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Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. It follows the Common Core Mathematics Standards (CCS) for 7th grade. Those standards are so constructed that students can continue to a traditional algebra 1 curriculum after studying this. However, you also have the option of following this course with Math Mammoth Grade 8, which provides a gentler and slower transition to high school math.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-B, covers ratios and proportions (chapter 6), percent (chapter 7), geometry (chapter 8), probability (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 7-A worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your student's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

In seventh grade, the first five chapters should be studied in order. Also, chapter 7 (Percent) should be studied before studying any of the chapters 8, 9, or 10 (Geometry, Probability, Statistics). You can be flexible with the rest of the scheduling.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your student!

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- The answer keys are included in the digital download version. They are sold as a separate book for the printed version.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for your student to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Worktext 7-A	
Chapter 1	8 days
Chapter 2	13 days
Chapter 3	9 days
Chapter 4	16 days
Chapter 5	16 days
TOTAL	**62 days**

Worktext 7-B	
Chapter 6	17 days
Chapter 7	12 days
Chapter 8	23 days
Chapter 9	10 days
Chapter 10	12 days
TOTAL	**74 days**

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 4 pages long, intended for one day. Some 5 and 6-page lessons take two days. There are also a few optional lessons.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page counts in the table below *include* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A	81	9	199	73	2.73	13.6
7-B	99	10	241	88	2.74	13.7
Grade 7 total	180	19	440	161	2.73	13.7

The table below is for you to use.

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A			199			
7-B			241			
Grade 7 total			440			

Let’s say you determine that your student needs to study about 2.5 pages a day, or 12-13 pages a week on average in order to finish the curriculum in a year. As the student studies each lesson, keep in mind that sometimes most of the page might be reserved for workspace to solve equations or other problems. You might be able to cover more than the average number of pages on such a day. Some other day you might assign the student only one page of word problems.

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, seventh graders might spend 45-90 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.

2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 6: Ratios and Proportions
Introduction

Chapter 6 reviews the concept, which has already been presented in grade 6, of the **ratio** of two quantities. From this concept, we develop the related concepts of a **rate** (so much of one thing per so much of another thing) and a **proportion** (an equation of two ratios).

When two quantities are in proportion, we can consider the quantities as variables, write an equation to describe the relationship between them, and graph that equation. This study of proportional relationships takes the concept of *ratio* to a new level, and paves the way to the study of linear functions in 8th grade.

The first lessons focus on the concepts of ratio, rate, and unit rate. Students use tables of equivalent ratios and unit rates to solve a variety of problems involving rates. We especially focus on calculating unit rates when the quantities involve fractions.

Then we study proportional relationships, using the familiar tables of equivalent rates as a starting point. Students write and graph equations relating the two quantities (seen as variables now). They find the unit rate and plot it on the graph as a single point, and relate the different representations of proportional relationships (graph, table of values, wording, and equation) to each other. We also spend some time analyzing whether a given relationship between variables is proportional or not.

The next topic is proportions — equations where one ratio is equal to another. Students learn to solve proportions with cross-multiplying and to set them up in the correct way to solve a word problem. They also learn and compare different ways to solve problems with rates. It is not always necessary to set up a proportion!

Then we turn our attention to an application of all this in geometry: scaled figures, scale drawings, floor plans, and maps. Students encounter scales such as 1:90 or 1 in = 2 ft. They calculate dimensions in reality from the scale drawing and vice versa, and redraw scale drawings at a different scale. Floor plans use a scale also, and are hopefully an interesting topic to students.

The lesson on maps is optional. In today's world, most of us are using online map services which calculate the distances for us, so there is much less need to figure out distances using physical maps, but some students (and teachers) might find the topic interesting.

In 8th grade, students encounter proportional relationships again, as they learn the connection between the unit rate and the slope of the graph, and compare different proportional relationships represented in different ways.

I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

There are free videos matched to the curriculum at **https://www.mathmammoth.com/videos/** (choose 7th grade).

Good Mathematical Practices

- The chapter gives students many opportunities to persevere in problem solving through various multi-step word problems. Remind your student(s) that making a mistake is part of normal problem solving, and when you think about your mistake, your brain actually grows. There is no brain growth when doing problems that you already know how to solve.
- The study of proportional relationships lays a big foundational piece for mathematical modeling. In this chapter, students encounter many real-life situations (e.g. speed, price per kg). They learn to see the quantities involved as variables, to write an equation relating the variables, and to graph the equations. These are all essential skills in mathematical modeling.
- In the lesson Scale Drawings 2, students will explore how the area of a scaled figure relates to the scale factor, and they have the opportunity to find the general rule through repeated reasoning.

Pacing Suggestion for Chapter 6

This table does not include the chapter test because it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Ratios and Rates	14	*3 pages*	1 day	
Solving Problems Using Equivalent Rates	17	*3 pages*	1 day	
Unit Rates	20	*4 pages*	1 day	
Proportional Relationships 1	24	*4 pages*	1 day	
Proportional Relationships 2	28	*4 pages*	1 day	
Proportional Relationship or Not?	32	*4 pages*	1 day	
Solving Proportions	36	*3 pages*	1 day	
Proportions and Problem Solving	39	*4 pages*	1 day	
More on Proportions	43	*4 pages*	1 day	
Scaling Figures	47	*4 pages*	1 day	
Scale Drawings 1	51	*3 pages*	1 day	
Floor Plans	54	*3 pages*	1 day	
Scale Drawings 2	57	*3 pages*	1 day	
Scale Drawings—More Practice (optional)	60	*2 pages*	1 day	
Maps (optional)	62	*6 pages*	2 days	
Chapter 6 Mixed Review	68	*3 pages*	1 day	
Chapter 6 Review	71	*5 pages*	2 days	
Chapter 6 Test (optional)				
TOTALS		*54 pages*	16 days	
with optional content		*(62 pages)*	(19 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr7ch6

SCAN ME

Ratios and Rates

A **ratio** is a comparison of two numbers, or quantities, using division.

For example, to compare the hearts to the stars in the picture, we say that the ratio of hearts to stars is 5:10 (read "five to ten").

The two numbers in the ratio are called the **first term** and the **second term** of the ratio. The order in which these terms are mentioned does matter! For example, the ratio of stars to hearts is *not* the same as the ratio of hearts to stars. The former is 10:5 and the latter is 5:10.

We can write this ratio in several different ways:

- The ratio of hearts to stars is 5:10.
- The ratio of hearts to stars is 5 to 10.
- The ratio of hearts to stars is $\frac{5}{10}$.
- For every five hearts, there are ten stars.

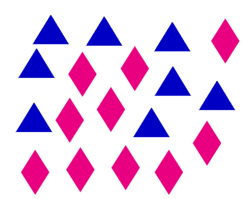
Note that we are not comparing two numbers to determine which one is greater (as in 5 < 10). The comparison is relative as in a multiplication problem. For example, the ratio 5:10 can be simplified to 1:2, and it indicates to us that there are twice as many stars as there are hearts.

We **simplify ratios** in exactly the same way we simplify fractions.

Example 1. In the picture at the right, the ratio of hearts to stars is 12:16. We can simplify that ratio to 6:8 and even further to 3:4. These three ratios (12:16, 6:8, and 3:4) are called **equivalent ratios**.

The ratio that is simplified to lowest terms, 3:4, tells us that for every three hearts, there are four stars.

1. Write the ratio and then simplify it to lowest terms.



The ratio of triangles to diamonds is _____ : _____ = _____ : _____ .

In this picture, there are ______ triangles to every _______ diamonds.

2. **a.** Draw a picture with pentagons and circles so that the ratio of pentagons to the total of all the shapes is 7:9.

b. What is the ratio of circles to pentagons?

3. **a.** Draw a picture in which (1) there are three diamonds for every five triangles, and (2) there is a total of 9 diamonds.

b. Write the ratio of all the diamonds to all the triangles, and simplify this ratio to lowest terms.

4. Write the equivalent ratios.

a. 5 to 45 = 1 to ________	**b.** 3 : _____ = 9 : 60	**c.** 280 : 420 = 2 : _____	**d.** $\frac{5}{13} = \frac{\ }{65}$

We can also form **ratios using quantities that have units**. If the units are the same, they cancel.

Example 2. Simplify the ratio 250 g : 1.5 kg.

First we convert 1.5 kg to grams and then simplify: $\frac{250\text{ g}}{1.5\text{ kg}} = \frac{250\text{ g}}{1{,}500\text{ g}} = \frac{250}{1{,}500} = \frac{1}{6}$.

5. Use a fraction line to write ratios of the given quantities as in the example. Then simplify the ratios.

a. 5 kg and 800 g $\frac{5\text{ kg}}{800\text{ g}} =$	**b.** 600 cm and 2.4 m
c. 1 gallon and 3 quarts	**d.** 3 ft 4 in and 1 ft 4 in

We can generally **convert** ratios with decimals or fractions **into ratios of whole numbers**.

Example 3. Because we can multiply both terms of the ratio by 10, $\frac{1.5\text{ km}}{2\text{ km}} = \frac{15\text{ km}}{20\text{ km}}$.

Then: $\frac{15\text{ km}}{20\text{ km}} = \frac{15}{20} = \frac{3}{4}$. So the ratio 1.5 km : 2 km is equal to 3:4.

You can also see that the ratio is 3:4 by noticing that both 1.5 km and 2 km are evenly divisible by 500 m.

Example 4. Simplify the ratio ¼ mile to 5 miles.

First, the units cancel: ¼ mi : 5 mi = ¼ : 5. Multiplying both terms of the ratio by 4, we get ¼ : 5 = 1:20.

6. Use a fraction line to write ratios of the given quantities. Then simplify the ratios to whole numbers.

a. 5.6 km and 3.2 km	**b.** 0.02 m and 0.5 m
c. 1.25 m and 0.5 m	**d.** 1/2 L and 7 1/2 L
e. 1/4 cup and 3 1/2 cups	**f.** 2/3 mi and 1 mi

If the two terms in a ratio have *different* units, then the ratio is also called a **rate.**
Example 5. The ratio "8 km to 40 minutes" is a rate that compares the quantities "8 km" and "40 minutes," perhaps for the purpose of giving us the speed at which a person is running. We can write this rate as 8 km : 40 minutes or $\frac{8\text{ km}}{40\text{ minutes}}$ or 8 km *per* 40 minutes. The word "per" in a rate signifies the same thing as a colon or a fraction line.
This rate can be simplified: $\frac{8\text{ km}}{40\text{ minutes}} = \frac{1\text{ km}}{5\text{ minutes}}$. The person runs 1 km in 5 minutes.
Example 6. Simplify the rate "15 pencils per 100¢." Solution: $\frac{15\text{ pencils}}{100¢} = \frac{3\text{ pencils}}{20¢}$.

7. Write each rate using a colon, the word "per," or a fraction line. Then simplify it.

a. Jeff swims at a constant speed of 400 meters : 15 minutes.

b. A car can travel 54 miles on 3 gallons of gasoline.

8. Fill in the missing terms to form equivalent rates.

a. $\frac{1/2\text{ cm}}{30\text{ min}} = \frac{\_\_\_\_}{1\text{ h}} = \frac{\_\_\_\_}{15\text{ min}}$	**b.** $\frac{\$88.40}{8\text{ hr}} = \frac{\_\_\_\_}{2\text{ hr}} = \frac{\_\_\_\_}{10\text{ hr}}$

9. Simplify these rates. Don't forget to write the units.

a. 280 km per 7 hours	**b.** 2.5 inches : 1.5 minutes

10. A car is traveling at a constant speed of 72 km/hour. Fill in the table of equivalent rates: each pair of numbers in the table (distance/time) forms a rate that is equivalent to the rate 72 km/hour.

Distance (km)							
Time (min)	10	30	40	50	60	90	100

11. Eight pairs of socks cost $20. Fill in the table of equivalent rates.

Cost ($)								
Pairs of socks	1	2	4	6	7	8	9	10

Solving Problems Using Equivalent Rates

Example 1. It took Liam 1 ½ hours to paint 8 meters of fence. Painting at the same speed, how long will it take him to paint the rest of the fence, which is 28 meters long?

In this problem, we see a rate of 8 m per 1 ½ hours. There is another rate, too: 28 m per an unknown amount of time. These two are equivalent rates. We can use a table of equivalent rates to solve the problem.

Amount of fence (m)	8	4	28
Time (minutes)	90	45	315

(1) We figure that Liam can paint 4 m of fence in 45 minutes (by dividing the terms in the original rate by 2).

(2) Next we multiply both terms in the rate of 4 m/45 min by seven to get the rate 28 m/315 min.

It will take Liam 315 minutes, or 5 hours 15 minutes, to paint the rest of the fence.

Example 2. Sofia rides her bike 20 miles in 45 minutes. Riding at the same speed, how far will she go in 1 hour?

We can multiply or divide both terms of a rate by the same number to form another, equivalent rate. (You have used this same idea in the past with equivalent fractions.)

$$\frac{20\text{ miles}}{45\text{ minutes}} \overset{\div 3}{=} \frac{\boxed{}\text{ miles}}{15\text{ minutes}} \overset{\times 4}{=} \frac{\boxed{}\text{ miles}}{60\text{ minutes}}$$

It's not easy to go directly from 45 minutes to 60, but we can use 15 as a "stepping stone" in between.

Recall that 20 ÷ 3 is easy to solve when you think of it as a fraction: 20/3 = 6 2/3. Sofia can ride 6 2/3 miles in 15 minutes.

Then, we multiply both terms of that rate by 4. Again, don't be intimidated by the fraction: 4 · (6 2/3) = 4 · (20/3) = 80/3 = 26 2/3. So, Sofia can ride 26 2/3 miles in 1 hour.

1. Fill in the tables of equivalent rates.

a.

Distance	15 km			
Time	3 hr	1 hr	15 min	45 min

b.

Pay	$15			
Time	45 min	15 min	1 hr	1 hr 45 min

2. Fill in the missing terms in these equivalent rates.

a. $\frac{3\text{ pies}}{8\text{ boys}} = \frac{\underline{\qquad}}{2\text{ boys}} = \frac{\underline{\qquad}}{12\text{ boys}} = \frac{\underline{\qquad}}{20\text{ boys}}$

b. $\frac{115\text{ words}}{2\text{ min}} = \frac{\underline{\qquad}}{1\text{ min}} = \frac{\underline{\qquad}}{3\text{ min}}$

3. Aiden can ride his bicycle 8 miles in 28 minutes. At the same constant speed, how long will he take to go 36 miles?

$$\frac{8\text{ miles}}{28\text{ minutes}} = \frac{4\text{ miles}}{\boxed{}\text{ minutes}} = \frac{\boxed{}\text{ miles}}{\boxed{}\text{ minutes}}$$

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Review: Percent

Percent (or **per cent**) means *per hundred* or "divided by a hundred." (The word "cent" means one hundred.) So, percent means the rate per hundred, or a hundredth part.

To convert percentages into fractions, simply read the "per cent" as "per 100." Thinking of hundredths, you can also easily write them as decimals.

Therefore, 8% = 8 per cent = 8 per 100 = 8/100 = 0.08.

Similarly, 167% = 167 per 100 = 167/100 = 1.67.

$\frac{5}{100}$ five per cent = 5%

1. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
	0.07	
52%		
		$\frac{59}{100}$

Percent	Decimal	Fraction
109%		
200%		
		$\frac{382}{100}$

A number with two decimal digits has hundredths, so it can easily be written as a percentage. For example, 0.56 = 56%. But we can write numbers with more decimal digits as percents, also.

Example 1. As a percentage, the number 0.5642 is 56.42%. Compare this to 0.56 = 56%. The digits "42" simply follow the digits "56", and become the decimal digits for the percentage.

Decimal	Percent	Fraction
0.09	9%	$\frac{9}{100}$
0.091	9.1%	$\frac{91}{1000}$
0.09146	9.146%	$\frac{9146}{100{,}000}$

2. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
0.9%		
		$\frac{282}{1000}$
	0.8914	

Percent	Decimal	Fraction
		$\frac{91}{10{,}000}$
2.391%		
	0.94284	

Writing fractions as percentages

Example 2. Sometimes you can convert a fraction into an equivalent fraction with a denominator of 100, 1000, or some other power of 10. After that it is easy to write it as a decimal and then as a percentage.

$$\frac{46}{25} \underset{\cdot 4}{\overset{\cdot 4}{=}} \frac{184}{100} = 1.84 = 184\%$$

Example 3. For most fractions, we need to use *division* to convert the fraction to a decimal first, and then to a percentage.

Simply treat the fraction line as a division symbol and divide (using long division or a calculator), to get a decimal. Then write it as a percentage.

$$\frac{8}{9} = 0.888... \approx 0.889 = 88.9\%$$

$$\begin{array}{r} 0.8888 \\ 9\overline{)8.0000} \\ \underline{-72} \\ 80 \\ \underline{-72} \\ 80 \\ \underline{-72} \\ 80 \\ \underline{-72} \\ 8 \end{array}$$

3. Fill in the table. First write each fraction as an equivalent fraction where the denominator is a power of ten.

Fraction	Fraction (denominator is a power of ten)	Decimal	Percent
$\frac{8}{25}$	$\frac{\square}{100}$		
$\frac{142}{200}$	$\frac{\square}{100}$		
$\frac{24}{20}$			
$\frac{31}{250}$			
$\frac{3}{8}$			

4. Write as percentages. Use long division. Round your answers to the nearest tenth of a percent.

a. 11/8

b. 11/24

5. Write the fractions as decimals and percentages. Round the decimals to four decimal digits. Use a calculator.

a. $\frac{2}{3} \approx$ ____________ = ____________%	**b.** $\frac{11}{6} \approx$ ____________ = ____________%
c. $\frac{17}{23} \approx$ ____________ = ____________%	**d.** $\frac{304}{57} \approx$ ____________ = ____________%

6. Match the fractions and percentages.

$\frac{3}{4}$	$\frac{17}{20}$	$\frac{4}{5}$	$\frac{18}{25}$	$\frac{5}{4}$	$\frac{7}{10}$	$\frac{9}{8}$	$\frac{23}{20}$
125%	80%	75%	115%	70%	72%	85%	112.5%

7. Remember mental math? Fill in the shortcuts for finding these easy percentages of a number.

To find 50% of a number, divide it by _______.	To find 10% of a number, divide it by ________.
To find 25% of a number, divide it by _______.	To find 1% of a number, divide it by ________.

To find 30% of a number, first find __________% of the number, then multiply that by _____.

To find 75% of a number, first find __________% of the number, then multiply that by _____.

8. Find various percentages of the number 360.

Percentage	Value
5%	
10%	
20%	
25%	
50%	
60%	
75%	
80%	
100%	360
125%	
150%	

9. Solve using mental math.

a. What is 25% of $84.00?

b. 17 is 25% of what number?

c. Find 60% of 300.

d. 8 is 1% of what number?

e. Find 3% of 2,000 km.

f. 9 is 3% of what number?

g. What is 20% of $45?

h. 24 is 60% of what number?

i. Find 150% of $60.

j. 36 is 200% of what number?

Solving Basic Percentage Problems

All percentages are fractions. Recall that "percent" means "per 100". For example, 34% is 34 per 100 or 34/100 — a fraction. Stated differently, a percentage is a "rate per 100".

All percentage problems have to do with a **part** versus **total**. As a fraction, we write $\frac{part}{total}$.

As a percent, it is still a fraction, and has the same value, but we want the denominator to be 100.

For example, to find what percentage 2 is of 5, we can write: $\frac{2}{5} = \frac{40}{100}$, and then write 40/100 as 40%.

Example 1. What percentage is 14 km of 75 km?

We know the total (75 km) and we know the part (14 km). This is essentially asking what fraction 14 km is of 75 km, but we need to express the answer as a percentage, not as a fraction. Therefore:

1. We write the fraction $\frac{part}{total}$: it is $\frac{14\text{ km}}{75\text{ km}}$ but the units "km" cancel out so it becomes just $\frac{14}{75}$.

2. Then we use a calculator to divide 14/75 = $0.18\overline{6}$ and write that as a percentage: $0.18\overline{6}$ = $18.\overline{6}$%.

Normally, we round the result and say that 14 km is about 19% of 75 km.

Example 2. Find 59.2% of $2,600.

Here we know the percentage — which means we know the fraction — and the total. We don't know the *part* as a quantity. This is the same as asking for 592/1000 of $2,600.

Recall that the word "of" translates into multiplication. Thus, we could use fraction multiplication (we could calculate (592/1000) · 2,600) but often, the quickest way to do these types of calculations is to convert the percentage into a decimal first, and then use decimal multiplication.

So, instead of (592/1000) · $2,600, we write 59.2% as 0.592, and calculate 0.592 · $2,600 = $1,539.20.

If the percentage is known and the total is known: (What is x% of y?)

This is the same as asking for a fraction of some total.

1. Write the percentage as a decimal.
2. Multiply that decimal by the total.

Or use mental math tricks for finding 1%, 10%, 20%, 30%, 25%, 50%, 75%, *etc*. of a number.

If you are asked the percentage:

Asking "what percentage" is essentially the same as asking "what part" or "what fraction."

1. Write the fraction $\frac{part}{total}$.

2. Write this fraction as a percentage. Often, you will do this in two steps: first write that fraction as a decimal, and then that as a percentage.

1. Fill in the tables. Use mental math.

a.

Amount			75					
Percentage	10%	20%	30%	50%	100%	120%	150%	200%

b.

Amount		12	20	32			60	
Percentage	10%				100%	120%	150%	200%

You may use a calculator for the rest of the problems in the lesson except question #8.

2. A shirt that cost $34 was discounted by $4. What is the discount percent?

3. Julia paid $325.08 of her $1,890 paycheck in taxes.
   What percentage of her paycheck did she pay in taxes?

4. At 7 years of age, Matthew was at 68.7% of his adult height, which is 182 cm.
   How tall was Matthew when he was 7?

5. On a certain Monday, 5.3% of a school's 980 students didn't show up.
   How many students were at school that day?

6. Harry has two roosters, named Captain and Chief. The weight of Captain is 7/5 of the weight of Chief.

   a. Write the second sentence above using a percentage instead of a fraction.

   b. If Chief weighs 6 lb, how much does Captain weigh?

7. The Carters live on a rectangular piece of land that measures 40 m × 35 m. The Joneses live on a rectangular piece of land that measures 42 m × 39 m.

   a. To the nearest hundredth of a percent, find what percentage the area of the Carters' land is of the area of the Joneses' land.

   b. If the Joneses' land is valued at $4,914, and the Carters' land is appraised at the same value per square meter, what would be the value of the Carters' land?

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Angle Relationships 1

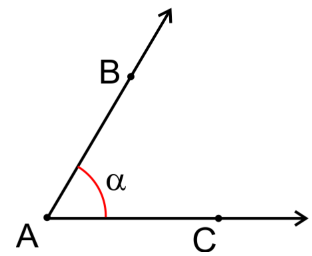
A **ray** has a starting point and continues indefinitely in one direction (indicated by one arrowhead).

In contrast to a ray, a **line** continues indefinitely in *two* directions (indicated by two arrowheads).

An **angle** consists of **two rays that start at the same point**, called the **vertex**. Each ray is called a **side** of the angle.

We can denote the angle on the right as angle BAC, or using the symbol "∠" for "angle," as ∠BAC.

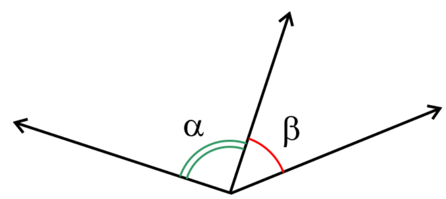
Note that we list the vertex point in the middle: it is ∠B**A**C, not ∠ABC. We could also name it ∠CAB.



In mathematics, we also often denote angles with the beginning letters of the Greek alphabet: α (alpha), β (beta), γ (gamma), and δ (delta). So ∠BAC can also be called "angle α."

Two angles are **adjacent** if they have a **common vertex and share one side.**

In the image on the right, ∠α and ∠β are adjacent (side-by-side) angles.



1. B is a point on line AD. Find the measures of the three angles, and also the angle sums. Do you notice any special numbers?

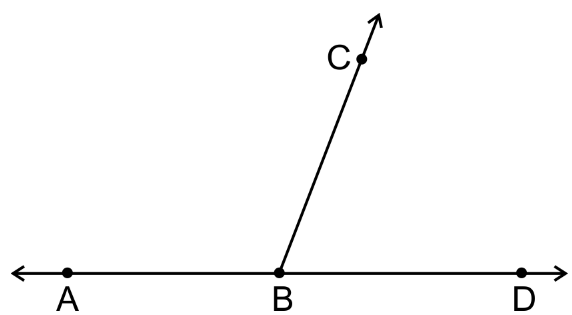
   ∠ABC = __________°

   ∠CBD = __________°

   ∠ABD = __________°

   sum of ∠ABC and ∠CBD : ____________°

   sum of all three angles: _____________ ° (This should be 360°.)

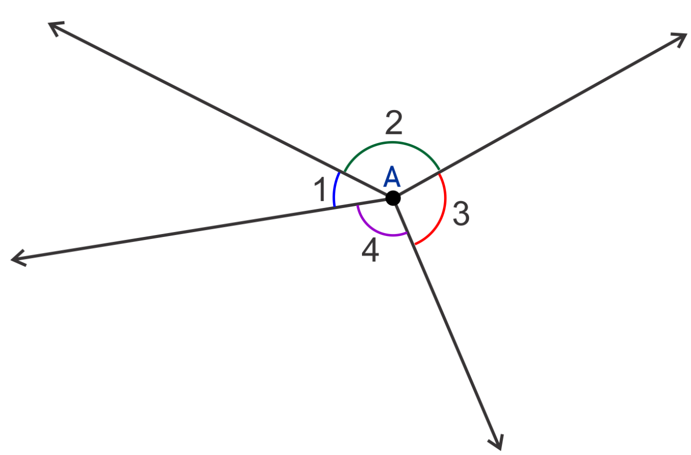


2. Several rays start at point A. Measure the angles. Calculate their sum.

   ∠1 = __________° ∠2 = __________°

   ∠5 = __________° ∠4 = __________°

   Sum of the angles = __________°

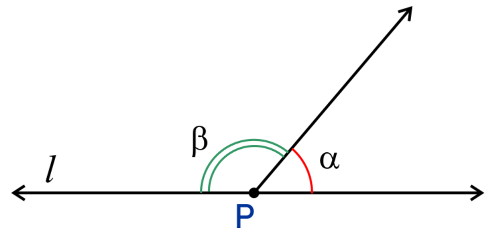


P is a point on line *l*. The angles ∠α and ∠β in this image are adjacent, and they form a straight angle (an angle of 180 degrees). They are called **supplementary angles**.

Two angles are supplementary if their **sum is 180 degrees:**

$$\angle\alpha + \angle\beta = 180°$$

We also say that α supplements β.

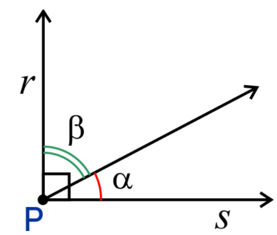


Rays *r* and *s* start at point P and form a right angle. The adjacent angles ∠α and ∠β form a right angle. They are called **complementary angles**.

Two angles are complementary if their **sum is 90 degrees:**

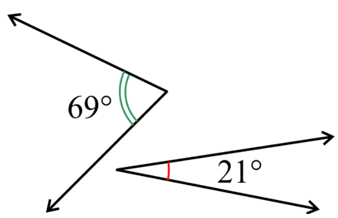
$$\angle\alpha + \angle\beta = 90°$$

We also say that α complements β.

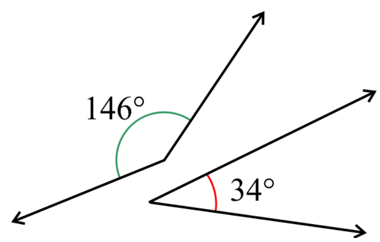


Here's a mnemonic to help you remember the difference: **S**upplementary angles form a **S**traight line, and **C**omplementary angles form a **C**orner (a right angle).

Supplementary angles don't have to be adjacent, and neither do complementary angles.



These are still complementary angles, because 21° + 69° = 90°.

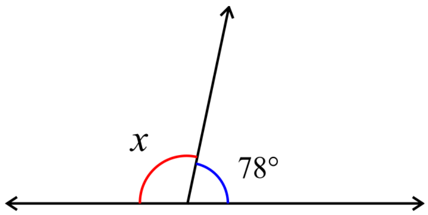


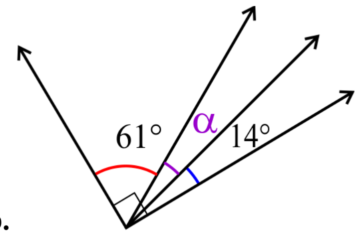
These are still supplementary angles, because 146° + 34° = 180°.

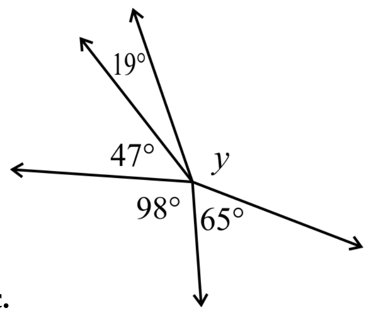
3. **a.** Draw a 38° angle. Then draw an adjacent angle that complements it.

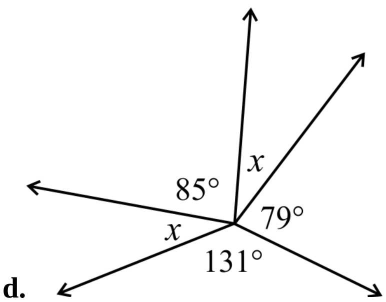
b. Draw an 82° angle. Then draw an adjacent angle that supplements it.

4. Write an equation for the unknown and solve it. Do not measure any angles.

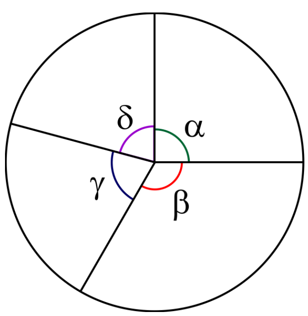
a.


b.


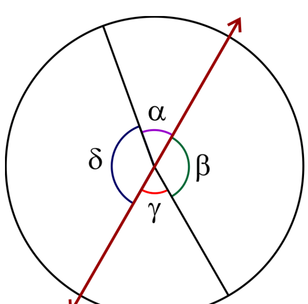
c.


d.


5. Figure out the missing entries in the tables without measuring any angles.

a.


Angle	Degrees	Fraction	Percentage
α		1/4	
β	120°		
γ			
δ	75°		

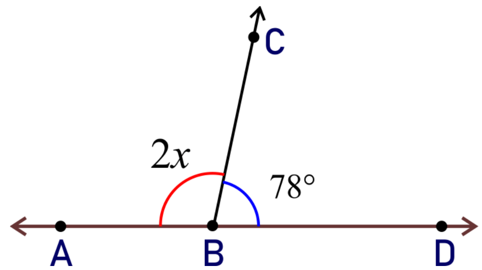
b.


Angle	Degrees	Fraction	Percentage
α	50°		
β			
γ		1/6	
δ			

Example. Point B is on line AD. Write an equation to solve for the unknown. What is the measure of angle ABC?

Since angle ABD is a straight angle (180°), the equation is:

$$\begin{aligned} 2x + 78 &= 180 \\ 2x &= 102 \\ x &= 51 \end{aligned}$$

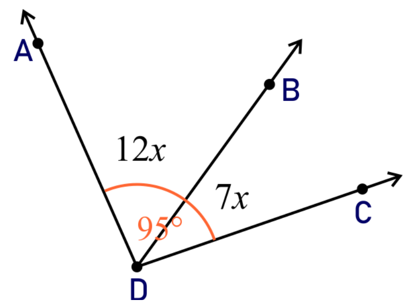


So, x is 51°. However, angle ABC does not measure 51° because its measure is $2x$, not x. So, we double the value of x to get that ∠ABC = 102°.

6. Angle ADC measures 95°.

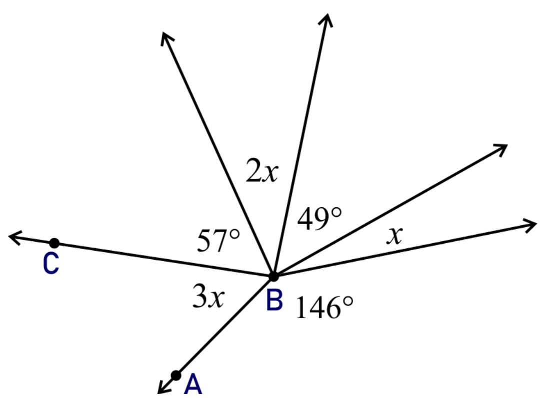
   a. Write an equation for the unknown and solve it.

   b. Find the measure of ∠BDC.



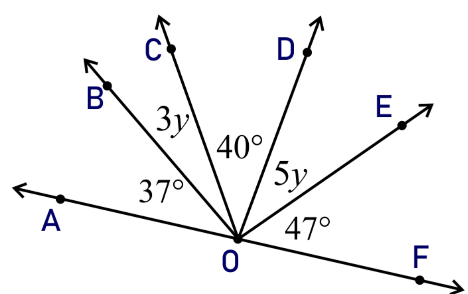
7. **a.** Write an equation for the unknown and solve it.

   b. Find the measure of ∠ABC.



8. **a.** Write an equation for the unknown and solve it.

   b. Find the measure of each angle in the image, excluding those whose angle measure is given.



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Proving the Formula for the Area of a Circle

We will now study an informal proof for the familiar formula A = πr^2. In the first part of the proof we prove that the area of the circle is ½C · r. In part 2 we will show that ½C · r is equal to the familiar πr^2.

Proof, part 1

Let's divide a circle into equal sectors, in this case into 12.

We can rearrange those sectors to form a figure that is very close to a parallelogram. Obviously, the area of this new shape is equal to the area of the circle. But since the area of this shape is very close to the area of a parallelogram, we can approximate its area with the formula for the area of the parallelogram.

The <u>base</u> of this parallelogram is approximately half of the circumference, or ½C.

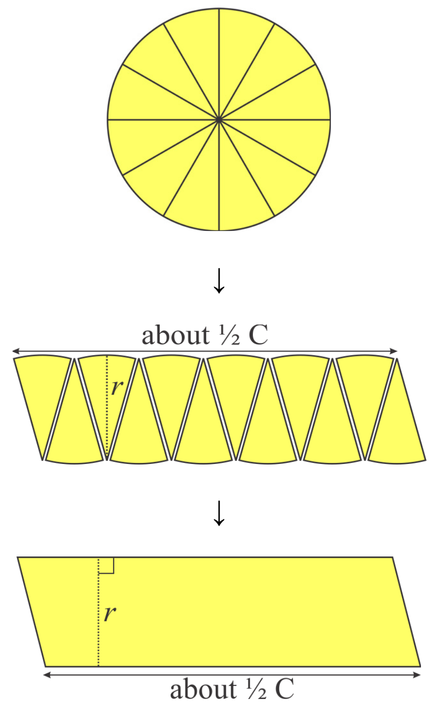
Why? The sectors alternate their orientations to put half of the original circumference on the top of the parallelogram and half on the bottom.

The <u>height</u> of the parallelogram is the radius of the circle, r (see the image at the right).

The area of a parallelogram is its base times its height, so the area of this parallelogram is

$$A = \tfrac{1}{2}C \cdot r$$

By dividing the circle into a larger number of sectors, the shape made of the sectors would get closer to a parallelogram. In fact, we can get as close to a parallelogram as we want by dividing the circle into a very large number of sectors. So the area of the circle is A = ½C · r.



Area = ½C · r

We have called this an "informal" proof because we have omitted a very important point: In order to prove that the area of the figure that looks like a parallelogram does indeed approach the area of a parallelogram as we increase the number of sectors, one would need to use the concept of a **limit** from calculus.

1. **a.** Study the proof above enough that you can explain it to another person.

   b. Draw a large circle on paper. Divide it carefully and as exactly as you can into 8 or 12 equal sectors, and cut the pieces out. Then explain the proof you just studied to a fellow student, friend, or your teacher.

2. Let's explore how much closer to a parallelogram you can get by dividing the circle into more sectors. You can do this exercise either in drawing software or on paper. Draw a large circle, and divide it carefully into 16 or 20 equal sectors (if drawing on paper, use a protractor). Arrange the sectors into a "parallelogram" shape. Compare your shape to the shape in this lesson, which was made from 12 sectors.

3. Estimate in your head (without a calculator) the area of a circle with a radius of 5.0 ft and circumference of 31.0 ft. Use A = ½C · *r*.

Proof, part 2

Let's use some algebra to transform our equation A = ½C · *r* into the familiar equation for the area of a circle, $A = \pi r^2$.

First, since we know that the circumference of a circle is $C = \pi d$, where *d* is the diameter, we can substitute πd in place of C to get

$$A = \tfrac{1}{2}\mathbf{C} \cdot r = \tfrac{1}{2}\boldsymbol{\pi d} \cdot r$$

In the final expression, we want to have the radius instead of the diameter, so let's substitute 2*r* in place of *d*:

$$A = \tfrac{1}{2}\,\pi \mathbf{d} \cdot r = \tfrac{1}{2}\,\pi \cdot \mathbf{2r} \cdot r$$

Lastly, the 1/2 and 2 cancel each other, and we are left with

$$A = \pi \cdot r \cdot r = \pi r^2$$

4. **a.** Estimate in your head (without a calculator) the area of a circle with a diameter of 12.0 m. Use A = ½C · *r* and the fact that the circumference of a circle is a little over 3 times its diameter.

   b. Use a calculator to find the exact area of this circle with the formula $A = \pi r^2$. Compare the result to your estimate from part (a).

5. Your jump rope is 10 ft long. You form it into a circle. What is the area of your circle? Use a calculator.

Problems Involving Circles

Sometimes, we might leave the answer in a format that includes π, and not give the answer as a rounded decimal. For example, if the diameter of a circle is 10 units, its circumference is 10π units, and its area is $\pi(5^2) = 25\pi$ square units. This format can be useful if there is a further calculation to be made using those values, and/or if we anticipate the π to cancel out in a further calculation.

Example. How much bigger in area is a circle with a radius 12 cm than a circle with a radius of 6 cm?

The former has an area of $\pi(12 \text{ cm})^2 = 144\pi \text{ cm}^2$. The latter has an area of $\pi(6 \text{ cm})^2 = 36\pi \text{ cm}^2$. To compare the two, we will write their ratio (using the fraction line):

$$\frac{\text{area of the smaller circle}}{\text{area of the bigger circle}} = \frac{36\pi \text{ cm}^2}{144\pi \text{ cm}^2} = \frac{36}{144} = \frac{1}{4}$$

Both the π and the unit "cm^2" cancel out, leaving the fraction 36/144 which then simplifies to 1/4. So, the area of the smaller circle is only 1/4 of the area of the larger.

If you do this calculation with decimals, you will have to round the intermediate answers, and thus the final answer will not be exactly 1/4 (though it will be close). Using π instead of rounded decimals will keep the calculations perfectly accurate.

You may use a calculator for all the problems in this lesson.

1. Tell from memory the formulas for the area of a circle and the circumference of a circle.
   If you have difficulty with this, work on memorizing the formulas.

2. **a.** The circumference of a certain circle is 8π units. What is its diameter? Radius?

   b. The area of a certain circle is 36π square units. What is its radius? Its diameter?

3. The table gives the radii of three circles.

	Circle 1	**Circle 2**	**Circle 3**
Radius	5 cm	10 cm	15 cm

   a. What fraction is the area of Circle 1 of the area of Circle 2?

   b. What fraction is the area of Circle 1 of the area of Circle 3?

   c. What fraction is the area of Circle 2 of the area of Circle 3?

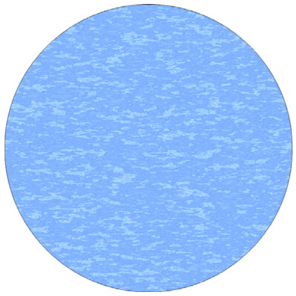
   d. (optional) How do the fractions above compare to the corresponding fractions formed from the radii of the circles?

4. The table shows the diameter and the circumference of some circular objects. One of the values is in error. Which?

   Explain how you know.

Object	Diameter	Circumference
Bicycle tire	70 cm	220 cm
Pot lid	9 in	28 in
A pond	62 ft	95 ft
A ring	12 mm	38 mm
Circle A	5	5π

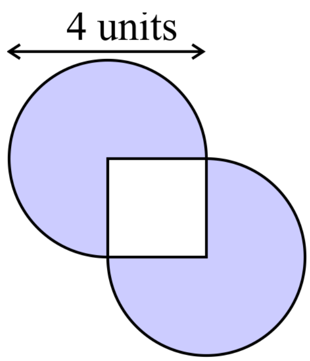
5. This is a scale drawing of a circular pond in a park, drawn here at the scale of 1:500.



   a. Find the area of the pond in reality, to the nearest 100 square feet.

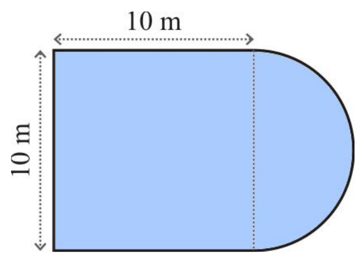
   b. There will be a fence built around the pond. How long will the fence be, to the nearest ten feet?

6. Find the area of the shaded part of this circle design, in terms of π.



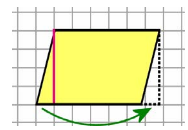
7. A swimming pool has the shape of the union of a square and a semicircle. The outer perimeter of the pool will be lined with decorative tiles that are 20 cm long. There will be 1 cm spaces between the tiles. The tiles are sold in packages of 20 that cost $24.90 per package.

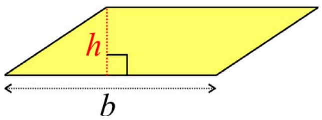
   Find how many packages of tiles must be purchased to line the perimeter of the pool and the total cost.



Area of Polygons and Compound Shapes

Recall that from any parallelogram we can cut off a triangular piece and move it to the other side to make it a rectangle. This shows us that we can calculate the area of a parallelogram the same way as the area of a rectangle.

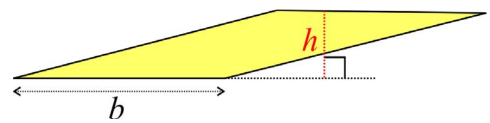




As a formula, the area of a parallelogram is $A = bh$

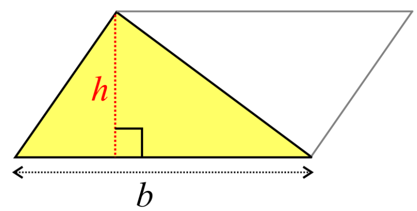
where b is the base and h is the altitude (height).

The **altitude** of a parallelogram is a perpendicular line segment from the base, or the extension of the base, to the top. Thus, the altitude might not be inside the parallelogram, if the parallelogram is very "slanted."

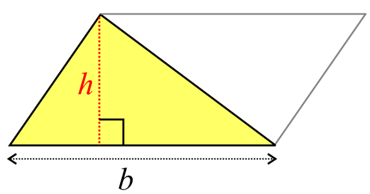


Since any triangle is half of its corresponding parallelogram, the area of a triangle is half the area of that parallelogram:

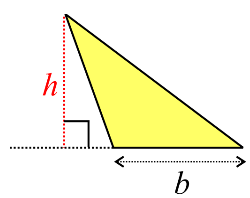
$A = \dfrac{bh}{2}$ where b is the base and h is the altitude of the triangle.



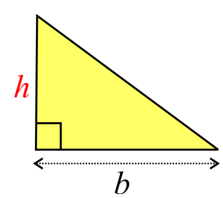
The **altitude** of a triangle is a line from one vertex to the opposite side that is perpendicular to that side. It can:



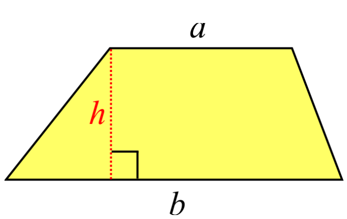
(1) fall inside the triangle;



(2) fall outside the triangle;



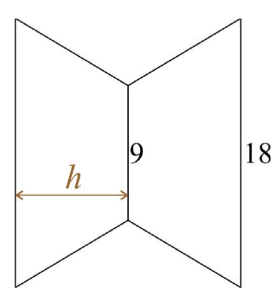
(3) be one of the sides of a right triangle.



The **area of a trapezoid** is given by the formula $A = \dfrac{(a + b)}{2}h$

where a and b are the lengths of the two parallel sides and h is the altitude. Essentially, we calculate the average of the lengths of the two parallel sides, and multiply that times the height.

Example. What should be the height of these two identical trapezoids so that their combined area would be 189 square units?

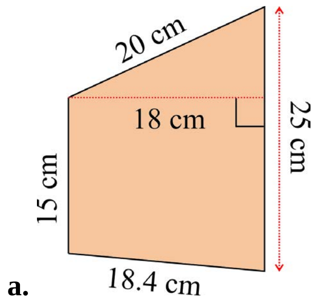


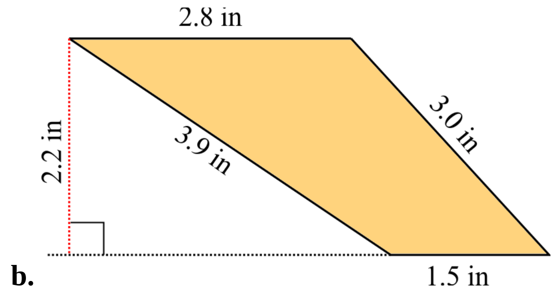
The area of each trapezoid is $(9 + 18)/2 \cdot h$ which simplifies to $13.5h$.
The area of the two trapezoids is $2 \cdot 13.5h$ which simplifies to $27h$.

So, we simply solve the equation $27h = 189$, from which $h = 189/27 = 7$ units.

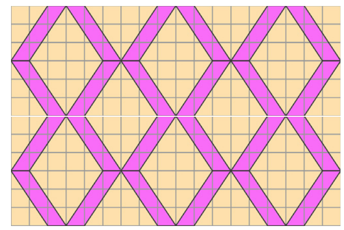
You may use a calculator for all the problems in this lesson.

1. Find the area of each trapezoid.



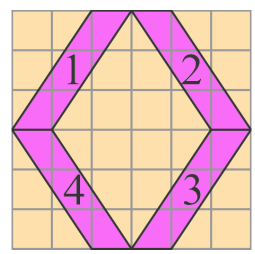


2. This is a pattern of light-colored rhombi with dark pink borders.
   The figure below shows the basic unit, or cell, of the pattern.
   The pink borders actually consist of parallelograms.



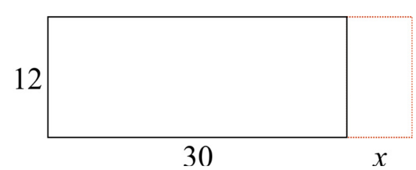
   a. Find the area, in square units, of one of the numbered parallelograms.

   b. Find the total area, in square units, of the lightly-colored parts of the cell.



   c. What percentage of the entire pattern do the rhombi (and parts of rhombi) cover?

3. A 12 × 30 rectangle is enlarged. How much longer should it be so that its area would be 444 square units?



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Probability

You *probably* already have an intuitive idea of what *probability* is. In this lesson we look at some simple examples in order to study probability from a mathematical point of view.

If we flip a coin, the chance, or **probability**, of getting “heads” is 1/2. The chance of getting “tails” is also 1/2. “Heads” and “tails” are the two possible **outcomes** when tossing a coin, and they are equally likely.

When rolling a six-sided number cube (a die), you have six possible **outcomes**: you can roll either 1, 2, 3, 4, 5, or 6. These are all equally likely (assuming the die is fair).

Thus the probability of rolling a five is 1/6. The probability of rolling a three is also 1/6. In fact, the probability of each of the six outcomes is 1/6.

The probability of rolling an even number is 3/6, or 1/2, because three of the six possible outcomes are even numbers.

Simple probability has to do with situations where each possible outcome is equally likely.

Then the **probability** of an event is the fraction $\dfrac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$

“Favorable outcomes” are those that make up the event you want. The examples will make this clear.

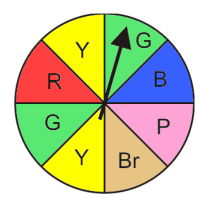
Example 1. What is the probability of getting a number that is less than 6 when tossing a fair die?

Count how many of the outcomes are “favorable” (less than 6). There are five: 1, 2, 3, 4, or 5.
And there are six possible outcomes in total.

Therefore, the probability is $\dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \dfrac{5}{6}$.

In math notation we write “P” for probability and put the event in parentheses: **P(less than 6) = 5/6.**

Example 2. On this spinner the number of possible outcomes is eight, because the arrow is equally likely to land on any of the eight wedges. What is the probability of spinning yellow?



There are TWO favorable outcomes (yellow areas) out of EIGHT possible outcomes.

P(yellow) = 2/8 = 1/4.

(Because green and yellow each have two wedges, there are only six possible colors that can result. When we list the possible outcomes, we list the six colors. However, when we figure the probabilities, we must use the eight equal-sized wedges to find the probability.)

By convention, the probability of an event is always at least 0 and at most 1. In symbols: $0 \le \text{P(event)} \le 1$.

A probability of 0 means that the event does not occur; it is impossible. Probability of 1 means that the event is sure to occur; it is certain. A probability near 1 (such as 0.85) means that the event is likely to occur. A probability of 1/2 means that an event is neither likely nor unlikely.

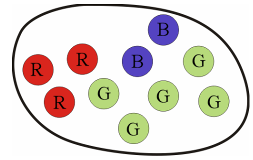
Example 3. What is the probability of rolling 8 on a standard six-sided die?

This is an impossible event, so its probability is zero: P(8) = 0.

Example 4. What is the probability of rolling a whole number on a die?

This is a sure event, so its probability is one. P(whole number) = 1.

1. There are three red marbles, two dark blue marbles, and five light green marbles in Michelle's bag. List all the possible outcomes if you choose one marble randomly from her bag.



2. Michelle chooses one marble at random from her bag. What is the probability that...

   a. the marble is blue?

   b. the marble is not red?

   c. the marble is neither blue nor green?

3. Make up an event with a probability of zero in this situation.

4. Suppose you choose one letter randomly from the word "PROBABILITY."

   a. List all the possible outcomes for this event.

   Now find the probabilities of these events:

   b. P(B)

   c. P(A or I)

   d. P(vowel)

   e. Make up an event for this situation that is likely to occur, yet not a sure event, and calculate its probability.

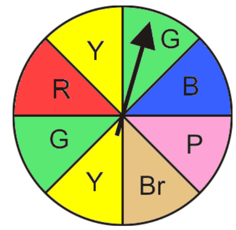
The complement of an event and the probability of "not"

The ***complement*** of any event A is the event that A does *not* occur.

If the probability of event A is a, then the probability of A not happening is simply $1 - a$.

5. The weatherman says that the chance of rain for tomorrow is 1/10. What is the probability of it not raining?

6. The spinner is spun once. Find the probabilities as simplified fractions.



   a. P(green) **b.** P(not green)

   c. P(not pink) **d.** P(not black)

   e. Make up an event for this situation that is not likely, yet not impossible either, and calculate its probability.

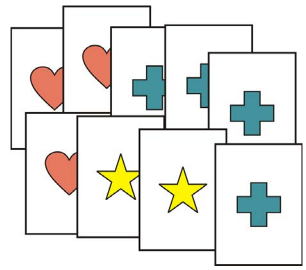
Probabilities are often given as percentages instead of fractions.

Example 5. Kimberly's sock bin contains 7 brown socks, 9 white socks, and 5 red socks. She picks one without looking. What is the probability that she gets a white sock?

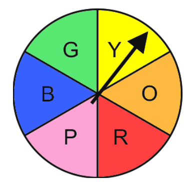
There are 9 white socks out of 21 socks in all. The probability is $9/21 = 3/7 \approx 0.42857 = 0.429 = 42.9\%$.

7. Suppose you were to draw one card from the set of cards on the right. Complete the table with the possible outcomes, and their probabilities both as fractions and as percentages (to the nearest tenth of a percent).

Possible outcomes	Probability (fraction)	Probability (percentage)



8. This "rainbow spinner" is spun once. Find the probabilities to the nearest tenth of a percent.



a. P(yellow)

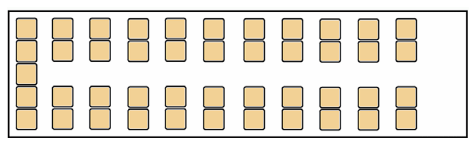
b. P(blue or green)

c. P(not orange)

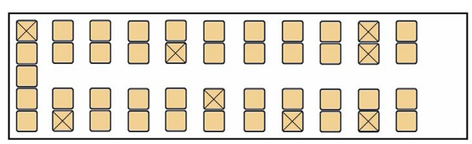
d. P(not red and not purple)

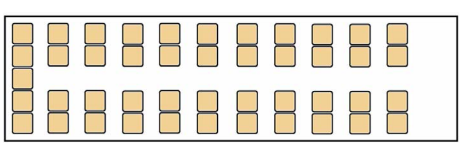
e. Make up an event for this situation with a probability of 1.

9. **a.** An empty bus has 45 seats, and 22 of them are window seats. If you are assigned a seat at random, what is the probability, to the nearest tenth of a percent, that you get a window seat?



b. Now each seat marked with an "x" is already occupied. If you choose a seat randomly, what is the probability, to the nearest tenth of a percent, that you get a window seat?





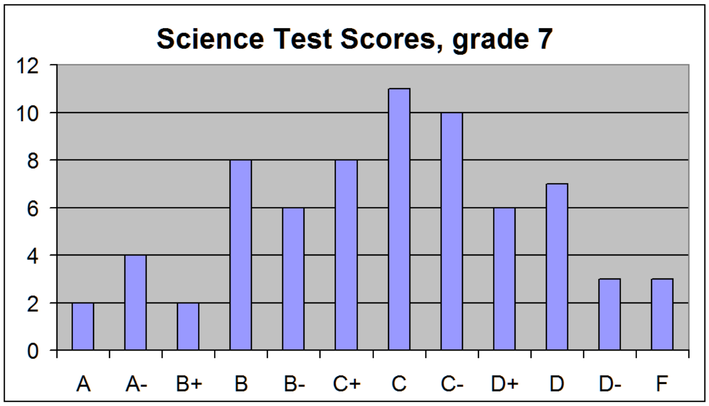
The chart shows you the seating arrangement of a bus. You enter the bus, and the driver informs you that fifteen seats are already occupied and that if you choose a seat randomly, the probability of getting a window seat is less than 25%.

How many window seats are occupied, at least?

Puzzle Corner

Probability Problems from Statistics

Example 1. The bar graph shows the science test scores of all seventy 7th graders in Westmont School. If you choose one of them at random, then what is the probability that the student's score was at least C− (in other words, C− or better)?



Sometimes when a probability question involves "at least," it is easier to look at the complement event — everything else — and find its probability first. The complement of "at least C−" is "at most D+" in other words, D+, D, D−, and F. From the graph, it is easier to sum the number of students who got the four low scores than to sum the number of students who got the eight high scores.

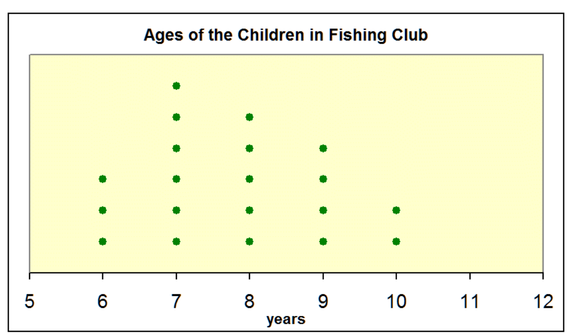
The number of students who got D+, D, D−, or F is 6 + 7 + 3 + 3 = 19 students. There are a total of 70 students, so P(at most D+) = 19/70. Now it's easy to calculate the original probability in question: P(at least C−) = 1 − 19/70 = 51/70.

1. You choose one student at random from the 7th graders in Westmont School.

   a. What is the probability that the student's score was at least D?

   b. What is the probability that the student's score was at most B+?

2. The dotplot shows the age distribution of a children's fishing club. One child is chosen randomly from the group.



   a. What is the probability that the child is at most 9 years of age?

   b. What is the probability that the child is at least 7 years of age?

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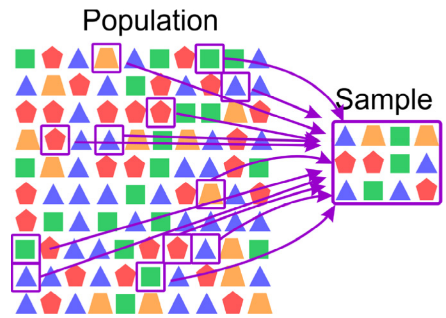
Random Sampling

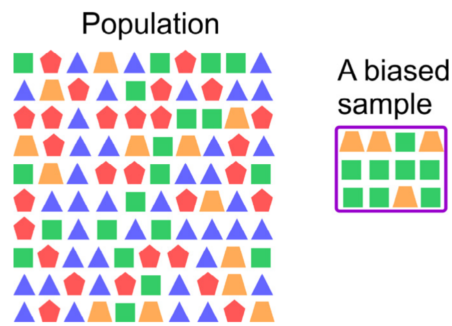
When researchers have a question concerning a large population, they obtain a **sample** (a part) of that population. That is because it is typically impossible to study the entire population.

For example, if you want to know how the citizens of France feel about climate change, you cannot just go and ask every person in France about it. You would choose for example 600 French citizens as your sample and ask them your question.



The way a sample is chosen is very important. Some methods of sampling may produce a sample that is *not* representative of the entire population. We call that a **biased sample**.

For example, if you are studying a student population of 630 in a school with close to an equal number of boys and girls, and you happen to choose a sample of 20 boys, then your sample is biased. It doesn't represent the entire population well.



We need to use **unbiased sampling methods** in order to get a sample that truly represents the population being studied. The best way to avoid biased samples is to select a **random sample.**

The main characteristics of a random sample are:

1. **Randomness:** each member of the population has an equal chance of being selected.

   Let's say a researcher is studying the types of cars Americans own. He decides to interview only people he finds at a local mall because that mall is close to where he lives, so it is convenient for him. His sample is biased because not every member of the US population even has a chance to be selected in his sample. Maybe the people at his local mall are predominantly rich people who own several cars per family, so in that respect those people would not be a good representation of the entire population of the US.

   We call this type of sample a **convenience sample** because it is convenient or easy to obtain.

2. **External selection:** respondents must be chosen by the researcher, not self-selected.

   If our researcher mails a questionnaire to various people across the US asking them to fill it out and return it, his sample is a **voluntary response sample**, which is a biased sample. Some people volunteer to return the questionnaire, but others don't. The people themselves decide whether or not to be a part of the sample.

   Why might this be a problem? Some of the people who would choose to take part may have an external reason to do so. They might want to show off how "good" they are in the particular aspect being studied, or they might just like to speak out about their opinions.

Our researcher could get a true random sample by choosing people randomly from a list of people living in the US and calling them. That way, each person has an equal chance of being selected in the sample (it is random), and the people cannot self-select to take part (the researcher chooses who takes part).

An unbiased sampling method is more likely to produce a representative sample.

1. You are studying whether students in a large college prefer to drink coffee black, with milk, with cream, or with sweetener, or whether they prefer not to drink coffee at all.

   a. Which of the six sampling methods listed below produce a voluntary response sample?

   b. Which methods don't give each member of the student population an equal chance to be selected for the sample?

   c. Which method is likely to produce a sample with only coffee drinkers, overlooking those who don't drink coffee?

   d. Which method will be the most likely to give you a representative (unbiased) sample?

   Sampling Methods

   (1) You interview 80 students in a cafe on the campus.

   (2) You interview 80 students who come in at the main door of the campus.

   (3) You interview the first 80 students you happen to meet on a certain day.

   (4) You choose 80 names randomly from a list of all the students. You call them to interview them.

   (5) You send an email to all the students in the college, asking them to fill in a form on a web page you have set up. You hope to get at least 80 responses.

   (6) You choose 80 names randomly from a list of all the students. You send them an email, asking them to fill in a form on a web page you have set up.

2. A recipe website posts a poll on their home page that any visitor to that website can take. In it, they ask if people are looking for a recipe for a dessert, a main dish, a side dish, bread, or salad. During the course of one Sunday, 4,600 people visit the page, and 252 of them fill in the poll. Explain why the poll results will be based on a biased sample.

Some common random sampling methods are:

1. **Simple random sampling.** Each individual in the sample is chosen randomly and entirely by chance, perhaps by using dice, through pulling names out of a hat, or with a random number generator.

2. **Systematic random sampling.** The individuals of the population are placed in some order, and then each individual at a certain specified interval is selected for the sample.

   For example, a supermarket might study the shopping habits of its customers by choosing every 15th customer who enters the store for the sample.

3. **Stratified random sampling.** The population is first divided into categories (strata) and then a random sample is obtained from each category.

   For example, to study how much sleep students in a particular school get, you might first divide the students into groups by grade levels (the stratification), then select a random sample from each of the grade levels.

3. A population to be studied doesn't have to be of people. A factory produces MP3 players. Out of the 500 units that the factory produces each day, a quality control inspector selects 25 for testing to study their quality and reliability. Which way should he choose those 25 so that his sample would best represent all the MP3 players that the factory produces?

   a. Choose the first 25 produced on a given day.

   b. First choose a number between 1 and 20 randomly. Select the player corresponding to that number, and after that, every 20th player, in the order they were produced that day.

   c. Choose 25 players that have just been finished around 1 PM when the inspector is touring the factory.

   d. Generate 25 random numbers between 1 and 500 and choose the corresponding 25 MP3 players in the order they were produced that day.

4. Ryan has two large fields planted with green beans. He wants to compare the bean plants in one field with the plants in the other. Design a practical sampling method for him to produce an unbiased sample.

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Chapter 10 Mixed Review

1. Solve. (Two-Step Equations, Part 2/Ch5)

a. $\dfrac{v - 6}{7} = -31$	**b.** $\dfrac{x}{4} - 1 = -5$

2. Find the value of each expression. If the result is a fraction, simplify it to lowest terms. (Various lessons/Ch2)

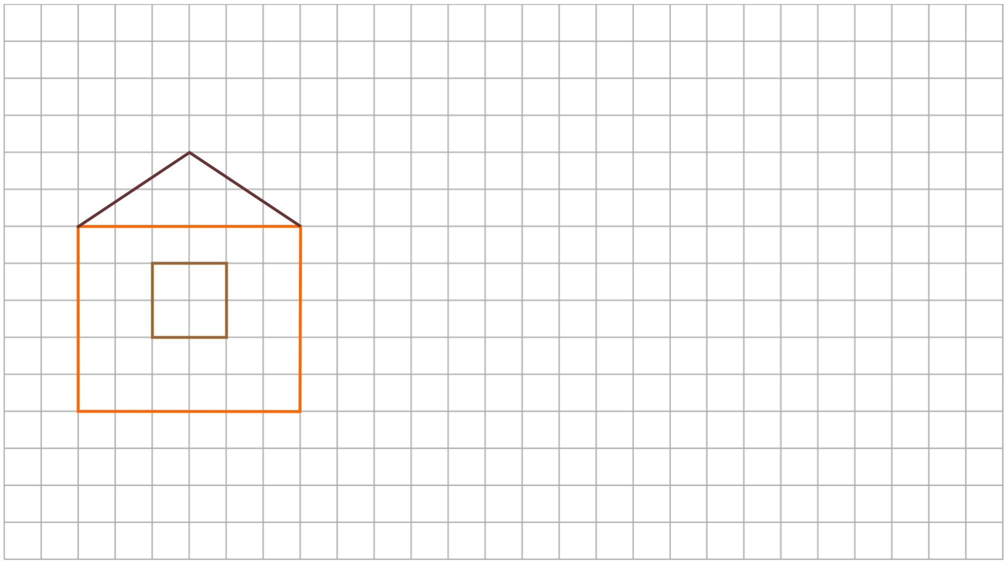
a. $3 \cdot (-3) \cdot (-1) =$	**b.** $(-6) \cdot 8 \div 16 =$
c. $-8 \cdot (-2) \cdot (-5) \cdot (-2) =$	**d.** $(-42) \div (-7) + 42 \div (-2) =$
e. $(-2 + 9) \div (-42) =$	**f.** $-8 \div (-2) + (-5) =$

3. A pair of rubber boots is discounted by 20%, and now they cost $36. What did they cost before? Write an equation for the situation and solve it. (Percent Equations and Price Changes, Part 1/Ch7)

4. **a.** Brazil nuts cost $11.85 for ¾ lb. Find the unit rate per one pound. (Unit Rates/Ch6)

b. Joel paints sceneries to sell to tourists. He can typically finish 2/3 of a painting in 1.5 hours. Find the unit rate per one painting.

5. The house in the grid is drawn at a scale of 1:60. Redraw it at the scale of 1:40. (Scale Drawings 1/Ch6)

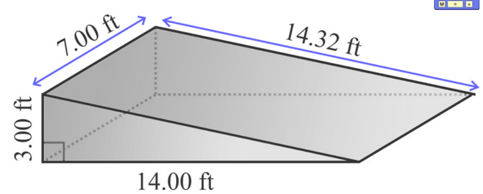


6. A house plan has the scale 1 in : 6 ft, and in the plan the house measures 5 ¼ in by 6 ¾ in. What are the true dimensions of the house? (Floor Plans/Ch6)

7. A cylindrical can of sardines has a bottom diameter of 6.6 cm and height of 8.5 cm. Another can has a diameter of 10 cm. If both cans have the same volume, what is the height of the second can? (Volume of Prisms and Cylinders/Ch8)

8. A rectangle with sides of 3.5 units and 2 units is enlarged using a similarity ratio of 5:2.
Find the area of the resulting rectangle. (Scale Drawings 2/Ch6)

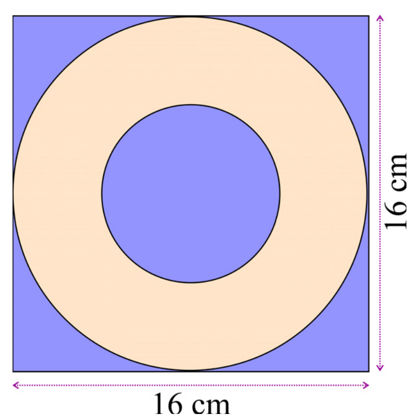
9. **a.** Find the volume of this ramp. (Various lessons/Ch8)



b. Find its surface area.

10. A circle is drawn inside a square. Then, another circle, with half the diameter of the first, is drawn with the same center point. (Area of a Circle/Ch8)

a. Find the area of the light-colored portion of the picture (the “ring”).



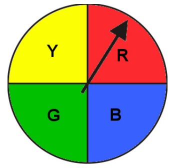
b. Find the area of the darker-colored portions of the picture.

11. Find the area of a 5-ft by 12-ft rectangle in square inches. (Conversions Between Customary Units of Area/Ch8)

12. Draw a triangle with sides 2 inches, 3 1/4 inches, and 3 3/4 inches long. (Basic Geometric Constructions/Ch8)

13. One side of a triangle measures 8.2 cm and it has 45° and 60° angles. (Drawing Triangles, Part 1/Ch8)
Do these conditions define a unique triangle? If so, draw it.
If not, draw at least two non-congruent triangles that fit the conditions.

14. A spinner with four colors is spun twice. (Counting the Possibilities/Ch9)



a. In the space on the right, make a table, a list, or a tree diagram showing all the possible outcomes of this experiment.

Then find the probabilities:

b. P(blue; blue)

c. P(green; not green)

d. P(not blue; yellow)

e. P(yellow or green; red or blue)

Sample space:

15. Tara tosses two coins. (Counting the Possibilities/Ch9)

a. Give an event in this experiment that has the probability of zero.

b. Give an event in this experiment that has the probability of 1/2.

16. John and Jim decided to check if a particular die was fair or not (in other words, if perhaps it was weighted on one side). They rolled that die 1000 times. Their results are at the right.

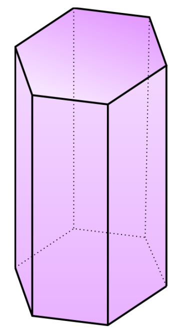
Based on the results, John said, “Yes, this die is indeed weighted, because we rolled ‘1’ many more times than we rolled ‘6’.”

Is his conclusion correct? Why or why not? (Experimental Probability/Ch9)

Outcome	Frequency
1	178
2	160
3	167
4	175
5	167
6	153

17. Sam is studying how well the people in his city like the paintings of the Romantic era. He is planning to stand on a certain street corner near his home and ask passersby if they would like to take part in his study. Explain why his sampling method is biased. (Random Sampling/Ch10)

18. Explain how to cut this hexagonal prism with a plane so that the resulting cross-section is a rectangle. (Slicing Three-Dimensional Shapes/Ch8)



19. Jason has a lemonade stand on a popular street where he sells lemonade to passersby. Let's say that the probability that a random passerby buys a lemonade is 0.1, and that 50 people pass by his stand during a particular hour. Design a simulation and carry it out, in order to find the following probabilities: (Using Simulations to Find Probabilities/Ch9)

a. P(exactly 6 people buy the lemonade)

b. P(at least 3 people buy the lemonade)

Puzzle Corner

Audrey wants to replace 2/3 of the amount of sugar in a cookie recipe with another sweetener. Right now, the recipe calls for 1 1/4 cups of sugar. The other sweetener is used to replace sugar in a ratio of 3:8. For example, you would replace 8 ounces of sugar with 3 ounces of this sweetener.

How much sugar and how much of the other sweetener will she use?