Contents

Chapter 4: Rational Numbers

Chapter 5: Equations and Inequalities

Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. It follows the Common Core Mathematics Standards (CCS) for 7th grade. Those standards are so constructed that students can continue to a traditional algebra 1 curriculum after studying this. However, you also have the option of following this course with Math Mammoth Grade 8, which provides a gentler and slower transition to high school math.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-A, covers the language of algebra (chapter 1), integers (chapter 2), one-step equations (chapter 3), rational numbers (chapter 4), and equations and inequalities (chapter 5). The rest of the topics are covered in the 7-B worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your student's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

In seventh grade, the first five chapters should be studied in order. Also, chapter 7 (Percent) should be studied before studying any of the chapters 8, 9, or 10 (Geometry, Probability, Statistics). You can be flexible with the rest of the scheduling.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your student!

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- The answer keys are included in the digital download version. They are sold as a separate book for the printed version.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for your student to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 5 pages long, intended for one day. Some 5-page lessons can take two days. There are also a few optional lessons.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Worktext 7-A	
Chapter 1	8 days
Chapter 2	13 days
Chapter 3	9 days
Chapter 4	16 days
Chapter 5	16 days
TOTAL	**62 days**

Worktext 7-B	
Chapter 6	17 days
Chapter 7	12 days
Chapter 8	23 days
Chapter 9	10 days
Chapter 10	12 days
TOTAL	**74 days**

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A	81	9	199	73	2.73	13.6
7-B	99	10	241	88	2.74	13.7
Grade 7 total	180	19	440	161	2.73	13.7

The table below is for you to use.

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A			199			
7-B			241			
Grade 7 total			440			

Let’s say you determine that your student needs to study about 2.5 pages a day, or 12-13 pages a week on average in order to finish the curriculum in a year. As the student studies each lesson, keep in mind that sometimes most of the page might be reserved for workspace to solve equations or other problems. You might be able to cover more than the average number of pages on such a day. Some other day you might assign the student only one page of word problems.

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, seventh graders might spend 45-90 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: The Language of Algebra
Introduction

In the first chapter of *Math Mammoth Grade 7* we both review basic algebra topics from sixth grade and also go deeper into them, plus study the basic properties of the four operations. Since a good part of this chapter is review, it serves as a gentle introduction to 7th grade math, laying a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers. When we study equations in chapters 3 and 5, and also in subsequent grade levels, students will use the skills from this chapter (such as simplifying expressions, using the distributive property) in solving equations.

The main topics are the order of operations, writing and simplifying expressions, and the properties of the four operations, including the distributive property. Students have studied most of these in 6th grade. The main principles are explained and practiced both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. See the user guide at the beginning of this book or at **https://www.mathmammoth.com/userguides/** for some further thoughts on using and pacing the curriculum.

You can find matching videos for topics in this chapter at **https://www.mathmammoth.com/videos/** (choose grade 7).

Good Mathematical Practices

- The student is embarking on a wonderful journey into algebra — learning to do arithmetic with letters. The familiar properties of the four operations still hold, just like they do with numbers. Algebra is such a wonderful tool because it allows us to abstract a given situation and represent it symbolically, and then manipulate the representing symbols as if they have a life of their own. It is the foundational tool that allows us to model real-world situations with mathematics.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Exponents and the Order of Operations	13	*4 pages*	1 day	
Expressions and Equations	17	*3 pages*	1 day	
Properties of the Four Operations	20	*4 pages*	1 day	
Simplifying Expressions	24	*4 pages*	1 day	
Growing Patterns 1	28	*3 pages*	1 day	
The Distributive Property	31	*5 pages*	2 days	
Chapter 1 Review	36	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*25 pages*	8 days	

Games at Math Mammoth Online Practice

Hexingo Game — Order of Operations
Practice the order of operations with the four basic operations, parentheses, and exponents.
https://www.mathmammoth.com/practice/order-operations#num=3&operations=add,sub,mult,div,exponents,parens

Expression Exchange
This online activity includes THREE separate work areas where you can explore how simple algebraic expressions work, and then one game. In the work areas, you can learn how to add and subtract simple algebraic terms in order to form an expression. In the game, you will go through practice exercises, forming the asked expressions from parts.
https://www.mathmammoth.com/practice/expression-exchange

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr7ch1

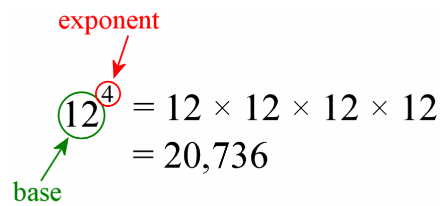
SCAN ME

Exponents and the Order of Operations

Let's review! Exponents are a shorthand for writing repeated multiplications by the same number.

For example, $0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$ is written 0.9^5.

The tiny raised number is called the **exponent**.
It tells us how many times the **base** number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 0.7^8 is read as "seven tenths to the eighth power" or "zero point seven to the eighth."

The "powers of 6" are simply expressions where 6 is raised to some power: for example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the <u>area of a square</u> with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the <u>volume of a cube</u> with an edge 1.5 units long.

1. Evaluate.

a. 4^3 **b.** 10^5 **c.** 0.1^2

d. 0.2^3 **e.** 1^{100} **f.** 100 cubed

2. Write these expressions using exponents. Find their values.

a. $0 \cdot 0 \cdot 0 \cdot 0 \cdot 0$ **b.** $0.9 \cdot 0.9$

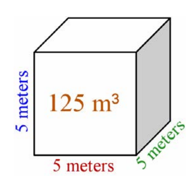
c. $5 \cdot 5 \cdot 5 \;+\; 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

d. $6 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \;-\; 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The expression $(5\text{ m})^3$ means that we multiply 5 meters by itself three times:

$(5\text{ m})^3 = 5\text{ m} \cdot 5\text{ m} \cdot 5\text{ m} = 125\text{ m}^3$

Notice that $(5\text{ m})^3$ is different from 5 m^3. The latter has no parentheses, so the exponent (the little 3) applies only to the unit "m" and not to the whole quantity 5 m.



3. Find the value of the expressions. Include the proper unit.

a. $(2\text{ cm})^3$ **b.** $(11\text{ ft})^2$ **c.** $(1.2\text{ km})^2$ **d.** $(6\text{ in})^2$

4. Match each of (a) and (b) with one expression on the right.

a. The volume of a cube with edges 2 cm long.

b. The volume of a cube with edges 8 cm long.

(i) 8 cm^3 **(ii)** $(8\text{ cm})^3$ **(iii)** 512 cm

The Order of Operations — PE[MD][AS]	**Example 1.** In $15 - 2 + 3 \cdot 3$, we do $3 \cdot 3$ first, then the subtraction, and lastly the addition.
1) Solve what is within parentheses **(P)**. 2) Solve exponents **(E)**. 3) Solve the multiplicative operations — this includes both multiplications **(M)** and divisions **(D)** — from left to right. 4) Solve the additive operations — this includes both additions **(A)** and subtractions **(S)** — from left to right.	You can remember PEMDAS with the silly mnemonic *Please Excuse My Dear Aunt Sally*. Or make up your own!

5. Find the value of each expression.

a. $120 - (9 - 4)^2$ **b.** $120 - 9 - 4^2$	**c.** $4 \cdot 5^2$ **d.** $(4 \cdot 5)^2$	**e.** $10 \cdot 2^3 \cdot 5^2$ **f.** $10 + 2^3 \cdot 5^2$
g. $(0.2 + 0.3)^2 \cdot (5 - 5)^4$	**h.** $0.7 \cdot (1 - 0.3)^2$	**i.** $20 + (2 \cdot 6 + 3)^2$

Example 2. Simplify $(10 - (5 - 2))^2$.	**Example 3.** Simplify $2 + \dfrac{1+5}{40-6^2}$.
Here we have double parentheses. First calculate what is within the *inner* parentheses: $5 - 2 = 3$. Then the expression becomes $(10 - 3)^2$. The rest is easy: $(10 - 3)^2 = 7^2 = 49$.	The fraction line works just like parentheses, as a grouping symbol, grouping both what is above the line and also what is below it. Therefore, first solve what is in the numerator and in the denominator (in either order). $2 + \dfrac{1+5}{40-6^2} = 2 + \dfrac{6}{4} = 2 + \dfrac{2}{3} = \dfrac{8}{3}$

6. Find the value of each expression.

a. $(12 - (9 - 4)) \cdot 5$	**b.** $12 - (9 - (4 + 2))$	**c.** $(10 - (8 - 5))^2$	**d.** $3 \cdot (2 - (1 - 0.4))$

7. Find the value of each expression.

a. $\dfrac{4 \cdot 5}{2} \cdot \dfrac{9}{3}$	**b.** $\dfrac{4 \cdot 5}{2} + \dfrac{9}{3}$	**c.** $\dfrac{4 + 5}{2} + \dfrac{9}{3-1}$

In algebra and beyond, the fraction line is preferred over the ÷ symbol, and it acts as a grouping symbol (just like parentheses).

Compare how each of these expressions looks when written either with the division symbol or with the fraction line. The latter usually makes the expressions easier to read.

$46 \div 2 + 50 \div 5$ vs. $\dfrac{46}{2} + \dfrac{50}{5}$ $\qquad$ $48 \div (1 + 5) \cdot 3$ vs. $\dfrac{48}{1 + 5} \cdot 3$

Notice how only what comes directly after the ÷ symbol, whether a single number or an expression in parentheses, goes to the denominator.

Example 4. Rewrite the expression $(10 + 8) \div 4 + 3$ using the fraction line.

The denominator is just 4, not 4 + 3. The 10 + 8 will not need parentheses anymore because the fraction line in itself is a grouping symbol. So, this is written as $\dfrac{10 + 8}{4} + 3$.

The additions and subtractions that are done last (*not* additions and subtractions in parentheses or in the numerator/denominator) separate the expression into subsections that we call *terms*.

Example 5. This expression has four terms, separated by a +, then a −, and lastly a + sign. $\quad 3^2 + \dfrac{2}{4} - \dfrac{30}{6 + 2} + 4 \cdot 8$

Example 6. Rewrite the expression $2 \div 4 + 3 \div (7 + 2)$ using the fraction line.

Now there are *two* divisions: the first by 4 and the second by (7 + 2), separated by an addition. This means we will use two fractions, or two terms, in the expression. It is written as $\dfrac{2}{4} + \dfrac{3}{7 + 2}$.

8. Match the expressions that are the same.

$2 \div 3 \cdot 4$ $\qquad$ $2 \div (3 \cdot 4)$ $\qquad$ $1 + 3 \div (4 + 2)$ $\qquad$ $1 + 3 \div 4 + 2$ $\qquad$ $(1 + 3) \div 4 + 2$ $\qquad$ $(1 + 3) \div (4 + 2)$

$1 + \dfrac{3}{4} + 2$ $\qquad$ $\dfrac{1 + 3}{4 + 2}$ $\qquad$ $\dfrac{2}{3} \cdot 4$ $\qquad$ $\dfrac{2}{3 \cdot 4}$ $\qquad$ $\dfrac{1 + 3}{4} + 2$ $\qquad$ $1 + \dfrac{3}{4 + 2}$

9. Rewrite each expression using the fraction line and then find its value.

a. $56 \div 7 + 6$	**b.** $7 \div (2 + 6)$	**c.** $16 \div (2 + 6) - 2$	**d.** $4 \div 5 - 1 \div 3$

To **evaluate an expression** means to find (calculate) its value.

Example 7. Evaluate the expression $x^2 - \dfrac{2+y}{y}$ when x is 10 and y is 3.

This means we substitute 10 for x and 3 for y in the expression and then calculate its value according to the order of operations:

$$x^2 - \frac{2+y}{y} = 10^2 - \frac{2+3}{3} = 100 - \frac{5}{3} = 98\ 1/3$$

However, in algebra and beyond, it is customary to *not* give answers as mixed numbers but as fractions, to avoid confusion. After all, 98 1/3 could easily be mistaken for 981/3. So let's go back to the expression 100 − (5/3) and simplify it so it becomes a fraction:

$$100 - \frac{5}{3} = \frac{300}{3} - \frac{5}{3} = \frac{295}{3}$$ (This is the final value as a fraction.)

10. Find the value of these expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $\dfrac{9^2}{9} \cdot 6$	**b.** $\dfrac{2^3}{3^2}$	**c.** $\dfrac{(5-3)\cdot 2}{8-1+2} + 3$

11. Evaluate the expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $2x^2 - x$, when $x = 4$	**b.** $\dfrac{3s}{5} - \dfrac{2t}{5}$, when $s = 10$ and $t = 4$
c. $\dfrac{x^2}{x+1}$, when $x = 3$	**d.** $\dfrac{a+b}{b} + 2$, when $a = 1$ and $b = 3$

12. Why is it wrong to write the expression $2 + 5 \cdot 2 \div 4$ as $\dfrac{2 + 5 \cdot 2}{4}$?

Expressions and Equations

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as x, y, a, T, and so on.

Note: Expressions do *not* have an “equals” sign!

Examples of expressions: 5 $\qquad \dfrac{xy^4}{2}$ $\qquad \mathrm{T} - 5 + \dfrac{x}{7}$

An **equation** has two expressions separated by an equals sign:

(expression 1) = (expression 2)

Examples: $0 = 0$ $\qquad 2(a - 6) = b$

$9 = -8$ (a false equation) $\qquad \dfrac{x+3}{2} = 1.5$

What do we do with expressions?

We can find the *value* of an expression (*evaluate* it). If the expression contains variables, we cannot find its value unless we know the value of the variables.

For example, to find the value of the expression $2x$ when x is 6/7, we simply substitute 6/7 in place of x.
We get $2x = 2 \cdot (6/7) = 12/7$.

Note: When we write $2x = 2 \cdot (6/7) = 12/7$, the equals sign is *not* signaling an equation to solve. (In fact, we already know the value of x!) It is simply used to show that the value of the expression $2x$ here is the same as the value of $2 \cdot (6/7)$, which is in turn the same as 12/7.

What do we do with equations?

If the equation has a variable (or several) in it, we can try to *solve* the equation. This means we find the values of the variable(s) that make the equation true.

For example, we can solve the equation $0.5 + x = 1.1$ for the unknown x.

The value 0.6 makes the equation true: $0.5 + 0.6 = 1.1$. We say $x = 0.6$ is the **solution** or the **root** of the equation.

1. This is a review. Write an expression.

To read the expression $2(x + y)$, use the word ***quantity***: “two times the quantity x plus y.”

There are other ways, as well, just not as common:

“two times the sum of x and y,” or “the product of 2 and the sum x plus y.”

a. $2x$ minus the sum of 40 and x.

b. The quantity 3 times x, cubed.

c. s decreased by 6

d. five times b to the fifth power

e. seven times the quantity x minus y

f. the difference of t squared and s squared

g. x less than 2 cubed

h. the quotient of 5 and y squared

i. 2 less than x to the fifth power

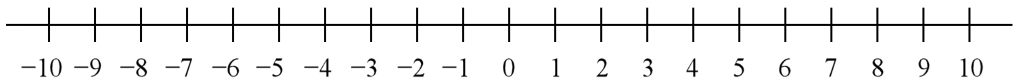
j. x cubed times y squared

k. the quantity $2x$ plus 1 to the fourth power

l. the quantity x minus y divided by the quantity x squared plus one

(This page intentionally left blank.)

Integers



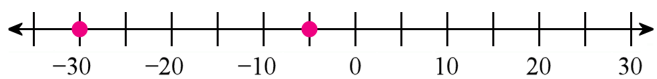
The **counting numbers** are 1, 2, 3, 4, 5, and so on. They work for addition. But counting numbers do not allow us to perform all possible subtractions; for example, the answer to the problem 2 − 7 is not any of them. That is when we come to the *negatives* of the counting numbers: −1, −2, −3, −4, −5, and so on.

Together with zero, all these form the set of **integers**: {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.
Note: Zero is neither positive nor negative.

Read −1 as “negative one” and −5 as “negative five.” Some people read −5 as “minus five.” That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

Often, we need to put parentheses around negative numbers in order to avoid confusion with other symbols. Therefore, $^{-}5$, −5, and (−5) all mean “negative five.”

Which is more, −30 or −5?



Which is *warmer*, −30°C or −5°C? Clearly −5°C is. Temperatures get colder and colder the more they move towards the negative numbers. We can write a comparison: −30°C < −5°C.

Similarly, we can write −\$500 < −\$200 to signify that to owe \$500 is a worse situation than to owe \$200.

Or, in elevation, −15 m > − 50 m means that 15 m below sea level is higher than 50 m below sea level.

1. Write comparisons using > , < , and integers. Don’t forget to include the units!

a. The temperature at 5 AM was 12°C below zero. Now, at 9 AM, it is 8°C below zero.

b. I owe my mom \$12, and my sister owes her \$25.

c. The bottom of the Challenger Deep trench is 11,033 m below sea level.
Mt. Everest reaches to a height of 8,848 m.

d. The total electric charge of five electrons is −5*e*. The total electric charge of 5 protons is +5*e*.
(The symbol *e* means elementary charge, or a charge of a single proton.)

e. Dean has \$55, whereas Jack owes \$15.

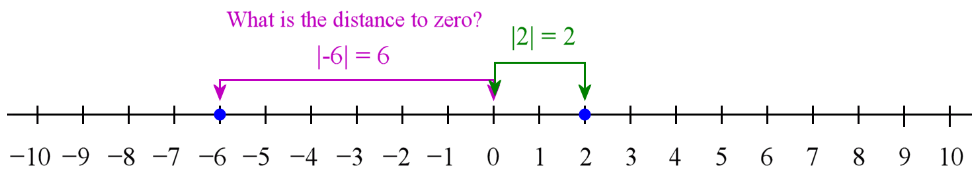
2. Which integer is...

a. 3 more than −7 **b.** 8 more than −3 **c.** 7 less than 2 **d.** 5 less than −11

3. Write the numbers in order from the least to the greatest.

a. −5 −56 51 −15	**b.** 3 0 −31 −13

The **absolute value** of a number is its distance to zero.



We denote the absolute value of a number by putting vertical bars on either side of it.
So $|-4|$ means "the absolute value of 4," which is 4. Similarly, $|87| = 87$. In an expression we treat the absolute value bars like parentheses and solve them first.

Example 1. Simplify $|-4| - |1|$. First simplify the absolute values. We get $4 - 1 = 3$.

Let's say someone's account balance is −$1,000. That person is in debt. The absolute value of the debt is written as |−$1000| and means that the *size* of the debt is $1,000.

If a diver is at a depth of −22 m, the absolute value |−22 m| tells us how far he is from the surface (22 m).

4. Simplify.

a. $\lvert -11 \rvert$	**b.** $\lvert +7 \rvert$	**c.** $\lvert 0 \rvert$	**d.** $\lvert -46 \rvert$
e. $\lvert -5 \rvert + \lvert -2 \rvert$		**f.** $\lvert -5 \rvert - \lvert 2 \rvert$	
g. $\lvert -5 \rvert + \lvert -2 \rvert + \lvert 8 \rvert$		**h.** $\lvert 5 \rvert + \lvert -2 \rvert - \lvert -1 \rvert$	

5. Answer, using the absolute value notation.

a. What is the distance between −153 and zero on a number line?

b. What is the distance between *x* and zero on a number line?

6. Interpret the absolute value in each situation.

a. A fishing net is at the depth of 15 feet. |−15 ft| = _______ft

Here, the absolute value shows ____

b. The temperature is −5°C. |−5°C | = _______°C

Here, the absolute value shows ____

c. Eric's balance is −$7. |−$7| = $_______

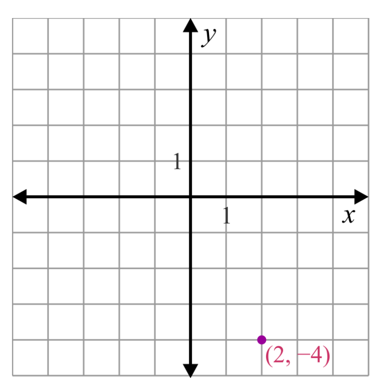
The absolute value shows ____________________________________

____

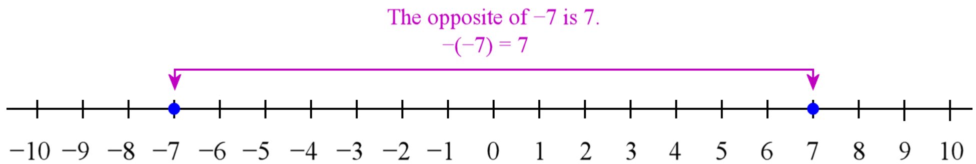
d. A point is drawn in the coordinate grid at (2, −4). |−4| = _______

Here, the absolute value shows _______________________________

____



The **opposite** of a number is the number that is on the opposite side of zero at the same distance from zero.



We denote the opposite of a number using the minus sign. For example, the opposite of 4 is written as −4. The opposite of −2 is written as −(−2), which is of course 2. So, −(−2) = 2.

The opposite of zero is zero itself. In symbols, −0 = 0.

"But wait," you might ask, "doesn't −4 mean 'negative four,' not 'the opposite of four'?"

It can mean either! Sometimes the context will help you tell which is which. Other times it isn't necessary to differentiate, because, after all, the opposite of four *is* negative four, or −4 = −4.

In the expression −(4 + 5), the minus sign means the opposite of the <u>sum</u> 4 + 5, which equals negative nine.

Example 2. −| 7 | means the opposite of the absolute value of seven. It simplifies to −7.

Notice that there are *three* different meanings for the minus sign:

1. To indicate subtraction, as in 7 − 2.
2. To indicate negative numbers: "negative 7" is written −7.
3. To indicate the opposite of a number: −(n + 1) is the opposite of n + 1.

7. Write using symbols, and simplify if possible.

a. the opposite of 6

b. the opposite of −11

c. the opposite of the absolute value of 12

d. the absolute value of negative 12

e. the opposite of the sum 6 + 8

f. the opposite of the difference 9 − 7

g. the absolute value of the opposite of 8

h. the absolute value of the opposite of −2

8. Simplify.

a. −| 8 | **b.** −(−9) **c.** −| −7 | **d.** −0 **e.** −(−(−100))

9. Write with symbols. Use a variable for "*a number*" or "*a certain number*".

a. The absolute value of a number is equal to 6.

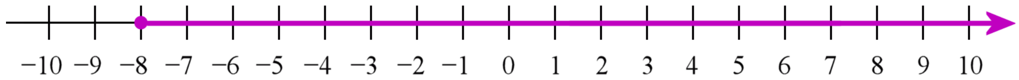
b. The opposite of a certain number is less than negative 2.

c. The absolute value of a certain number is greater than 15.

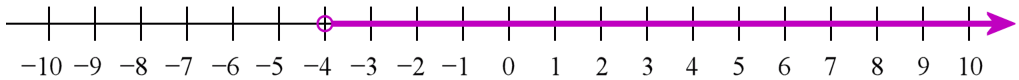
d. The opposite of n is equal to to the sum 56 + 5.

10. Daniel owed \$5. Then he borrowed \$10 more. Next, he paid off \$7 of his debts. Lastly he made yet another debt of \$4. Write one integer to express Daniel's money situation now.

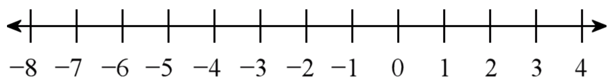
Remember **inequalities?** The number line below illustrates the inequality $x \geq -8$. Notice the arrow on the right, which shows that the ray continues to infinity. The closed circle denotes that −8 belongs to the solution set.



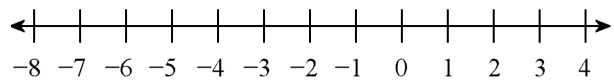
The inequality $x > -4$ is plotted on the number line below. The open circle indicates that −4 is not part of the solution set.



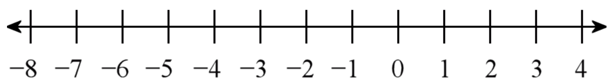
11. Plot these inequalities on the number line. Don’t forget the arrow on the open end.



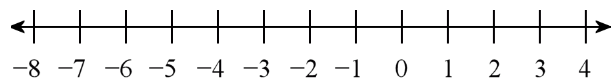
a. $x < -3$



b. $x > -1$



c. $x \geq -2$



d. $x \leq 2$

12. **a.** Solve the inequality $x < 2$ in the set {−3, −2, −1, 0, 1, 2, 3}.

b. Solve the inequality $x \geq -5$ in the set {−10, −8, −6, −2, −1, 0, 5}.

13. Write an inequality. Use negative integers where appropriate.

a. The pit is at most 10 m deep.

b. The pit is at least 12 m deep.

c. Tim’s debt is no more than $500.

d. Nora owes at least $100.

e. For the skiing contest to take place, the temperature has to be warmer than 15 degrees below zero.

f. The freezer temperature should be colder than 10 degrees below zero.

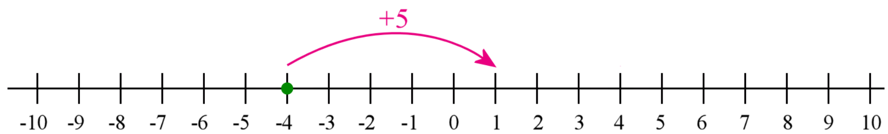
Puzzle Corner

Let a and b be two negative integers, with $b > a$.
What is the distance between them on the number line?
Write an expression.

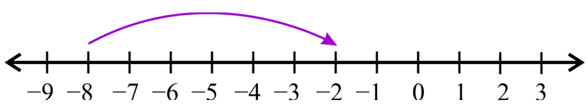
Addition and Subtraction on the Number Line 1

Addition can be modeled on the number line as a movement to the *right*.

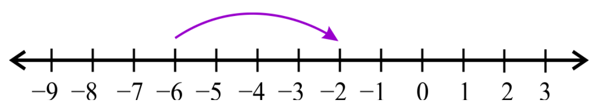
Suppose you are at −4, and you jump 5 steps to the right. You end up at 1. We write the addition −4 + 5 = 1.



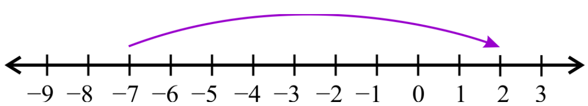
1. Write an addition equation to match each number line jump.



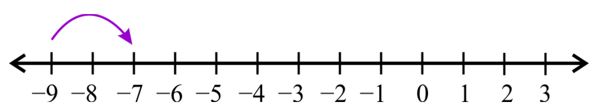
a. _______ + _______ = _______



b. _______ + _______ = _______

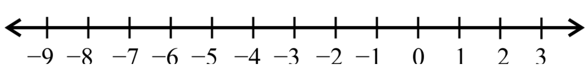


c. _______ + _______ = _______

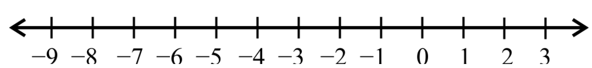


d. _______ + _______ = _______

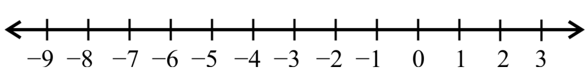
2. Draw a number line jump for each addition equation and solve.



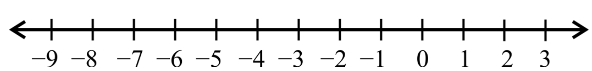
a. −8 + 3 = _______



b. −2 + 5 = _______



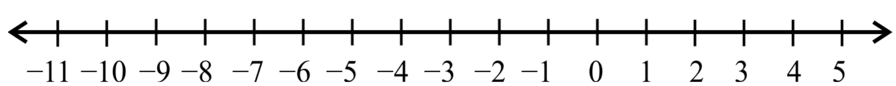
c. −4 + 4 = _______



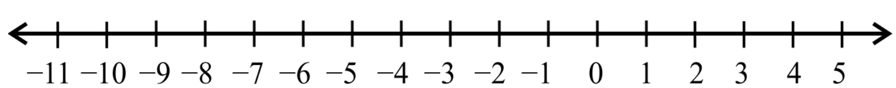
d. −10 + 12 = _______

3. What about adding more than one number? How could these additions be illustrated by number line jumps?

a. −4 + 2 + 3



b. −11 + 6 + 4



(This page intentionally left blank.)

Chapter 2 Review

1. Match the equations with the situations and complete the missing parts.

a. A ball was dropped from 18 ft above sea level; it fell 12 ft.
Now the ball is at _________ ft.

b. John had a $12 debt. He earned $18. Now he has _____________.

c. John had $12. He had to pay his dad $18. Now he has _____________.

d. A diver was at the depth of 18 ft. Then he rose 12 ft.
Now he is at _________ ft.

e. The temperature was −12° C and fell 18°. Now it is _____ ° C.

12 − 18 = ______

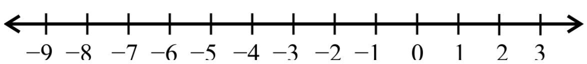
−12 + 18 = ______

−18 + 12 = ______

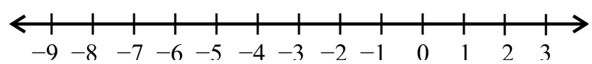
−12 − 18 = ______

18 − 12 = ______

2. Illustrate each addition on the number line.



a. −9 + 5 = ______



b. −1 + (−3) = ______

3. Add or subtract.

a. (−12) + (−1) = ______	**b.** −12 − (−1) = ______	**c.** 7 − 12 = ______
d. 21 + (−48) = ______	**e.** 41 + (−38) = ______	**f .**−610 + 900 = ______
g. (−2) + 7 + (−7) + (−1) = ______	**h.** 4 + (−10) + (−12) + 1 = ______	

4. Complete the equations, using <u>one positive</u> and <u>one negative</u> integer. There are many possible solutions.

a. _____ + _____ = −2 _____ + _____ = −2	**b.** _____ + _____ = 0 _____ + _____ = 0	**c.** _____ + _____ = 3 _____ + _____ = 3

5. Change each subtraction into an addition and solve.

a. 1 − (−7) ↓ ____ + ____ = ______	**b.** 2 − 11 ↓ ____ + ____ = ______	**c.** −20 − (−6) ↓ ____ + ____ = ______	**d.** −3 − 8 ↓ ____ + ____ = ______

6. Which king ruled the Persian Empire longer, Xerxes I, who ruled from 486 to 465 BC, or Darius II, who ruled from 424 to 404 BC?

7. Iodide is an ion with 53 protons and 54 electrons.
Write a sum to represent the total electric charge of this ion.

8. The chart shows you the high and low temperatures during two winter days. Which day saw a greater difference in the high and low temperatures?

Day	**Monday**	**Friday**
High temperature	−13° C	−5° C
Low temperature	−18° C	−8° C

9. Give a real-life context for each expression, and find its value.

a. $-6 + (-8)$

b. $5 - 12$

c. $4 \cdot (-20)$

d. $-50 \div 2$

10. Draw lines to connect the expressions that have the same *value*.

$-50 - 20$ | $50 - 20$ | $-50 - (-20)$

$50 - (-20)$ | $-50 + 20$ | $50 + (-20)$

$-50 + (-20)$ | $50 + 20$

11. The distance between b and -28 is fifteen units.
Find two possible values for b.

12. True or false?

a. Any integer more than 6 has an absolute value more than 6.

b. Any integer less than 6 has an absolute value less than 6.

c. A number and its opposite have the same absolute value.

d. The absolute value of the opposite of a number is the same as the opposite of the absolute value of the same number.

13. Multiply.

a. $-2 \cdot (-4) =$ _______ $-2 \cdot 4 =$ _______	**b.** $(-3) \cdot (-8) =$ _______ $7 \cdot (-12) =$ _______	**c.** $(-3) \cdot 3 \cdot (-1) =$ _______ $-7 \cdot (-2) \cdot (-2) =$ _______

14. Divide.

a. $-10 \div (-5) =$ _______ $24 \div (-3) =$ _______	**b.** $(-12) \div (-4) =$ _______ $21 \div (-3) =$ _______	**c.** $-56 \div 7 =$ _______ $-120 \div (-10) =$ _______

15. Find the missing numbers.

a. $-5 \cdot$ _______ $= -30$	**b.** $2 \cdot$ _______ $= -18$	**c.** $-8 \cdot$ _______ $= 48$
d. $-42 \div$ _______ $= 6$	**e.** $-64 \div$ _______ $= -8$	**f.** $81 \div$ _______ $= -9$

16. Solve the equations by thinking logically.

a. $5y = -100$ $y =$ _______	**b.** $-4b = -48$ $b =$ _______	**c.** $\frac{35}{y} = -5$ $y =$ _______

17. Divide and simplify if possible.

a. $1 \div (-6)$	**b.** $-3 \div 15$	**c.** $-6 \div (-7)$

18. Find the value of each expression.

a. $8 - (-2) \cdot 1 + 6$	**b.** $30 - 3 \cdot (-5)$	**c.** $-2 + 5 \cdot (2 - 3)$
d. $-17 + \frac{(-32)}{4}$	**e.** $\frac{8}{(-2)} - 15$	**f.** $2 + \frac{30}{-3 - 3}$

19. Find the value of the expressions when $x = -3$ and $y = 4$.

a. x^2	**b.** $-5xy$	**c.** $2 - (y + x)$

(This page intentionally left blank.)

Addition and Subtraction Equations

You can **keep track of the operations you're using in a couple of different ways**. One way is to write the operation underneath the equation on both sides. Another is to write it in the right margin, like we did in the last lesson. But in either case, always check your solution: does it solve the original equation?	**One way:** $x + 9 = 4$ $\underline{-9} \quad \underline{-9}$ $x = -5$	**Another way:** $x + 9 = 4 \quad \vert -9$ $x + 9 - 9 = 4 - 9$ (This step is optional.) $x = -5$
	Check: $-5 + 9 \stackrel{?}{=} 4$. Yes, it checks.	

1. Solve these one-step equations. Keep track of the operations either under the equation or in the margin, whichever way your teacher prefers.

a. $x + 5 = 9$	**b.** $x + 5 = -9$
c. $x - 2 = 3$	**d.** $w - 2 = -3$
e. $z + 5 = 0$	**f.** $y - 8 = -7$

2. In these equations, first simplify, separately on the right and left sides. Look at (a) for an example.

a. $x - 4 - 3 = 2 + 8$ $x - 7 = 10$	**b.** $x - 5 - 5 = -9 + 5$
c. $2 + s + 3 = 3 + (-9)$	**d.** $t + 10 - 4 = -3 - 5$

If the **unknown is on the right side of the equation**, you have two options:	**1. Flip the sides first:**	**2. Solve as is:**
• First, flip the two sides. Then solve as usual. • Or, solve as usual, isolating the unknown —this time on the right side of the equation. The solution will initially read as $-7 = x$. Flip the sides now and write the solution as $x = -7$.	$\begin{aligned} -9 &= x - 2 \\ x - 2 &= -9 \\ \underline{+2} \quad & \quad \underline{+2} \\ x &= -7 \end{aligned}$	$\begin{aligned} -9 &= x - 2 \\ \underline{+2} \quad & \quad \underline{+2} \\ -7 &= x \\ x &= -7 \end{aligned}$

3. Solve. Check your solutions.

a. $-8 = s + 6$	**b.** $-2 = x - 7$
c. $4 = s + (-5)$	**d.** $2 - 8 = y + 6$
e. $1 + x + 4 = -9$	**f.** $-6 - 5 = z - (-1)$
g. $y - (-700) = 100 - (-50)$	**h.** $60 + (-12) = x + 16 - 8$
i. $13 - (-19) = -27 + x + 12$	**j.** $2 - 8 = 5 + w + (-3)$

Example 1. Solve $-2 + 8 = -x$.

Our first step is to simplify the sum $-2 + 8$.

The equation becomes $-x = 6$ (or $6 = -x$). What does that mean? It means that the <u>opposite of x</u> is 6. So x must equal -6!

Lastly we check the solution $x = -6$:
$-2 + 8 \stackrel{?}{=} -(-6)$, which simplifies to $6 \stackrel{?}{=} 6$, so it checks.

1. Flip the sides first:

$$\begin{aligned} -2 + 8 &= -x \\ -x &= -2 + 8 \\ -x &= 6 \\ x &= -6 \end{aligned}$$

2. Solve as is:

$$\begin{aligned} -2 + 8 &= -x \\ 6 &= -x \\ -6 &= x \\ x &= -6 \end{aligned}$$

4. Solve for x. Check your solutions.

a. $-x = 6$	**b.** $-x = 5 - 9$
c. $400 + 3 = -y$	**d.** $-72 - 6 = -z$

5. Which equation best matches the situation?

a. The sides of a square playground were shortened by 1/2 m, and now its perimeter is 12 m.

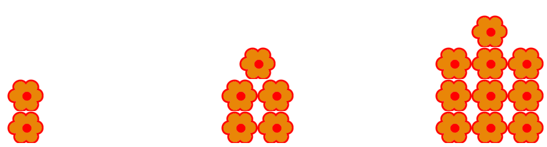
$4s - 1/2 = 12$ | $4(s - 1/2) = 12$ | $4s - 50 = 12$

$s - 1/2 = 4 \cdot 12$ | $s - 1/2 = 12$ | $4(s - 0.5) = 12$

b. How long were the sides before they were made shorter? Solve the problem using mental math.

c. *Challenge*: Solve the same problem using the equation. Compare the steps of this formal solution to the way you reasoned it out in your head. Are the steps similar?

6. Here is another "growing pattern." Draw steps 4 and 5 and answer the questions.



Step 1 2 3

a. How do you see this pattern grow?

b. How many flowers will there be in step 39?

c. In step n?

Example 2. Solve $8 - x = -2$.

As usual, think about what you need to do to isolate the x on one side. Since there is an 8 on the side with the x, we need to subtract 8 from both sides.

However, note that x is being *subtracted*, or in other words, there is a negative sign in front of the x. This negative sign does not disappear when you subtract 8 from both sides.

Writing the operation underneath each side:

$$\begin{aligned} 8 - x &= -2 \\ \underline{-\,8} \quad & \quad \underline{-\,8} \\ -x &= -10 \\ x &= 10 \end{aligned}$$

Writing the operation in the right margin:

$$\begin{aligned} 8 - x &= -2 \quad \Big| -8 \\ 8 - x - 8 &= -2 - 8 \\ -x &= -10 \\ x &= 10 \end{aligned}$$

If this is confusing, think of it this way: The equation $8 - x = -2$ can also be written as $8 + (-x) = -2$. When we subtract 8 from both sides, the left side becomes $8 + (-x) - 8$. The positive 8 and negative 8 will cancel each other and leave $-x$.

So we end up with the equation $-x = -10$. This equation says that the opposite of x is negative 10, so x must be 10. (Why?)

Lastly, check your solution by substituting $x = 10$ back into the original equation: $8 - \underline{10} \stackrel{?}{=} -2$ ✓

7. Solve. Check your solutions.

a. $2 - x = 6$	**b.** $8 - x = 7$
c. $-5 + (-x) = 5$	**d.** $2 - x = -3 - 3$
e. $-15 + 16 = 10 - 15 - x$	**f.** $22 + (-29) = 54 - z + 14$
g. $-32 - 16 + r = -50 + (-27)$	**h.** $20 - (-50) = 20 + 50 + t$

Multiplication and Division Equations

Remember **how to show simplification**? Just cross out the numbers and write the new numerator above the fraction and the new denominator below it.

Notice that the number you divide by (the 5 at the right) isn't indicated in any way!

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

We can simplify expressions involving variables in exactly the same way.

In the examples on the right, we cross out the *same number or variable* from the numerator and the denominator. That is based on the fact that a number divided by itself is 1. We could write a little "1" beside each number that is crossed out, but that is usually omitted.

$$\frac{\cancel{2}x}{\cancel{2}} = x \qquad \frac{\cancel{-5}s}{\cancel{-5}} = s \qquad \frac{4\cancel{x}}{\cancel{x}} = 4$$

$$\frac{\overset{1}{\cancel{3}}x}{\underset{2}{\cancel{6}}} = \frac{1}{2}x \quad \text{or} \quad \frac{x}{2}$$

On the left, we simplify the fraction 3/6 into 1/2 the usual way.

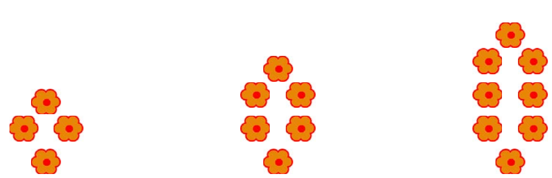
Here, we divide both the numerator and the denominator by 8. **Notice:** this leaves <u>−1 in the denominator</u>. Therefore, the whole expression simplifies to $-z$ instead of z.

$$\frac{\cancel{8}z}{-\cancel{8}} = \frac{z}{-1} = -z$$

1. Simplify.

a. $\frac{8x}{8}$	**b.** $\frac{8x}{2}$	**c.** $\frac{2x}{8}$
d. $\frac{-6x}{-6}$	**e.** $\frac{-6x}{6}$	**f.** $\frac{6x}{-6}$
g. $\frac{2w}{-2}$	**h.** $\frac{6w}{w}$	**i.** $\frac{6w}{-2}$

2. Draw the fourth and fifth steps of the pattern and answer the questions.



Step 1 2 3

a. How would you describe the growth of this pattern?

b. How many flowers will there be in step 39?

c. In step n?

Now you should be ready to use multiplication and division to solve simple equations.

Example 1. In this equation, the unknown is being multiplied by −2. To isolate it, we need to divide both sides by −2.

$-2x = 68$	(This is the original equation.)
$\dfrac{-2x}{-2} = \dfrac{68}{-2}$	We divide both sides by −2.
$\dfrac{\cancel{-2}x}{\cancel{-2}} = \dfrac{68}{-2}$	Now we simplify. We cross out the negative twos on the left side. On the right side, we do the division.
$x = -34$	(This is the final answer.)

Most people combine the first 3 steps into one when writing the solution. Here they are written out for clarity.

Lastly we check the solution by substituting −34 in place of x in the original equation:

$$-2(-34) \stackrel{?}{=} 68$$

$$68 = 68$$ It checks.

Example 2. In this equation, we first need to simplify on the left and on the right side, separately.

The left side, $2x - 7x$, simplifies to $-5x$. The right side, 36 + 19, simplifies to 55.

$2x - 7x = 36 + 19$	
$-5x = 55$	
$\dfrac{-5x}{-5} = \dfrac{55}{-5}$	We divide both sides by −5.
$\dfrac{\cancel{-5}x}{\cancel{-5}} = \dfrac{55}{-5}$	Now we simplify.
$x = -11$	

Check: $2(-11) - 7(-11) \stackrel{?}{=} 36 + 19$

$-22 - (-77) \stackrel{?}{=} 55$

$55 = 55$ ✓

3. Solve. Check your solutions.

a. $5x = -45$	**b.** $-3y = -21$	**c.** $-4 = 4s$	**d.** $72 = -6y$

4. Solve. Simplify the right and/or left side first.

a. $-5q = -40 - 5$	**b.** $2 \cdot 36 = -2y - 4y$
c. $x + 2x = -4 + 3 + (-2)$	**d.** $5 \cdot (-4) = 2z - 5z - 7z$

Example 3. Solve $\dfrac{x}{-6} = -5$.

Here the unknown is divided by −6. To undo that division, we need to *multiply* both sides by −6. (See the solution on the right.)

We get $x = 30$. Lastly we check the solution:

$$\frac{30}{-6} \stackrel{?}{=} -5$$

$$-5 = -5$$

$$\frac{x}{-6} = -5$$

(This is the original equation.)

$$\frac{x}{-6} \cdot (-6) = -5 \cdot (-6)$$

We multiply both sides by −6.

$$\frac{x}{\cancel{-6}} \cdot (\cancel{-6}) = 30$$

Now it is time to simplify. We cross out the −6 factors on the left side, and multiply on the right.

$$x = 30$$

(This is the final answer.)

When writing the solution, most people would combine steps 2 and 3. Here both are written out for clarity.

5. Solve. Check your solutions.

a. $\dfrac{x}{2} = -45$	**b.** $\dfrac{s}{-7} = -11$	**c.** $\dfrac{c}{-7} = 4$
d. $\dfrac{a}{-13} = -9 + (-11)$	**e.** $30 + 50 = \dfrac{s}{-12}$	**f.** $-30 \cdot 4 = \dfrac{x}{11}$

6. Write an equation for each situation. Then solve it. Do not write the answer only, as the main purpose of this exercise is to practice writing equations.

a. A submarine was located at a depth of 500 ft.
There was a shark swimming at 1/6 of that depth.
At what depth is the shark?

b. Three towns divided highway repair costs equally.
Each town ended up paying $21,200.
How much did the repairs cost in total?

(This page intentionally left blank.)

Rational Numbers

If you can write a number as a *ratio of two integers*, it is a **rational number.**

For example, 4.3 is a rational number because we can write it as the ratio $\frac{43}{10}$ or 43:10.

Note: To represent rational numbers, we usually indicate the ratio with a fraction line rather than a colon.

Examples of rational numbers

Since −10 can be written as $\frac{-10}{1}$, it is a rational number. It can also be written as $\frac{10}{-1}$.

Since 0.1 can be written as $\frac{1}{10}$, it is a rational number.

Since 3.24 can be written as $\frac{324}{100}$, it, too, is a rational number.

Negative fractions

The ratio of the integers 7 and −10 gives us the fraction $\frac{7}{-10}$. As we studied earlier, we usually write this as $-\frac{7}{10}$ and read it as “negative seven tenths.”

Obviously, all fractions, whether negative or positive, are rational numbers.

Negative fractions give us negative decimals.

For example, $-\frac{8}{10}$ is written as a decimal as −0.8, and $-5\frac{21}{100} = -5.21$.

You can write a rational number as a ratio of two integers in many ways.

For example, the decimal −1.4 can be written as a ratio of two integers in all these ways (and more!):

$$-1.4 = \frac{-14}{10} = \frac{-28}{20} = \frac{28}{-20} = \frac{42}{-30} = \frac{-42}{30} = \frac{-7}{5}$$

So −1.4 is *definitely* a rational number! ☺ But the same holds true for all rational numbers—you can always write them as a ratio of two integers in multitudes of ways.

1. Write these numbers as a ratio (fraction) of two integers.

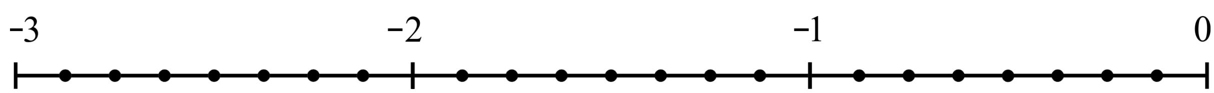
a. 6	**b.** −100	**c.** 0	**d.** 0.21
e. −1.9	**f.** −5.4	**g.** −0.56	**h.** 0.022

2. Are all percents, such as 34% or 5%, rational numbers? Justify your answer.

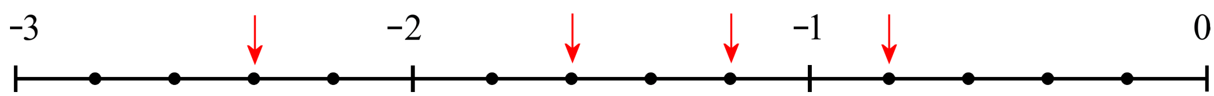
3. Form a fraction from the two given integers. Then convert it into a decimal.

a. 8 and 5	**b.** −4 and 10	**c.** 89 and −100
d. −5 and 2	**e.** 91 and −1000	**f.** −1 and −4

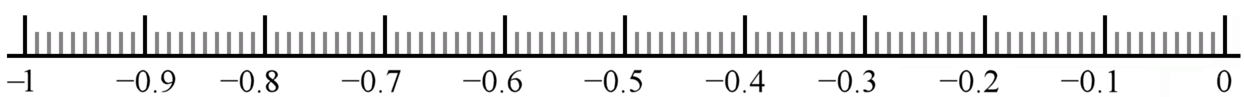
4. Mark the fractions and mixed numbers on the number line below: $-\frac{1}{2}$, $-\frac{7}{8}$, $-1\frac{5}{8}$, $-\frac{9}{4}$, $-2\frac{3}{4}$



5. Write the fractions marked by the arrows.



6. Mark the decimals on the number line: −0.11, −0.58, −0.72, −0.04



7. Sketch a number line from −3 to 0, with tick marks at every tenth. Then mark the following numbers on your number line: −0.2, −1.5, −2.8, −3/5, and −5/2.

8. Write these rational numbers as ratios of two integers (fractions) in a lot of different ways.

a. $-2 = -\frac{2}{1} =$
b. $0.6 = \frac{6}{10} =$

9. Compare, writing < or > in between the numbers.

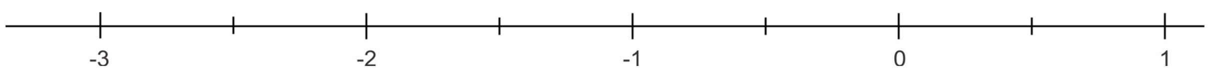
a. $-\frac{7}{8}$ $\quad$ -1	**b.** $-\frac{3}{4}$ $\quad$ $\frac{1}{2}$	**c.** $-\frac{15}{2}$ $\quad$ -7	**d.** -0.98 $\quad$ -1.4

10. Order these rational numbers in order, from the smallest to the greatest.

$2.1 \quad -\frac{1}{8} \quad -1 \quad -\frac{7}{3} \quad -2.01 \quad 1 \quad \frac{1}{3} \quad -0.5$

11. Mark the decimals *and* the fractions on the number line, approximately.

$0.3 \quad -\frac{2}{5} \quad -0.8 \quad -\frac{10}{4} \quad -2.1 \quad -1\frac{1}{2} \quad -\frac{17}{10} \quad 0.95$



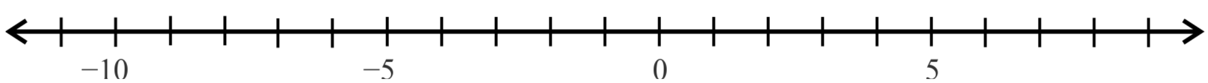
Recall that the absolute value of a number is its distance from zero.

Below, the thickened line shows the set of numbers that are greater than −3 and at the same time, less than 3. We can write it like this: the set of numbers x so that $-3 < x < 3$.

These are the numbers whose absolute value is less than 3, in other words the set of numbers for which $|x| < 3$. Their distance to zero is less than 3. For example, −2.8 and 0.492 and −6/5 belong to this set.

Note that 3 and −3 are not part of this set; that is why we use an open circle at 3 and −3.

12. **a.** Show on the number line the set of numbers x for which $|x| < 1.5$



b. List three rational numbers in that set that are not integers.

13. List three rational numbers r so that $|r| < 2$ and $r > -1$.

Repeating Decimals

As you already know, sometimes it is easy to write a fraction as a decimal. For example, 3/10 = 0.3 and 1/4 = 0.25. However, if you don't know of any other way to to find the decimal equivalent of a fraction, the technique that works all the time is to **treat the fraction as a division** and divide.

Example 1. Write $\frac{31}{40}$ as a decimal.

We will use long division. Note how we add many decimal zeros to the dividend (31) so that we can continue the division into the decimal digits.

This division **terminates** (comes out even) after just three decimal digits.

We get $\frac{31}{40}$ = 0.775. This is a **terminating decimal**.

```
      0 0.7 7 5
40) 3 1.0 0 0 0
   -2 8 0
      3 0 0
   -  2 8 0
        2 0 0
      - 2 0 0
            0
```

1. Write as decimals, using long division. Continue the division until it terminates.

a. $\frac{3}{16}$	**b.** $\frac{51}{32}$	**c.** $\frac{17}{80}$

2. Use long division to write these fractions as decimals. Continue the division to at least 6 decimal digits. Notice what happens!

a. $\frac{2}{3}$	**b.** $\frac{7}{11}$	**c.** $\frac{8}{9}$

Example 2. Write $\frac{18}{11}$ as a decimal.

$$
\begin{array}{r}
01.6363 \\
11\,\overline{)\,18.0000} \\
\underline{-11} \\
70 \\
\underline{-66} \\
40 \\
\underline{-33} \\
70 \\
\underline{-66} \\
40 \\
\underline{-33} \\
7
\end{array}
$$

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient and the remainders 40 and 70 start repeating.

So $\frac{18}{11}$ = 1.636363... . We can use an ellipsis (three dots, or "…") to indicate

that the decimal is non-terminating. A better notation is to draw a **bar** (a line) over the digits that repeat: 1.636363… = $1.\overline{63}$.

This number is called a **repeating decimal** because the digits "63" repeat forever!

The decimal form of ANY rational number is either a terminating decimal or a repeating decimal.

This is an important fact. It says that when you write any fraction as a decimal, there are only two possibilities: either the decimal terminates or it repeats.

The converse is also true: if a decimal terminates or is a repeating decimal, it *can* be written as a fraction, thus is a rational number.

Example 3. The repeating decimal 1.9051050505… is written as $1.9051\overline{05}$. Notice that the bar marks only the digits that repeat ("05"). The digits "9051" that don't repeat are not included under the bar.
(If you're curious, as a fraction, this number is 1,886,054/990,000.)

Example 4. A calculator gives the decimal expansion of 5/13 as 0.384615384615384615384615384615384615...

The repeating part is the digits "384615". So, 5/13 = $0.\overline{384615}$.

Example 5. The decimal 0.095 is a terminating decimal, but we *can* write it with an unending decimal expansion if we write zeros for all the decimal places after thousandths:

0.095 = 0.095000000000...

In other words, we can think of it as repeating the digit zero. In that sense, 0.095 = $0.095\overline{0}$.
However, as you know, we normally write terminating decimals without the extra zeros.

3. Write each decimal using a line over the repeating part.

a. 0.09090909… **b.** 5.6843434343… **c.** 0.19866666666…

4. Do it the other way around: write the repeating digits several times followed by an ellipsis (three dots).

a. $0.0\overline{887}$ **b.** $0.245\overline{6}$ **c.** $2.1\overline{7234}$

5. Which decimal is greater?

a. Which is more, $0.\overline{3}$ or 0.3? How much more?	**b.** Which is more, $0.\overline{55}$ or $0.\overline{5}$? How much more?
c. Which is more, $0.45\overline{0}$ or 0.45? How much more?	**d.** Which is more, $0.\overline{12}$ or 0.12? How much more?

(This page intentionally left blank.)

Chapter 4 Mixed Review

1. Jeremy purchased *x* pairs of gloves for \$3 each and one pair of rubber boots for \$9.
(Expressions and Equations/Ch1)

a. Write an expression for the total cost of his purchases.

b. The total cost of Jeremy's purchases was \$57.
Write an equation for this situation.

c. How many pairs of gloves did he buy?
You can solve the equation or figure it out using mental math.

2. Solve. (Various lessons/Ch2)

a. $-8 \cdot (-7) \cdot (-1) =$	**b.** $(-2)^4 =$
c. $-45 \div (2 - 3)$	**d.** $-8 \cdot (-13 + 1) =$

3. Find the value of the expressions using the correct order of operations. (Exponents and the Order of Operations/Ch1)

a. $5 \cdot \dfrac{2}{-10}$	**b.** $-\dfrac{12}{-4} + 7$	**c.** $-1 + \dfrac{24}{12 + (-6)}$
d. $-2 + 7 \cdot 2 - 6$	**e.** $-8 \cdot (-7) - 11$	**f.** $(-3 + 9) \cdot 8$

4. Light travels at a speed of 299,792.458 kilometers per second. That is quite fast! (Constant Speed/Ch3)

a. A beam of light is sent from the earth to the moon. How long will it take to reach the moon?
The average distance between the earth and the moon is 384,403 km.
Give your answer to five decimal digits.

b. (Optional.) Choose some other object in our solar system, and calculate how long it will take a radio message sent from the earth to reach that object. Radio waves travel at the speed of light.
Use an encyclopedia to find the distances you need to solve the problem.

5. Solve. Check your solutions. (Addition and Subtraction Equations/Ch3)

a. $2 - x = -6$	**b.** $-10 - x = 7$
c. $2 \cdot 3 = 9 - 6y$	**d.** $2 + (-11) = 8 + z$

6. Divide and simplify if possible. (Negative Fractions/Ch2)

a. $1 \div (-3)$	**b.** $-16 \div 20$	**c.** $-45 \div (-36)$

7. Write an addition or subtraction using integers to match the situation.
(Addition and Subtraction on the Number Line 2/Ch2)

a. A shark was swimming at the depth of 4 m. Then it rose 2 m.

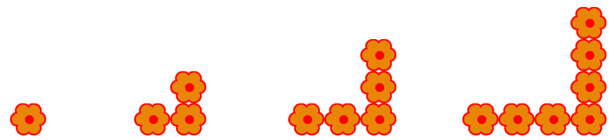
b. Michael owed $250. He then purchased a computer on his credit card for $500 (adding to his debt).

8. Solve. Check your solutions. (Multiplication and Division Equations/Ch3)

a. $\dfrac{x}{-13} = 4$	**b.** $\dfrac{w}{-3} = -11 + 5$
c. $-31 = \dfrac{1}{6}x$	**d.** $1 = -5x$

(This page intentionally left blank.)

Growing Patterns 2



Step 1 2 3 4 5 6

How do you think this pattern is growing?

How many flowers will there be in step 39?

This pattern adds 2 flowers in each step, except in step 1. This means that by step 39, we have added 2 flowers 38 times. Therefore, there are $1 + 2 \cdot 38 = 77$ flowers in step 39.

Write a formula for the number of flowers in step *n*.

There are several ways to do this. And the three ways explained below are not the only ones!

1. Let's view the pattern as adding 2 flowers in each step after the first one. By step n, the pattern has added one less than n times 2 flowers, because we need to exclude that first step. This means that $(n - 1) \cdot 2$ flowers were added to the one flower that we started with.

   This gives us the expression $1 + (n - 1) \cdot 2$. Since we customarily put the variable first and the constant last, we can rewrite that expression as $1 + 2(n - 1)$ and then as $2(n - 1) + 1$.

2. Another way to think about this pattern is as two legs. One leg includes the flower in the corner, so it has the same number of flowers as the step number. The other leg doesn't have the corner flower, so it has one flower less than the step numbers. In other words, in step 3, we have 3 + 2 flowers. In step 4, we have 4 + 3 flowers. In step 5, we have 5 + 4 flowers.

   This gives us a formula for the number of flowers in step n: there are $n + (n - 1)$ flowers in step n.

3. Yet another way is that, in each step, there are twice as many flowers as the step number, minus one for the flower that is shared. For example, in step 4, we have twice 4 minus 1, which is seven flowers.

   This also gives us a formula: there are $2n - 1$ flowers in step n.

All of the formulas are equivalent (just as we would expect!) and simply represent different ways of thinking about the number of flowers in each step. On the right, you can see how the first two formulas can be simplified to the third one.

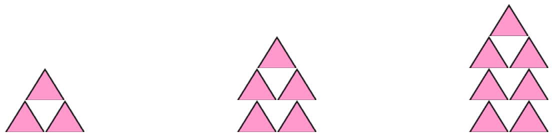
$$\begin{aligned} & n + (n - 1) \\ &= n + n - 1 \\ &= 2n - 1 \end{aligned}$$

$$\begin{aligned} & 2(n - 1) + 1 \\ &= 2n - 2 + 1 \\ &= 2n - 1 \end{aligned}$$

In which step are there 583 flowers?

We can use our formula to write an equation to answer this question. In the question, the step number n is unknown, but the total number of flowers in that step is 583. Since we know from our formula that there are $2n - 1$ flowers in step n, we get the equation $2n - 1 = 583$.

$$\begin{aligned} 2n - 1 &= 583 && \mid +1 \\ 2n &= 584 && \mid \div 2 \\ n &= 292 \end{aligned}$$



1 2 3 4 5

1. **a.** How is this pattern growing?

b. How many triangles will there be in step 39?

c. Write a formula for the number of triangles in step *n*.
Check your answer with your teacher before going on to part (d).

d. In which step will there be 311 triangles?
Write an equation and solve it.
Notice, this question is <u>different</u> from the one in part (c).

1 2 3 4 5

2. **a.** How do you think this pattern is growing?

b. How many snowflakes will there be in step 39?

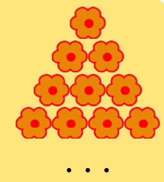
c. Write a formula for the number of snowflakes in step *n*.
Check your answer with your teacher before going on to part (d).

d. In which step will there be 301 snowflakes?
Write an equation and solve it.

Instead of showing the steps of the pattern horizontally, like in the previous exercises, we can also show them like the illustration on the right:

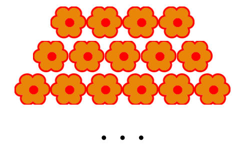
Now, each row of flowers is one step of the pattern.

1
2
3
4



3. A section of a flower garden has rows of flowers as shown on the right.

1
2
3



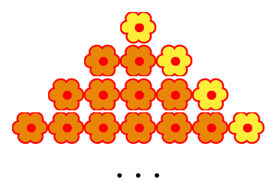
a. Write a formula that tells the gardener the number of flowers in row *n*.

b. How many flowers are in the 28th row?

c. In which row will there be 97 flowers? Write an equation and solve it.

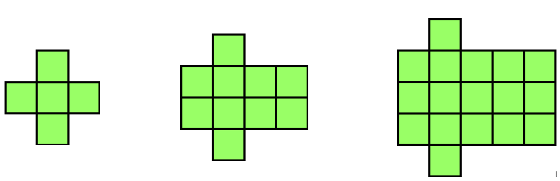
4. Here, we have labeled the first row as "row 0."

0
1
2
3



a. Write a formula that tells the gardener the number of flowers in row *n*.

b. In which row will there be 97 flowers? Write an equation and solve it.



Step 1 2 3 4 5

What is the pattern of growth here?
How many squares will there be in step 59?

Puzzle Corner

Using the Distributive Property

Sometimes we need to use the distributive property to remove parentheses.

Depending on the context, this answer might also be given as the decimal 6.2.

Example 1.

$$\begin{aligned} 5(x + 9) &= 76 \\ 5x + 45 &= 76 && \mid -45 \\ 5x &= 31 && \mid \div 5 \\ x &= 31/5 \end{aligned}$$

1. Solve. Give your answers as fractions or whole numbers.

a. $5(x + 2) = 85$	**b.** $9(y - 2) = 66$	**c.** $2(x - 3) = 13$
d. $70 = 8(z + 9) - 3z$	**e.** $300 = 20(s - 12) + 15s$	**f.** $10(x - 9) - 2 = 18$

2. Solve. Now give your answers as decimals, rounded to three decimal digits.

a. $0.4(x + 3) = 1.2$	**b.** $30(v - 0.4) = -1$	**c.** $0.98 = 3(x - 0.07) + 0.9x$

<table>
<tr><td colspan="2">Example 2. Sometimes there are two different ways that you can start to solve an equation.</td></tr>
<tr><td>First divide by a common factor:
$$\begin{aligned} 3(x+5) &= 18 && \mid \div 3 \\ \frac{\cancel{3}(x+5)}{\cancel{3}} &= \frac{18}{3} \\ x+5 &= 6 && \mid -5 \\ x &= 1 \end{aligned}$$</td><td>First distribute the multiplication:
$$\begin{aligned} 3(x+5) &= 18 \\ 3x+15 &= 18 && \mid -15 \\ 3x &= 3 && \mid \div 3 \\ x &= 1 \end{aligned}$$</td></tr>
</table>

3. Solve in two ways: (i) by dividing first and (ii) by distributing the multiplication over the parentheses first.

Divide first:	Distribute the multiplication first:
a. $6(x-7) = 72$	**a.** $6(x-7) = 72$
b. $10(q-5) = 60$	**b.** $10(q-5) = 60$
c. $1.5(v+2) = 4.5$	**c.** $1.5(v+2) = 4.5$
d. $3.2 = 8(x+0.7)$	**d.** $3.2 = 8(x+0.7)$

4. Matthew and Madison got a different solution to the same equation. Check their work. Who is correct? Explain why.

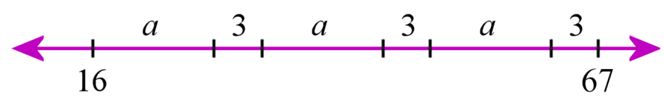
Matthew:

$$\begin{aligned} 3(y-4) &= 2 \\ 3y-4 &= 2 \\ 3y &= 6 \\ y &= 2 \end{aligned}$$

Madison:

$$\begin{aligned} 3(y-4) &= 2 \\ 3y-12 &= 2 \\ 3y &= -10 \\ y &= -10/3 \end{aligned}$$

5. **a.** Write an equation for this number line diagram, using <u>parentheses</u>. Then find the value of the unknown *a*.



b. Draw a number line diagram for the equation $75 + 3(x + 2) = 156$, and solve it.

6. (optional) Solve a few equations with negative numbers.

a. $-4(x + 3) = -76$	**b.** $-4(x - 7) = 51$	**c.** $-3(x + 18) = 111$

Puzzle Corner

Select two expressions at the right to make an equation that has −3 as its root. See the answer key for a hint.

$2(x + 4)$ -15 -12 -9 $6(x + 1)$ $6(x - 1)$ 12 $2(x - 4)$

(This page intentionally left blank.)

Two-Step Equations, Part 2

Example 1. Again, the unknown is "tangled up" with two different operations (division and addition), so to isolate it, we need two steps.

$$\begin{aligned} \frac{x}{4} + 5 &= -2 & &\quad \Big| -5 \\ \frac{x}{4} &= -7 & &\quad \Big| \cdot 4 \\ x &= -28 \end{aligned}$$

Check:

$$\frac{-28}{4} + 5 \stackrel{?}{=} -2$$

$$-7 + 5 \stackrel{?}{=} -2$$

$$-2 = -2 \checkmark$$

Example 2. The unknown is multiplied by a fraction. Fill in the missing portions and finish the solution. Check it too! Ask your teacher for help as necessary.

$$\begin{aligned} \frac{3}{2}x - 7 &= 26 & &\quad \Big| +7 \\ \frac{3}{2}x &= \end{aligned}$$

Note that $\frac{x}{4}$ is the same as $\frac{1}{4}x$. Similarly, $\frac{3x}{8}$ is the same as $\frac{3}{8}x$.

1. Solve.

a. $\frac{x}{10} + 3 = -2$	**b.** $\frac{x}{7} - 8 = 5$	**c.** $\frac{1}{6}x - 7 = -3$
d. $\frac{2}{3}x + 1 = 11$	**e.** $\frac{5}{8}x + 15 = -100$	**f.** $\frac{6x}{5} - 17 = 43$

2. John bought three ceiling fans at half price and one standing fan for $55. His total came to $253.

   a. Find the <u>regular</u> price of one ceiling fan using arithmetic and logical reasoning.

   b. Now write an equation for the problem and solve it.
   Compare the steps of the two kinds of solutions.

3. Hannah spent 12 days at a camp during her summer vacation, which was just three days less than 1/4 of her entire vacation.

   a. Find the length of her vacation using logical reasoning.

   b. Now write an equation to find the same thing and solve it. Compare the steps of solving the equation to the steps of the arithmetic solution.

4. Three-sevenths of a number is 321. What is the number?

Example 3. Could the equation from Example 1 be solved by multiplying first, and subtracting last?

Yes, that is possible. See the solution on the right.

Note how *both* terms on the left side ($x/4$ and 5) have to be multiplied by 4! It is a common error to multiply only the first term of an expression by a number. The *entire* left side is multiplied by 4. That is why we enclose it in parentheses.

Compare the two (Example 1 and this example). Which do you feel is the easier way?

$$\begin{aligned} \frac{x}{4} + 5 &= -2 & &\Big|\cdot 4 \\ 4\left(\frac{x}{4} + 5\right) &= 4 \cdot (-2) & & \\ \cancel{4} \cdot \frac{x}{\cancel{4}} + 4 \cdot 5 &= -8 & & \\ x + 20 &= -8 & &\Big| - 20 \\ x &= -28 & & \end{aligned}$$

5. Check each solution below. If it is incorrect, find the error, and correct it.

a.

$$\begin{aligned} \frac{x}{3} + 16 &= 109 \\ x + 48 &= 327 \\ x &= 279 \end{aligned}$$

b.

$$\begin{aligned} \frac{3x}{2} - 32 &= 40 \\ 3x - 32 &= 80 \\ 3x &= 112 \\ x &= 112/3 \end{aligned}$$

c.

$$\begin{aligned} \frac{1}{5}x - 1.8 &= 3.4 \\ \frac{1}{5}x &= 1.6 \\ x &= 8 \end{aligned}$$

6. Ashley says that the first step in starting to solve the equation on the right is to subtract 32 from both sides. Do you agree? Why or why not?

$$\frac{y}{10} - 32 = 20$$

Solve the equation.

7. Aaron paid 15/100 of his salary in taxes. Then he paid off some various bills for a total of \$750, and now he has \$2,820 left.

a. Write an equation to find Aaron's salary before taxes, and solve the equation.

b. Now find Aaron's salary before taxes using logical reasoning. Compare the steps of both types of solution.

Example 4. (optional) Here, we have 7 in the denominator. Therefore, we will first multiply both sides by 7.

In the second step, the 7s cancel out.

Finish solving the equation. Then check your solution.

$$\frac{x+2}{7} = 12 \quad \Big| \cdot 7$$

$$\frac{\cancel{7} \cdot (x+2)}{\cancel{7}} = 84$$

$$=$$

$$=$$

Check:

$$\frac{\boxed{} + 2}{7} \stackrel{?}{=} 12$$

8. (optional) Solve. Check your solutions (as always!).

a. $\dfrac{x+6}{5} = 14$	**b.** $\dfrac{y-16}{8} = 30$	**c.** $\dfrac{t-20}{2} = -3$

9. (optional) Solve. Compare the equations. They are similar, yet different!

a. $\dfrac{x}{7} + 2 = 6$	**b.** $\dfrac{x+2}{7} = 6$	**c.** $\dfrac{2x}{7} = 6$
d. $\dfrac{8x}{7} = -5$	**e.** $\dfrac{x-8}{7} = -5$	**f.** $\dfrac{x}{7} - 8 = -5$

Word Problems and Equations, Part 1

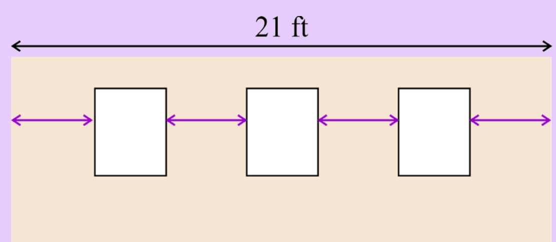
You may use a calculator in all the problems of this lesson.

1. Solve the problem below in two ways: write an equation, and use logical reasoning. Then compare the steps of each way.

a. Mr. Sanchez spent \$80.30 to treat eleven people in his bicycling club to a cup of coffee and an ice cream cone each. Each coffee cost \$3.50. How much did one ice cream cone cost?

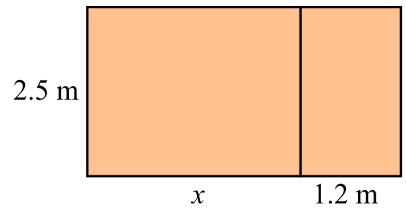
Equation:	Arithmetic and logical thinking:

b. You want to hang three framed pictures on a wall that is 21 feet long, in an equidistance manner (see the illustration). Each picture is 24 inches wide. How long will the spaces between the pictures need to be?



Equation:	Arithmetic and logical thinking:

2. The total area of a sandbox with two compartments is 8 square meters. Solve for x.



3. Matt says that the first step in starting to solve the equation on the right is to add 0.4 to both sides. Do you agree? Why or why not?

$15(p - 0.4) = 12.20$

Solve the equation.

4. Samantha walks her dog in a city. She goes a certain number of blocks due north, then one block due west, then the same number of blocks south as she did north, and lastly one block to the east. She varies the number of blocks she walks north and south according to weather, available time, and other factors.

Each block is 140 meters. If she walked a total distance of 2.24 km, how many blocks north did she go?

Puzzle Corner Solve.

a. $-3(z - 7) + 21 = -10$	**b.** $5(x - 2) + 2(x + 9) = 67$	**c.** $-2(x + 9) - 5(x + 1) = -21$