Math Mammoth Grade 7, 2025 edition
Alignment to the Common Core Standards

This document lists the Common Core Standard(s) relevant to each lesson in the two student worktexts, 7-A and 7-B (2025 edition). For each chapter, I also list the main standards covered in the chapter, discuss what students have learned in previous grades that is foundational to those, and what they are learning in future grades that ties in with the major topics of the chapter.

Contents

Math Mammoth Grade 7-A

Chapter 1: The Language of Algebra

In the first chapter of *Math Mammoth Grade 7* we both review basic algebra topics from sixth grade and also go deeper into them, plus study the basic properties of the four operations. Since a good part of this chapter is review, it serves as a gentle introduction to 7th grade math, laying a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers.

The main topics are the order of operations, writing and simplifying expressions, and the properties of the four operations, including the distributive property. Students have studied most of these in 6th grade. The main principles are explained and practiced both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

What we are coming from

In 6th grade, students wrote, read, and evaluated expressions in which letters stand for numbers. They simplified expressions using the properties of the operations and learned about equivalent expressions. They also solved very simple one-step equations with nonnegative numbers, and wrote inequalities of the form $x > c$ or $x < c$.

What we are going towards

When we study equations in chapters 3 and 5, and also in subsequent grade levels, students will use the skills from this chapter (such as simplifying expressions, using the distributive property) in solving equations.

Relevant 6th grade standards for this chapter

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Relevant 7th grade standards

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Lesson	page number	Standards
Exponents and the Order of Operations	13	6.EE1, 6.EE2
Expressions and Equations	17	6.EE.2, 6.EE.5, 6.EE.6
Properties of the Four Operations	20	7.EE.1
Simplifying Expressions	24	6.EE.2, 7.EE.1, (7.EE.4)
Growing Patterns 1	28	7.EE.1, 7.EE.2
The Distributive Property	31	7.EE.1, 7.EE.2, (7.EE.4)
Chapter 1 Review	36	6.EE2, 7.EE.1, (7.EE.4)

Standards for Mathematical Practice

- (MP.5) The student is embarking on a wonderful journey into algebra — learning to do arithmetic with letters. The familiar properties of the four operations still hold, just like they do with numbers. Algebra is such a wonderful tool because it allows us to abstract a given situation and represent it symbolically, and then manipulate the representing symbols as if they have a life of their own. It is the foundational tool that allows us to model real-world situations with mathematics.

Chapter 2: Integers

This chapter deals with integers, which are signed (positive or negative) whole numbers. In 6th grade, students have learned various basic integer concepts, such as the absolute value of integers, how to order integers, and how to plot pairs of integers on a coordinate plane. In this chapter, we begin with a review of those concepts in the lesson *Integers*. Then we study in detail addition, subtraction, multiplication, and division of integers.

What we are coming from

In 6th grade, students learned various basic integer concepts, such as the absolute value of integers, how to order integers, and how to plot pairs of integers on a coordinate plane.

What we are going towards

The last topic of the chapter, negative fractions, is a starting point for Chapter 4 where students work with rational numbers (fractions and decimals, including negative ones). This chapter and chapter 4 together conclude the study of rational numbers and their operations in middle school. In 8th grade, we expand the number system beyond rational numbers into *irrational* numbers.

After working with integers and being introduced to negative fractions in this chapter, students will also be ready to work with equations that involve integers (chapters 3 and 5). Being able to calculate with negative numbers is actually a foundational skill for algebra: they are encountered universally when working with equations, graphing, and functions.

Relevant 7th grade standards

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Lesson	page number	Standards
Integers	43	6.NS.5, 6.NS.6, 6.NS.7
Addition and Subtraction on the Number Line 1	47	7.NS.1.b
Addition and Subtraction on the Number Line 2	50	7.NS.1.a, 7.NS.1.b
More Addition of Integers	53	7.NS.1.a, 7.NS.1.b
Subtraction of Integers	56	7.NS.1.c
Adding and Subtracting Several Integers	60	7.NS.1.d, 7.NS.1.b, 7.NS.3
Distance Between Numbers	62	7.NS.1.c, 7.NS.3
Multiplying Integers 1	66	7.NS.2.a, 7.NS.3
Multiplying Integers 2	70	7.NS.2.a, 7.NS.2.c, 7.NS.3
Dividing Integers	73	7.NS.2.b, 7.NS.3
Negative Fractions	76	7.NS.2.b, 7.NS.3
Mixed Review Chapter 2	80	6.EE.2, 6.EE.6, 7.EE.1
Chapter 2 Review	82	7.NS.1, 7.NS.2, 7.NS.3

Standards for Mathematical Practice

- (MP.3) The topics of multiplying and dividing integers provide opportunities for students to encounter and also to construct mathematical arguments. The lessons on multiplication bring up several possible justifications for the shortcut “negative times negative is positive,” including the use of patterns and the structure of the distributive property. Students also have the opportunity to discover the shortcut for the sign of the product when several integers are multiplied.

Chapter 3: One-Step Equations

In 6th grade, students have solved one-step linear equations (of the forms $x + p = q$ and $px = q$) that only involved nonnegative rational numbers. In this chapter, we extend that to involve similar one-step equations with integers. Next, in chapter 4, students encounter these one-step equations with fractions and decimals, and in chapter 5, they solve to two-step equations involving rational numbers.

The topic of solving linear equations is finished in grade 8 or in high school, when students encounter equations where the variable is on both sides and practice solving linear equations of varied complexity.

The first lesson of the chapter reviews the concept of an equation and how to model equations using a pan balance (scale). Recall that the basic principle for solving equations is that, when you perform the same operation on both sides of an equation, the two sides remain equal.

The chapter presents two alternatives for keeping track of the operations to be performed on an equation. The one method, writing the operation under each side of the equation, is common in the United States. The other method, writing the operation in the right margin, is common in parts of Europe. Either is adequate, and the choice is just a matter of the teacher's personal preference.

The introduction to solving equations is followed by a lesson on addition and subtraction equations and another on multiplication and division equations. All the equations are easily solved in only one step of calculations. The twofold goal is to make the student proficient in manipulating negative integers and also to lay a foundation for handling more involved equations in Chapter 5.

In the next lesson, students write equations to solve simple word problems. Even though they could solve most of these problems without using the equations, the purpose of the lesson is to make the student proficient in writing simple equations before moving on to more complex equations from more difficult word problems.

The last topic, in the lesson *Constant Speed*, is solving problems with distance (d), rate or velocity (v), and time (t). Students use the equivalent formulas $d = vt$ and $v = d/t$ to solve problems involving constant or average speed. They learn an easy way to remember the formula $v = d/t$ from the unit for speed that they already know: "miles per hour."

Many of the lessons in the chapter may seem simple and easy. However, the topics we go through are fundamental, preparing students to use equations to solve word problems, which is at the heart of being able to model with mathematics. By taking time for one-step equations on this level, we ensure that students are able to fluently work with integers in equations and that they are thoroughly familiar with the mechanics of solving equations before working on more complex equations. The lesson *Constant Speed* presents some more challenging problems to solve in this chapter, and there are more in chapters to come.

What we are coming from

In 6th grade, students have solved one-step linear equations (of the forms $x + p = q$ and $px = q$) that only involved nonnegative rational numbers. (6.EE.7).

What we are going towards

Next, in chapter 4, students encounter these one-step equations with fractions and decimals, and in chapter 5, they solve to two-step equations involving rational numbers.

The topic of solving linear equations is finished in grade 8 or in high school, when students encounter equations where the variable is on both sides and practice solving linear equations of varied complexity.

Relevant 7th grade standards

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Lesson	page number	Standards
Solving Equations	87	6.EE.5, 6.EE.7, 7.EE.4
Addition and Subtraction Equations	94	7.EE.4.a
Multiplication and Division Equations	98	7.EE.4.a
Word Problems 1	102	7.EE4a
Constant Speed	105	7.EE.3, 7.EE.4, 7.RP.2.c
Chapter 3 Mixed Review	112	7.NS.1, 7.NS.2, 7.NS.3, 7.EE.1
Chapter 3 Review	115	7.EE.3, 7.EE.4.a.

Chapter 4: Rational Numbers

In this chapter we study *rational* numbers, which are numbers that can be written as a *ratio* of two integers. All fractions and whole numbers are rational numbers, and so are percents and decimals (except non-ending non-repeating decimals). Obviously, students already know a lot about rational numbers and how to calculate with them, from all their schooling thus far. Our focus in this chapter is to extend that knowledge to *negative* fractions and *negative* decimals. Then in 8th grade and in high school, students will learn about irrational numbers, including square roots and other roots, and how to do calculations with them.

The first lesson presents the definition of a rational number as a ratio of two integers, and students practice plotting rational numbers on number lines and comparing them. In the next lesson, they learn about repeating decimals and how all rational numbers are either terminating or repeating decimals when written in their decimal form.

Then we turn our attention to calculations. Adding, subtracting, multiplying and dividing with negative and positive rational numbers happens the same way as doing those calculations with integers, so this chapter effectively reviews and reinforces what was learned in Chapter 2.

Students first practice adding and subtracting rational numbers, with an emphasis on adding and subtracting negative fractions and decimals. The next two lessons are about multiplying and dividing rational numbers. The first of the two focuses on basic multiplication and division with negative fractions and decimals. The second of the two presents complex fractions (fractions that contain another fraction, either in the numerator, in the denominator, or in both), and problems that mix decimals, fractions, and percents.

Then students continue on to practice more complex calculations with rational numbers, dealing with real-life contexts for products and quotients of rational numbers, and learning the importance of pre-estimating what a reasonable answer would be.

The instructional portion of the chapter concludes with two lessons on solving simple equations that involve fractions and decimals, reviewing the topic of solving equations from Chapter 3.

What we are coming from

Students have calculated with rational numbers ever since they learned to add and subtract, but in particular, they have learned to calculate with fractions and decimals in grades 4-6.

What we are going towards

In 8th grade and in high school, students will learn about irrational numbers, including square roots and other roots, and how to do calculations with them.

Relevant 7th grade standards

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p – q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $–(p/q) = (–p)/q = p/(–q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Lesson	page number	Standards
Rational Numbers	120	6.NS.6, 6.NS.7, (7.NS.1)
Repeating Decimals	123	7.NS.2.d
Fractions and Decimals (optional)	126	7.NS.2.d
Adding Rational Numbers	129	7.NS.1.b, 7.NS.1.c
Adding and Subtracting Rational Numbers	133	7.NS.1.d, 7.NS.1.b
Multiply and Divide Rational Numbers 1	137	7.NS.2.c
Multiply and Divide Rational Numbers 2	141	7.NS.2.c
Calculations with Rational Numbers	145	7.NS.2.a, 7.NS.2.b, 7.NS.3
More Practice with Calculations	149	7.NS.3
Problems to Solve	151	7.EE.3
Equations with Fractions	154	7.EE.3, 7.EE.4a
Equations with Decimals	159	7.EE.4a
Chapter 4 Mixed Review	162	7.EE.3, 7.EE.4, 7.NS.2.b, 7.NS.1.b
Chapter 4 Review	164	7.NS.1, 7.NS.2, 7.NS.3, 7.EE.3, 7.EE.4

Standards for Mathematical Practice

- (MP.6) The chapter includes a lot of calculations, including some challenging ones. Throughout the chapter, students are learning and reviewing strategies that help them to calculate more efficiently. Students are often asked to check their answer by comparing it to an estimate made with rounded numbers and mental math. Encourage your student(s) to check that their answers are reasonable, at all times. These things are all a part of learning to be precise and accurate with calculations.
- (MP.4) Being able to calculate with rational numbers allows students to model many more real-life situations with mathematics than before. They can now deal with situations involving debt, elevation, electric charge, and more. These skills will also be needed when working with equations in upcoming chapters.

Chapter 5: Equations and Inequalities

In this chapter we delve deeper into our study of (linear) equations. In sixth grade, and earlier in seventh grade, students have solved simple one-step equations (for example $-6x = 24$ or $x + 9 = -2$). In Math Mammoth curriculum, they have also seen simple two-step equations modeled with a bar diagram and a pan balance. Now in 7th grade, students solve equations that typically require two steps to solve and may contain parentheses. The expectation is that they learn to solve equations of the types $px + q = r$ and $p(x + q) = r$ fluently.

The chapter contains several lessons where we focus writing an equation for a word problem. Students solve these word problems both using arithmetic and using an equation, comparing the solution steps.

There is also another lesson on patterns of growth, which may seem to be simply a fascinating topic, but in reality presents the fundamentals of a very important concept in algebra—that of linear functions (although they are not mentioned by that name).

After the section about equations, the text presents the basics of inequalities and how to graph them on a number line. Students apply the principles for solving equations to solve simple inequalities and word problems that involve inequalities.

The equations in this grade are limited in complexity, with the variable on one side only, and usually only requiring two (sometimes three) steps to solve. In 8th grade, or in an algebra course, students solve multi-step linear equations, including those with the variable on both sides of the equation, thus completing their understanding of how to solve linear equations.

What we are coming from

In sixth grade, and earlier in seventh grade, students have solved simple one-step equations (for example $-6x = 24$ or $x + 9 = -2$).

What we are going towards

In 8th grade (or in an algebra 1 course), students will solve multi-step linear equations; yet the beauty of mathematics is that those complex equations will be able to be reduced to these simpler forms that they are learning to solve now.

Relevant 7th grade standards

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Lesson	page number	Standards
Two-Step Equations, Part 1	172	(7.EE.4.a)
Problems to Solve	176	7.EE.4.a
More Problems to Solve	180	7.EE.4.a
Growing Patterns 2	183	7.EE.2, 7.EE.4
Using the Distributive Property	186	(7.EE.4.a)
Two-Step Equations, Part 2	189	7.EE.4.a
Word Problems and Equations, Part 1	193	7.EE.4.a
Word Problems and Equations, Part 2	195	7.EE.4.a
Equations with Fractions, Part 1	197	(7.EE.4.a)
Equations with Fractions, Part 2	200	7.EE.4.a
Word Problems and Equations, Part 3	203	7.EE.4.a
Inequalities	205	7.EE.4.b
Word Problems and Inequalities	209	7.EE.4.b
More on Inequalities	213	7.EE.4.b
Mixed Review Chapter 5	218	7.EE.1, 7.EE.4.a, 7.NS.1, 7.NS.2, 7.NS.3
Review Chapter 5	221	7.EE.4

Standards for Mathematical Practice

- (MP.5) Many of the word problems ask students to compare an algebraic solution to a word problem with an arithmetic solution. This allows them to see the differences and similarities in the two approaches, and to wisely choose later on which approach or "tool" is more efficient.
- (MP.6) Encourage students to check each solution to an equation that they find. Checking and reflecting on your own work is an important habit, and is especially important in mathematics, where we often seek a precise answer to a question.
- (MP.4) Choosing a variable for the unknown in a word problem and writing an equation for the situation (in other words, representing a situation abstractly and symbolically) is an important skill practiced in this chapter. It is, in fact, an important subskill when modeling with mathematics.

Math Mammoth Grade 7-B

Chapter 6: Ratios and Proportions

Chapter 6 reviews the concept, which has already been presented in grade 6, of the **ratio** of two quantities. From this concept, we develop the related concepts of a **rate** (so much of one thing per so much of another thing) and a **proportion** (an equation of two ratios).

When two quantities are in proportion, we can consider the quantities as variables, write an equation to describe the relationship between them, and graph that equation. This study of proportional relationships takes the concept of *ratio* to a new level, and paves the way to the study of linear functions in 8th grade.

The first lessons focus on the concepts of ratio, rate, and unit rate. Students use tables of equivalent ratios and unit rates to solve a variety of problems involving rates. We especially focus on calculating unit rates when the quantities involve fractions.

Then we study proportional relationships, using the familiar tables of equivalent rates as a starting point. Students write and graph equations relating the two quantities (seen as variables now). They find the unit rate and plot it on the graph as a single point, and relate the different representations of proportional relationships (graph, table of values, wording, and equation) to each other. We also spend some time analyzing whether a given relationship between variables is proportional or not.

The next topic is proportions — equations where one ratio is equal to another. Students learn to solve proportions with cross-multiplying and to set them up in the correct way to solve a word problem. They also learn and compare different ways to solve problems with rates. It is not always necessary to set up a proportion!

Then we turn our attention to an application of all this in geometry: scaled figures, scale drawings, floor plans, and maps. Students encounter scales such as 1:90 or 1 in = 2 ft. They calculate dimensions in reality from the scale drawing and vice versa, and redraw scale drawings at a different scale. Floor plans use a scale also, and are hopefully an interesting topic to students.

The lesson on maps is optional. In today's world, most of us are using online map services which calculate the distances for us, so there is much less need to figure out distances using physical maps, but some students (and teachers) might find the topic interesting.

What we are coming from

In sixth grade, students learned the concepts of ratio and unit rate, and they used ratio reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, or equations.

What we are going towards

In 8th grade, students encounter proportional relationships again, as they learn the connection between the unit rate and the slope of the graph, and compare different proportional relationships represented in different ways.

Relevant 7th grade standards

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

7.RP.2. Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Chapter 7: Percent

In this chapter we study all the basic concepts relating to percentages. Students have already encountered percentages in 6th grade, and we start the chapter by reviewing the concept of percent as "per hundred" or as hundredth parts, and how to convert between fractions, decimals, and percents. The lesson *Solving Basic Percentage Problems* is also intended for review of sixth grade topics, focusing on finding a known percentage of a number (such as 21% of 56) and finding the percentage when you know the part and the total.

The lesson *Circle Graphs* is optional, and allows students to apply the concept of percent in a somewhat familiar context.

We take a little different perspective of these concepts in the two lessons about *Percent Equations and Price Changes*. Students write simple equations for situations where a price increases or decreases (discounts) and for sales tax. This lesson also explains what a percent proportion is—though personally, I prefer writing the percentage as a decimal and then writing an equation. I feel that approach adapts better to solving complex problems than using percent proportion.

Here is a quick example to show the difference between the two methods. Let's say an item is discounted by 22% and it now costs \$28. Then, the new price is 78% of the original. If we let p be the price of the item before the discount, we can write the percent proportion $\$28/p = 78/100$ and solve for p. If we write the percentage 78% as the decimal 0.78, we get the equation $0.78p = \$28$. Personally, I consider percent proportion to be an optional topic, and the reason I have included it here is to make this curriculum fully meet the Common Core Standards for seventh grade.

Next, we delve into the percentage of change. Students sometimes view the percentage of change as a totally different concept as compared to other percentage topics, but it is not that at all. To calculate the percentage of change, we still use the fundamental idea of *percentage = part/total*, only this time, the "part" is how much the quantity in question changes (the difference) and the "total" is the original quantity.

Simple Interest is a lesson on the important topic of interest, using as a context both loans and savings accounts. Students learn to use the formula $I = prt$ in a great variety of problems and situations.

The text concludes with a review lesson of all of the concepts taught in the chapter.

What we are coming from

In sixth grade, students learned to find a percent of a quantity (e.g. 30% of \$120) and found the whole, given a part and the percent.

What we are going towards

Percent is not a topic in itself in 8th grade or high school math, but naturally there are many applications and mathematical and real-world situations where that concept is used (e.g. exponential functions, statistics). Therefore, this chapter concludes this topic for school mathematics.

Relevant 7th grade standards

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
7.EE.2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*
7.EE.4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities

Lesson	page number	Standards
Review: Percent	79	6.RP.3, 7.NS.2.d
Solving Basic Percentage Problems	82	6.RP.3, 7.RP.3
Circle Graphs (optional)	86	7.RP.3
Percent Equations and Price Changes, Part 1	88	7.EE.2, 7.EE.4
Percent Equations and Price Changes, Part 2	91	7.EE.4, 7.RP.3
Percentage of Change	94	7.RP.3
Percentage of Change: Applications	97	7.RP.3
Simple Interest	101	7.RP.3
Chapter 7 Mixed Review	107	7.RP.2, 7.NS.1,7.NS.2, 7.NS.3, 7.EE.1, 7.EE.4
Chapter 7 Review	111	7.RP.3, 7.EE.4

Standards for Mathematical Practice

- (MP.1) The lesson “Percentage of Change: Applications” includes nice problems that allow students to make sense of problems and to persevere in problem solving (other lessons do, too). Encourage students to ask themselves often, “Does this make sense?” as they are solving the problems.

Chapter 8: Geometry

The geometry topics for 7th grade are varied:

- some angle relationships
- drawing geometric figures (especially triangles), and some basic geometric constructions
- pi and the area and circumference of a circle
- slicing 3-dimensional solids
- surface area and volume.

Angles were previously studied in detail in 4th grade, and briefly reviewed in Math Mammoth Grade 5-B. Now students examine angle relationships that are formed when several rays originate from the same starting point, writing and solving simple equations for an unknown angle in such figures.

Next, students practice drawing geometric figures. The foundation for this work has been laid all through the earlier grades, when students have learned to categorize various two-dimensional shapes according to their angles and sides.

In this chapter, students draw figures both using modern tools (normal ruler and protractor), as well as using only a compass and straightedge (a ruler without measurement units) in the lessons about geometric constructions. These constructions help students to think about the main defining features of a figure — angles and side lengths. Personally I have always enjoyed geometric constructions because they are like little puzzles to solve.

We especially focus on determining whether the given information (angles and/or sides) defines and allows one to draw a unique triangle. This topic directly prepares students for studying triangle congruency theorems in high school geometry. Learning to pay attention to the angles and sides of a figure is also helpful towards 8th grade, when students will rotate, move, reflect, and scale geometric figures, noting how the angles and sides of the figure possibly change under such transformations.

Then we turn our attention to pi. Students first learn the definition of pi as a ratio of a circle's circumference to its diameter in the lesson *Circumference of a Circle*. Then they learn and practice how to calculate the area of a circle in a wide variety of word problems and applications. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorize that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

After this, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modeling clay. Some Internet links (provided in the lesson) will also help students to visualize what happens when a solid is cut with a plane.

Lastly, students solve a variety of problems concerning surface area and volume and practice converting between various units of area and volume. Students learned to find the surface area of some solids in 6th grade. In this chapter, they continue this work, expanding into pyramids, right prisms, and cylinders. In 6th grade, they found the volume of a rectangular prism with fractional edge lengths. Here in 7th grade, we study the volume of right prisms and cylinders, and lastly in 8th grade, the volume of cones, cylinders, and spheres.

While the topics of surface area and volume tend to involve lots of calculations and less possibilities for hands-on activities, they are very important in real life.

What we are coming from

Students studied basic angle concepts in 4th grade. In 5th, they learned to classify triangles and quadrilaterals, and to calculate the volume of rectangular prisms. In 6th, they studied the area of triangles, parallelograms, and polygons, and were introduced to surface area.

What we are going towards

In 8th grade, the focus is on geometric transformations, but students also encounter more angle relationships (angle sum of a triangle, corresponding angles, alternate interior angles, etc.), and calculate the volume of cones, cylinders, and spheres. In high school, they spend an entire year on geometry, coming back to all basic geometry topics and learning much more.

Relevant 7th grade standards

7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
7.G.4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
7.G.5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Standards for Mathematical Practice

- (MP.7) The geometry problems in the lessons provide multiple opportunities for looking closely to find **a pattern** or **a structure**. For example, in the lessons about angle relationships, students need to look for supplementary, complementary, or vertical angles in the figures, in order to be able to solve for the unknown (s). Another example is how geometric constructions build upon each other, so that one construction often uses a basic pattern from a previous one. The same is true of the patterns students find as they draw triangles from the given information.

- (MP.3) Logical reasoning and providing proofs of statements is very important in mathematics. In the lessons about angle relationships and about drawing triangles students practice constructing arguments and justifying their conclusions. In the lesson *Proving the Formula for the Area of a Circle* students also see an example of an actual mathematical proof.
- (MP.1) Various lessons include multi-step problems with some challenge, allowing students to persevere in problem solving.

Chapter 9: Probability

Probability is new to Math Mammoth students, as the topic doesn't appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms "probably" and "likely," listening to weather forecasts, and so on.

In the past, probability wasn't taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field of math, it was felt that it should be introduced sooner, so during the 1990s and 2000s it "crept" down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by 7th grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle school and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favorable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, which include flipping a coin, tossing a pair of dice, picking marbles, or spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase "at least," which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a die hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations at the web page https://www.mathmammoth.com/lessons/probability_simulations.php

Next, we study compound events, which combine two or more individual simple events. Tossing a die twice or choosing first a girl then a boy from a group of people are compound events. Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or simply by using logical thinking to list all the possible outcomes.

The last major topic in this chapter is simulations. Students design simulations to find the probabilities of events. For example, we let heads represent "female" and tails represent "male," so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at https://www.random.org/integers or a spreadsheet program on a computer.

In the last lesson of the chapter, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other). This topic exceeds the Common Core Standards for 7th grade and thus is optional. I have included it here because the idea studied in the lesson is very simple and I feel many students will enjoy it.

What we are coming from

Probability has not been studied in earlier grade levels, but students have learned about fractions and percentages, which are the foundation for probability.

What we are going towards

In high school, students learn about conditional probability and probability of compound events.

Relevant 7th grade standards

7.SP.5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

7.SP.8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Note: If a standard is mentioned in parentheses, it means the lesson contains material that approaches that standard or prepares students for it.

Lesson	page number	Standards
Probability	193	7.SP.5, 7.SP.7
Probability Problems from Statistics	196	7.SP.7
Experimental Probability	198	7.SP.6, 7.SP.7
Count the Possibilities	201	7.SP.7, 7.SP.8
Using Simulations to Find Probabilities	207	7.SP.8
Probabilities of Compound Events (optional)	213	(S-CP.7)
Chapter 9 Mixed Review	217	7.EE.1, 7.EE.4, 7.EE.3, 7.RP.3, 7.G.4, 7.G.6, 7.G.5, 7.G.2, 7.G.1
Chapter 9 Review	222	7.SP.6, 7.SP.7, 7.SP.8

Standards for Mathematical Practice

- (MP4) The lesson about simulations shows an interesting example of mathematical modeling. We use a simulation to model a real-life scenario and to find probabilities of events connected with it.

Chapter 10: Statistics

Math Mammoth Grade 7 ends with a study of statistics. In 6th grade, students learned some basics of analyzing data distributions. This time, we focus on two major topics: **random sampling** and **comparing two populations**. The former is a new topic, but the latter is an extension of what students have learned in 6th grade: now, instead of looking at one data distribution, they look at *two*, and compare them.

The first two lessons of the chapter focus on random sampling. Students learn that sampling methods vary and that random sampling is a method that is likely to produce an *unbiased sample*—a sample that represents the population well.

In the lesson *Using Random Sampling*, students choose several random samples from a population of 100 geometric shapes, and they see firsthand that random samples can vary—even a lot. Yet, if the sample size is sufficiently large, or if we have several random samples, we can be relatively confident in concluding something about the population itself. Students will also practice making inferences about populations based on several random samples.

Then we turn our attention to comparing two populations or two samples, using statistical measures and graphs. The first lesson on these topics reviews these topics from 6th grade: how to make a boxplot, and how to calculate the mean absolute deviation.

Then students will compare two populations. They describe the overlap in the distributions based on the graphs, and make some basic notions about how the populations differ, based on the measures of center and variability. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences.

Lastly, students again compare two populations, but this time indirectly by using *samples* from the populations. This lesson also includes an optional project where students gather sample data on their own, and compare the populations based on those samples.

What we are coming from

In 6th grade, students learned some basics of analyzing data distributions.

What we are going towards

In 8th grade, students learn some basics concerning bivariate data (data involving two variables), as they study scatter plots and two-way tables. Then in high school, they will study in more detail the statistical topics they have already encountered in grades 6-8.

Relevant 7th grade standards

7.SP.1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
7.SP.2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

7.SP.4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Lesson	page number	Standards
Random Sampling	227	7.SP.1
Using Random Sampling	231	7.SP.2
Some Review	238	6.SP.4, 6.SP.5
Comparing Two Populations	244	7.SP.3
Comparing Two Samples	251	7.SP.3, 7.SP.4
Chapter 10 Mixed Review	260	7.EE.4, 7.NS.3, 7.RP.1, 7.G.1, 7.G.6, 7.G.4, 7.G.2, 7.SP.8, 7.SP.5, 7.SP.6, 7.SP.1, 7.G.3
Chapter 10 Review	266	7.SP.1, 7.SP.2, 7.SP.3, 7.SP.4

Standards for Mathematical Practice

- (MP4) Statistics is an area of math that connects very closely with real life and with mathematical modeling. Taking a sample of a population is essentially a *model* of that population. Once we have that sample or a model, we use statistical graphs and statistical measures (such as mean, median, IQR and others) to allow us to infer facts about the populations and to answer questions about them.