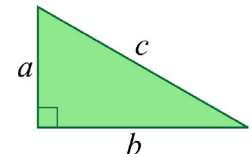
The Pythagorean Theorem

You will now learn a very famous mathematical result, the Pythagorean Theorem, which has to do with the lengths of the sides in a right triangle. First, we need to study some terminology.

In a right triangle, the two sides that are perpendicular to each other are called **legs**. The third side, which is always the longest, is called the **hypotenuse**.

In the image on the right, the sides *a* and *b* are the legs, and *c* is the hypotenuse.

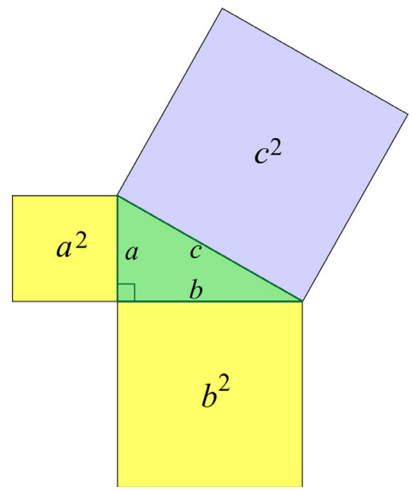


Note: We don't use the terms "leg" and "hypotenuse" to refer to the sides of an acute or obtuse triangle — this terminology is restricted to *right* triangles.

The Pythagorean Theorem states that **the sum of the squares of the legs equals the square of the hypotenuse.**

In symbols it looks much simpler:

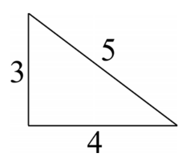
$$a^2 + b^2 = c^2$$



The picture shows squares drawn on the legs and on the hypotenuse of a right triangle. Verify visually that the total area of the two yellow squares drawn on the legs looks about equal to the area of the blue square on the hypotenuse.

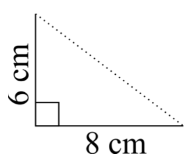
We will prove this theorem in another lesson.
For now, let's get familiar with it and learn how to use it.

1. Below you see the famous 3-4-5 triangle: its sides measure 3, 4, and 5 units and it is a right triangle. Check that the Pythagorean Theorem holds for it by filling in the numbers on the right.



$$\square^2 + \square^2 \overset{?}{=} \square^2$$
$$\square + \square \overset{?}{=} \square$$
$$\square = \square$$

2. If we double the lengths from the 3-4-5 triangle, we get the 6-8-10 triangle. Use a ruler and a protractor to draw line segments that are 6 cm and 8 cm long and are perpendicular to each other.



Then draw in the third side to complete the triangle. Does the hypotenuse of your triangle measure 10 cm? Do the lengths 6, 8, and 10 fulfill the Pythagorean Theorem?

Example 3. The two legs of a right triangle measure 7 and 10 units. How long is the hypotenuse?

Let x be the length of the unknown side, which is the hypotenuse. From the Pythagorean Theorem, we get:

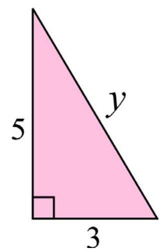
$$7^2 + 10^2 = x^2$$
$$49 + 100 = x^2$$
$$x^2 = 149$$
$$x = \sqrt{149} \quad \cancel{\text{or } x = -\sqrt{149}}$$

We ignore the negative root as the length of a side cannot be negative!

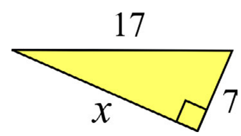
The answer is left in the root form since this is not a real-life problem.

3. Solve for the unknown side of each right triangle. Leave your answer in root form if the radicand (number under the radical) is not a perfect square.

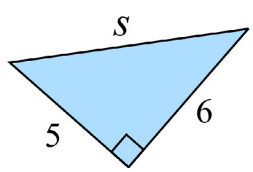
a.



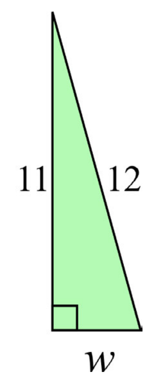
b.



c.



d.



4. The sides of a square measure 6 units. How long is the diagonal of the square?