Math Mammoth Statistics & Probability

Contents

Introduction

Math Mammoth Statistics & Probability is a worktext covering statistics and probability topics for 6th-7th grades. A worktext contains both the instructions and exercises.

The statistics lessons in the first section are for about 6th grade level, starting with the lesson, *Understanding Distributions*. The concept of distribution is a fundamental theme in the study of statistics. A distribution is basically *how* the data is distributed. The distribution can be described by its center, spread, and overall shape. The shape is read from a graph, such as a dot plot or bar graph.

Two major concepts when summarizing and analyzing distributions are its center and its variability. We study the center in the next lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of center to the shape of the data distribution and the type of data.

In the lesson *Measures of Variation* we study range, interquartile range, and mean absolute deviation. The last one takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program, such as Excel.

Then in the next lessons, students learn to make several different kinds of graphs: histograms, boxplots, and stem-and-leaf plots. In those lessons, students continue summarizing distributions by giving their shape, a measure of center, and a measure of variability.

The book also includes an optional statistics project, in which the student can develop investigative skills.

The next focus area of the book is random sampling. Students learn that sampling methods vary and that random sampling is likely to produce an *unbiased sample*—a sample that represents the population well. In the lesson "*Using Random Sampling*," students choose several random samples from a population of 100 geometric shapes, and they see first hand that random samples can vary, sometimes a lot. Yet if the sample size is sufficiently large, or if we have several random samples, we can relatively confidently conclude something about the population itself. Students will also practice making inferences about populations based on several random samples.

Another major topic is comparing two populations, either directly or by using samples from the populations. Students learn to use the overall distributions and the measures of center and variability to compare two sets of data in various ways. While some of the ways in which we compare the data are informal only, all of the concepts presented are fundamental to the use of statistics in various sciences. Students also do a project where they gather data on their own from two populations and compare them.

Probability is a new topic to Math Mammoth™ students, as the topic does not appear at all in grades 1-6. However, most students have an intuitive understanding of probability based on hearing the terms "probably" and "likely," listening to weather forecasts, and so on.

In the past, probability was not taught until high school—for example, I personally encountered it for the first time in 12th grade. However, since probability is such a useful and easily accessible field of math, it was felt that it should be introduced sooner, so during the 1990s and 2000s it "crept" down the grade levels until many states required probability even in elementary school. The Common Core Standards include probability starting in 7th grade. I feel that is good timing because by seventh grade students have studied fractions, ratios, and proportions, so they have the tools they need to study probability. Moreover, they will need an understanding of the basic concepts of probability in order to understand the statistical concepts that they will study in middle and high school.

In this chapter we start with the concept of simple (classic) probability, which is defined as the ratio of the number of favorable outcomes to the number of all possible outcomes. Students calculate probabilities that involve common experiments, to include flipping a coin, tossing dice, picking marbles, and spinning a spinner.

The lesson *Probability Problems from Statistics* introduces probability questions involving the phrase "at least," which are often solved by finding the probability of the complement event. For example, it might be easier to count the number of students who got at most D+ on a test than to count the number of students who got at least C-.

In the next lesson, *Experimental Probability*, students conduct experiments, record the outcomes, and calculate both the theoretical and experimental probabilities of events, in order to compare the two. They will draw a card from a deck or roll a die hundreds of times. The download version of this curriculum includes spreadsheet files for some of the lengthier probability simulations. You can also access those simulations at the web page http://www.mathmammoth.com/lessons/probability_simulations.php

Next, we study compound events, which combine two or more simple events. Tossing a die twice or choosing first a girl, then a boy from a group of people are compound events. They construct the sample space in several ways: Students calculate the probabilities of compound events by using the complete sample space (a list of all possible outcomes). They construct the sample space in several ways: by drawing a tree diagram, by making a table, or by simply using logical thinking to list all the possible outcomes.

The last major topic in this book is simulations. Students design simulations to find the probabilities of events. For example, we let the heads represent "female" and tails represent "male," so we can toss a coin to simulate the probability of choosing a person of either sex at random. Later in the lesson, students design simulations that use random numbers. They generate those numbers by using either the free tool at http://www.random.org/integers or a spreadsheet program on a computer.

In the last lesson of the book, *Probabilities of Compound Events*, we learn to calculate the probability of a compound event by *multiplying* the probabilities of the individual events (assuming the outcomes of the individual events are independent of each other).

Wishing you success in all your endeavors to teach math,
Maria Miller, the author

Helpful Resources on the Internet

Statistics videos by Maria
These video lessons cover topics that have been chosen to complement the lessons in this chapter.
http://www.mathmammoth.com/videos/statistics/statistics_lessons.php

Graphs and Plots

Statistics Interactive Activities from Shodor
A set of interactive tools for exploring and creating boxplots, histograms, dot plots, and stem-leaf plots. You can enter your own data or explore the examples.
http://www.shodor.org/interactivate/activities/BoxPlot/
http://www.shodor.org/interactivate/activities/Histogram/
http://www.shodor.org/interactivate/activities/PlopIt/
http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/

PlotLy
A comprehensive, collaborative data analysis and graphing tool. Bring data in from anywhere, do the math, graph it with interactive plots (scatter, line, area, bar, histogram, heat map, box, and more), and export it.
http://plot.ly

How to make a box plot using Plotly
Easy-to-follow instructions.
https://plot.ly/how-to-make-a-box-plot-online/

Analyzing and Displaying Data Gizmos from ExploreLearning
Gizmos are interactive online tools with lesson plans that allow you to explore and learn about the topic in a virtual, dynamic environment. This page includes gizmos for box-and-whisker plots, histograms, stem-and-leaf plots, and more. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=231

Create A Box and Whisker Chart
An online tool for creating a box-and-whisker plot from your own data. Includes lots of options, such as colors, fonts, and more.
https://www.meta-chart.com/box-whisker

How to create a BoxPlot (Box-and-Whisker Chart) in Excel
Detailed instructions for how to create a boxplot in different versions of Excel.
https://support.microsoft.com/en-us/kb/155130

Make Your Own Stem-and-Leaf Plot
Enter values from your own data, and this web page creates your stem-and-leaf plot for you.
http://www.mrnussbaum.com/graph/sl.htm

Stem-and-Leaf Plots Quiz
An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4

How to Read a Dot Plot
This lesson explains how to answer questions from a dot plot.
http://www.mathbootcamps.com/how-to-read-a-dotplot/

Statistical measures

Measures of Center and Quartiles Quiz from ThatQuiz.org
An online quiz about the measures of center and quartiles in boxplots, stem-and-leaf plots, and dot plots.
http://www.thatquiz.org/tq-5/?-jr0t0-l1-p0

Mean, Median, and Mode
A lesson that explains how to calculate the mean, median, and mode for a set of data given in different ways. It also has interactive exercises.
http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median
Lessons with simple explanations and examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragerev1.shtml

Measures Activity
Enter your own data and the program will calculate mean, median, mode, range, and some other statistical measures.
http://www.shodor.org/interactivate/activities/Measures

Train Race Game
Calculate the median and range of travel times for four different trains, then choose a good train to take based on your results.
http://www.bbc.co.uk/schools/mathsfile/shockwave/games/train.html

Measures of Variation - Self-Check Quiz
An online multiple-choice quiz about range, quartiles, and interquartile range that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=2&&headerFile=4

Mean Deviation
A simple explanation about what the mean absolute deviation is, how to find it, and what it means.
http://www.mathsisfun.com/data/mean-deviation.html

Mean Absolute Deviation
Several videos explaining how to calculate the mean absolute deviation of a data set.
http://www.onlinemathlearning.com/measures-variability-7sp3.html

Working with the Mean Absolute Deviation (MAD)
A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.
http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

Sampling

Random and Biased Sampling
A comprehensive lesson to read that explains about unbiased types of sampling.
http://www.ck12.org/na/Random-and-Biased-Sampling-in-a-Population---7.SP.1,2-1/lesson/user%3Ac2ZveDJAb3N3ZWdvLm9yZw../Random-and-Biased-Sampling-in-a-Population---7.SP.1%252C2/

How many fish in the pond?
This problem is based on the capture-recapture method scientists use to estimate the size of a population. The page includes a hint and a complete solution.
http://figurethis.nctm.org/challenges/c52/challenge.htm#hint

Capture-Recapture
If you are interested in learning more about the capture-recapture method, this web page provides a complete lesson plan and activity where students take a sample of beans to explore this concept.
http://illuminations.nctm.org/Lesson.aspx?id=2528

Polling: Neighborhood Gizmo
Conduct a phone poll of citizens in a small neighborhood to determine their response to a yes-or-no question. Use the results to estimate the sentiment of the entire population. Investigate how the error of this estimate becomes smaller as more people are polled. Compare random versus non-random sampling. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=507

Identify a Random Sample
A video lesson about what a sample is and what makes a sample random. On the left are links to three other videos about representative (unbiased) samples and biased samples.
https://learnzillion.com/student/lessons/1844

Valid Claims
Multiple-choice questions to practice figuring out whether we took a random sample and whether we are able to draw valid conclusions from the data.
https://www.khanacademy.org/math/probability/statistical-studies/statistical-questions/e/valid-claims

Random Sampling
Multiple-choice questions that test your understanding of the basics of random sampling.
https://www.khanacademy.org/math/recreational-math/math-warmup/random-sample-warmup/e/random-sample-warmup

Make inferences about a population by analyzing random samples
A video lesson that teaches how to make inferences about a population based on random samples.
https://learnzillion.com/student/lessons/1848

Making Inferences from Random Samples
Multiple-choice questions about what can reasonably be inferred, from a random sample, about an entire population
https://www.khanacademy.org/quetzalcoatl/content-improvement/middle-school-content/e/making-inferences-from-random-samples

Comparing two populations

Compare Populations Using the Mean
Video lessons that teach how to informally compare two populations using the mean.
https://learnzillion.com/student/lessons/1445

Compare Two Populations using the Range and the Interquartile range
A video lesson that teaches you how to informally compare the ranges and the interquartile ranges of two populations.
https://learnzillion.com/lessons/1453-compare-two-populations-using-range-and-interquartile-range

Comparing Populations - Khan Academy
Multiple-choice questions to practice comparing centers of distributions in terms of their spread.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-population-sampling/e/comparing-populations

Displaying Univariate Data Practice
Interactive practice questions about double stem-and-leaf plots.
http://www.ck12.org/statistics/Displaying-Univariate-Data/asmtpractice/Displaying-Univariate-Data-Practice/

Double Box-and-Whisker Plots Practice
Interactive practice questions about double box-and-whisker plots.
http://www.ck12.org/statistics/Double-Box-and-Whisker-Plots/asmtpractice/Double-Box-and-Whisker-Plots-Practice/

Mean and Median with BoxPlots
This interactive tool allows you to create several box-and-whisker plots. The data sets may contain up to 15 integers, each with a value from 0 to 100.
http://illuminations.nctm.org/Activity.aspx?id=3576

Facts & figures

GapMinder
Visualizing human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.
http://www.gapminder.org/data/

WorldOdometers
World statistics updated in real time. Useful for general educational purposes — for some stunning facts.
http://www.worldometers.info

Simple probability

Probability videos by Maria
Videos on probability topics that are helpful for the lessons in this book.
http://www.mathmammoth.com/videos/probability/probability_lessons.php

Probability Fair
Choose the probability that has the best chance when the colorful spinner is spun.
http://www.mrnussbaum.com/probfair/index.html

"Data Analysis & Probability Games" from MathWire
A list of board and dice games to help to teach topics appropriate for beginners in probability.
http://mathwire.com/games/datagames.html

Probability Game with Coco
A multiple-choice online quiz on simple probability.
http://www.math-play.com/Probability-Game.html

Math Goodies Probability Lessons
Tutorials with lots of worked-out examples and online interactive exercises. Topics include independent and dependent events, addition rules, conditional probability, and more.
www.mathgoodies.com/lessons/toc_vol6.html

Cross the Bridge
This is a printable board game based on throwing two dice and the probabilities for the sum of the dice.
http://www.mathsphere.co.uk/downloads/board-games/board-game-17-crossing-the-river.pdf

Rocket Launch
A three-stage rocket is about to be launched. In order for a successful launch to occur, all three stages of the rocket must successfully pass their pre-takeoff tests. By default, each stage has a 50% chance of success, however, this can be altered by dragging the bar next to each stage. Observe how many tries it takes until there is a successful launch.
http://mste.illinois.edu/activity/rocket/

How could I send the check and not pay the bill?
What is the probability that Tessellation will put each of the three checks into the correct envelopes if she does it randomly? The page includes a hint and a complete solution (click "answer" at the bottom of page).
http://figurethis.nctm.org/challenges/c69/challenge.htm

She always wins. It's not fair!
Is this game with two dice fair? We can use basic probability concepts to analyze the situation. The page includes a hint and a complete solution (click "answer" at the bottom of page).
http://figurethis.nctm.org/challenges/c26/challenge.htm

Sticks and Stones
Play the game "Sticks and Stones," gather data from the game play, and then consider the probabilities for the four possible moves in the game.
http://illuminations.nctm.org/Lesson.aspx?id=1915

Flippin' Discs
In this interactive activity, you throw two discs. You win if they both show the same color. You can run the game 100 times and see the detailed results. Can you explain why you win approximately half the time? The questions below the activity lead the student to explore the situation with 3, 4, and even 5 discs. The solution is found in a link near the top left of the page.
http://nrich.maths.org/4304

At Least One...
The tree diagram and related discussion on this page guides students' thinking to help them answer probability questions like, "What is the probability of getting at least one head by flipping a coin ten times?" A link near the top left of the page leads to the solution.
http://nrich.maths.org/7286

Same Number
Imagine you are in a class of thirty students. The teacher asks everyone to secretly write down a whole number between 1 and 225. How likely is it for everyones numbers to be different? The web page provides an interactive simulation so you can experiment with this problem. The following discussion also leads students to the classic birthday problem. The solution is found in a link near the top left of the page.
http://nrich.maths.org/7221

What Does Random Look Like?
This problem challenges our thinking about randomness. Make up a sequence of twenty Hs and Ts that *could* represent a sequence of heads and tails generated by a fair coin. Then use the animation to generate truly random sequences of 20 coin flips. Can you learn how to spot fakes?
http://nrich.maths.org/7250

Interactivate: *Fire*!! and *Directable Fire*!!
In these two activities, you first set the probability that a fire will spread from tree to tree in a forest of 100 trees. Then you click the tree where the fire starts, and watch it spread. In the Directable Fire activity, you can set the probabilities for each direction to be different. Repeat the activity several times to see that the amount of forest that burns varies (for any set probability of fire spreading).
http://www.shodor.org/interactivate/activities/Fire/
http://www.shodor.org/interactivate/activities/DirectableFire/

The Monty Hall Problem
Try this interactive version of the famous Monty Hall problem. The page includes a simple explanation of the solution. In the Monty Hall Problem you choose one of three closed doors. Behind one door is a big prize and behind the others something else, such as a goat. After you choose a door, Monty opens one of the doors with no prize, and you have a chance to switch.
www.nytimes.com/2008/04/08/science/08monty.html?_r=1

Virtual tools for experimental probability

Virtual Experimental Probability
Virtually roll one or two dice, spin a spinner, choose a card, or toss a coin virtually many times. The results are recorded so you can compare them to the theoretical probability.
http://staff.argyll.epsb.ca/jreed/math9/strand4/probability_display.htm

Coin Flip
This virtual coin toss shows the results numerically and can generate at least 100,000 flips.
http://www.btwaters.com/probab/flip/coinmainD.html

Coin Toss Simulation
Another virtual coin toss. This one shows the results both using images of coins and numerically.
http://syzygy.virtualave.net/multicointoss.htm

Coin Tossing - National Library of Virtual Manipulatives
This coin-toss tool not only lets you set the number of virtual tosses, but also the probability of heads, which means that you can observe what happens when a coin is weighted.**http://nlvm.usu.edu/en/nav/frames_asid_305_g_3_t_5.html**

Theoretical and Experimental Probability Gizmo
Experiment with spinners and compare the experimental probability of a particular outcome to the theoretical probability. Select the number of spinners, the number of sections on a spinner, and a favorable outcome of a spin. Then tally the number of favorable outcomes. The gizmo works for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=310

Dice Roll
Choose the number of virtual dice to roll and how many times you want to roll them. The page shows both the actual results and expected (theoretical) probabilities, and the simulation works for very large numbers of rolls.
http://www.btwaters.com/probab/dice/dicemain3D.html

Interactive: Spinner
You can adjust the number of regions (each is a different color) and choose the number of spins. The results show the frequencies for each color, the experimental probabilities, and the theoretical probabilities.
http://www.shodor.org/interactivate/activities/BasicSpinner/

Adjustable Spinner
Create a virtual spinner with variable-sized sectors to compare experimental results to theoretical probabilities. You can choose the sizes of the sectors, the number of sectors, and the number of trials.
http://www.shodor.org/interactivate/activities/AdjustableSpinner/

Experimental Probability
Experiment with probability using virtual spinners or dice. The sections on the spinner can be of the same size or of different sizes, and the dice can be regular 6-sided dice or customized.
http://www.shodor.org/interactivate/activities/ExpProbability/

Box Model - Random Drawings
First, select some numbers from which the computer draws. You can even select the same number multiple times. Tick the box for "Quick draw" and enter a number of repetitions. Then press "Start" to have the program draw a number randomly from among the numbers you selected. A bar graph shows the relative frequencies for your experiment.
http://nlvm.usu.edu/en/nav/frames_asid_146_g_3_t_5.html

Marbles
Run repeated experiments where you draw 1, 2, or 3 marbles from a set of blue, red, purple, and green marbles. You choose the number of each kind of marble and how many repetitions there are. The results shown include the frequencies of each possible outcome, the experimental probabilities, and the theoretical probabilities. This activity can be used not only to explore probabilities but also to perform simulations.
http://www.shodor.org/interactivate/activities/Marbles/

Racing Game with One Die
Explore how experimental probability relates to fair and unfair games with this two-car race. You choose which and how many numbers of the die make each of the cars move. Other options include the number of runs (trials) and the length of the race (in segments). The program calculates the percentage of wins for each car and draws a pie chart.
http://www.shodor.org/interactivate/activities/RacingGameWithOneDie/