Square Roots

The **square** of a number is that number multiplied by itself. For example, six squared = $6^2 = 6 \cdot 6 = 36$. (Recall that the square of 6 tells us the area of a square with sides 6 units long.)

Taking a **square root** is the opposite operation to squaring: the square root of 36 is the number that when squared, gives you 36.

There are actually two such numbers: 6 and −6. The positive one, 6, is **the principal square root** of 36. We use the "$\sqrt{\ }$" symbol (called the "radical sign" or "radix") to signify the principal square root of a number. For example, $\sqrt{25} = 5$ because $5^2 = 25$.

The words "radish" and "radical" both come from the Latin word *radix*, meaning **root**.

Taking a square root allows us to find the side length of a square when its area is given.

Here is a way to remember what a square root is. In the picture on the right, the area of a square is written inside the square and the length of the side is written to the side:

49 7

Now, imagine the square is a radical sign that "houses" the number for the area:

$\sqrt{49} = 7$

To find the (principal) **square root of a number, think of a square with that area, and find the side length of that square.**

1. Find the (principal) square roots.

a. $\sqrt{100}$	**b.** $\sqrt{64}$	**c.** $\sqrt{4}$	**d.** $\sqrt{0}$
e. $\sqrt{81}$	**f.** $\sqrt{144}$	**g.** $\sqrt{1}$	**h.** $\sqrt{10{,}000}$

2. It is especially easy to find square roots of numbers that are **perfect squares:** numbers we get by squaring whole numbers. For example, 49 is a perfect square because it is 7^2. Fill in the list of perfect squares from 1^2 to 20^2 at the right:

x	x^2
1	1
2	4
3	9
4	_____
_____	_____
_____	_____
_____	49
8	_____
9	_____
_____	_____

x	x^2
11	_____
12	_____
13	_____
14	_____
15	_____
_____	256
_____	289
_____	324
_____	_____
_____	_____

3. Find the square roots of these perfect squares.

a. $\sqrt{169}$ **b.** $\sqrt{900}$

c. $\sqrt{225}$ **d.** $\sqrt{121}$

e. $\sqrt{441}$ **f.** $\sqrt{8{,}100}$

g. $\sqrt{324}$ **h.** $\sqrt{400}$

i. $\sqrt{6{,}400}$ **j.** $\sqrt{25{,}600}$

k. $\sqrt{16{,}900}$ **l.** $\sqrt{1{,}000{,}000}$

Most whole numbers are *not* perfect squares, and their square roots are unending decimals. (In fact, their square roots are **irrational numbers**, which means they cannot be written as a fraction, and their decimal expansions are unending decimals without any repeating patterns in the digits.)

We can handle that situation in at least three ways:

1. We can find an approximate value of such square roots **with a calculator**, rounding the answer to a reasonable accuracy. This is necessary if we're dealing with a real-life application.
2. We can find an approximate value using **guess and check**, and decimal multiplication. For example, we know that the value of $\sqrt{17}$ will be between 4 and 5 (since $\sqrt{16} = 4$ and $\sqrt{25} = 5$). We can also tell that it will be closer to 4 than 5, since 17 is very close to 16. So, we could guess that it is 4.1, square that, and based on the result, refine our guess.
3. We can **indicate such values using the square root symbol**, and not find a decimal approximation. For example, the side of a square with an area of 2 square units is $\sqrt{2}$ units. This is the preferred way in pure mathematics, and any time you want to convey an accurate value.

4. Between which two whole numbers do the following square roots lie? Do not use a calculator. Tell also which of those whole numbers the root is closer to.

a. $\sqrt{5}$ **b.** $\sqrt{24}$ **c.** $\sqrt{47}$ **d.** $\sqrt{83}$

5. Tell the side of the square (exact value) when its area is given. Indicate the side length using the square root symbol, if the area is not a perfect square. Note: u^2 signifies square units, and *u* signifies a unit.

a. area = 25 u^2 side = __________	**b.** area = 1,600 u^2 side = __________	**c.** area = 5 u^2 side = __________	**d.** area = 11 u^2 side = __________

6. **a.** What is the area of a square, if its side measures $\sqrt{8}$ units?

b. What is the value of $(\sqrt{7})^2$?

c. What is the side of a square with an area of 130 square meters? Give an exact value.

Example 1. Since $0.5 \cdot 0.5 = 0.25$, then $\sqrt{0.25} = 0.5$.

Example 2. Since $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$, then $\sqrt{\frac{4}{9}} = \frac{2}{3}$.

7. Find the square roots.

a. $\sqrt{0.16}$ **b.** $\sqrt{0.01}$ **c.** $\sqrt{1.21}$

d. $\sqrt{\frac{16}{25}}$ **e.** $\sqrt{\frac{100}{9}}$ **f.** $\sqrt{\frac{49}{36}}$