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Foreword

Math Mammoth Grade 4 comprises a complete math curriculum for the fourth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

The main areas of study in Math Mammoth Grade 4 are:

1. Students develop understanding and fluency with multi-digit multiplication, and use efficient multiplication procedures to solve problems.
2. They develop understanding of division to find quotients involving multi-digit dividends (long division), and they solve word problems involving division, including division with a remainder.
3. Students develop an understanding of fraction equivalence and some operations with fractions. They learn to add and subtract fractions with same denominators, and to multiply a fraction by a whole number.
4. Students learn the concept of angle. They draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Additional topics we study are place value, time, measuring, graphs, and decimals.

This book, 4-B, covers division (chapter 5), geometry (chapter 6), fractions (chapter 7), and decimals (chapter 8). The rest of the topics are covered in the 4-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. Chapters 1, 2, and 3 should be studied in order, and Chapter 3 (multiplication) should be studied before Chapter 5 (division). However, you can be flexible with chapters 4 (time and measuring) and 6 (geometry), and schedule them earlier or later. Also, most lessons from chapters 7 and 8 (fractions and decimals) can be studied earlier; however the topic of finding parts with division should naturally be studied only after mastering division.

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your child study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for your child.

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your child’s needs. You can use the skipped exercises later for review. For most children, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at <https://www.mathmammoth.com/faq-lightblue.php>

I heartily recommend that you view the full user guide for your grade level, available at <https://www.mathmammoth.com/userguides/>

And lastly, you can find free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>

I wish you success in teaching math!

Maria Miller, the author

Sample worksheet from
<https://www.mathmammoth.com>

Chapter 5: Division

Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, average, divisibility, and problem solving. It is a long chapter, because division and long division are “in focus” in fourth grade. Therefore, feel free to mix the lessons from this chapter with lessons from some other chapter, essentially using the curriculum in a somewhat spiral manner. This is especially advisable if your student has difficulties retaining the material or starts feeling bored with these topics.

For further help in teaching these topics, check out the free videos matched to the curriculum at <https://www.mathmammoth.com/videos/>. Remember not to automatically assign all the exercises. Instead, adjust the amount of exercises according to the student’s needs. The rest can be used later for review.

We start out by reviewing basic division facts by single-digit numbers (such as $24 \div 4$ or $56 \div 7$). After that, we study terminology of division and dividing numbers by whole tens and hundreds (such as $400 \div 20$).

The lesson *Finding Fractional Parts with Division* shows an important relationship between fractions and division. For example, we can find $\frac{3}{4}$ of a number by first finding $\frac{1}{4}$ (dividing by 4) and then multiplying the result by 3.

Next students practice the order of operations again—this time with division as one of the operations.

Then we study the concept of remainder, preparing students for the upcoming lessons on long division. At first, the concept of remainder is presented visually. Soon, students solve simple division problems with a remainder, written with the long division symbol (or long division “corner”, as I like to call it).

Next comes a set of lessons intended to teach long division in several small steps. We start with divisions where each of the digits in the dividend (thousands, hundreds, tens, and ones) can be divided evenly by the divisor (for example, $3096 \div 3$). As the next step, there is a remainder in the ones. Then, the divisions have a remainder in the tens. Finally, there is a remainder in the hundreds and in the thousands, and this completes the step-by-step learning process for long division. The lessons also include lots of word problems to solve.

After long division, we study the concept of average, which is a nice application of division, and problems that involve a fractional part of a quantity (such as $\frac{3}{4}$ of \$600). Students get help from visual bar models to solve the problems.

The last section deals with elementary number theory. We study basic divisibility rules (though not all of them), prime numbers, and finding all factors of a given two-digit number.

The Lessons in Chapter 5

	page	span
Review of Division	10	3 pages
Division Terms and Division with Zero	13	2 pages
Dividing with Whole Tens and Hundreds	15	3 pages
Order of Operations and Division.....	18	2 pages
The Remainder, Part 1	20	3 pages
The Remainder, Part 2	23	2 pages
The Remainder, Part 3	25	2 pages
Long Division 1	27	4 pages
Long Division 2	31	3 pages
Long Division 3	34	4 pages
Long Division with 4-Digit Numbers	38	4 pages

Sample worksheet from
<https://www.mathmammoth.com>

More Long Division	42	3 pages
Remainder Problems	45	4 pages
Long Division with Money	49	2 pages
Long Division Crossword Puzzle	51	1 page
Average	52	3 pages
Finding Fractional Parts with Division	55	3 pages
Problems with Fractional Parts	58	2 pages
Problems to Solve	60	3 pages
Divisibility	63	4 pages
Prime Numbers	67	3 pages
Finding Factors	70	2 pages
Mixed Review Chapter 5	72	2 pages
Review Chapter 5	74	2 pages

Helpful Resources on the Internet

DIVISION CONCEPT AND DIVISION FACTS

The Forty Frogs Game

Learn to find fractions of sets by dividing a set of baby frogs into equal groups.

<http://www.mathactivities.net/frogs.htm>

Patty's Paints Division

Help Patty paint cars by solving basic division questions. Lastly, drive your newly painted car in a fun race!

<http://www.multiplication.com/games/play/pattys-paints-division>

Flying High Division

Fly your plane safely through the storm clouds by answering the division facts correctly.

<http://www.multiplication.com/games/play/flying-high-division>

Times or Divide Bingo

A useful class teaching resource on division and multiplication by 10 and 100. It includes decimals and is suitable for use on an interactive whiteboard.

<http://www.topmarks.co.uk/Flash.aspx?f=bingotimesordivide>

Leftovers—game with beads

Practice division with this fun dice game!

<http://www.learn-with-math-games.com/long-division-games-for-the-classroom.html>

Fraction of a Number

Practice finding a fraction of a given number.

http://www.mathplayground.com/fractions_fractionof.html

Multiply & Divide Whole Numbers by 10, 100, 1000

Practice multiplying and dividing by 10, 100, and 1,000 with this interactive online quiz.

<https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-place-value-decimals-top/cc-5th-mult-div-whole-num-10-100-1000/e/mult-div-whole-numbers-by-10-100-1000>

Division with remainders (mental math) — online practice

Practice division with remainders with this ad-free online practice program at MathMammoth.com website. Also available on Khan Academy in most browsers. Includes the option for both timed and non-timed practice.

Sample worksheet from
<https://www.mathmammoth.com/practice/division-remainder.php>

Order of Ops

Save seven members of a Royal Family from prison by using your order of operation skills. The program uses a visual representation of a stairway to show how the mathematical expression gets shorter at each step.

<https://mrnussbaum.com/order-ops-online-game>

Free customizable worksheets for the order of operations

Choose from five operations and parentheses. You can choose the number range, number of problems, and more.

http://www.homeschoolmath.net/worksheets/order_of_operations.php

ITP Remainders

This ITP sets up an empty grid into which you can place counters. Removing or highlighting extra counters will change the calculation displayed.

http://mathsframe.co.uk/en/resources/resource/67/itp_remainders

Moving Remainders Division Game

Practice your division skills with this printable board game for two or more players.

<https://www.lauracandler.com/wp-content/uploads/2018/06/MovingRemaindersGame.pdf>

Division with Remainders

Practice modeling division with remainders in the quotients in this interactive online activity.

<http://www.harcourtschool.com/activity/elab2004/gr4/5.html>

LONG DIVISION

MathFrog Dividerama!

Interactive long division practice. Guided help available.

<http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml>

Mr. Martini's Classroom: Long Division

An interactive long division tool.

<http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html>

Drag-and-Drop Math

Practice division interactively. Choose “Division”, 2-digit dividend, and 1-digit divisor.

<https://mrnussbaum.com/drag-n-drop-math-online>

Long Division Millionaire Game

Learn to divide large numbers up to thousands. Can you answer all 15 questions?

<http://www.kidsmathtv.com/free/math-games/sixth-grade/long-division/millionaire/game.html>

Bike Racing Math Average

Race your motorcycle against others while answering questions about average. Correct answers speed you up!

<http://www.mathnook.com/math/bike-racing-math-average.html>

Division Jump — board game

Practice division of one-digit numbers into two, three, and four-digit numbers.

<http://www.learn-with-math-games.com/division-activities.html>

Long Division Quiz

Practice dividing four-digit numbers by single-digit numbers in this online quiz.

<http://i4c.xyz/nmenbdv>

Double-Division.org

Double-division is a form of the long division algorithm that takes away the guesswork of finding how many times the divisor goes into the number to be divided. Also called 1-2-4-8 division.

<http://www.double-division.org/>

Sample worksheet from
<https://www.mathmammoth.com>

Short Division

This is a web page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in your head and not written down.

<http://www.themathpage.com/ARITH/divide-whole-numbers.htm>

FACTORS AND PRIMES

Arrays and Factors

Drag rectangles to show the factorizations of a given number on a grid.

<http://www.shodor.org/interactivate/activities/FactorizeTwo/>

Find all the factors of a given number — online practice

An ad-free online practice program at MathMammoth.com. Also works as an offline program in most browsers.

Choose the minimum and maximum numbers and the amount of practice problems.

<https://www.mathmammoth.com/practice/factorfind.php>

Factor Game

Choose a number from the game board, and your opponent gets all the numbers that are its proper factors. Adjust the number of rows and columns on the board to get a more challenging (and interesting) game. The game can be adapted to be played offline.

<http://illuminations.nctm.org/Activity.aspx?id=4134>

Sliding Tile Factorization Game

Slide a number over another to capture it, but you can only do this if the number you slide is a factor of the other.

Number 1 is only supposed to be used to capture prime numbers.

http://www.visualmathlearning.com/Games/sliding_factors.html

Octopus Factors

Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.

<https://web.archive.org/web/20171024183705/http://www.counton.org/games/map-numbers/octopus/>

Not a Factor

Choose a number that is *not* a factor of the given number.

http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

Product Game

Choose factors, and the product of those gets colored in on the game board. The player who gets four products in a row wins. This game can easily be adapted to be played offline, with paper and colored pencils.

<http://illuminations.nctm.org/Activity.aspx?id=4213>

Primes, Factors and Divisibility—Explorer at CountOn.org

Lessons explaining divisibility tests, primes, and factors.

<https://web.archive.org/web/20180319072651/http://www.counton.org:80/explorer/primes/>

Factoring Calculator

This tool lists all the factors of a given number and shows an interesting visual that pairs the various factors of the number. You can even find all the factors of very large numbers, and it is fun to experiment with!

<http://www.dadsworksheets.com/factoring-calculator.html>

Sample worksheet from
<https://www.mathmammoth.com>

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Long Division 1

Divide hundreds, tens, and ones separately.

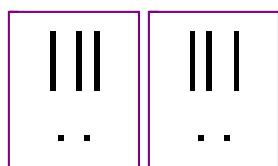
Write the dividend inside the long division “corner”, and the quotient on top.

$$\underline{64} \div 2 = ?$$

Divide tens and ones separately:

$$6 \text{ tens} \div 2 = 3 \text{ tens (t)}$$

$$4 \text{ ones} \div 2 = 2 \text{ ones (o)}$$



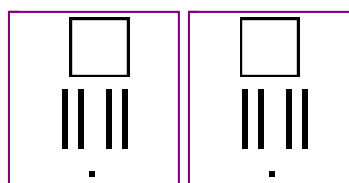
$$\begin{array}{r} \text{t o} \\ 3 \ 2 \\ 2 \overline{) 6 \ 4} \end{array}$$

$$\underline{282} \div 2 = ?$$

$$2 \text{ hundreds} \div 2 = 1 \text{ hundred (h)}$$

$$8 \text{ tens} \div 2 = 4 \text{ tens (t)}$$

$$2 \div 2 = 1 \text{ (o)}$$



$$\begin{array}{r} \text{h t o} \\ 1 \ 4 \ 1 \\ 2 \overline{) 2 \ 8 \ 2} \end{array}$$

1. Make groups. Divide. Write the dividend inside the “corner” if it is missing.

a. Make 2 groups



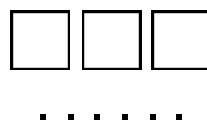
$$2 \overline{) 6 \ 2}$$

b. Make 3 groups



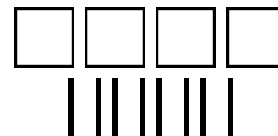
$$3 \overline{) \quad}$$

c. Make 3 groups



$$3 \overline{) \quad}$$

d. Make 4 groups



$$4 \overline{) \quad}$$

2. Divide thousands, hundreds, tens, and ones separately.

a. $4 \overline{) 8 \ 4}$

b. $3 \overline{) 3 \ 9 \ 3}$

c. $3 \overline{) 6 \ 6 \ 0}$

d. $4 \overline{) 8 \ 0 \ 4 \ 0}$

e. $3 \overline{) 6 \ 6}$

f. $2 \overline{) 6 \ 0 \ 4 \ 2}$

g. $3 \overline{) 3 \ 3 \ 0}$

h. $4 \overline{) 4 \ 8 \ 0 \ 4}$

<div style="text-align: center;"> h t o 0 $4 \overline{) 248}$ </div> <p>Four does not go into 2. You can put zero in the quotient in the hundreds place or omit it. Four does go into 24, six times. Put 6 in the quotient.</p>	<div style="text-align: center;"> h t o 0 6 2 $4 \overline{) 248}$ </div> <p>Five does not go into 3. You can put zero in the quotient. Five does go into 35, seven times.</p>
<p style="text-align: center;"><u>Explanation:</u></p> <p>The 2 of 248 is 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient will not have any whole hundreds.</p> <p>Then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result, 6 tens goes as part of the quotient.</p> <p>Check the final answer: $4 \times 62 = 248$.</p>	<p style="text-align: center;"><u>Explanation:</u></p> <p>$3,000 \div 5$ will not give any whole thousands to the quotient because the answer is less than 1,000.</p> <p>But 3 thousands and 5 hundreds make 35 hundreds together. You can divide $3,500 \div 5 = 700$, and place 7 as part of the quotient in the hundreds place.</p> <p>Check the final answer: $5 \times 701 = 3,505$.</p>
<p>If the divisor does not “go into” the first digit of the dividend, look at the <u>first two digits</u> of the dividend.</p>	

3. Divide. Check your answer by multiplying the quotient and the divisor.

a.
$$\begin{array}{r} 04 \\ 3 \overline{) 123} \end{array}$$

b.
$$\begin{array}{r} \\ 4 \overline{) 284} \end{array}$$

c.
$$\begin{array}{r} \\ 6 \overline{) 360} \end{array}$$

d.
$$\begin{array}{r} \\ 8 \overline{) 248} \end{array}$$

e.
$$\begin{array}{r} \\ 2 \overline{) 184} \end{array}$$

f.
$$\begin{array}{r} \\ 7 \overline{) 427} \end{array}$$

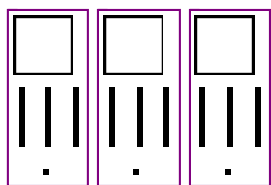
g.
$$\begin{array}{r} \\ 3 \overline{) 1833} \end{array}$$

h.
$$\begin{array}{r} \\ 4 \overline{) 2404} \end{array}$$

i.
$$\begin{array}{r} \\ 7 \overline{) 4970} \end{array}$$

j.
$$\begin{array}{r} \\ 5 \overline{) 4505} \end{array}$$

The ones division is not even. There is a remainder.



$$\underline{395 \div 3 = 131 \text{ R}2}$$

$$\begin{array}{r} \text{h t o} \\ 13 \\ 3 \overline{) 395} \end{array}$$

3 goes into 3 one time.
3 goes into 9 three times.

$$\begin{array}{r} \text{h t o} \\ 131 \text{ R}2 \\ 3 \overline{) 395} \end{array}$$

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

Four does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160). Four goes into 16 four times. Four goes into 5 once, with a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

Eight does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200). Eight goes into 32 four times ($3,200 \div 8 = 400$). Eight goes into 0 zero times (tens). Eight goes into 7 zero times, with a remainder of 7.

4. Divide into groups. Find the remainder.

 a. $2 \overline{) 63}$	 b. $2 \overline{) \quad}$	 c. $3 \overline{) \quad}$	 d. $2 \overline{) \quad}$
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5. Divide. Indicate the remainder if any.

a. $4 \overline{) 847}$

b. $2 \overline{) 69}$

c. $3 \overline{) 367}$

d. $4 \overline{) 89}$

e. $2 \overline{) 121}$

f. $6 \overline{) 1805}$

g. $7 \overline{) 215}$

h. $8 \overline{) 2482}$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 6 \quad 1 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th} \quad \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 0 \quad 2 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

6. Practice some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3 \overline{) 128}$

b. $3 \overline{) 95}$

c. $6 \overline{) 4267}$

d. $4 \overline{) 2845}$

e. $5 \overline{) 5507}$

f. $2 \overline{) 8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. $440 \div 4 =$

$820 \div 2 =$

b. $3600 \div 400 =$

$369 \div 3 =$

c. $824 \div 2 =$

$560 \div 90 =$

(This page intentionally left blank.)

Divisibility

A number n is **divisible** by another number m , if the division $n \div m$ is exact (no remainder).

For example, $18 \div 3 = 6$, so 18 is divisible by 3.

Also, 18 is divisible by 6, because we can write the other division $18 \div 6 = 3$.

So, 18 is divisible by *both* 6 and 3. We say 6 and 3 are **divisors** of 18.

You can use long division to check if a number is divisible by another.

$$\begin{array}{r} 16 \\ 4 \overline{) 67} \\ \underline{-4} \\ 27 \\ \underline{-24} \\ 3 \end{array}$$

For example, $67 \div 4 = 16$, R3. There is a remainder, so 67 is *not* divisible by 4.

Also, from this we learn that neither 4 nor 16 are divisors of 67.

1. Divide and determine if the number is divisible by the other number.

a. $21 \div 3 = \underline{\hspace{2cm}}$ Is 21 divisible by 3?	b. $40 \div 6 = \underline{\hspace{2cm}}$ Is 40 divisible by 6?	c. $17 \div 5 = \underline{\hspace{2cm}}$ Is 5 a divisor of 17?	d. $84 \div 7 = \underline{\hspace{2cm}}$ Is 7 a factor of 84?
--------------------------------------------------------------------	--------------------------------------------------------------------	--------------------------------------------------------------------	-------------------------------------------------------------------

2. Answer the questions. You may need long division.

<p>a. Is 98 divisible by 4?</p> <div style="border: 1px solid black; height: 200px; width: 100%;"></div>	<p>b. Is 603 divisible by 7?</p> <div style="border: 1px solid black; height: 200px; width: 100%;"></div>	<p>c. Is 3 a factor of 1,256?</p> <div style="border: 1px solid black; height: 200px; width: 100%;"></div>
----------------------------------------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------	------------------------------------------------------------------------------------------------------------

In any multiplication, the numbers that are multiplied are called **factors** and the result is called a **product**.

factor		factor		product
7	\times	6	$=$	42

For example, since $6 \times 7 = 42$, 6 and 7 are **factors** of 42.

From this multiplication fact we can write two divisions: $42 \div 6 = 7$ and $42 \div 7 = 6$. So, this means that 6 and 7 are also divisors of 42.

From this we can notice the following:

If a number is a factor of another number, it is also its divisor.

There is yet one more new word to learn that ties in with all of this: **multiple**.

We say **42 is a multiple of 6**, because 42 is some number times 6 (namely 7×6).

And of course 42 is also a multiple of 7, because 42 is some number times 7 (namely, 6×7)!

3. Fill in.

We know that $8 \times 9 = 72$. So, 8 is a _____ of 72, and so is 9.

Also, 72 is a _____ of 8, and 72 is a _____ of 9.

And, 72 is _____ by 8 and by 9.

4. Fill in.

a. Is 5 a factor of 55? Yes, because ____ \div ____ = _____.	b. Is 8 a divisor of 45? No, because ____ \div ____ = _____.
c. Is 36 a multiple of 6? _____, because ____ \div ____ = _____.	d. Is 34 a multiple of 7? _____, because ____ \div ____ = _____.
e. Is 7 a factor of 46? _____, because _____.	f. Is 63 a multiple of 9? _____, because _____.

Multiples of 6 are all those numbers we get when we multiply 6 by other numbers. For example, we can multiply 0×6 , 7×6 , 11×6 , 109×6 , and so on. The resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, and so on.

5. **a.** Make a list of multiples of 11, starting at 0 and continue at least to 154.

Sample worksheet from

<https://www.mathmammoth.com> **b.** Make a list of multiples of 111, starting at 0. Continue as long as you can in this space!

Divisibility by 2

Numbers that are divisible by 2 are called **even** numbers.
 Numbers that are NOT divisible by 2 are called **odd** numbers.
 Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

Divisibility by 5

Numbers that end in 0 and 5 are divisible by 5.
 For example, 10, 35, 720, and 3,675 are such numbers.

6. Mark an “x” if the number is divisible by 2 or by 5.

number	divisible		number	divisible		number	divisible		number	divisible	
	by 2	by 5		by 2	by 5		by 2	by 5		by 2	by 5
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

Divisibility by 10

Numbers that end in 0 are divisible by 10.
 For example, 10, 60, 340, and 2,570 are such numbers.

7. Mark an “x” if the number is divisible by 2, by 5, or by 10.

number	divisible			number	divisible			number	divisible		
	by 2	by 5	by 10		by 2	by 5	by 10		by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in a zero, so it is ALSO divisible by ____ and ____.

8. a. Write a list of numbers that are divisible by 2, from 0 to 60.

This is also a list of _____ of 2.

- b. In the list above, *underline* those numbers that are divisible by 4.
What do you notice?
- c. In the list above, *color* those numbers that are divisible by 6.
What do you notice?
- d. Which numbers are divisible by both 4 and by 6?

9. a. Write a list of numbers that are divisible by 3, from 0 to 60.

This is also a list of _____ of 3.

- b. In the list above, *underline* those numbers that are divisible by 6.
What do you notice?
- c. In the list above, *color* those numbers that are divisible by 9.
What do you notice?

10. Use the lists you made in (8) and (9). Find numbers that are divisible by *both* 2 and 9.

11. What number is a factor of every number?

12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers.
Which ones?

Who am I?

(Hint: I am less than 50.)

Mystery Number
38 2 1 99
47 101

Divided by 9, I leave a remainder of 6.
Divided by 4, I leave a remainder of 1.
Divided by 10, I leave a remainder of 3.

Who am I?

(Hint: I am less than 100.)

Mystery Number
38 2 1 99
47 101

I am a multiple of 3, 4, 5, and 6.
I am a factor of 120.
Divided by 7, I leave a remainder of 4.

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Chapter 6: Geometry

Introduction

We start our study of geometry by reviewing the third grade concepts of area and the perimeter of rectangles. Students also apply these concepts in various problems, including problems where they write simple equations and a problem where they explore all possible perimeters for a given area.

Note: Students will need a ruler and a protractor throughout the chapter.

The focus of the chapter is angles. Students learn about lines, rays, and angles; and about acute, right, obtuse, and straight angles. They learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations. The lesson *Estimating Angles* has an optional section on turning in an angle, which can be challenging, so feel free to omit it if you wish.

The lesson *Parallel and Perpendicular Lines* also ties in with the topic of angles, because two lines are perpendicular if they form a right angle. After that, we study parallelograms and other quadrilaterals in more detail, paying attention to their angles and lengths of sides.

We also study triangles and classify them according to their angles (acute, obtuse, or right triangles). Classifying triangles according to their sides (equilateral, isosceles, or scalene) will be studied in 5th grade. The last (and easy) topic in this chapter is line symmetry.

The lessons include quite a few drawing exercises which can be done on blank paper, in a notebook, or in the worktext (for most). Please stress to the student to always use a ruler and other proper tools, such as a protractor or a triangular ruler, so the drawings will be as accurate as possible. Some exercises may mention to only sketch something, in which case it is okay to not use any drawing tools.

Geometry is full of strange-sounding words. I suggest that student(s) keep a geometry notebook, where they draw picture(s) and text to explain every new concept or term. This will help them to remember those terms. They can also do the drawing exercises in the notebook. Encourage the students to be creative so that the notebook becomes their own special work. You can even give them credit for it.

The Lessons in Chapter 6

	page	span
Review: Area of Rectangles	81	3 pages
Problem Solving: Area of Rectangles	84	2 pages
Review: Area and Perimeter	86	4 pages
Lines, Rays, and Angles	90	3 pages
Measuring Angles	93	7 pages
Drawing Angles	100	2 pages
Estimating Angles	102	5 pages
Angle Problems	107	5 pages
Parallel and Perpendicular Lines	112	5 pages
Parallelograms	117	3 pages
Triangles	120	4 pages
Line Symmetry	124	3 pages
Mixed Review Chapter 6	127	2 pages
Review Chapter 6	129	4 pages

Sample worksheet from
<https://www.mathmammoth.com>

Helpful Resources on the Internet

AREA AND PERIMETER

Free Worksheets for Area and Perimeter

Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property.

https://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php

Area and Perimeter Dominoes

Match your domino tiles with domino tiles on the “table” that have the same value in this fun, interactive two-player game

<https://www.turtlediary.com/game/calculating-area-and-perimeter.html>

Shape Explorer

Find the perimeter and area of odd shapes on a rectangular grid.

<http://www.shodor.org/interactivate/activities/ShapeExplorer/>

Math Playground: Party Designer

You need to design areas for the party, such as a crafts table, food table, seesaw, and so on, so that they have the given perimeters and areas.

<https://www.mathplayground.com/PartyDesigner/index.html>

Area and Perimeter Builder

Create your own rectangular shapes using colorful blocks and explore the relationship between perimeter and area. You can choose to show the side lengths to understand how a perimeter works. You can also use two work areas (grids) to compare the area and perimeter of two shapes side-by-side. Lastly, challenge yourself in the game screen to build shapes or find the area of various figures.

http://phet.colorado.edu/sims/html/area-builder/latest/area-builder_en.html

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz, about the area and perimeter of rectangles, triangles, and trapezoids. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when the perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1200b-lc-p0>

Area: Missing Side Length Quiz

Practice finding the value of the unknown side in this 10-question quiz.

<http://www.thatquiz.org/tq-4/?-j8001-lc-p0>

Area and Perimeter of Rectangles and Squares

A 10-question quiz with varying questions concerning the area and perimeter of rectangles and squares.

<https://www.ck12.org/assessment/ui/?test/view/practice/geometry/square-and-rectangle-area-and-perimeter-practice>

ANGLES

Angles and Their Measures Matching Game

Practice matching angles to their angle measures in this interactive online game.

<https://www.mathmammoth.com/practice/angles-matching>

Sample worksheet from

<https://www.mathmammoth.com>

Online Protractor

Investigate angles and the use of protractors.

<https://web.archive.org/web/20180219193040/http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html>

Measuring Angles

Practice measuring angles with a protractor.

<https://www.mathplayground.com/measuringangles.html>

Draw Angles - Khan Academy

Use a protractor to construct angles.

<https://www.khanacademy.org/math/on-sixth-grade-math/on-geometry-spatial-sense/on-angles-polygons/e/drawing-angles>

Topmarks - Angles activity

Squirt the dog with water by dragging the correct angle onto the screen.

<http://www.topmarks.co.uk/Flash.aspx?b=maths/angles>

Fruit Picker

Try to pick 6 apples in 6 shots by turning at the correct angle.

<http://www.fruitpicker.co.uk/activity/>

Estimating Angles

Estimate the target angle. The closer you get to the target angle the more points you will score.

<https://nrich.maths.org/1235>

Draw Perpendicular and Parallel Lines - Interactive

Learn about perpendicular and parallel lines and practice drawing them.

<https://www.mathsisfun.com/perpendicular-parallel.html>

Parallel and Perpendicular Lines in Shapes Quiz

Classify the lines as parallel, perpendicular, or neither.

<https://www.studyladder.com/games/activity/parallel-and-perpendicular-lines-in-shapes-28459>

Turtle Pond

Guide a turtle to a pond using commands, which include turning him in certain angles or moving him a specific distance.

<http://illuminations.nctm.org/Activity.aspx?id=3534>

SHAPES / POLYGONS

Polygon Matching Game

Learn all the common polygons by playing this fun, timed matching game.

https://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary

A matching game.

<http://www.quia.com/cc/2758.html>

Interactive Quadrilaterals

See all the different kinds of quadrilaterals “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.

<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

Sample worksheet from

<https://www.mathmammoth.com>

Interactive Parallelogram

Drag the parallelogram and learn about its properties, angles, and sides.

<http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php>

Interactive Triangles Tool

Read about triangles, and then play with them to become familiar with them from all angles.

<https://www.mathsisfun.com/geometry/triangles-interactive.html>

Triangles Splat

“Shoot” the triangles as their names appear on the screen. Choose “Right, Acute, Obtuse”.

http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/triangles_shoot.htm

Classifying Triangles Drag-and-Drop Game

Drag each triangle in the correct basket before the time runs out!

<http://www.math-play.com/classifying-triangles/Triangles-Drag-and-Drop-Game.html>

Lines of Symmetry

Match the corresponding lines of symmetry.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/SymmetryLinesShapesShoot.htm>

Symmetry Shapes Shoot

Practice identifying symmetrical shapes by clicking on them.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/SymmetryShapesShoot.htm>

Line Shoot

Learn about lines of symmetry the fun way in this line-shoot geometry math game.

http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/line_shoot.htm

GENERAL

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

Patch Tool

An online activity where the student designs a pattern using geometric shapes.

<http://illuminations.nctm.org/Activity.aspx?id=3577>

Polygon Playground

Drag various colorful polygons to the work area to make your own creations!

<http://www.mathcats.com/explore/polygons.html>

Geometry Worksheets

Worksheets about complementary and supplementary angles, parallel, perpendicular, and intersecting lines, types of angles, basic shapes, area and perimeter of rectangles, and parts of a circle.

<http://www.dadsworksheets.com/worksheets/basic-geometry.html>

Space Logic

Write a program that will guide the space rover to its target. Watch out for the boulders!

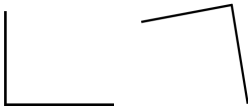
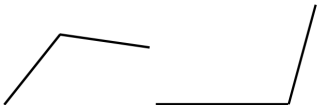
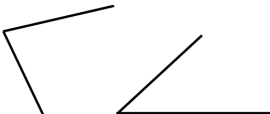
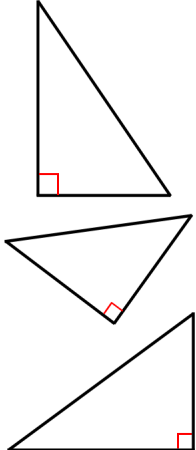
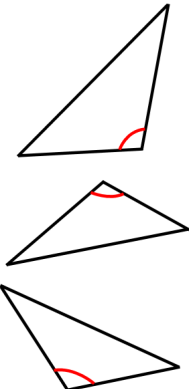
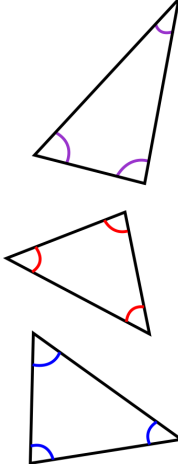
https://www.mathplayground.com/space_logic.html

Sample Worksheet from

<https://www.mathmammoth.com>

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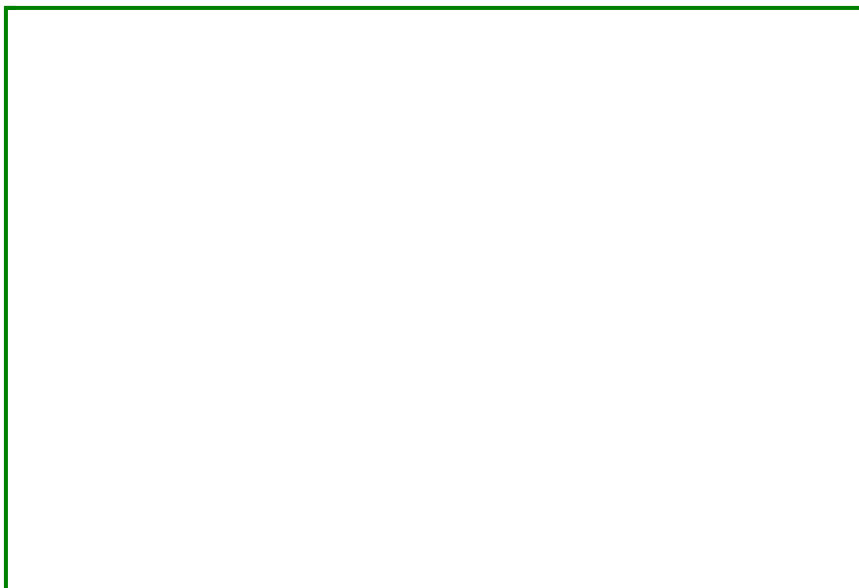
Triangles

 <p>right angles (exactly 90°)</p>	 <p>obtuse angles (more than 90°, less than 180°)</p>	 <p>acute angles (less than 90°)</p>
 <p>Right triangles have exactly one right angle.</p>	 <p>Obtuse triangles have exactly one obtuse angle.</p>	 <p>Acute triangles have three acute angles. In other words, <i>all</i> the angles are acute.</p>

1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.

b. Draw another, different right triangle.

c. A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?



Sample worksheet from <https://www.mathmammoth.com>
 A right triangle has one right angle. The other two angles are _____.

2. **a.** Draw an obtuse angle.
Then make it into an obtuse triangle by drawing in the third side.



- b.** Draw another, different obtuse triangle.
- c.** An obtuse triangle has one obtuse angle. Are the other two angles in an obtuse triangle acute, right, or obtuse?



An obtuse triangle has one obtuse angle. The other two angles are _____.

3. **a.** Draw any acute triangle.
- b.** Measure its angles.

They measure _____ $^{\circ}$,
_____ $^{\circ}$, and _____ $^{\circ}$.



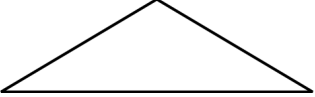
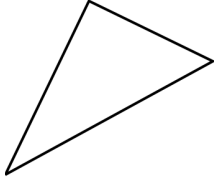
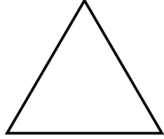
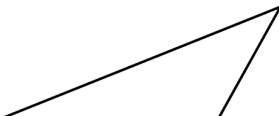
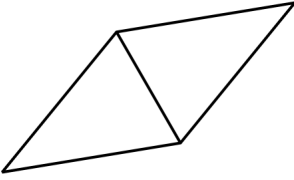
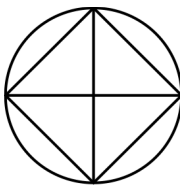
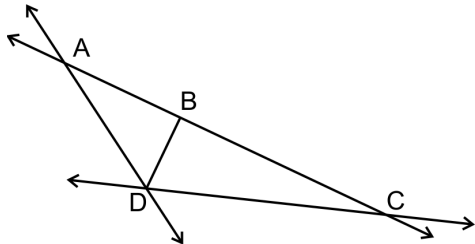
4. Observe all that you have done in this lesson thus far, and fill in the blanks below.

Right triangles have exactly one _____, _____,
and the other two angles are _____.

Obtuse triangles have exactly one _____, _____,
and the other two angles are _____.

Acute triangles have _____ angles.

5. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p> 	<p>b.</p> 	<p>c.</p> 	<p>d.</p> 
<p>e.</p> 	<p>f.</p> 	<p>g.</p>  <p>triangle ABD: triangle ACD: triangle BCD:</p>	

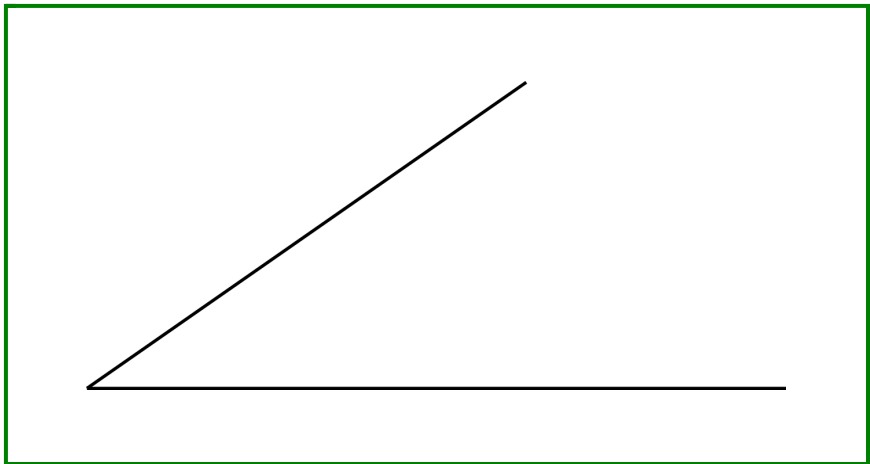
6. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p>  <p>National Museum of Marine Corps Photo by Ron Cogswell</p> <p>_____</p>	<p>b.</p>  <p>The City Center of Woburn, MA Photo by Randy Robertson</p> <p>_____</p>	<p>c.</p>  <p>The White House Photo by Glyn Lower</p> <p>_____</p>
<p>d.</p>  <p>Waterlily Photo by Francis Chung</p> <p>_____</p>	<p>e.</p>  <p>Warning of a steep hill ahead Photo by www.flickr.com/photos/shirokazan/</p> <p>The black triangle is _____</p> <p>The red triangle is _____</p>	

Find acute, obtuse, and right triangles outside in nature, in buildings, in signs, and so on!

<https://www.mathmammoth.com>

7. **a.** Draw a triangle with 35° and 40° angles.
The 35° angle is already drawn for you.
- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)



8. **a.** Draw a triangle with 125° and 40° angles.
- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)



9. **a.** Draw a triangle with 55° and 35° angles.
- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)

**New Terms**

- an acute triangle
- an obtuse triangle
- a right triangle

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Chapter 7: Fractions

Introduction

In third grade, students have studied equivalent fractions and compared some easy fractions. In fourth grade, it is time to expand their knowledge of fraction topics. We study:

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (sums where the denominators are the same, such as $\frac{5}{6} + \frac{3}{6}$ or $1\frac{2}{3} + 2\frac{1}{3}$)
- equivalent fractions (for example, $\frac{2}{3} = \frac{8}{12}$)
- comparing fractions
- multiplying a fraction by a whole number (for example $5 \times \frac{1}{2}$)

Then in fifth grade, students tackle *all* four operations with fractions. This chapter is laying groundwork for that. The lessons here are important also because they are the basis for understanding decimal numbers, which is the topic of the next chapter.

In this grade, we continue studying fractions and their operations with the help of visual models. In addition to the visuals in the lessons, you can optionally also use fraction manipulatives, but they are not required.

Visual models help children build a strong conceptual understanding of fraction operations. While we do study some actual rules of fraction arithmetic in this chapter, we also want to avoid presenting fraction math as a list of computational rules to be learned by rote memory. If students only memorize these rules, then they will also easily confuse them (eventually), because there are so many of them. The rules become *shortcuts* for ideas that are already understood, but we don't want to start with them. The goal is to let the ideas and concepts “sink in” first, and then study the shortcuts.

A friendly reminder: don't automatically assign all the exercises. As always, use your judgment.

The Lessons in Chapter 7

	page	span
One Whole and Its Fractional Parts	137	3 pages
Mixed Numbers	140	4 pages
Mixed Numbers and Fractions	144	3 pages
Adding Fractions	147	2 pages
Adding Mixed Numbers	149	3 pages
Equivalent Fractions	152	5 pages
Subtracting Fractions and Mixed Numbers	157	4 pages
Comparing Fractions	161	4 pages
Multiplying Fractions by Whole Numbers	165	3 pages
Practicing With Fractions	168	2 pages
Mixed Review Chapter 7	170	2 pages
Review Chapter 7	172	2 pages

Sample worksheet from
<https://www.mathmammoth.com>

Helpful Resources and Games on the Internet

FRACTIONS AND MIXED NUMBERS

Identifying Fractions at Conceptua Fractions

A tool that shows fractions or mixed numbers using a pie, a bar, dots, and a number line.

<https://www.conceptuamath.com/app/tool/identifying-fractions>

Number Bonds - Fractions

Combine balls whose fractions add to one.

https://www.mathplayground.com/number_bonds_fractions.html

Puzzle Pics Fractions

Reveal the mystery picture by dragging each puzzle piece to the number line that illustrates the fraction.

http://www.mathplayground.com/puzzle_pics_fractions.html

Animal Rescue: Fractions Number Line Game

Find and free the trapped animals by moving the arrow to the correct place on the number line.

<http://www.sheppardsoftware.com/mathgames/fractions/AnimalRescueFractionsNumberLineGame.htm>

Clara Fraction's Ice Cream Shop

Convert improper fractions into mixed numbers, and scoop the right amount of ice cream flavors onto the cone for your customers.

<https://mrnussbaum.com/clara-fraction-s-ice-cream-shop-online-game>

Mixed Numbers and Improper Fractions

Practice converting mixed numbers to improper fractions.

https://www.mathplayground.com/fractions_mixed.html

ADDITION AND SUBTRACTION

Adding of Like Fractions with Circle Models

Practice adding fractions with the help of a visual model.

<http://www.visualfractions.com/AddEasyCircle/addcircles.html>

Fractions Workshop

Choose “Add mixed fractions with like denominators” in order to practice adding mixed numbers.

<https://mrnussbaum.com/fraction-workshop-online>

Action Fraction

A racing game with several levels where you add and subtract fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.

http://solvemymath.com/math_games/arithmetic_games/action_fraction/

Add Mixed Numbers: 10-Question Quiz

Practice adding mixed numbers. Express the answer as a mixed fraction in lowest terms.

<http://www.thatquiz.org/tq-3/?-j301-l1-p0>

Fraction Game

Move the markers on the fraction number line from left to right according to the given fraction cards.

<http://illuminations.nctm.org/Activity.aspx?id=4148>

Sample worksheet from
<https://www.mathmammoth.com>

Four-Sum Fractions Board Game

Practice adding and simplifying fractions with common denominators. Be the first player to score four in a row!

<http://www.learn-with-math-games.com/fraction-math-games.html>

Subtracting Mixed Numbers with Borrowing

Perform subtraction calculations using borrowing with mixed number fractions.

<https://www.wisc-online.com/learn/formal-science/mathematics/abm701/subtracting-mixed-number-fractions-with-borro>

Subtracting Mixed Fractions Quiz

Practice subtracting mixed fractions in this multiple-choice quiz. Drag and drop corresponding answers.

<http://www.fractions4kids.com/subtracting-mixed-fractions-quiz/>

Fruit Shoot Fractions Addition

Click the fruit with the correct answer. To match the topics students learn in this section, choose adding 2 or 3 fractions with like denominators. You can also choose your mode (untimed or timed) and speed (slow versus fast fruit).

<http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.htm>

Fraction Worksheets: Addition and Subtraction

Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose “Like Fractions” for this level.

<https://www.homeschoolmath.net/worksheets/fraction.php>

EQUIVALENT FRACTIONS

Equivalent Fractions

Draw two equivalent fractions for the given fraction. Choose either a square or a circle for the shape.

<http://illuminations.nctm.org/Activity.aspx?id=3510>

Fresh Baked Fractions

Practice equivalent fractions by clicking on a fraction that is not equal to the others.

<http://www.funbrain.com/fract/index.html>

Fishy Fractions

Feed Ulani the pelican by choosing the fish with the correct equivalent fraction.

<https://web.archive.org/web/20190901123002/http://streaming.discoveryeducation.com/braingames/iknowthat/Fractions/FractionGame.cfm?Topic=namematch>

Free Equivalent Fractions Worksheets

Create custom-made worksheets for equivalent fractions that can either include pie images or not.

https://www.homeschoolmath.net/worksheets/equivalent_fractions.php

COMPARING FRACTIONS

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.

<http://www.fuelthebrain.com/games/comparison-shootout/>

Comparing Fractions—XP Math

Simple timed practice for comparing two fractions.

<http://xpmath.com/forums/arcade.php?do=play&gameid=8>

Sample worksheet from

<https://www.mathmammoth.com>

Ordering Fractions at Conceptua Fractions

An interactive tool where students place numbers, visual models, and decimals on a number line.

<https://www.conceptuamath.com/app/tool/comparing-fractions>

Ordering Fractions

Drag the fractions into the right order, from lowest to highest.

<http://www.topmarks.co.uk/Flash.aspx?b=maths/fractions>

Dirt Bike Comparing Fractions

Play tug of war while practicing comparing fractions in this interactive online game.

<https://www.arcademics.com/games/dirt-bike-comparing-fractions>

MULTIPLYING FRACTIONS

Multiplying Fractions with Circle Models

This page illustrates fraction multiplication with circle models.

<http://www.visualfractions.com/MultiplyCircle/>

Multiply Fractions by Whole Numbers

Practice multiplying fractions by whole numbers in this simple online exercise.

<http://www.mathgames.com/skill/4.67-multiply-fractions-by-whole-numbers>

Multiplying Fractions Concentration

Practice multiplying fractions by whole numbers with this interactive online matching game.

<https://www.quia.com/cc/2740524.html>

GENERAL

Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated with a visual model. Also includes a couple of games.

<http://www.visualfractions.com/>

Conceptua Math Fraction Tools

Interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, finding common denominators and more. Each activity uses several visual models, such as fraction circles, horizontal and vertical bars, number lines, etc.

<https://www.conceptuamath.com/app/tool-library>

Who Wants Pizza?

This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.

<http://math.rice.edu/~lanius/fractions/index.html>

Fractioncity

Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.

<http://www.teachnet.com/lesson/math/fractioncity.html>

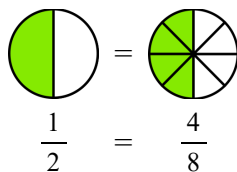
Sample worksheet from

<https://www.mathmammoth.com>

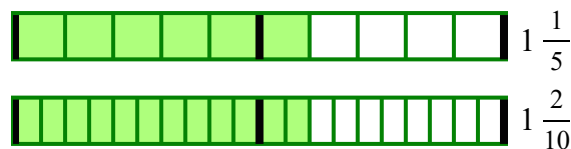
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Equivalent Fractions

If you eat half of a pizza, or if you eat $\frac{4}{8}$ of a pizza, you have eaten the same amount.



$\frac{1}{2}$ and $\frac{4}{8}$ are **equivalent fractions**.



The two fraction strips show an equal amount. So, we can write an equal sign between the

two mixed numbers: $1\frac{1}{5} = 1\frac{2}{10}$.

1. Color the first fraction. Shade the same amount of pie in the second picture. Write the second fraction.

<p>a. = =</p> <p>$\frac{1}{2} =$</p>	<p>b. = =</p> <p>$\frac{3}{4} =$</p>	<p>c. = =</p> <p>$\frac{6}{10} =$</p>	<p>d. = =</p> <p>$\frac{8}{12} =$</p>
<p>e. = =</p> <p>$\frac{1}{3} =$</p>	<p>f. $1\frac{2}{3} =$ =</p>		<p>g. $1\frac{10}{12} =$ =</p>

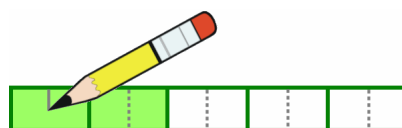
2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.

	a. $\frac{3}{3} =$	b. $\frac{4}{3} =$	c. $\frac{7}{3} =$
	d. $2\frac{1}{3} =$	e. $1\frac{2}{3} =$	f. $2\frac{2}{3} =$

3. Mark the equivalent fractions on the number lines.

<p>a. $\frac{4}{5} =$ =</p>	<p>b. $\frac{3}{9} =$ =</p>
-----------------------------------------	-----------------------------------------

Example 1. The fraction strip illustrates $\frac{2}{5}$. If you split each piece (both the colored and white pieces) into *two* new pieces, what fraction do you get?



You get $\frac{4}{10}$: four colored pieces, and ten pieces total.

You have *two* times as many colored pieces, and *two* times as many total pieces as before.

4. Split both the colored and white pieces as instructed. Write the fraction after you change it.

a. Split all the pieces into two new ones.



$$\frac{1}{2} = \frac{\boxed{}}{\boxed{}}$$

b. Split all the pieces into four new ones.



$$\frac{1}{2} = \frac{\boxed{}}{\boxed{}}$$

c. Split all the pieces into three new ones.



$$\frac{1}{2} = \frac{\boxed{}}{\boxed{}}$$

d. Split all the pieces into three new ones.



$$\frac{1}{3} = \frac{\boxed{}}{\boxed{}}$$

e. Split all the pieces into two new ones.



$$\frac{5}{6} = \frac{\boxed{}}{\boxed{}}$$

f. Split all the pieces into three new ones.



$$\frac{2}{5} = \frac{\boxed{}}{\boxed{}}$$

Do you notice a *shortcut* for finding the second fraction?

g. Split all the pieces into four new ones.



$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

h. Split all the pieces into two new ones.



$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

i. Split all the pieces into three new ones.



$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

If you found the shortcut, explain how it works in these problems:

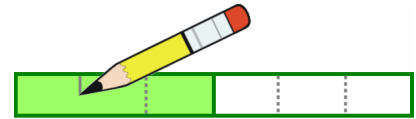
Split all the pieces into three new ones.

$$\frac{1}{3} = \frac{\boxed{}}{\boxed{}}$$

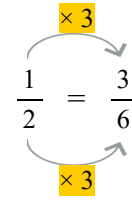
Split all the pieces into two new ones.

$$\frac{3}{5} = \frac{\boxed{}}{\boxed{}}$$

Example 2. The fraction strip illustrates $\frac{1}{2}$. If we split each piece into *three* new pieces, we get $\frac{3}{6}$.



Now we have *three* times as many colored pieces, and *three* times as many pieces in total as we had before. Look at the right side of this box, to see how we can illustrate it this way →



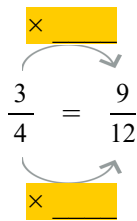
We multiply both the top and bottom number in a fraction by 3.
We get an equivalent fraction—it is the same amount, just cut into more pieces. *This does not mean we multiply the whole fraction by 3.*

5. Split the pieces. Fill in the missing parts.



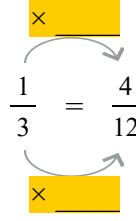
a. This is $\frac{3}{4}$. Make it $\frac{9}{12}$.

Each piece is split
into ____ new ones.



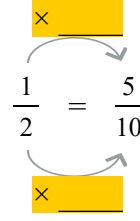
b. This is $\frac{1}{3}$. Make it $\frac{4}{12}$.

Each piece is split
into ____ new ones.

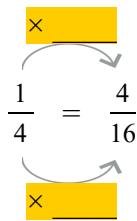


c. This is $\frac{1}{2}$. Make it $\frac{5}{10}$.

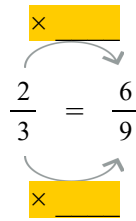
Each piece is split
into ____ new ones.



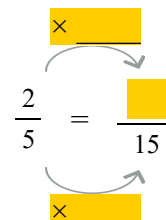
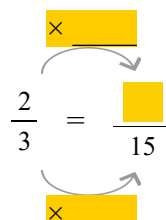
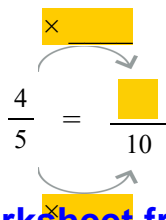
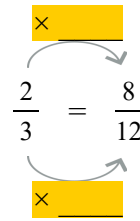
d. This is $\frac{1}{4}$. Make it $\frac{4}{16}$.



e. This is $\frac{2}{3}$. Make it $\frac{6}{9}$.



f. This is $\frac{2}{3}$. Make it $\frac{8}{12}$.



6. Write the equivalent fraction. Use multiplication.

a. Split all the pieces into three new ones. $\frac{5}{6} = \frac{\text{■}}{\text{■}}$	b. Split all the pieces into five new ones. $\frac{3}{4} = \frac{\text{■}}{\text{■}}$	c. Split all the pieces into four new ones. $\frac{2}{5} = \frac{\text{■}}{\text{■}}$	d. Split all the pieces into ten new ones. $\frac{9}{10} = \frac{\text{■}}{\text{■}}$
--------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------

7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.

a. Pieces were split into ____ new ones. $\frac{1}{2} = \frac{\text{■}}{6}$	b. Pieces were split into ____ new ones. $\frac{3}{10} = \frac{30}{\text{■}}$	c. Pieces were split into ____ new ones. $\frac{2}{5} = \frac{\text{■}}{30}$	d. Pieces were split into ____ new ones. $\frac{7}{8} = \frac{35}{\text{■}}$
e. $\frac{2}{3} = \frac{\text{■}}{6}$	f. $\frac{3}{5} = \frac{9}{\text{■}}$	g. $\frac{5}{6} = \frac{\text{■}}{12}$	h. $\frac{1}{3} = \frac{\text{■}}{9}$

8. Write the fractions that have tenths with hundredths instead.

a. $\frac{1}{10} = \frac{\text{■}}{100}$	b. $\frac{3}{10} =$	c. $\frac{6}{10} =$	d. $\frac{4}{10} =$	e. $\frac{13}{10} =$
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9. Connect the equivalent fractions with a line.

a.	$\frac{2}{3}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{10}$	$\frac{2}{8}$
	$\frac{2}{6}$	$\frac{6}{9}$
b.	$\frac{1}{2}$	$\frac{2}{10}$
	$\frac{3}{4}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{6}{12}$
	$\frac{4}{12}$	$\frac{9}{12}$
c.	$\frac{3}{6}$	$\frac{3}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$

10. Write chains of equivalent fractions!



a. $\frac{1}{2} = \frac{\text{■}}{4} = \frac{\text{■}}{6} = \frac{\text{■}}{8} = \frac{\text{■}}{\text{■}} = \frac{\text{■}}{\text{■}} = \frac{\text{■}}{\text{■}}$	b. $\frac{1}{3} = \frac{\text{■}}{6} = \frac{\text{■}}{9} = \frac{\text{■}}{12} = \frac{\text{■}}{\text{■}}$
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We can use equivalent fractions to add fractions that have different denominators.

Example 3. Add $\frac{2}{10} + \frac{17}{100}$. First, write $\frac{2}{10}$ as $\frac{20}{100}$ (an equivalent fraction).

Then you can add, because the fractions now have the same denominator: $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$.

11. Add.

a. $\frac{1}{10} + \frac{8}{100}$ $\downarrow \quad \downarrow$  $+$ $\frac{8}{100} =$	b. $\frac{7}{10} + \frac{3}{100}$ $\downarrow \quad \downarrow$  $+$ $\frac{3}{100} =$	c. $\frac{45}{100} + \frac{3}{10}$
d. $\frac{9}{10} + \frac{9}{100}$	e. $\frac{7}{10} + \frac{23}{100}$	f. $\frac{24}{100} + \frac{9}{10}$
g. $\frac{7}{100} + 1\frac{4}{10}$	h. $2\frac{28}{100} + 1\frac{5}{10}$	i. $\frac{6}{10} + \frac{35}{100} + \frac{7}{100}$

12. Draw a picture showing that $\frac{1}{3}$ and $\frac{4}{12}$ are equivalent fractions.

Puzzle Corner

Add. This is challenging. Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.

a. $\frac{3}{4} + \frac{1}{2}$

b. $\frac{1}{5} + \frac{3}{10}$

c. $\frac{2}{3} + \frac{2}{9}$

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Chapter 8: Decimals

Introduction

In fourth grade, students learn about decimal numbers that have one or two decimal digits, and they learn to add and subtract them. It is important to grasp these simple topics well because we are laying a groundwork for fifth and sixth grades where decimal operations take “center stage.”

The focus is, first of all, on understanding that decimals are simply fractions with a denominator of 10 or 100. Then with that in mind, we study comparing, adding, and subtracting them.

Take note of this common misconception that students have. Many students add $0.5 + 0.9 = 0.14$. The correct way to view $0.5 + 0.9$ is as 5 tenths plus 9 tenths, which is 14 tenths = 1.4.

An example of another misconception is when a student adds $0.5 + 0.11 = 0.16$. This student is thinking of the decimal parts as if they were “whole numbers” and adding $5 + 11 = 16$. To solve $0.5 + 0.11$ correctly, students can rewrite 0.5 as 0.50, and then the problem becomes $0.50 + 0.11 = 0.61$.

In the lesson *Using Decimals with Measuring Units*, students encounter decimals in connection with metric units, such as 0.1 km or 2.4 kg, and they also convert between the units, such as writing 0.5 km as 500 m. This topic will be studied further in 5th grade.

The Lessons in Chapter 8

	page	span
Decimal Numbers—Tenths	177	2 pages
Adding and Subtracting with Tenths	179	2 pages
Two Decimal Digits—Hundredths	181	4 pages
Add and Subtract Decimals in Columns	185	3 pages
Add and Subtract Decimals Mentally	188	4 pages
Using Decimals with Measuring Units	192	2 pages
Mixed Review Chapter 8	194	2 pages
Review Chapter 8	196	2 pages

Helpful Resources on the Internet

Fractions & Decimals Matching Game

Practice converting fractions to decimals while also uncovering a hidden picture in this fun matching game!

<https://www.mathmammoth.com/practice/fractions-decimals>

Decimal Place Value - Hundredths

Practice identifying numbers that have two decimal digits with this interactive multiple-choice quiz.

http://www.henryanker.com/Math/Number_Sense/Writing_Numbers/Writing_Decimals_100ths.swf

Sample worksheet from

<https://www.mathmammoth.com>

Modeling Decimals (Area and Grid Models)

An interactive “gizmo” for modeling decimals in a grid or on a number line. It is by subscription, but you can try the gizmo for 5 minutes for free.

<http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007>

Decimals on a Number Line

This multiple-choice quiz asks questions about the position of letters on the number line.

http://www.henryanker.com/Math/Fractions/Number_Line_Fractions_Decimals/Decimals_on_Number_Line_1.swf

Printable Math Puzzles

This page has several brain teasers and puzzles which will help the student apply and practice their math skills to solve a range of challenges and number problems.

<https://www.math-salamanders.com/printable-math-puzzles.html>

Decimal Challenge

Guess the decimal number between 0 and 10. Each time you get a response that tells whether your guess was too high or too low.

<http://www.interactivestuff.org/sums4fun/decchall.html>

Switch

Switch the decimals around until they are in ascending order. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/switch.html>

Scales

Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/scales.html>

Fraction/Decimal Worksheets

Change fractions to decimal numbers or decimal numbers to fractions.

<http://www.homeschoolmath.net/worksheets/fraction-decimal.php>

Fraction Snake Game

Arrange the numbers on the snake in order from the largest on the head to the smallest at the tail.

http://www.transum.org/software/SW/fracorder/fraction_order.asp

Rock Hopper

Help the frog reach the other side of the pond by clicking on the rocks that add up to the correct answer.

http://www.eduplace.com/kids/mw/swfs/rockhopper_grade4.html

Bubble Burst

Burst the bubble that has the correct answer to the decimal addition.

<http://www.mathnook.com/math/bubbleburst.html>

Decimals Quiz

Practice adding and subtracting decimals in this 10-question quiz.

<https://www.thatquiz.org/tq-3/?-j163-l5-p0>

Decimal Subtraction - Matching

Match each decimal subtraction with the correct answer.

<http://www.mathmammoth.com/mathgames/decimals/matchingDecimalsMinus.htm>

<https://www.mathmammoth.com>

Decimal Mania - Addition and subtraction

Practice decimal addition and subtraction with this interactive exercise.

<http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/addsubdec.shtml>

Adding Decimals: Hundredths

Practice adding numbers that have two decimal digits in this interactive online activity.

https://www.khanacademy.org/math/arithmetic/arith-decimals/arith-review-add-decimals/e/adding_decimals

Get to the (Decimal) Point Addition and Subtraction card games (pp. 60-63 of the PDF file)

This is a card game with four different variations that practice decimal addition and subtraction.

http://www.pepnonprofit.org/uploads/2/7/7/2/2772238/acing_math.pdf

Decimals Magic Square

Add and subtract decimals to make a “magic square”. Add to find a magic sum and subtract to find the missing addends.

<http://www.hbschool.com/activity/elab2004/gr4/14.html>

Convert units (metrics)

Practice converting between metric units of measurement in this interactive online exercise.

<https://www.khanacademy.org/math/cc-fifth-grade-math/imp-measurement-and-data-3/imp-unit-conversion/e/converting-units>

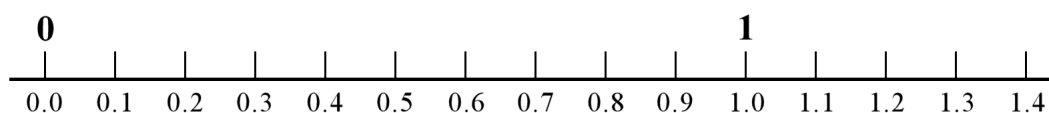
Sample worksheet from
<https://www.mathmammoth.com>

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Adding and Subtracting with Tenths

<p>You already know how to add or subtract decimals that have tenths, such as $0.8 + 0.5$. They are just fractions with a denominator of 10.</p> <p>Compare the two additions in each box. One of them is written with decimals and the other with fractions.</p>	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
<p>There is one tricky thing: $0.6 + 0.7$ is <u>NOT</u> 0.13!</p> <p>To see why, add the corresponding fractions. Notice that six-tenths and seven-tenths makes thirteen-tenths, which is more than one!</p>	$0.6 + 0.7 = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each “number-line jump.”



- a. You are at 0.7, and you jump *five tenths* to the right. _____
- b. You are at 0.6, and you jump *eight tenths* to the right. _____
- c. You are at 1.1, and you jump *eight tenths* to the left. _____
- d. You are at 1.3, and you jump *four tenths* to the left. _____
- e. You are at 0.2, and you jump *eleven tenths* to the right. _____

2. Solve the fraction additions, and then write them using decimals.

<p>a. $\frac{2}{10} + \frac{7}{10} =$</p> <p>$0.2 +$</p>	<p>b. $\frac{5}{10} + \frac{6}{10} =$</p>	<p>c. $\frac{9}{10} + \frac{8}{10} =$</p>
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3. Add or subtract.

a.	b.	c.	d.
$0.9 + 0.2 =$ _____	$0.5 + 0.7 =$ _____	$0.8 + 0.7 =$ _____	$1.8 - 0.9 =$ _____
$1.9 + 0.2 =$ _____	$2.5 + 0.7 =$ _____	$0.8 + 2.7 =$ _____	$5.8 - 0.9 =$ _____

4. Calculate.

a.	b.	c.	d.
$2.3 + 0.9 =$ _____	$1.5 + 0.7 =$ _____	$6.6 - 0.5 =$ _____	$4.7 - 1.7 =$ _____

5. Write the numbers.

a. 3 tenths, 5 ones

b. 7 tens, 8 ones, 4 tenths

c. 4 tenths, 3 ones, 6 tens

T	O	te
4	7	5

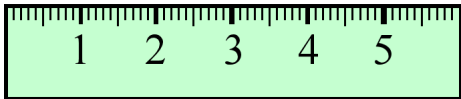
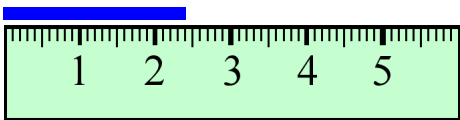
In this place value chart, “T” means tens, “O” means ones, and “te” means tenths.

We can see that the number 47.5 has 4 tens, 7 ones, and 5 tenths.

6. Continue the patterns by adding or subtracting the same number repeatedly.

a. 0.1	b. 1.1	c. 2.5	d. 3.6
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____
$+ 0.2 =$ _____	$+ 0.5 =$ _____	$+ 0.3 =$ _____	$- 0.4 =$ _____

7. Remember: **1 millimeter is one-tenth of a centimeter.** Or, $1 \text{ mm} = 0.1 \text{ cm}$.

<p>a. Draw a line that is 4.7 cm long.</p> 	<p>b. Measure the line in centimeters. Use a decimal.</p> 
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8. In (a) and (b), convert. In (c), add and give your answer in centimeters.

a. $0.5 \text{ cm} =$ _____ mmb. $7 \text{ mm} =$ _____ cmc. $5 \text{ mm} + 0.9 \text{ cm} =$ _____ cm $1.2 \text{ cm} =$ _____ mm $35 \text{ mm} =$ _____ cm $4 \text{ cm} + 3.4 \text{ cm} =$ _____ cm

9. The two sides of a rectangle measure 6.5 cm and 3.6 cm.
Draw the rectangle on blank paper. What is its perimeter?