths makes a tenth (or 10 × 0,01 = 0,1).

Using the place value chart, the digit one (signifying one hundredth) *moves* in the chart one slot to the left.

What if 0,01 was multiplied by 100?

10 × 0,01 = 0,1

Th	H	T	O		t	h	th
				,	1 ←	**1**	

Example 3. Since 10 × 0,01 = 0,1, it follows that 10 times *seven* hundredths equals seven tenths. The digit 7 moves in the place value chart one step to the left.

What if seven hundredths was multiplied by 100? By 1000?

What if there were other digits?

10 × 0,07 = 0,7

Th	H	T	O		t	h	th
				,	7 ←	**7**	

1. **a.** Using this technique, what happens to 7 thousandths when it is multiplied by 100? Explain, using the place value chart.

Th	H	T	O		t	h	th

b. What happens to 0,35 when it is multiplied by 1000? Explain.

Th	H	T	O		t	h	th

When you multiply a number by a power of ten (10, 100, 1000, etc.), each digit of the number *moves* in the place value chart as many steps as there are zeros in the power of ten.

The same thing happens when *dividing* a number by a power of ten. This time, the number moves to the *right* — again, as many steps as there are zeros in the power of ten.

See the examples on the right.

0,47 ÷ 10 = 0,047

H	T	O		t	h	th
		0	**,**	**4**	**7**	

becomes

		0	**,**	**0**	**4**	**7**

21,5 ÷ 100 = 0,215

H	T	O		t	h	th
	2	**1**	**,**	**5**		

becomes

		0	**,**	**2**	**1**	**5**

2. Fill in the missing numbers. Use the place value charts to help.

Th	H	T	O		t	h	th

a. 100 × 0,208 = __________

Th	H	T	O		t	h	th

b. 7,5 ÷ 100 = __________

Th	H	T	O		t	h	th

c. ________ × 0.915 = 9,15

Th	H	T	O		t	h	th

d. 16 ÷ ________ = 0,016

3. Multiply and divide. Notice the patterns. You can use the place value charts to help.

a. 10 × 0,04 = _________

100 × 0,04 = _________

1000 × 0,04 = _________

10 000 × 0,04 = _________

b. 450 ÷ 10 = _________

450 ÷ 100 = _________

450 ÷ 1000 = _________

450 ÷ 10 000 = _________

c. 0,5 ÷ 10 = _________

0,5 ÷ 100 = _________

d. 10 × 0,056 = _________

100 × 0,056 = _________

e. 2 ÷ 100 = _________

2 ÷ 1000 = _________

f. 100 × 2,3 = _________

1000 × 2,3 = _________

g. _______ × 0,89 = 89

_______ × 0,209 = 2,09

h. 78,6 ÷ _________ = 0,786

24 ÷ __________ = 0,024

Th	H	T	O		t	h	th
				,			

Th	H	T	O		t	h	th
				,			

Th	H	T	O		t	h	th
				,			

Th	H	T	O		t	h	th
				,			

Th	H	T	O		t	h	th
				,			

Th	H	T	O		t	h	th
				,			

A neat trick that makes it easy!

Example 4. What is 100 × 2,105?

Instead of thinking of the movement of the digits, consider what happens to the digit 2 when it is multiplied by 100. It becomes 200. So, you can simply write the digits “2105”, and then place the decimal comma in such a manner that the answer ends up being 200-something: **210,5**.

Example 5. What is 5460 ÷ 100?

When divided by 100, the 400 in 5460, becomes 4. That fixes the decimal comma: it has to come right after the digit 4. So, the answer is 54,60, which simplifies to 54,6.

4. Solve. You can use any method.

a. 100 × 5,439 = _________	**c.** 1000 × 3,06 = _________	**e.** 30,73 ÷ 10 = _________
b. 100 × 4,03 = _________	**d.** 100 × 30,54 = _________	**f.** 9608 ÷ 100 = _________

Reminder: Finding one tenth of a number is the same as dividing it by 10. Finding one-hundredth of a number is the same as dividing it by 100.

For example, one tenth of 4,5 kg is the same as 4,5 kg ÷ 10 = 0,45 kg.

5. Find one-tenth of... **a.** R8 **b.** R25,50 **c.** 126 km

6. **a.** Find one-tenth of 45 kg.

b. Find three-tenths of 45 kg.

7. A 10-kg sack of nuts costs R272,50.
How much does one kilogram cost?

8. Find one-hundredth of... **a.** R78 **b.** 4 kg **c.** R390

9. Find the price of 100 ping-pong balls if one ball costs R5,89.

10. One-hundredth of a certain number is 0,03. What is the number?

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Multiply Decimals by Decimals 2

The shortcut to decimal multiplication:

1) Multiply as if there were no decimal commas.

2) Place the decimal comma in the answer. The **number of decimal digits** in the answer is the **SUM** of the number of decimal digits in the factors.

Example 1. To solve 0,81 × 2,5, multiply **as if it was 81 × 25**, ignoring the decimal commas. (Also, “081” is the same as as 81 so we can ignore the zero, too.)

Since 0,81 has *two* decimal digits, and 2,5 has *one*, the answer will have *three*. The final answer is therefore 2,025.

$$\begin{array}{r} 8\ 1 \\ \times\quad 2\ 5 \\ \hline 4\ 0\ 5 \\ +\ 1\ 6\ 2\ 0 \\ \hline 2\ 0\ 2\ 5 \end{array}$$

Does that make sense?

We can estimate the product as 0,8 × 3 = 2,4 or as 0,8 × 2,5 = 2.

Yes, a final answer of 2,025 makes sense, since it is close to our estimates.

Example 2. This time, we include the decimal commas when calculating 1,49 × 0,7, but even so, we multiply **as if it was 149 × 7**. Imagine the decimal commas are not there! And there is NO need to align the decimal commas like in addition and subtraction.

$$\begin{array}{rl} {}^{3\ 6} & \\ 1,4\ 9 & \leftarrow \textbf{two} \text{ decimal places} \\ \times\quad 0,7 & \leftarrow \textbf{one} \text{ decimal place} \\ \hline 1,0\ 4\ 3 & \leftarrow \textbf{three} \text{ decimal places} \end{array}$$

Estimate: 1,49 × 0,7 ≈ 1,5 × 0,7 = 1,05. If the estimate was *not* close to our final answer, there would probably be an error somewhere.

1. Solve with long multiplication. Also, estimate. Write the longer number on top.

a. 0,3 × 1,19

Estimate: ___________

b. 0,9 × 51,7

Estimate: ___________

c. 204,5 × 0,4

Estimate: ___________

d. 2,2 × 0,72

Estimate: ___________

e. 5,6 × 2,8

Estimate: ___________

f. 3,34 × 4,2

Estimate: ___________

Example 3. Potatoes cost R5,15/kg, and you buy 0,7 kg. What is the total cost?

If you were to buy 3 kilograms of potatoes, you would multiply 3 × R5,15 to find the total price. When you buy 0,7 kg, you do the SAME: **multiply the price by 0,7.** The total cost is **0,7 × R5,15 = 3,605** (see the multiplication on the right).

```
   1 3
   5 1 5
×      7
--------
 3 6 0 5
```

Since this is a money amount, it needs rounded to two decimals: R3,61.

Does the answer make sense? Yes: 0,7 kg of potatoes should cost less than 1 kg of potatoes, which was R5,15.

2. Find the total cost. Write a multiplication.

a. Ribbon costs R10,10 per metre, and you buy 0,4 metres.

Cost: ________ × ________ = ______________

b. Bananas cost R25/kg. You buy 0,3 kg.

Cost: ________ × ________ = ______________

c. A phone call costs R70 per hour. You talk for 1,2 hours.

Cost: ________ × ________ = ______________

d. Lace costs R25,20 per metre, and you buy 1,5 metres.

Cost: ________ × ________ = ______________

3. Lauren went to a farmer’s market, and bought 0,4 kg of onions at R18,40/kg, 3,5 kg of oranges at R12,20/kg, and 1,2 kg of tomatoes at R20,45/kg. Find the total cost.

4. You're trying to find out how much it costs to feed your pet parrot for a <u>year</u>.

   You have figured that your parrot eats about 90 grams, which is 0,09 kg, of parrot feed in a day. Also, parrot feed costs R70,80/kg. So, what is the cost of feeding your parrot for a year (not just a day)? Round your answer to the nearest rand.

5. Multiply.

   a. 0,7 × 1,1 = ________

   b. 0,02 × 0,5 = ________

   c. 0,002 × 9 = ________

   d. 1,1 × 0,3 = ________

   e. 0,4 × 4 × 0,2 = _________

   f. 6 × 0,06 × 0,2 = _________

6. (optional) Solve in your head or by long multiplication (use extra paper). Place the letter of each problem under the right answer, and solve the riddle.
   Hint: Sometimes you can figure out the correct answer by estimating.

0,8 × 2,7	U	2,09 × 1,7	R
1,6 × 2,05	D	70 × 0,05	S
10 × 0,04	L	0,05 × 2,4	E
10 × 0,042	S	15,2 × 0,18	B
1,5 × 1,5	E	200 × 0,008	O
3,45 × 0,7	I	0,5 × 0,4 × 0,7	Y
1,06 × 0,4	E	0,01 × 600 × 0,2	F

Riddle: *What did number 22 say to 21,21? "YOU'RE...*

2,736	0,424	0,42	2,415	3,28	2,25

0,14	1,6	2,16	3,553	3,5	0,12	0,4	1,2

Multiplication as Scaling

Scaling means expanding or shrinking something by some factor.
Multiplication can be thought of as scaling.

This red stick is 40 pixels long. Let's **scale** it to be four times as long: → We can write a multiplication "equation": 4 × = Using pixels, 4 × 40 px = 160 px.	Now let's scale the same red stick to be 0,4 (four-tenths) as long as it is at first: → Notice, it shrank! We write a multiplication: 0,4 × = In pixels, 0,4 × 40 px = 16 px.

The number we multiply by (4 and 0,4 above) is called the **scaling factor**.

- If the scaling factor is more than 1, such as 2,3, the resulting stick is *longer* than the original one.
- If the scaling factor is less than 1, such as 0,5 or 0,66, the resulting stick is *shorter*.

1. The stick is being *shrunk*. How long will it be in pixels? Compare the problems.

a. 0,1 × = 0,1 × 40 px = ________ px	**b.** 0,3 × = 0,3 × 40 px = ________ px	**c.** 0,6 × = 0,6 × 40 px = ________ px
d. 0,2 × = 0,2 × 40 px = ________ px	**e.** 0,5 × = 0,5 × 40 px = ________ px	**f.** 0,9 × = 0,9 × 40 px = ________ px

Let's **expand** this stick (40 px) to be 1,2 times as long, like this:

1,2 × =

In pixels, it is now 1,2 × 40 = 48 pixels long.

2. The red stick is 50 pixels long. It is being *expanded*. Fill in the blanks.

a. 1,5 × = 1,5 × 50 px = ________ px	**b.** 1,3 × = 1,3 × 50 px = ________ px
c. 2,2 × = 2,2 × 50 px = ________ px	**d.** 3,3 × = 3,3 × 50 px = ________ px

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Fraction Terminology

As we study fraction operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can post it on your wall or even make your own fraction poster based on it. Some of the terms below you already know; some we will study in this chapter.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.

The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

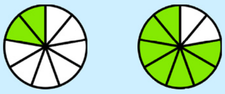
A mixed number has two parts: a whole-number part and a fractional part.

For example, in $2\frac{3}{7}$, the whole-number part is 2, and the fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

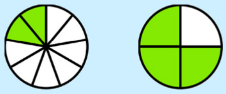
$2\frac{3}{7}$

Like fractions have the same denominator. They have the same kind of parts. It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts. It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

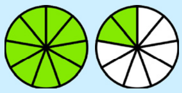


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

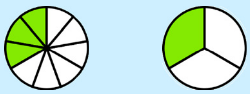
An improper fraction is more than 1 (more than a whole pie). Being a *fraction*, it is written as a fraction and *not* as a mixed number.

$\frac{2}{9}$ is a proper fraction.



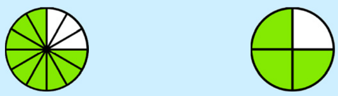
$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

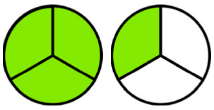


$\frac{9}{12}$ simplifies to $\frac{3}{4}$

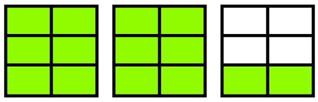
Revision: Mixed Numbers

1. Write the mixed numbers that these pictures illustrate.

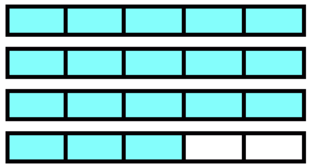
a.



b.



c.



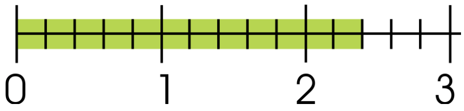
2. Draw pictures that illustrate these mixed numbers.

a. $3\,\frac{2}{6}$

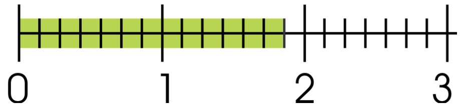
b. $4\,\frac{7}{8}$

3. Write the mixed number that is illustrated by each number line.

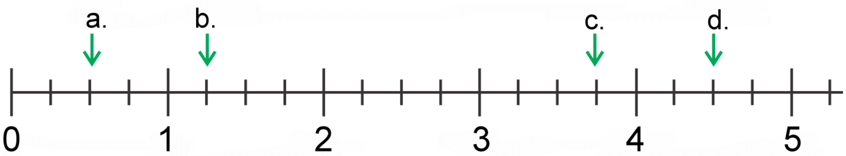
a.



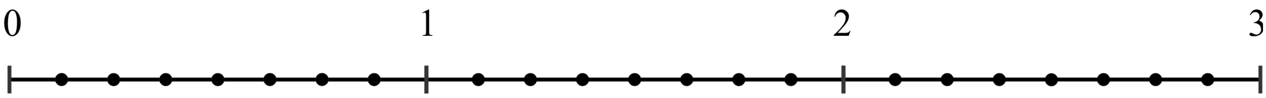
b.



4. Write the fractions and mixed numbers that the arrows indicate.

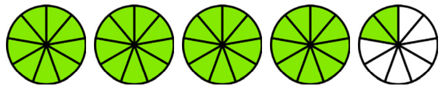


5. Mark the fractions on the number line. $\frac{9}{8}$, $\frac{22}{8}$, $\frac{13}{8}$, $\frac{24}{8}$, $\frac{11}{8}$



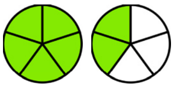
Mixed numbers to fractions

Example 1. To write $4\frac{2}{9}$ as a fraction, we *count* all the ninths:

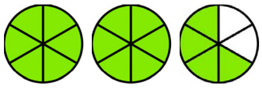


- Each pie has nine ninths, so the four complete pies have 4 × ____ = _____ ninths.
- Additionally, the incomplete pie has ____ ninths.
- The total is _____ ninths or $\frac{}{}$.

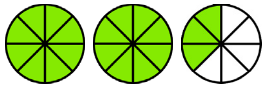
6. Write as mixed numbers and as fractions.



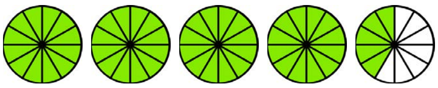
a. $1\frac{2}{5} = \frac{}{5}$



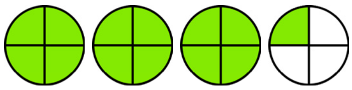
b. $\square\frac{}{} = \frac{}{}$



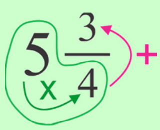
c. $\square\frac{}{} = \frac{}{}$



d. $\square\frac{}{} = \frac{}{}$



e. $\square\frac{}{} = \frac{}{}$

Shortcut: 

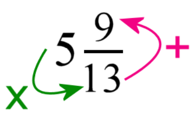
$= \frac{23}{4}$

Numerator: 5 × 4 + 3 = 23

Denominator: 4

Multiply the whole number times the denominator, then add the numerator, to get the number of fourths, or the numerator for the fraction. The denominator will remain the same.

7. Explain how the shortcut works, and why. Use the image on the right as an example.



8. Write as fractions. Think of the shortcut.

a. $7\frac{1}{2}$	**b.** $6\frac{2}{3}$	**c.** $8\frac{3}{9}$	**d.** $6\frac{6}{10}$
e. $2\frac{5}{11}$	**f.** $8\frac{1}{12}$	**g.** $2\frac{5}{16}$	**h.** $4\frac{7}{8}$

Fractions to mixed numbers

Example 2. To write a fraction, such as 58/7, as a mixed number, we need to figure out:

- how many *whole* "pies" there are, and
- how many *individual slices* are left over.

In the case of 58/7, each whole "pie" will have 7 sevenths. (How do you know?) So we ask:

- How many 7s are there in 58? (Those make the whole pies!)
- After the 7s are gone, how many "slices" are left over?

Finish this example in the next exercise.

9. Refer to the example above. How many 7s are there in 58? ________

After that, how many "slices" are left over? ________

What maths operation helps you with the above?

Now, use the answers to the above questions to write 58/7 as a mixed number.

10. Rewrite the "division problems with remainders" as "fractions changed to mixed numbers."

a. 47 ÷ 4 = 11, remainder 3 $\frac{47}{4} = 11\frac{3}{4}$	**b.** 35 ÷ 8 = 4, remainder 3 $\frac{\square}{\square} = \square\frac{\square}{\square}$	**c.** 19 ÷ 2 = $\frac{\square}{\square} = \square\frac{\square}{\square}$
d. 35 ÷ 6 = $\frac{\square}{\square} = \square\frac{\square}{\square}$	**e.** 72 ÷ 10 = $\frac{\square}{\square} = \square\frac{\square}{\square}$	**f.** 22 ÷ 7 = $\frac{\square}{\square} = \square\frac{\square}{\square}$

The Shortcut: Think of the fraction bar as a *division* symbol, and DIVIDE. The quotient tells you the whole number part, and the remainder tells you the numerator of the fractional part.

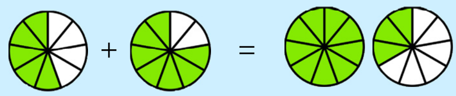
11. Write these fractions as mixed numbers (or as whole numbers, if you can).

a. $\frac{62}{8}$ =	**b.** $\frac{16}{3}$ =	**c.** $\frac{27}{5}$ =	**d.** $\frac{32}{9}$ =
e. $\frac{7}{2}$ =	**f.** $\frac{25}{4}$ =	**g.** $\frac{50}{6}$ =	**h.** $\frac{32}{5}$ =
i. $\frac{24}{11}$ =	**j.** $\frac{39}{3}$ =	**k.** $\frac{57}{8}$ =	**l.** $\frac{87}{9}$ =

Adding Mixed Numbers

Revision: adding like fractions

Example 1. Here, 5/9 and 7/9 are *like* fractions: they have the same denominator (same kind of parts). To add them, simply add the numerators.



$$\frac{5}{9} + \frac{7}{9} = \frac{12}{9} = 1\,\frac{3}{9}$$

Why does the denominator not change?

Because the *kind* of parts they are is not changing.

The answer, 12/9, is an improper fraction (more than one whole), so we write the final answer as a mixed number.

1. The calculation on the right shows a common student error. What is the error?

   Fix the calculation.

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{16}$$

2. Add. If your final answer is more than one, write it as a mixed number.

a. $\frac{5}{10} + \frac{3}{10} =$	**b.** $\frac{7}{8} + \frac{5}{8} =$	**c.** $\frac{7}{12} + \frac{5}{12} =$
d. $\frac{1}{4} + \frac{3}{4} + \frac{3}{4} =$	**e.** $\frac{4}{5} + \frac{3}{5} + \frac{2}{5} =$	

In this lesson we only deal with mixed numbers that have *like* fractional parts (the same denominator.) To add them, simply **add the whole number parts and the fractional parts separately:**

$$1\,\frac{1}{7} + 5\,\frac{3}{7} = 6\,\frac{4}{7}$$

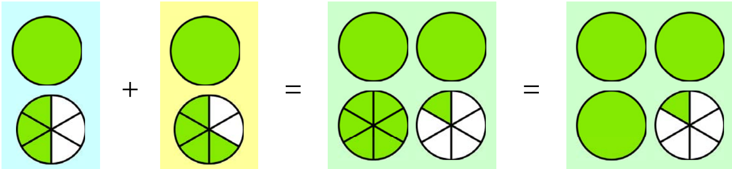
You can also add in columns →

$$\begin{array}{r} 2\,\frac{5}{8} \\ +\ 3\,\frac{2}{8} \\ \hline 5\,\frac{7}{8} \end{array}$$

3. Add.

a. $3\,\frac{2}{4} + 7\,\frac{1}{4} =$	**c.** $\begin{array}{r} 2\,\frac{1}{4} \\ +\ 11\,\frac{2}{4} \\ \hline \end{array}$	**d.** $\begin{array}{r} 28\,\frac{6}{12} \\ +\ 4\,\frac{4}{12} \\ \hline \end{array}$
b. $15\,\frac{3}{9} + 3\,\frac{5}{9} =$		

Often the sum of the fractional parts is more than one whole pie. Look at this example carefully:



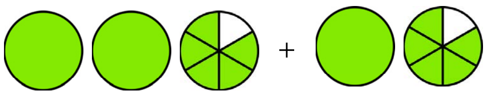
$1\frac{3}{6} + 1\frac{4}{6} = 2\frac{7}{6} = 3\frac{1}{6}$

Here, the sum of the fractional parts is 7/6. Think of that as 1 1/6, and add the one whole to the sum of the whole numbers (which was 2) to get 3. The final answer is 3 1/6.

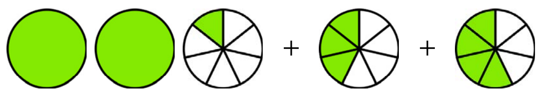
4. These mixed numbers have a fractional part that is more than one "pie." Write them in such a way that the fractional part is less than one. The first one is done for you. You can use manipulatives or draw fraction pictures to help.

a. $3\frac{3}{2} = 4\frac{1}{2}$	**b.** $1\frac{11}{9} =$	**c.** $6\frac{7}{4} =$	**d.** $3\frac{13}{8} =$

5. Write the addition sentences that the pictures illustrate and then add.



a.



b.

6. Add.

a. $3\frac{2}{3} + 8\frac{1}{3} =$	**b.** $4\frac{4}{5} + 1\frac{3}{5} =$	**c.** $6\frac{8}{9} + 1\frac{2}{9} =$

7. The sides of a triangle measure 7 3/8 units, 5 7/8 units, and 3 4/8 units. What is its perimeter?

8. Add.

a. $4\frac{3}{7} + 5\frac{5}{7}$ $9\frac{8}{7} = 10\frac{1}{7}$	**b.** $3\frac{3}{5} + 3\frac{4}{5}$ $=$	**c.** $4\frac{6}{9} + 2\frac{7}{9}$	**d.** $7\frac{6}{8} + 2\frac{7}{8}$

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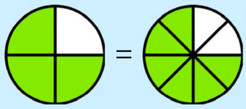
Simplifying Fractions 1

You have learned how to convert a fraction into an equivalent fraction:

Each slice is **split two ways**.



$\times 2$

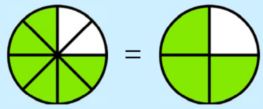
$$\frac{3}{4} = \frac{6}{8}$$

$\times 2$

What happens if we *reverse* the process?

Then it is called **SIMPLIFYING** or **REDUCING** a fraction:

Every two slices are **joined together**.



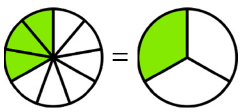
$\div 2$

$$\frac{6}{8} = \frac{3}{4}$$

$\div 2$

1. Simplify the following fractions, filling in the missing parts.

a. Every *three* slices are joined together.

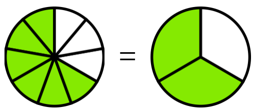


$\div$ 3

$$\frac{3}{9} = \frac{\square}{\square}$$

$\div$ ☐

b. Every ________ slices are joined together.

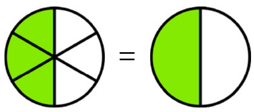


$\div$ ☐

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$\div$ ☐

c. Every ________ slices are joined together.

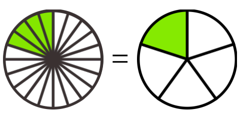


$\div$ ☐

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$\div$ ☐

d. Every ________ slices were joined together.

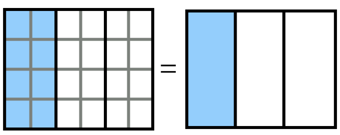


$\div$ ☐

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$\div$ ☐

e. Every ________ parts were joined together.

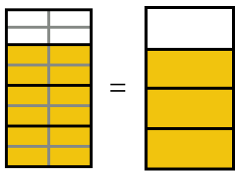


$\div$ ☐

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$\div$ ☐

f. Every ________ parts were joined together.



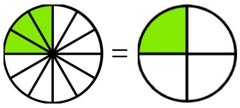
$\div$ ☐

$$\frac{\square}{\square} = \frac{\square}{\square}$$

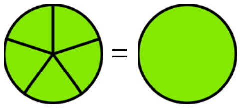
$\div$ ☐

2. Write the simplifying process. You can write the arrows and the divisions to help you.

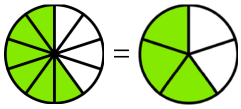
a. Every _________ slices were joined together.



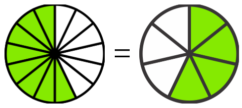
b. Every _________ slices were joined together.



c. Every _________ slices were joined together.

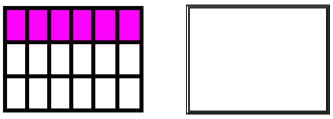


d. Every _________ slices were joined together.



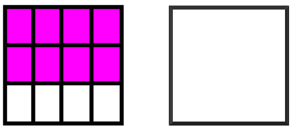
3. Draw a picture and reduce the fractions.

a. Join together every six parts.



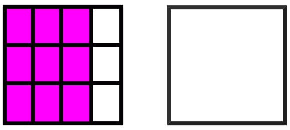
$\frac{\square}{\square} = \frac{\square}{\square}$

b. Join together every four parts.



$\frac{\square}{\square} = \frac{\square}{\square}$

c. Join together every three parts.



$\frac{\square}{\square} = \frac{\square}{\square}$

4. Reduce the fractions.

a. $\frac{6}{16} = \frac{\square}{\square}$ (÷ ☐)	**b.** $\frac{15}{25} = \frac{\square}{\square}$ (÷ ☐)	**c.** $\frac{28}{32} = \frac{\square}{\square}$ (÷ ☐)	**d.** $\frac{12}{42} = \frac{\square}{\square}$ (÷ ☐)	**e.** $\frac{18}{27} = \frac{\square}{\square}$ (÷ ☐)

5. **a.** What happens to the *value* of the fraction in simplification?

b. Why do you think it's called *simplifying* or *reducing* a fraction?

6. Could there ever be a fraction that cannot be simplified? (Why or why not?)
Give examples.

In simplification, we need to *divide* the numerator and the denominator by some number, which means the numerator and the denominator both have to be **divisible by this number**—a number that is a **common factor** to both.

Example 1. Since both 28 and 40 are divisible by 4, we can divide the numerator and denominator by four. (This means that each four slices are joined together.)

$$\frac{28}{40} \overset{\div 4}{=} \frac{7}{10}$$

Example 2. We cannot find any number that would go into 6 *and* 17 (except 1). So 6/17 is already as simplified as it can be. It is already in its **lowest terms**.

$$\frac{6}{17} \overset{\div 1}{=} \frac{6}{17}$$

7. Simplify.

a. $\frac{12}{20} =$	**b.** $\frac{24}{32} =$	**c.** $\frac{3}{15} =$	**d.** $\frac{15}{18} =$	**e.** $\frac{16}{20} =$

8. Simplify the fractional parts of these mixed numbers. The whole number does not change.

a. $1\frac{4}{16} =$	**b.** $5\frac{3}{27} =$	**c.** $7\frac{5}{20} =$	**d.** $3\frac{14}{49} =$

9. You cannot simplify some of these fractions because they are already in lowest terms. Cross out the ones that are already in lowest terms, and simplify the rest.

a. $\frac{2}{3}$	**b.** $3\frac{4}{28}$	**c.** $\frac{6}{13}$	**d.** $\frac{6}{33}$
e. $\frac{11}{22}$	**f.** $1\frac{4}{7}$	**g.** $\frac{5}{11}$	**h.** $\frac{9}{21}$

10. Lucky is on the track team. He spends 10 minutes warming up before practice, and 10 minutes stretching after practice. All together, he spends a total of one hour for the warm-up, the practice, and the stretching.

What part of the total time is the warm-up time?
Give your answer in lowest terms.

What part of the total time is he actually practising?
Give your answer in lowest terms.

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Multiply Fractions by Fractions 1

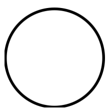
We have studied how to find a fractional part of a whole number using multiplication.

For example, $\frac{3}{5}$ of 80 is written as the multiplication $\frac{3}{5} \times 80$.

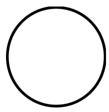
REMINDER: The word "of" in this context translates into multiplication.

Now let's examine how we can use the same idea to find **a fractional part of a fraction.**

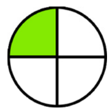
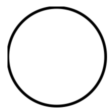
1. First find a fractional part of the given fraction **visually**. You can think of a leftover piece of pizza, which you are sharing equally with some other people. Then write a multiplication.

a. $\frac{1}{2}$ of  is 

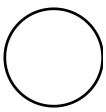
$\frac{1}{2} \times \frac{1}{2} =$

b. $\frac{1}{2}$ of  is 

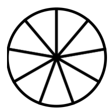
$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

c. $\frac{1}{2}$ of  is 

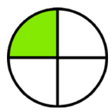
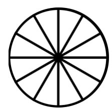
$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

d. $\frac{1}{3}$ of  is 

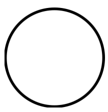
$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

e. $\frac{1}{3}$ of  is 

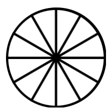
$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

f. $\frac{1}{3}$ of  is 

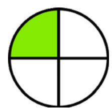
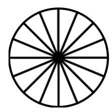
$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

g. $\frac{1}{4}$ of  is 

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

h. $\frac{1}{4}$ of  is 

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

i. $\frac{1}{4}$ of  is 

$\frac{\_\_}{\_\_} \times \frac{\_\_}{\_\_} = \frac{\_\_}{\_\_}$

2. Did you notice a shortcut? If so, write it here. Use examples, such as (1/5) × (1/2) and (1/4) × (1/6).

Shortcut for multiplying a *unit fraction* by another *unit fraction*:
(A unit fraction is of the form 1/*n* where *n* is a whole number.)

Shortcut: multiplying unit fractions

To multiply fractions of the form 1/n where n is a whole number, simply multiply the denominators to get the new denominator.

Example 1. $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ and $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

3. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$	**b.** $\frac{1}{13} \times \frac{1}{3}$	**c.** $\frac{1}{5} \times \frac{1}{20}$

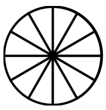
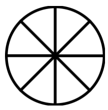
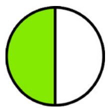
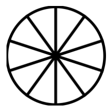
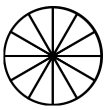
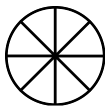
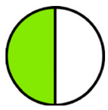
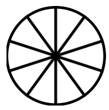
What about finding some other kind of fractional part? Let's again compare this to whole numbers.

Revision: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16$ becomes 3 × (¼ of 16) = 12.

We can use the same idea when finding a fractional part of another fraction.

4. Colour in the answer. Compare the problems in each box.

a. $\frac{1}{3}$ of  is 	**b.** $\frac{1}{4}$ of  is 	**c.** $\frac{1}{5}$ of  is 
$\frac{2}{3}$ of  is 	$\frac{3}{4}$ of  is 	$\frac{4}{5}$ of  is 

5. Two-thirds of a pizza is left from last night. You eat half of what is left.

a. Write a fraction multiplication to match this situation.
You can also draw a picture to help you.

b. Is your final answer a fraction of the *original* pizza, or of the portion that was left over?

Example 2. To find $\frac{4}{5}$ of $\frac{1}{7}$, first find $\frac{1}{5}$ of $\frac{1}{7}$ = $\frac{1}{35}$, and then take that four times to get $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction.
It is just like taking a certain part of the leftovers, when what is left over is a fraction.

6. Solve. You can find the answer to the bottom problem based on the top problem in each box.

a. $\frac{1}{5} \times \frac{1}{7} =$ $\frac{2}{5} \times \frac{1}{7} =$	**b.** $\frac{1}{6} \times \frac{1}{4} =$ $\frac{5}{6} \times \frac{1}{4} =$	**c.** $\frac{1}{8} \times \frac{1}{3} =$ $\frac{3}{8} \times \frac{1}{3} =$

What about generic fraction multiplication problems? For example, how can we do $\frac{5}{8} \times \frac{6}{7}$?

Mathematically, we can treat this as $5 \times \frac{1}{8} \times 6 \times \frac{1}{7}$, and then change the order of the factors to get $5 \times 6 \times \frac{1}{8} \times \frac{1}{7}$, which is equal to $5 \times 6 \times \frac{1}{56} = \frac{30}{56}$.

Essentially, the numerators get multiplied, and the denominators get multiplied.

A shortcut for fraction multiplication:	Multiply the numerators to get the numerator for the product. Multiply the denominators to get the denominator for the product.

Example 3. Give your final answer simplified and as a mixed number.

$$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1\frac{1}{10}$$

Example 4. Notice how we can write the whole number 5 as 5/1:

$$\frac{3}{7} \times 5 = \frac{3}{7} \times \frac{5}{1} = \frac{3 \times 5}{7 \times 1} = \frac{15}{7} = 2\frac{1}{7}$$

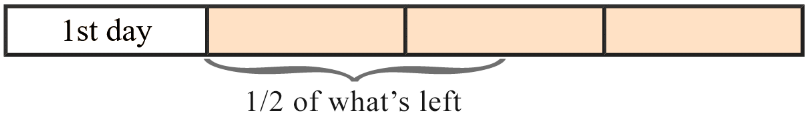
7. Multiply. Give your answers in the lowest terms and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$	**b.** $\frac{11}{12} \times \frac{1}{6}$
c. $\frac{1}{3} \times \frac{3}{13}$	**d.** $9 \times \frac{2}{3}$
e. $\frac{2}{9} \times \frac{6}{7}$	**f.** $10 \times \frac{5}{7}$

Multiply Fractions by Fractions 2

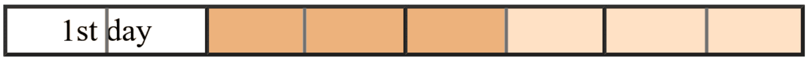
Example 1. Sibusiso finished 1/4 of a job he was given in one day. The next day, he finished half of what was left. Now what part of the task is left to do?

After the first day of work, he has 3/4 of the job left:



Then he finished *half* of that. This means we need to figure out half of 3/4. This is found with fraction multiplication: $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$. What does this 3/8 signify?

It is 3/8 of the original whole:



1. There was 1/4 of the pizza left. Junior ate 2/3 of that.

   a. Write a fraction multiplication. You can also draw a picture.

   b. What part of the *whole* (original) pizza did he eat?

   c. What part of the *whole* (original) pizza is left now?

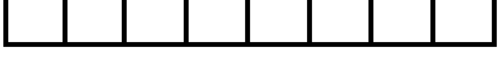
2. Chloe has painted 5/8 of the room.

   a. What part is still left to paint?

   b. Now, Chloe has painted *half* of what was still left.

   Write a fraction multiplication.

   Use the bar model on the right to help you.
   What part of the room is still left to paint?



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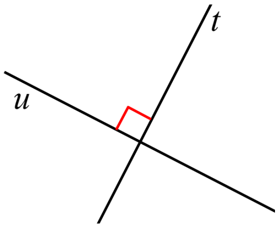
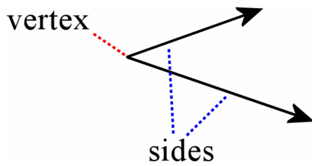
Geometry Vocabulary Reference Sheet

I encourage you to draw pictures to illustrate the terms, or even make your own geometry notebook!

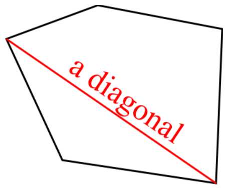
Two lines are **perpendicular** if they form a right angle.



An **angle** consists of two rays that start at the same point, called vertex. The two rays form the sides of the angle.



- A **polygon** is a flat, two-dimensional figure that consists of line segments, and is closed.
- A **regular polygon** is one with congruent sides and angles.
- A **vertex** is a "corner" of a polygon.
- A **diagonal** is a line segment drawn from one vertex of a polygon to another.



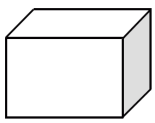
- A **quadrilateral** – a polygon with *four* sides
- A **pentagon** – a polygon with *five* sides.
- A **hexagon** – a polygon with *six* sides.
- A **heptagon** – a polygon with *seven* sides.
- An **octagon** – a polygon with *eight* sides.

- A **right triangle** is a triangle with one right angle.
- An **obtuse triangle** is a triangle with one obtuse angle.
- An **acute triangle** is a triangle with all three angles acute.

- An **equilateral triangle** is a triangle with three congruent sides.
- An **isosceles triangle** is a triangle with two congruent sides.
- A **scalene triangle** is a triangle where none of the sides are congruent.

- A **trapezium** is a quadrilateral with at least one pair of parallel sides.
- A **parallelogram** is a quadrilateral with two pairs of parallel sides.
- A **rhombus** is a parallelogram with four congruent sides.
- A **kite** is a quadrilateral that has two pairs of congruent sides, and the congruent sides are adjacent (neighbouring each other).
- A **rectangle** is a quadrilateral with four right angles.
- A **square** is a rectangle with four congruent sides.
- A **scalene quadrilateral** has no congruent sides.

- A **rectangular prism** is a box-shaped solid (three-dimensional shape) with edges that meet at right angles.

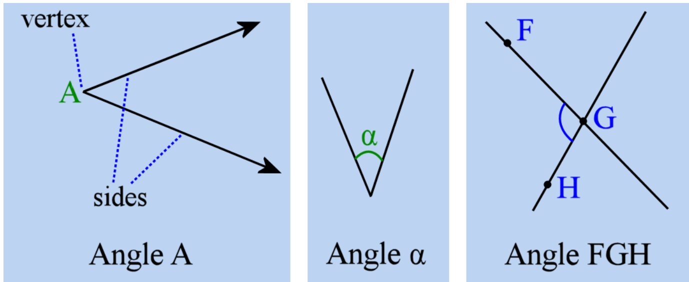


Revision: Angles

An angle is a figure formed by two rays that have the same beginning point. That point is called the **vertex**. The two rays are called the **sides** of the angle.

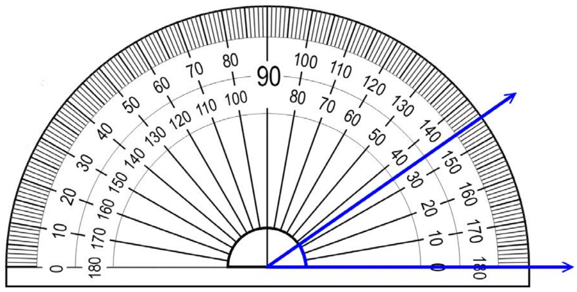
Imagine the two sides as being like two sticks that open up a certain amount. The more they open, the bigger the angle.

An angle can be named (1) after the vertex point, (2) with a letter inside the angle, or (3) using one point on the ray, the vertex point, and one point on the other ray.

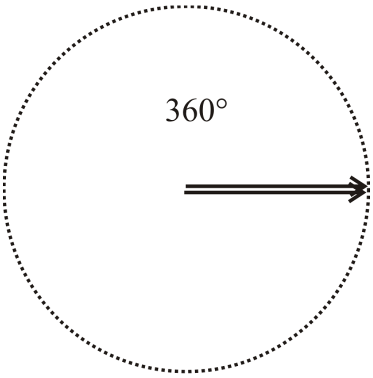


We use **a protractor** to measure angles. The vertex of the angle has to be placed in the middle of the protractor, and ONE side of the angle has to line up with the "zero line" of the protractor.

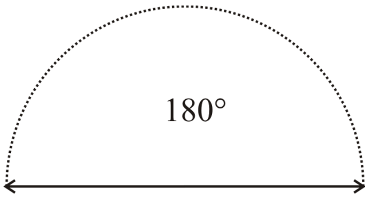
The angle on the right measures 35 degrees.



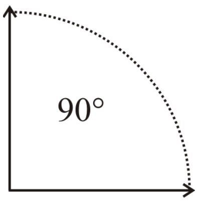
A full angle = 360°



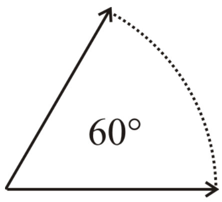
A straight angle = 180°

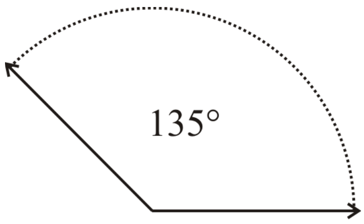


A right angle = 90°

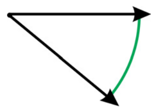


A zero angle = 0°

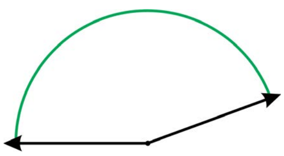




Angles that are more than 0° but less than 90° are called **acute** ("sharp") angles.

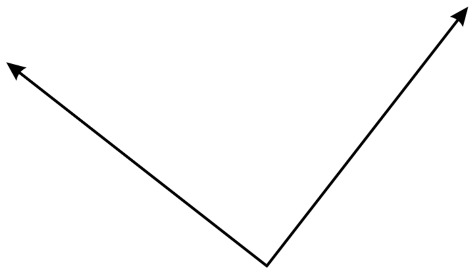


Angles that are more than 90° but less than 180° are called **obtuse** ("dull") angles.

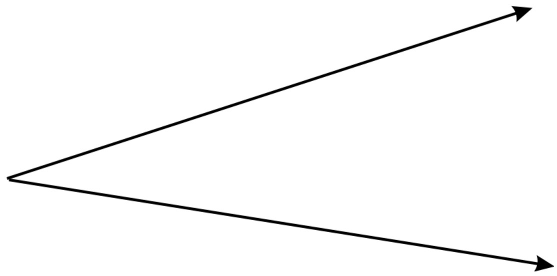


Angles that are more than 180° but less than 360° are called *reflex* angles.

1. Measure these angles with a protractor. If necessary, continue the sides of the angle with a ruler.



a. __________°

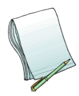


b. __________°

2. **a.** Draw any acute angle, and measure it.

b. Draw any obtuse angle, and measure it.

3. Draw three dots on a blank paper and join them to form a triangle.
Draw the dots far enough apart so that the triangle nearly fills the page.
Then, measure the angles of your triangle.



The angles of my triangle are: ________°, ________°, and ________°.

Classify each angle as acute, right, or obtuse.

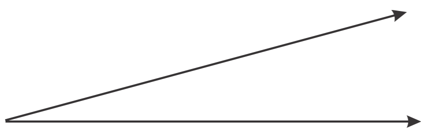
4. You see a line and a point on it. The point will be the vertex of an angle. Draw the other side of the angle from the vertex so that the angle measures 76°. Use a protractor.

5. Follow the procedure above to draw acute angles with the following measures:
a. 30° **b.** 60° **c.** 45°

6. Draw obtuse angles with these measures:
a. 135° **b.** 100° **c.** 150°

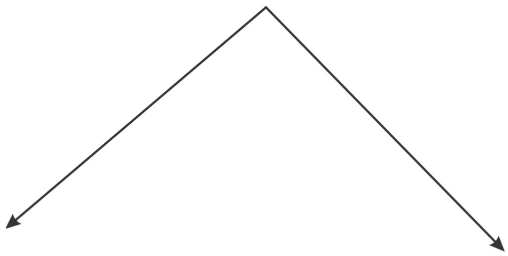
7. *Estimate* the measure of these angles. Measure to check (you may need to continue the sides).

a.



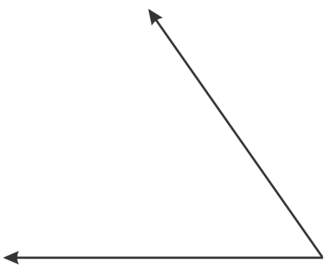
Estimate: ________° Measured: ________°

b.



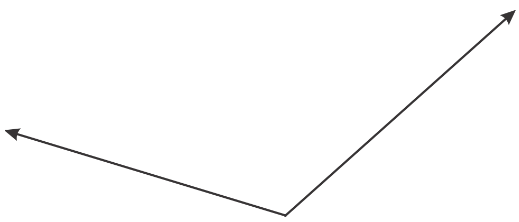
Estimate: ________° Measured: ________°

c.



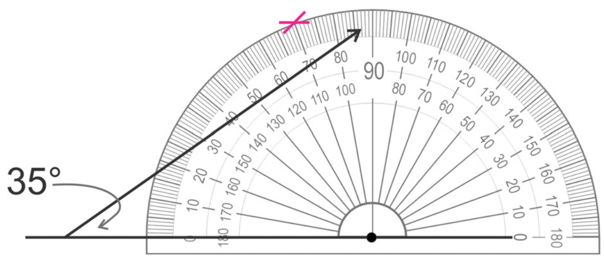
Estimate: ________° Measured: ________°

d.

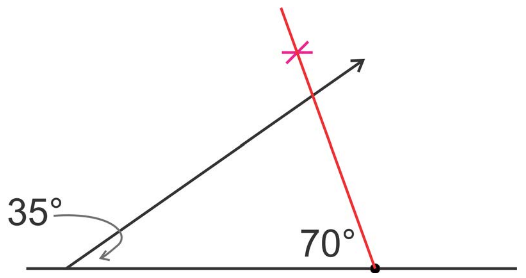


Estimate: ________° Measured: ________°

How to draw a triangle with two given angle measurements (optional)



Let’s say you have already drawn a 35° angle, and the second angle is supposed to be 70°. The image shows you how to place your protractor so you can measure and mark the 70° angle.



Then remove the protractor and draw the third side of the triangle.

8. (optional)

a. Draw a triangle with 50° and 75° angles. It can be of any size — smaller or bigger.
Hint: Start out by drawing a (long) horizontal line, and two dots on it which mark the two vertices of the triangle.

b. Measure the third angle. It measures ________°.

c. Label each angle in the triangle as acute, obtuse, or right.

9. (optional)

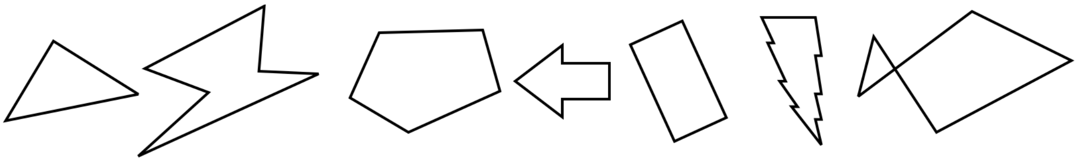
a. Draw a triangle with 110° and 35° angles.

b. Measure the third angle. It measures ________°.

c. Label each angle in the triangle as acute, obtuse, or right.

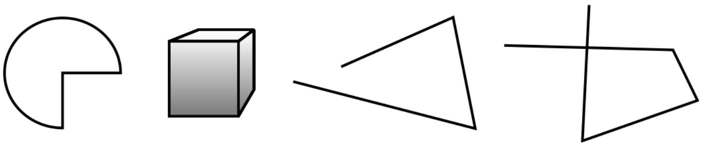
Polygons

A **polygon** is a flat, two-dimensional figure that consists of line segments, and is *closed*.



The boundary of a polygon is allowed to cross itself, like in the polygon above at the right. However, in this chapter we will mostly deal with *simple* polygons where such does not happen.

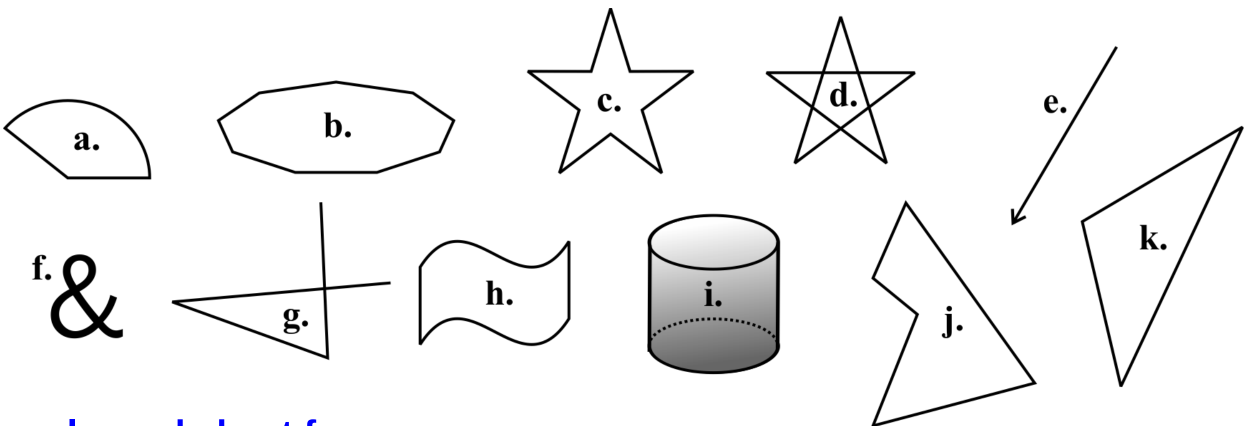
These figures are not polygons. Notice how each figure either is not closed, does not consist of line segments, or is not a flat, two-dimensional figure:



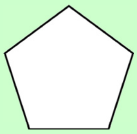
Polygons are named after the number of vertices they have. Most of the names for polygons in English have their roots in Greek, using a number and the Greek word "*gonia*" which means "angle".

Vertices	Name	Greek/Latin
3	triangle	tri = three
4	quadrilateral	quadri (Latin) = four
5	pentagon	pente = five
6	hexagon	hex = six
7	heptagon	hepta = seven
8	octagon	okto = eight

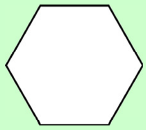
1. Classify each figure as a polygon, or not a polygon.



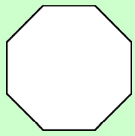
A **regular polygon** is one with congruent sides and angles. In other words, the sides are of equal length, and the angles have the same measure.



A regular pentagon



A regular hexagon



A regular octagon

2. What is the common name for a *regular quadrilateral*?

3. Name the figures. Include the word “regular” when fitting.

a. ____

b. ____

c. ____

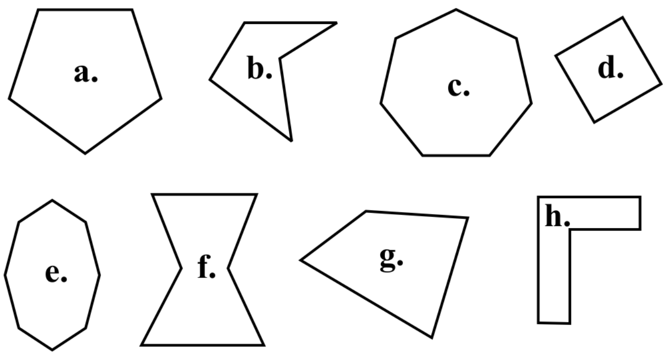
d. ____

e. ____

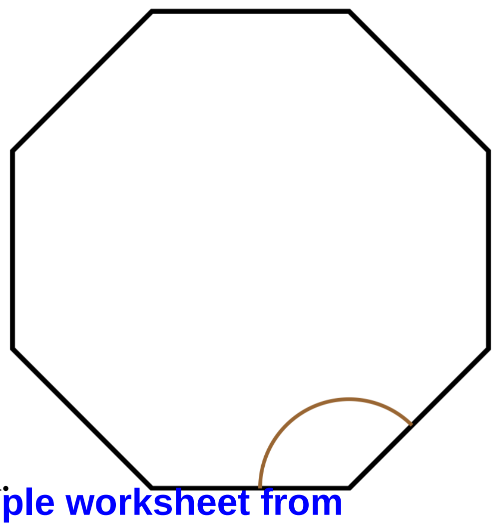
f. ____

g. ____

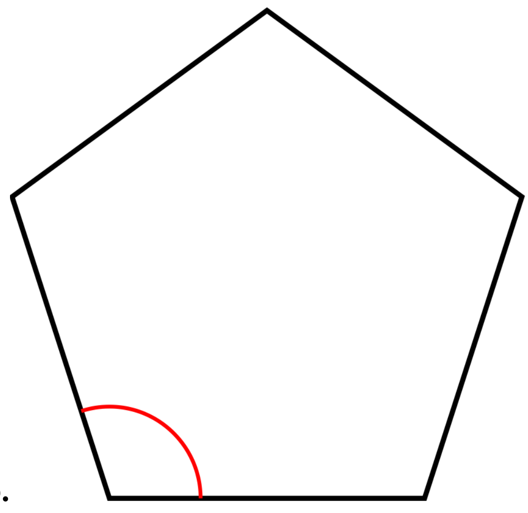
h. ____



4. For the regular octagon and regular pentagon below, measure one of the angles to find the measure of *all* the angles in each one (since it is *regular*!).

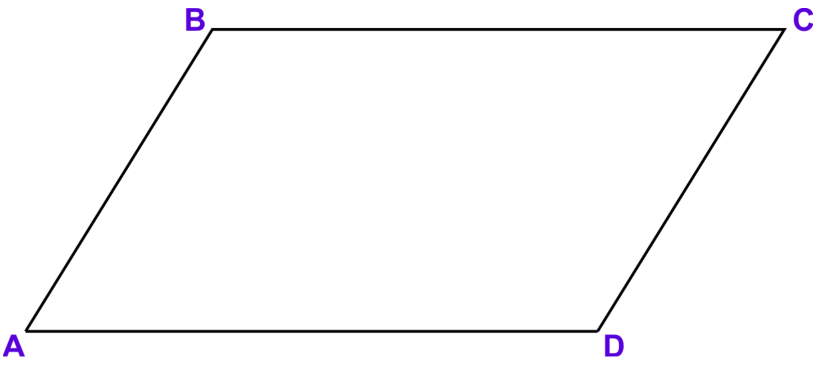


a.



b.

5. Find the measures of all the angles in these quadrilaterals. What do you notice?



a. A parallelogram

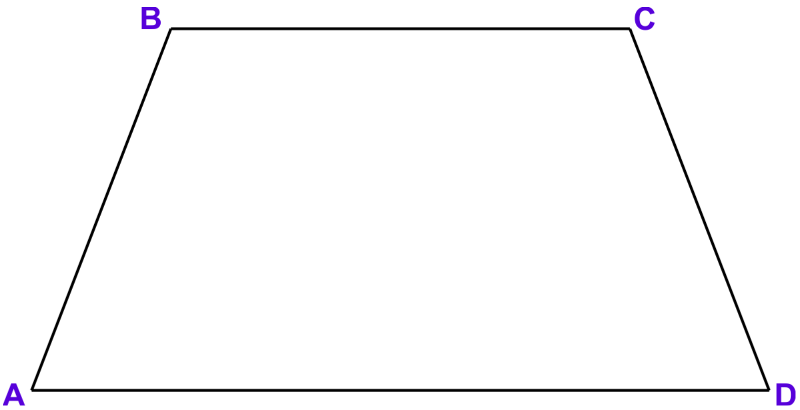
Angle A = ________°

Angle B = ________°

Angle C = ________°

Angle D = ________°

I notice that ____.



b. A trapezium

Angle A = ________°

Angle B = ________°

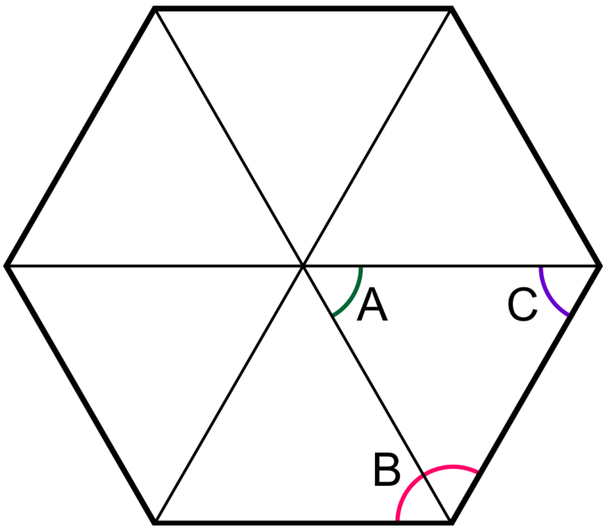
Angle C = ________°

Angle D = ________°

I notice that ____.

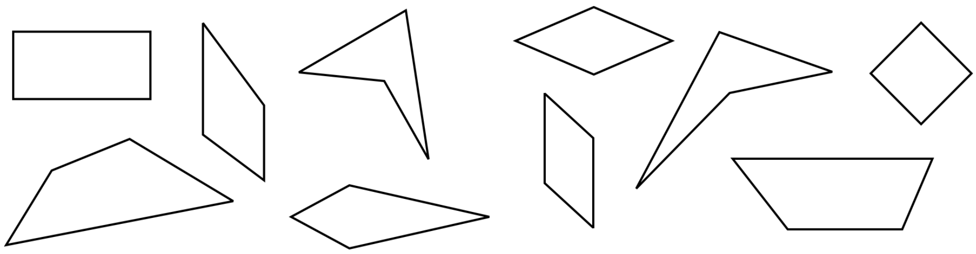
Puzzle Corner

This is a regular hexagon.
Figure out the angles A, B, and C.
Measure to check.



Classifying Quadrilaterals 1

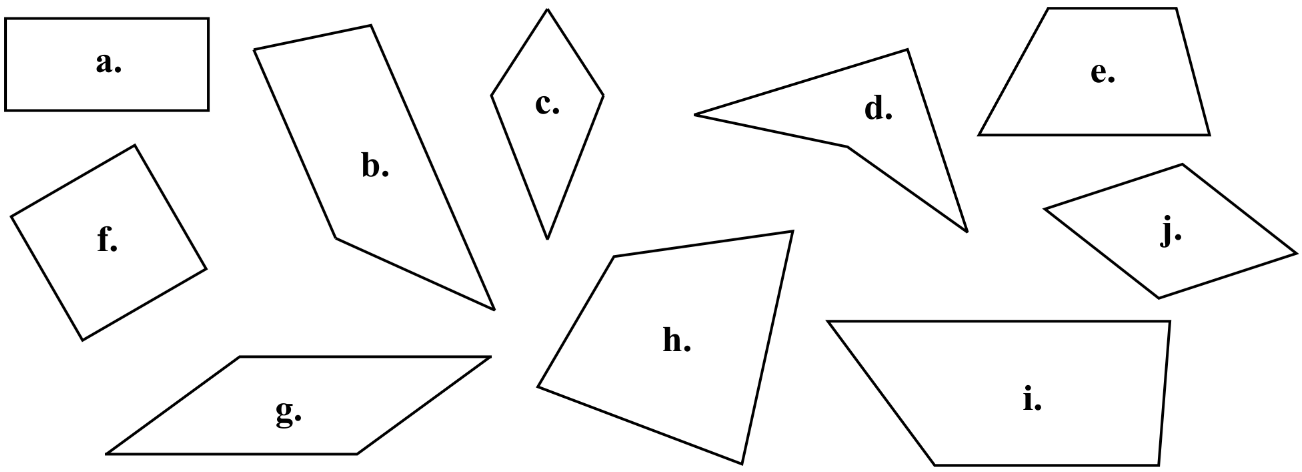
Quadrilaterals are polygons with four sides (*quadri-* = four, *lateral* = referring to a side).



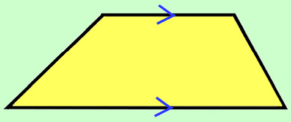
There are many different kinds of quadrilaterals. We could classify, or sort, them in various ways. The sorting could be done on the basis of the types of angles, or on the basis of the lengths of sides — and these characteristics are used. But the main way that we will *start* the classification of quadrilaterals has to do with whether or not the quadrilateral has **any parallel sides**.

1. Sort the quadrilaterals into groups according to the number of parallel sides each has. You can write the letter of each in the box below, or cut them out. Downloadable version here:

   https://www.mathmammoth.com/download/sort_quadrilaterals.pdf

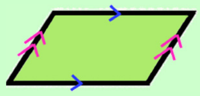


No parallel sides	One pair of parallel sides	Two pairs of parallel sides



A **trapezium** has at least one pair of parallel sides.

The arrowheads mark the pair of parallel sides.



A **parallelogram** has two pairs of parallel sides.

The single arrowheads mark the one pair of parallel sides, and the double arrowheads mark the other.

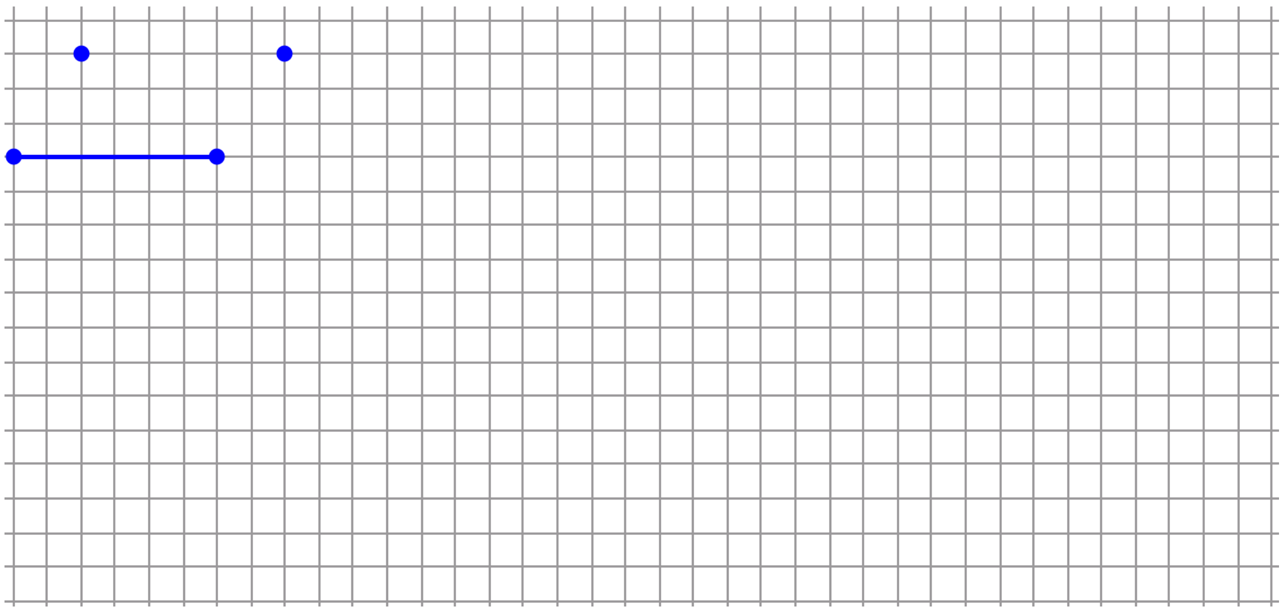
2. In the grid below:

a. Join the dots to make a parallelogram.

b. Draw one other, differently shaped parallelogram.

c. Draw two different trapeziums.

d. Draw a quadrilateral with no parallel sides.



3. **a.** Is a rectangle always a parallelogram? In other words, does a rectangle fulfil the definition of a parallelogram?

b. Is a parallelogram always a trapezium? In other words, does a parallelogram fulfil the definition of a trapezium?

c. Are all trapeziums parallelograms? Explain.

d. Are all rectangles trapeziums? How about vice versa?