United States Population

Using Quadratic Models

When am I ever going to use this?

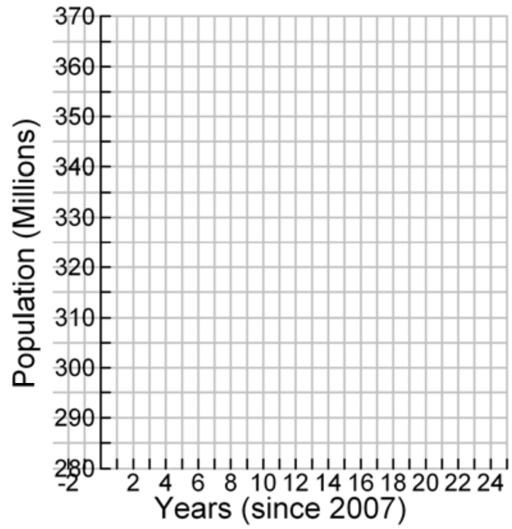
Using the concepts in this worksheet, you will be able to use quadratic models of real world data sets to forecast unknown values.

Based on Census Bureau projections for 2007 – 2050, the population of the United States may be modeled by $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ where P is the population (in millions) and t is the number of years since 2007 (Source: Modeled from Statistical Abstract of the United States 2008, Table 3).

1. Use the Quadratic Formula to solve $P(t) = 350$. Then explain what the solution means.

2. According to the population model, what will be the population of the United States in 2010? Round your answer to the nearest hundred thousand.

3. Graph $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ and $y = 320$ on the axes below.



4. Referring to the graph in the solution of Exercise 3, estimate the coordinates of the point of intersection of the two graphs. Then interpret the practical meaning of the point of intersection.

5. The graph of $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ in Exercise 3 appears to be linear not quadratic. Referring to the equation of P, explain why this is.

6. Give three examples of groups who benefit from population models such as the one above and explain why the model would be of value to them.

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Based on Census Bureau projections for 2007 – 2050, the population of the United States may be modeled by $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ where P is the population (in millions) and t is the number of years since 2007 (Source: Modeled from Statistical Abstract of the United States 2008, Table 3).

1. Use the Quadratic Formula to solve $P(t) = 350$. Then explain what the solution means.

$$350 = 0.0022675t^2 + 2.6804t + 300.760$$
$$0 = 0.0022675t^2 + 2.6804t - 49.240$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2.6804 \pm \sqrt{(2.6804)^2 - 4(0.0022675)(-49.240)}}{2(0.0022675)} \\ &\approx -1200 \ \textit{and} \ 18.09 \end{aligned}$$

Since the domain of the original data set used to create the model is $0 \le t \le 43$, *the value of* $t = -1200$ *is not reasonable. Since* $t = 18$ *is 2025, we predict that near 2025 the population of the United States will reach 350 million (350,000 thousand).*

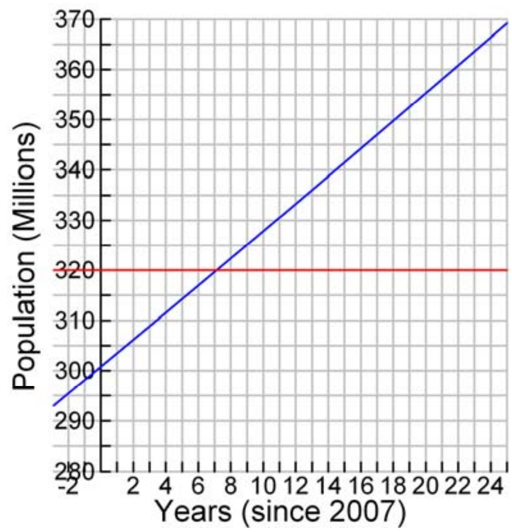
2. According to the population model, what will be the population of the United States in 2010? Round your answer to the nearest hundred thousand.

Since $t = 3$ *corresponds with 2010, we evaluate the function at* $t = 3$.

$$\begin{aligned} P(3) &= 0.0022675(3)^2 + 2.6804(3) + 300.760 \\ &= 308.822 \end{aligned}$$

We estimate that the population in the United States will be 308.8 million in 2010.

3. Graph $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ and $y = 320$ on the axes below.



4. Referring to the graph in the solution of Exercise 3, estimate the coordinates of the point of intersection of the two graphs. Then interpret the practical meaning of the point of intersection.

The point of intersection is approximately $(7, 320)$. *In 2014, the population of the United States is predicted to be 320 million.*

5. The graph of $P(t) = 0.0022675t^2 + 2.6804t + 300.760$ in Exercise 3 appears to be linear not quadratic. Referring to the equation of P, explain why this is.

Since $a = 0.0022675$ *is close to 0, the term* $0.0022675t^2$ *will have little effect on the value of* P *for small values of* t. *For example,* $0.0022675(25)^2 \approx 1.42$. *In contrast, the term* $2.6804t$ *changes the value of* P *much more dramatically. For example,* $2.6804(25) \approx 67.01$

6. Give three examples of groups who benefit from population models such as the one above and explain why the model would be of value to them.

Internal Revenue Service – Based on the population model, the IRS can forecast tax revenue.

Congressional Budget Planners – Based on the model, budget planners can forecast spending requirements for government programs such as Medicare, Medicaid, and similar programs.

Military Planners – Based on the model, military planners can predict the number of citizens who are eligible to defend their country in time of war.

Worksheet Title	United States Population: Using Quadratic Models				*Filename:*	m5053
Keywords	United States, population, quadratic, modeling, quadratic formula, graphing, intersection					
NCTM Standard		Content Standards			Process Standards	
		Number and Operations		x	Problem Solving	
	x	Algebra			Reasoning and Proof	
		Geometry			Communication	
		Measurement		x	Connections	
		Data Analysis and Probability		x	Representations	
Grade Band		PreK – 2				
		3 – 5				
		6 – 8				
	x	9 – 12				
Data Type	Equation					