| Lesson | page number | Standards |
|---|---|---|
| Rounding to the Nearest Ten, Part 1 | 47 | 3.NBT.1 |
| Rounding to the Nearest Ten, Part 2 | 49 | 3.NBT.1 |
| Regrouping in Addition | 52 | 3.NBT.2, (3.OA.8) |
| How to Check Addition Problems | 55 | 3.NBT.2, 3.OA.8 |
| Review: Regrouping in Subtraction | 58 | 3.NBT.2, (3.OA.8) |
| Regrouping Twice in Subtraction | 61 | 3.NBT.2 |
| Regrouping Twice in Subtraction, Part 2 | 65 | 3.NBT.2, 3.OA.8 |
| Regrouping with Zero Tens | 68 | 3.NBT.2 |
| Regrouping with Zero Tens, Part 2 | 71 | 3.NBT.2, (3.OA.8) |
| Addition, Subtraction, and Parentheses | 74 | (3.OA.8) |
| Word Problems Practice | 76 | 3.OA.8 |
| Mileage Chart | 79 | |
| Mixed Review Chapter 2 | 81 | 2.NBT.5, 3.OA.8, 3.OA.9 |
| Review Chapter 2 | 83 | 3.OA.8, 3.NBT.1, 3.NBT.2 |

**Standards for Mathematical Practice**

- (MP.6) A focus topic of this chapter is to check one's work in addition and subtraction problems and to use estimation to evaluate whether an answer is reasonable. This allows students to develop the good habit of attending to precision in mathematical problems.
- (MP.1) The chapter also contains lots of word problems. Once again, some of them may be challenging, but challenges are good for students as long as they are not too challenging. The goal is that the challenge is something the student can attain to and is within their reach, or is at the edge of their understanding and skill level.

  Keep in mind also that problem solving is a learned skill. By doing lots of word problems, children will learn to solve them better and better and persevere in solving them — and then they don't have to be afraid of word problems! My oldest daughter felt that the word problems were the best part of math lessons.

## Chapter 3: Multiplication Concept

The third chapter of *Math Mammoth Grade 3* covers the concept of multiplication and related concepts. Working on memorizing the times tables is in the following chapter, Chapter 4.

The first lessons introduce multiplication as repeated addition of groups of the same size and the array as a model for multiplication. Then, students learn how multiplication is illustrated on the number line as consecutive jumps or skips of the same size (skip-counting).

The following lesson has to do with the order of operations. We only focus on multiplication and addition. Students write number sentences involving those two operations, such as $2 \times 6 + 4 \times 5$. This is preparing them for the next lesson, *Understanding Word Problems 1*, where they use these two operations to write the total cost of buying so many of one thing and so many of another thing.

The lesson *Understanding Word Problems 1* shows how multiplication problems involve the idea of "each," "every," or "all." For example, *each* item has the same number of something. If your student finds these problems difficult, encourage them to draw a picture of the situation.

*Zero and One in Multiplication* is a lesson that practices multiplying by zero and by one, using illustrations of same-size groups and the number line.

*Understanding Word Problems, Part 2* introduces multiplicative situations where the question is not asking for the total, but for the size of the groups or for the number of groups. While division could be used, at this stage we are not doing so, but the student learns to write a multiplication sentence with a missing number, such as $3 \times \_\_\_\_ = 18$. This is an important skill and prepares students for algebra.

*Multiplication in Two Ways* focuses on the thought that it does not matter in which order the numbers to multiply appear (the commutative property of multiplication). Objects in an array illustrate this fact nicely: either the row or the column can be taken as the group being multiplied.

The last lesson in the chapter circles back to the order of operations, this time including parentheses, multiplication, and addition/subtraction. Students write expressions of the form $3 \times (2 + 5)$ to match real-life situations, and practice solving two-step and multi-step word problems. This is, in fact, one of the major goals of this chapter: to practice solving word problems that use multiplication and some other operation. It is to that end that we need to study the order of operations here in third grade.

**What we are coming from**

In 2nd grade, students have worked with equal groups of objects to gain foundations for multiplication. They have worked with odd and even numbers (2.OA.3) and used addition to find the total number of objects arranged in rectangular arrays (2.OA.4).

**What we are going towards**

In 4th grade, students learn multi-digit multiplication in some cases (one-digit by up to four-digit number, and two-digit by two-digit number). (4.NBT.5)

**Relevant 3rd grade standards**

| |
|---|
| **3.OA.1** Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as $5 \times 7$.* |
| **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.* |

| |
|---|
| **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* |
| **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. |

| Lesson | page number | Standards |
|---|---|---|
| Many Times the Same Group | 89 | 3.OA.1 |
| Multiplication as an Array | 92 | 3.OA.1, 3.OA.3 |
| Multiplying on a Number Line | 94 | 3.OA.1, 3.OA.4 |
| Order of Operations 1 | 97 | (3.OA.8) |
| Understanding Word Problems, Part 1 | 99 | (3.OA.8), 3.OA.3 |
| Zero and One in Multiplication | 102 | 3.OA.1, 3.OA.5 |
| Understanding Word Problems, Part 2 | 104 | 3.OA.3, (3.OA.8) |
| Multiplication in Two Ways | 107 | 3.OA.1, 3.OA.3, 3.OA.5 |
| Order of Operations 2 | 110 | 3.OA.3, 3.OA.8 |
| Mixed Review Chapter 3 | 113 | (3.OA.8), 3.NBT.1, 3.NBT.2 |
| Review Chapter 3 | 115 | 3.OA.1, 3.OA.3, 3.OA.5 |

**Standards for Mathematical Practice**

- (MP.2) Understanding which situations in word problems (and in real life) involve multiplication is a focus topic of this chapter. This is very foundational skill. Students begin to learn to decontextualize—to think of a given situation in an abstract way and to represent it symbolically. This skill will be much more needful in later grades and especially in algebra.
- (MP.4) Learning to write down the calculations they do when solving word problems is also the beginning step in learning to model with mathematics. So, don’t let students to skip that step.

## Chapter 4: Multiplication Tables

In this chapter, the focus is on memorizing the times tables. The lessons include lots of repetition, drill, and practice. The lessons also point out patterns in the tables, and various ways to use properties of the operations to find products (e.g. to find 4 times a number, you can double a number twice).

**Relevant 3rd grade standards**

| |
|---|
| **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |
| **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.* |
| **3.OA.5** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* |
| **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. |
| **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* |

**Standards for Mathematical Practice**

- (MP.7) The chapter contains several opportunities for students to look for and make use of structure. Students notice and learn about patterns in the multiplication tables, such as in the table of 2 and the table of 9. They compare two different times tables, noticing that the products (answers) of one are multiples of the other (e.g. the tables of 5 and 10). They also learn about the distributive property and how it can be used to find products, such as finding 9 times a number as 10 times that number minus the number.

## Chapter 5: Time

This chapter covers reading the clock to the minute, finding time intervals (elapsed time), and using the calendar.

First, we review the 2nd grade topic of reading the clock to the five-minute intervals. In the first lesson, students use numbers to tell the time, such as 6:45 or 12:15. Then, they learn and review the expressions "quarter till", "quarter after", and "half past". In the third lesson of the chapter, the focus is on using "till" and "after" in telling time, such as in "20 till 6" or "25 after 11".

After that, students learn to tell time to the minute, and that finishes the topic of telling time. From now on, the focus switches to finding time intervals and other time-related calculations.

Next, we study elapsed time. The goal is for the student to determine elapsed time in two or more parts when the ending time has the next hour compared to the starting time, such as from 5:49 to 6:15. The lessons provide both clock faces and number lines as visual aids to help students figure out how much time passes. You can also use a practice (analog) clock. The last lesson (*Elapsed Time 4*) also covers finding the elapsed time in full hours when the minutes for the starting and ending times are the same. The most complex situation of elapsed time, where the hours differ by more than one (e.g. from 9:13 AM to 1:45 PM), is left for Math Mammoth Grade 4.

Lastly, students practice using the calendar for one lesson.

**What we are coming from**

In 2nd grade, students learned to tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

**What we are going towards**

In 4th grade, students will be converting measurement units going from a larger unit to a smaller one, including hours, minutes, and seconds. They will also solve word problems involving intervals of time. In Math Mammoth Grade 4, students will still practice finding elapsed time, when the hours differ by more than one.

**Relevant 3rd grade standard**

> **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by

# Chapter 6: Money

This chapter of *Math Mammoth Grade 3* has to do with counting money, making change, and solving simple problems about money.

The first lesson, *Counting Coins*, is a review lesson from 2nd grade. We review counting quarters, dimes, nickels, and cents. The lesson also briefly introduces the half-dollar. In the following lesson, *Dollars*, the student counts both bills and coins, and writes dollar amounts using the "$" symbol and the decimal point.

The next lesson focuses on how to count up to make change. I view this as an important skill because it so clearly ties together addition and subtraction, and helps children with mental math skills. We are essentially finding the difference between two numbers, such as $10.00 and $6.35, but we do so by adding (counting up).

The following lesson, *Making Change*, shows the two methods for finding change: subtraction or counting up. We use mental math throughout this lesson also.

Next, students solve simple money problems using mental math, such as finding totals and solving various word problems. The last topic in this chapter is adding and subtracting amounts of money vertically in columns, and using that technique in solving problems.

The Common Core Standards don't include any money-related topics for 3rd grade and most of these lessons can be delayed till 4th grade if desired. I prefer not to skip a grade level but rather present a continuum of the topic of money across the first three grade levels in particular.

**What we are coming from**

In 2nd grade, students counted coins and bills, and solved word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

**What we are going towards**

In 4th grade, students will use the four operations to solve word problems involving money, including problems involving simple decimals.

| Lesson | page number | Standards |
|---|---|---|
| Counting Coins | 203 | 2.MD.8 |
| Dollars | 206 | 2.MD.8 |
| Counting Up to Make Change | 209 | |
| Making Change | 212 | (4.MD.2) |
| Using Mental Math to Solve Money Problems | 214 | 4.MD.2 |
| Finding the Total and the Change | 216 | 4.MD.2 |
| More Problem Solving | 218 | 4.MD.2 |
| Mixed Review Chapter 6 | 220 | 3.OA.1, 3.OA.3, 3.OA.4, 3.OA.7, 3.OA.8 |
| Review Chapter 6 | 222 | 2.MD.8, 4.MD.2 |

**Standards of Mathematical Practice**

- (MP.5) Several lessons in the chapter deal with counting up to find change and the usage of mental math. Students have an opportunity to experience firsthand how these techniques are often more efficient and less prone to errors than using subtraction in columns (because it often involves regrouping over many zeros). Mental math is a valuable tool, particularly when it comes to shopping.

# Math Mammoth Grade 3-B

## Chapter 7: Four-Digit Numbers

This chapter focuses on four-digit numbers. The first lessons cover place value and comparing four-digit numbers. The rest of the chapter deals with addition, subtraction, and word problems.

First, students learn to write and read four-digit numbers. Then they learn about the concept of place value, write numbers in expanded form, and place them on number lines. One lesson is spent comparing numbers.

Then we turn our attention to addition and subtraction, first using mental math. The lessons on mental math stress the similarities between adding and subtracting four-digit numbers and adding and subtracting smaller numbers. This helps build number sense.

Several lessons are spent on adding and subtracting four-digit numbers in columns and practicing regrouping. Students also solve a variety of word problems involving four-digit numbers along the way.

Overall, this chapter is probably fairly easy for most students in the fact it mostly has to do with place value, addition, and subtraction. I advise that you do not assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

**Note Concerning the Common Core Standards (CCS)**

The CCS specify that in grade 2, students learn numbers up to 1,000, and that in grade 4, students learn numbers up to 1 million. There is no mention of studying numbers and place value in grade 3; however I have decided to include 4-digit numbers here (numbers up to 10,000). I feel a more gradual progression towards large numbers is easier for students.

The topics of this chapter therefore technically belong to fourth grade. This entire chapter can be safely omitted without affecting the subsequent chapters in third grade.

However, keep in mind the following. Chapter 1 of Math Mammoth Grade 4 curriculum (2020 edition) has a few exercises that use four-digit numbers. Also, and the coverage of large numbers in Chapter 2 of Math Mammoth Grade 4 (2020 edition) picks up where this chapter leaves off. If you want or need to delay this topic till fourth grade, you could use some or all of these lessons as part of studying chapter 2 in grade 4.

**Standards of Mathematical Practice**

- (MP.6) This chapter has lots of opportunities to focus on accuracy (precision) and checking one's own work. Addition problems can be checked by adding each column in a different order. Subtraction problems can be checked by adding. Sometimes children hurry through their work and thus make mistakes. Don't assign so much work that it creates time-related stress. Less can be more! Emphasize to your student(s) that they can take time to compute carefully, and to check their own work.

## Chapter 8: Division

In this chapter, students learn the concept division, find whole-number quotients within 0-100 (essentially, the basic division facts), and are introduced to bar graphs and pictographs.

Here in third grade, students will find quotients by thinking of the multiplication tables. In fourth and fifth grades, they will go on to divide larger numbers, primarily using long division, though thinking of multiplication will still be used for some types of division problems. The basic concept of division is also the foundation for finding factors of a number and prime factorization, which will be studied in 4th and 5th grades.

The concept of division is intimately tied in with multiplication, being the opposite operation to multiplication. Several lessons in the chapter focus on this basic concept that division problems can be solved by thinking of multiplication. This means that the student needs to know the multiplication tables fairly well before this chapter.

There are basically two ways to interpret division, and thus to illustrate it with concrete objects. The first way has to do with grouping. The problem 12 ÷ 3 is interpreted as, "How many groups of three items can you make out of 12 items?" This is also called measurement division.

The second way to interpret division is equal sharing: we divide or share items equally between people or other items. For example, the problem 12 ÷ 3 can be interpreted as, "If you share 12 bananas equally between 3 people, how many bananas does each one get?"

Both of these interpretations of division are important to understand so that the student can solve real-life and mathematical problems involving division.

We also study division by zero. In that lesson, students should recognize that division by zero "does not work." I realize that in higher forms of mathematics, division by zero may be defined (such as 1 ÷ 0 = infinity), but for now, this is how we present it for third graders.

The latter part of the chapter deals with bar graphs and pictographs. In second grade, students drew pictographs and bar graphs with single-unit scale. Now, they draw scaled pictographs and bar graphs from the given data, choosing the scale, or how many items one block in the bar graph or one picture in the pictograph signifies. This process provides practice in using both division and multiplication.

Lastly, the chapter includes an optional lesson on the concept of remainder. It focuses on finding the remainder by using visual models. This concept will be studied in more detail in fourth grade.

**What we are coming from**

The concept of division is rooted in the student's understanding of multiplication, since both have to do with equal-size groups and since division is the opposite operation to multiplication. Once students learn to interpret multiplication as the total number of objects in equal-size groups (3.OA.1.), they can go on to learning division, and interpret it as finding either the number of groups or the size of groups in multiplicative situations.

**What we are going towards**

In 4th grade, students will use division in several ways. They will learn long division, find factors of whole numbers, and solve multistep word problems using the four operations, including problems in which remainders must be interpreted. In 5th grade, they will finalize fluency with long division and start learning how to divide decimals and fractions.

**The relevant standards for third grade**

| |
|---|
| **3.OA.2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* |
| **3.OA.3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |

**3.OA.4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

**3.OA.5.** Apply properties of operations as strategies to multiply and divide. *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

**3.OA.6.** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

**3.OA.7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

**3.OA.8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**3.MD.3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

| Lesson | page number | Standards |
|---|---|---|
| Division as Making Groups | 47 | 3.OA.2 |
| Division and Multiplication | 50 | 3.OA.5, 3.OA.6 |
| Division and Multiplication Fact Families | 53 | 3.OA.5, 3.OA.6 |
| Dividing Evenly into Groups | 56 | 3.OA.2, 3.OA.3, 3.OA.7 |
| Multiplication and Division Word Problems | 59 | 3.OA.3, 3.OA.4 |
| More Word Problems | 61 | 3.OA.3, 3.OA.4 |
| Zero in Division | 63 | 3.OA.3, 3.OA.5 |
| Division Practice | 65 | 3.OA.3, 3.OA.7 |
| Missing Numbers | 68 | 3.OA.4 |
| Bar Graphs | 70 | 3.MD.3 |
| Pictographs | 74 | 3.MD.3 |
| When Division is not Exact (optional) | 77 | (4.OA.3) |
| Mixed Review Chapter 8 | 80 | 3.OA.4, 3.NBT.2, 3.MD.1 |
| Review Chapter 8 | 82 | 3.OA.2, 3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7 |

### Standards of Mathematical Practice

- (MP.4) This chapter will be relatively easy for students who have mastered the multiplication tables, since division facts are just multiplication tables "backwards". So, students can focus on another important skill: recognizing real-life situations in word problems that mathematically relate to either division or multiplication. This is very important to master so that the student can later on translate more complex situations into equations (modeling with mathematics).

# Chapter 9: Measuring

In this chapter we delve into both customary and metric measuring units. In second grade, students measured lengths of objects using units of inches, feet, centimeters, and meters. Now, they will also measure liquid volumes and masses / weights of objects.

First, students learn about measuring lengths to the nearest quarter of an inch. Since most rulers measure to the eighth or sixteenth part of an inch, it is helpful to cut out a ruler from the lesson that only has tick marks for every fourth of an inch, and tape that onto an existing ruler. If your student has trouble with the fractions, consider studying some lessons from chapter 11 (Fractions) first.

In the following lesson, students measure many objects (e.g. pencils) to the nearest quarter inch, and create line plots from the data. Then we cover the corresponding metric units, as students measure objects using centimeters and millimeters.

Next, we turn our attention to mass (weight), using grams and kilograms. Students find the mass of objects using both a kitchen scale and a bathroom scale. Then they learn about liquid volume, milliliters, liters.

The next two lessons cover customary units for weight and volume: pounds and ounces, and cups, pints, quarts, and gallons. Again, the focus is on doing actual measurements using appropriate tools, so students become familiar with the units.

The last lesson of the chapter focuses on word problems. While the Common Core Standards only dictate one-step word problems using measurement units, I have included some multi-step problems in this lesson, because I feel Math Mammoth users are ready for them. Some of the word problems in the lessons also require a conversion between units, but the conversion factor is always given, and the questions are simple. Measurement conversions are practiced in detail in 4th and 5th grades.

We all use various measuring units in our everyday lives, and using them is the key to remembering what they are, how big they are, and what the conversion factors are. People in the United States do not use the metric system a lot, while people in most other countries mainly use the metric system. The units that you do not use are likely to be forgotten. So encourage the student(s) to have free play time with measuring devices such as scales, measuring cups, measuring tapes, and rulers.

**What we are coming from**

In 2nd grade, students measured the length of objects and also estimated lengths using units of inches, feet, centimeters, and meters. Students using Math Mammoth curriculum also used scales to weigh items, using pounds and kilograms.

**What we are going towards**

In 4th grade, students will continue learning about measurement units in general, and will convert a larger unit of measurement into a smaller unit. They also solve word problems involving distances, intervals of time, liquid volumes, masses of objects, which may require a conversion of a larger unit into smaller units. Students will continue learning more about measurement unit conversions both in 5th and 6th grades.

**The relevant standards for third grade**

| |
|---|
| **3.MD.2.** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. |
| **3.MD.4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. |

| Lesson | page number | Standards |
|---|---|---|
| Measuring to the Nearest Fourth-Inch | 87 | (3.MD.4) |
| Line Plots | 90 | 3.MD.4 |
| Centimeters and Millimeters | 92 | 2.MD.1 |
| Grams and Kilograms | 95 | 3.MD.2 |
| Milliliters and Liters | 99 | 3.MD.2 |
| Pounds and Ounces | 101 | (4.MD.1) |
| Cups, Pints, Quarts, and Gallons | 104 | |
| Word Problems and More | 106 | 3.MD.2 |
| Mixed Review Chapter 9 | 108 | 3.OA.7, 3.NBT.1, 3.NBT.2, 3.NBT.3, 3.MD.7 |
| Review Chapter 9 | 111 | 3.MD.2, 3.MD4 |

# Chapter 10: Geometry

This chapter covers, first of all, some shapes and their attributes. The big emphasis is on two new concepts: area and perimeter.

In second grade, students learned to recognize and draw shapes having specified attributes, such as a given number of angles. Now in third grade, in the first three lessons of the chapter, students continue and extend this work, sorting shapes into categories based on their attributes, and learning, in particular, about quadrilaterals as a category. Students observe attributes such as the number of sides, presence of right angles, and whether the shape has equal sides or not. They learn about rhombuses, rectangles, and squares, and how a square is *also* a rectangle and a rhombus.

Their work here prepares students to classify quadrilaterals in great detail in grade 5. Students classify figures in grade 4 also, based on angles, and focusing on triangles in particular.

Next, students delve into the topic of area of rectangular shapes. The foundation for finding the area of a rectangle has been laid in second grade, when students partitioned a rectangle into rows and columns of same-size squares and counted to find the total number of them. Here in third grade, we focus on the area of rectangles and shapes composed of rectangular areas. How to calculate the area of other common shapes, such as triangles, polygons, and circles, is studied in 6th and 7th grades.

We start the topic with the lesson *Getting Started with Area*, where students tile shapes with unit squares cut out of paper, thus learning the foundation of how area is measured. From there, the lessons gradually lead to the thought that area of a rectangle can be found by multiplying its side lengths. Students learn about different units for area (both metric and customary) and get hands-on experience on finding the area of rectangles by measuring the sides in inches or centimeters.

Next, the lessons deal with the area of compound (rectilinear) shapes, whose area can be found by decomposing the shape into (non-overlapping) rectangles.

In this context, we also study the distributive property "in disguise", in the lesson *Area of Decomposed Rectangles*. A rectangle is divided into two parts, and its area is found in two different ways: either as the area of the entire rectangle, or by adding the areas of the two parts.

From this area model, we can see that $a \times (b + c)$ is equal to $a \times b + a \times$ c. The expression $a \times (b + c)$ is the area of the entire rectangle, with side lengths $a$ and $(b + c)$, and this is equal to the sum of the areas of two rectangles, one with sides $a$ and $b$, and the other with sides $a$ and $c$. (The images in the lesson make it more clear.)

The distributive property is studied in its algebraic form in 6th grade, but students will use it again in 4th grade in the context of finding common factors of two whole numbers. For example, they will express the sum $36 + 8$ as $4 (9 + 2)$.

In the lesson *Multiplying by Whole Tens* students learn to multiply a single-digit number by a multiple of ten, for example $3 \times 40$ or $90 \times 7$. After that, students use this skill to calculate areas of bigger rectangles in the rest of the lessons in the chapter.

The last major topic in the chapter is perimeter. Students learn perimeter as the "go-around measure": the total distance when you go around a shape. Then, several lessons focus on both area and perimeter, to make sure students learn to distinguish between these two concepts. We investigate rectangles that have the same area but different perimeters, and also rectangles that have the same perimeter but different areas.

**What we are coming from**

In 2nd grade, students learned to recognize and draw shapes having specified attributes, such as a given number of angles. They also partitioned a rectangle into rows and columns of same-size squares and counted to find the total number of them.

### What we are going towards

In both 4th and 5th grades, students will be classifying figures based on their properties (triangles in 4th grade, quadrilaterals in 5th). In 4th grade, students revisit the topics of area and perimeter when they use solve real-world and mathematical problems related to area and perimeter. Then in 6th and 7th grades, students learn how to find the area of triangles, polygons, and circles.

### The relevant standards for third grade

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|---|
| **3.G.1.** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. |
| **3.MD.5.** Recognize area as an attribute of plane figures and understand concepts of area measurement.<br>a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.<br>b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. |
| **3.MD.6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). |
| **3.MD.7.** Relate area to the operations of multiplication and addition.<br>a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.<br>b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.<br>c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.<br>d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. |
| **3.MD.8.** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. |
| **3.NBT.3.** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., $9 \times 80$, $5 \times 60$) using strategies based on place value and properties of operations. |

| Lesson | page number | Standards |
|---|---|---|
| Area of Rectangles 2 | 140 | 3.MD.7.b |
| More Units for Measuring Area | 143 | 3.MD.7.b |
| Area of Compound Shapes 1 | 145 | 3.MD.7.d |
| Area of Decomposed Rectangles | 147 | 3.MD.7.c |
| Multiplying by Multiples of Ten | 151 | 3.NBT.3 |
| Area of Compound Shapes 2 | 153 | 3.MD.7.d |
| Perimeter | 155 | 3.MD.8 |
| More About Perimeter | 158 | 3.MD.8 |
| Area and Perimeter Problems | 160 | 3.MD.8 |
| Same Area, Different Perimeter | 162 | 3.MD.8 |
| Same Perimeter, Different Area | 164 | 3.MD.8 |
| More Practice | 166 | 3.MD.8, 3.MD.7.b, 3.MD.7.c |
| Mixed Review Chapter 10 | 168 | 3.MD.1, 3.MD.7, 3.MD.4, 3.MD.3 |
| Review Chapter 10 | 172 | 3.G.1., 3.MD.7.b, 3.MD.7.c, 3.MD.7.d, 3.NBT.3, 3.MD.8 |

**Standards of Mathematical Practice**

- (MP.6) Encourage and require students to include the proper unit for area and for perimeter in their answers. Area is measured in square units of some kind (e.g. square meters, square inches), whereas perimeter is measured in linear units (e.g. meters, inches). Giving a numerical answer only (e.g. “24”) is not totally accurate. Remembering to include the proper unit is part of learning to convey mathematical results in a precise manner.
- (MP.5) While the geometric word problems in this chapter are still very simple, it is a good practice for students to learn to draw a sketch of the figure in question. With more complex problems in later grade levels, drawing a picture of the shape(s) becomes an essential tool in one’s “mathematical toolbox”.
- (MP.5) The area of a rectangle is found by *multiplying* the side lengths. The area model becomes an important tool in subsequent grade levels in the context of multiplying multi-digit whole numbers, multiplying mixed numbers, and even in multiplying polynomials in algebra. We lay the foundation for all this in this chapter.
- (MP.7) The lessons *Polygons* and *Multiplying by Whole Tens* give students opportunities to look for structure in sorting shapes and for numerical patterns. Mathematics has been called the “science of patterns”, so, these types of tasks have to do with the essence of mathematics.
- (MP.5) Note that when a problem asks for the area or a perimeter of a figure in centimeters, millimeters, or inches, and the figure does not indicate the side lengths, the student is supposed to use a ruler to measure. By not including that instruction in the problem, we are giving the student opportunity to reason out the need for a ruler as a tool.

# Chapter 11: Fractions

The last chapter of *Math Mammoth Grade 3* deals with several elementary fraction topics: the concept of a fraction, fractions on a number line, equivalent fractions, and comparing fractions. A major goal is to develop the student's understanding of fractions as *numbers* — a foundational concept that will allow students to do arithmetic with fractions in grades 4-6.

In second grade, students worked with halves, thirds, and fourths. Now, this is extended, mainly to sixths and eighths. The curriculum also includes fifths, tenths, and a few other types of parts in some instances.

In the first two lessons, students learn that when a whole is divided into equal parts, then one of those parts is a unit fraction (of the form $1/n$), and other fractions are formed from the unit fractions. (In general, a fraction $m/n$ is formed by $m$ parts of size $1/n$.)

To help students identify fractions, you can also use manipulatives or the fraction cutouts provided. In the digital version, they are found in their separate folder, and in the printed version, they are appended to the answer key.

Next, students represent fractions on a number-line diagram by partitioning the interval from 0 to 1 into equal parts. They also study fractions on number lines that extend past 1 whole, and learn to write whole numbers as fractions.

The following topic is equivalent fractions. Students recognize and generate simple equivalent fractions using visual models and number lines. In fourth grade, they will extend this work beyond visual models and learn how to generate equivalent fractions using multiplication.

Lastly, students start building their understanding on how to compare fractions. In this grade, we only deal with a few special cases, such as when the fractions have the same numerator or the same denominator, or when the comparison can be made from visual models. They also learn that comparisons are valid only when the two fractions refer to the same whole. In fourth grade, students will learn how to use benchmark fractions or converting the fractions so they have the same denominator, in order to compare them.

**What we are coming from**

In 2nd grade, students partitioned circles and rectangles into two, three, or four equal shares, and worked with halves, thirds, and fourths.

**Relevant standards for third grade**

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|---|
| **3.NF.1** Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$. |
| **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.<br>a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.<br>b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. |
| **3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.<br>a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.<br>b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.<br>c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. |

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|---|
| d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols> , =, or <, and justify the conclusions, e.g., by using a visual fraction model. |
| **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. |

**What we are going towards**

In 4th grade, students will learn how to form equivalent fractions using multiplication, compare fractions, work with fraction addition and subtraction (the fractions will have the same denominator), and learn to multiply a fraction by a whole number.

## The lessons in Chapter 11

| Lesson | page number | Standards |
|---|---|---|
| Understanding Fractions 1 | 178 | 3.NF.1, 3.G.2 |
| Understanding Fractions 2 | 182 | 3.NF.1 |
| Fractions on a Number Line 1 | 186 | 3.NF.2 |
| Fractions on a Number Line 2 | 189 | 3.NF.2 |
| Equivalent Fractions 1 | 193 | 3.NF.3 |
| Equivalent Fractions 2 | 195 | 3.NF.3 |
| Equivalent Fractions 3 (optional) | 197 | 3.NF.3 |
| Comparing Fractions 1 | 199 | 3.NF.3 |
| Comparing Fractions 2 | 202 | 3.NF.3 |
| Comparing Fractions 3 | 205 | 3.NF.3 |
| Mixed Review Chapter 11 | 207 | 3.OA.5, 3.OA.7, 3.OA.6, 3.OA.5, 3.MD.7, 3.G.1, 3.MD.8, 3.NF.2, 3.NF.3 |
| Review Chapter 11 | 211 | 3.NF.1, 3.NF.2, 3.NF.3 |

**Standards of mathematical practice**

- (MP.5) In this chapter, students encounter several fraction models (the pie model, the area model, and number lines). Questions throughout the chapter give them opportunities to choose which is the best model or tool to use.
- (MP.4) Students practice modeling with mathematics in a simple way when they write comparison inequalities about fractions (e.g. $1/5 < 1/3$).
- (MP.3) The chapter has opportunities for students to construct arguments and to critique the reasoning of others, as they explain why fractions are equivalent or they justify their conclusions about fraction comparisons.