

math

MAMMOTH

Grade 5-B Worktext

Graphing and statistics

Fractions: add
and subtract

Fractions:
multiply and
divide

Geometry



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Foreword

Math Mammoth Grade 5-B Worktext comprises a complete math curriculum for the second half of fifth grade mathematics. Fifth grade focuses on fractions and decimals, in particular. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, and decimal arithmetic. In this part, 5-B, we study common statistical graphs, fraction arithmetic, and geometry.

This book starts with chapter 5, where we study graphing in a coordinate grid, line and bar graphs, and average and mode. Today's world has become increasingly complex with lots of data presented in the media, so our children need a good grasp of statistical graphs to be able to make sense of all of that information.

Chapter 6 is about the addition and subtraction of fractions—another topic of focus for 5th grade, besides decimals. The most difficult topic of this chapter is adding and subtracting unlike fractions, which is done by first converting them to equivalent fractions with a common denominator.

In chapter 7, we study the multiplication and division of fractions (division only in special cases), relying first on visual models, and then proceeding to the abstract shortcuts.

Chapter 8 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success with teaching math!

Maria Miller, the author

Chapter 5: Statistics and Graphing

Introduction

This chapter starts out with a study of the coordinate grid, but only in the first quadrant. Besides learning how to plot points, students also plot ordered pairs (points) from number patterns or rules. This is actually the beginning of the study of *functions*.

Practicing the use of the coordinate grid is a natural “prelude” to the study of line graphs, which follows next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories (“bins”). The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study average (also called the *mean*) and mode, and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is plenty of time to learn that concept in subsequent grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all a lot of students are able to grasp from that is only the calculation procedures. I feel it is better to introduce and contrast initially only the two concepts, the mean and the mode, in order to give the student a solid foundation. We will introduce the median later, and then compare and contrast it with the other two.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 5

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Coordinate Grid	10	3 pages
Number Patterns in the Coordinate Grid	13	4 pages
More Number Patterns in the Coordinate Grid	17	4 pages
Line Graphs	21	4 pages
Reading Line Graphs	25	2 pages
Double and Triple Line Graphs	27	2 pages
Making Bar Graphs	29	2 pages
Making Histograms	31	2 pages
Double Bar Graphs	33	2 pages

Average (Mean)	35	3 pages
Mean, Mode, and Bar Graphs	38	2 pages
Statistics Project (optional)	40	1 page
Mixed Review	41	3 pages
Review	44	2 pages

Helpful Resources on the Internet

COORDINATE GRID

Billy Bug Game

Click on the arrow keys to guide Billy to the coordinates of the hidden grub. How long will it take you to feed Billy ten times?

<http://resources.oswego.org/games/BillyBug/bugcoord.html>

Soccer Coordinates Game

Plot the coordinates on the coordinate grid correctly to block the soccer ball from entering the goal.

<http://www.xpmath.com/forums/arcade.php?do=play&gameid=90>

Coordinate Grid Quiz from ThatQuiz.org

Practice plotting a point and giving the coordinates of a given point (in the first quadrant).

<https://www.thatquiz.org/tq-7/?-j48-l5-p0>

Function Machine

Enter a rule in the function machine. Then enter various x-values to find their corresponding y-values, and generate a table and a graph of the function.

http://hotmath.com/util/hm_flash_movie.html?movie=/learning_activities/interactivities/function_machine.swf

Function Machine

What's the rule? Enter your own values or let the computer decide for you.

<http://www.mathplayground.com/functionmachine.html>

Number Pattern Tables

Apply the rule to find the missing number in the table.

<https://www.studyladder.com/games/activity/number-pattern-tables--20584>

Interpret Relationships Between Number Patterns

Generate patterns using given rules, identify relationships between terms, and graph ordered pairs consisting of corresponding terms from the patterns.

<https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/number-patterns/e/visualizing-and-interpreting-relationships-between-patterns>

Graph a Two-Variable Relationship

Practice identifying relationships between variables with this interactive exercise.

<https://www.ixl.com/math/grade-5/graph-a-two-variable-relationship>

GRAPHING AND GRAPHS

Easy Practice Problems for Reading Bar Graphs

First, customize your bar chart. Then, click on the buttons on the left side to get questions to answer.

<http://www.topmarks.co.uk/Flash.aspx?f=barchartv2>

Graphs Quiz from That Quiz.org

Questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, box plot, scatter graph). You can modify the quiz parameters to your liking.

<http://www.thatquiz.org/tq-5/math/graphs>

Survey Game

First, ask children their favorite hobby or color. Then, make a frequency table, a bar graph, and a pictogram from the results.

<http://www.kidsmathgamesonline.com/numbers/mathdata.html>

Line Graphs Quiz

Answer the questions about the line graph in this multiple-choice quiz.

<http://www.studyzone.org/testprep/math4/d/linegraph4p.cfm>

Line Graphs Quiz

Answer the questions about the line graph in this interactive 10-question quiz.

<http://www.thatquiz.org/tq-5/?-j10f14-l5-p0>

Creating Histograms at Khan Academy

Use the given data to create a histogram in this interactive exercise.

<https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/histograms/e/creating-histograms>

Create Double Bar Graphs Using Tables

Use the data in the table to complete the bar graph in this interactive exercise.

<http://www.mathgames.com/skill/6.96-create-double-bar-graphs-using-tables>

Interpret Double Bar Graphs

Read the double bar graphs and answer the questions in this interactive quiz.

<http://www.mathgames.com/skill/6.95-interpret-double-bar-graphs>

Statistics Interactive Activities from Shodor

A set of interactive tools for exploring and creating different kinds of graphs and plots. You can enter your own data or explore the examples.

<http://www.shodor.org/interactivate/activities/BarGraph/>

<http://www.shodor.org/interactivate/activities/Histogram/>

<http://www.shodor.org/interactivate/activities/CircleGraph/>

<http://www.shodor.org/interactivate/activities/MultiBarGraph/>

Math Goodies Interactive Data and Graphs Lessons

Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs, and exercises.

http://www.mathgoodies.com/lessons/toc_vol11.html

Interactive tool for creating graphs

Customize your own bar graph, line graph, or pie chart using this interactive tool.

<https://www.mathsisfun.com/data/data-graph.php>

Create a Graph

Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.

<http://nces.ed.gov/nceskids/createagraph/default.aspx>

Data Grapher

Use this tool to create bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.

<http://illuminations.nctm.org/Activity.aspx?id=4098>

MEAN, MEDIAN, MODE, AND RANGE

Math - Elephants - Line Graphs & Mean

Interactive exercises for interpreting a line graph, drawing a line graph, and calculating the mean.

<http://www.e-learningforkids.org/math/lesson/elephants-plant-line-graphs-mean/>

Mean/Mode Quiz

A 10-question quiz about calculating the mode and the mean.

<http://www.thatquiz.org/tq-p-z1/?-j6g00-l5-p0>

The Mean Machine

Use this interactive tool to see how average is calculated.

<http://www.mathsisfun.com/data/mean-machine.html>

Quiz: Finding the Mean of a Set of Numbers

Practice calculating the mean for a simple data set. This quiz helps to clarify the definition of mean as it relates to median, mode and range.

<http://www.turtlediary.com/game/finding-the-mean-of-set-of-numbers.html>

Study Jams: Mode

This site gives step-by-step illustrations of how to find the mode for a set of data.

<http://studyjams.scholastic.com/studyjams/jams/math/data-analysis/mode.htm>

Mean, Median, and Mode

How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.

http://www.cimt.org.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Using and Handling Data

Simple explanations for finding the mean, median, or mode.

<http://www.mathsisfun.com/data/central-measures.html>

Measures Activity

Enter your own data and the program will calculate the mean, median, mode, range, and some other statistical measures.

<http://www.shodor.org/interactivate/activities/Measures>

Coordinate Grid

This is a **coordinate grid**.

The long black line across the bottom, with the “x” near its end, is called the **x-axis**.

The vertical line that has “y” near the top is called the **y-axis**.

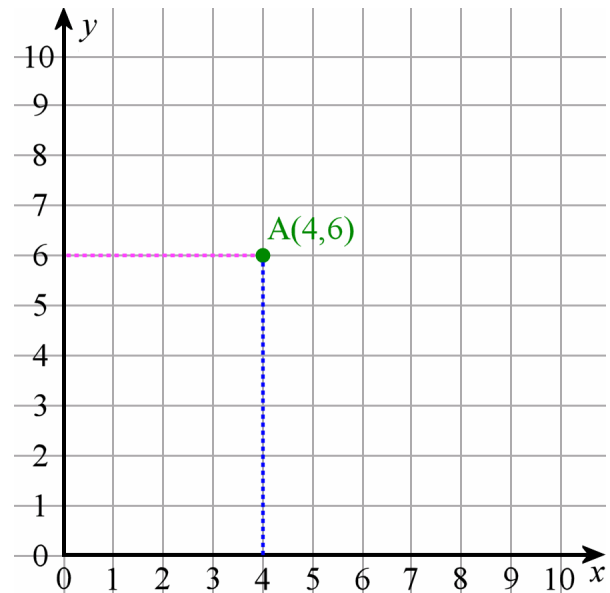
You can see one point, called “A,” that is drawn or *plotted* on the grid.

It has two numbers *associated*, or matched, with it. Those two numbers are called the **coordinates** of the point A.

The first number is called the **x-coordinate** of the point A, and the second number is called the **y-coordinate** of the point A.

The x-coordinate of the point A is 4 because if you drew a line straight down from A, it would *intersect*, or “hit,” the x-axis at 4. The y-coordinate of the point A is 6 because if you drew a line straight left from A, it would intersect the y-axis at 6.

We write the two coordinates of a point inside parentheses, separated by a comma.



Note: The order of the two coordinates matters. The *first* number is ALWAYS the x-coordinate, and the *second* number is ALWAYS the y-coordinate, not the other way around. So (5, 8) means that the x-coordinate is 5 and the y-coordinate is 8.

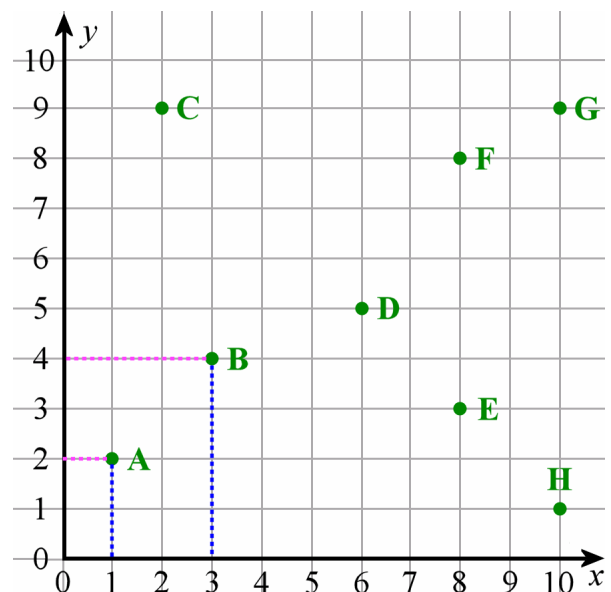
1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in.

A (__, __) B (__, __)

C (__, __) D (__, __)

E (__, __) F (__, __)

G (__, __) H (__, __)

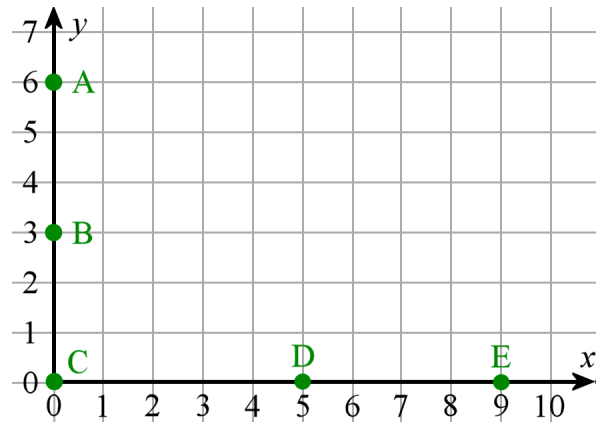


Notice especially the points that are located on the two axes.

If a point lies on the y -axis, its x -coordinate is zero.
A is $(0, 6)$ and B is $(0, 3)$.

If the point lies on the x -axis, its y -coordinate is zero.
D is $(5, 0)$ and E is $(9, 0)$.

The point C has the coordinates $(0, 0)$.
This point $(0, 0)$ is called the **origin**.

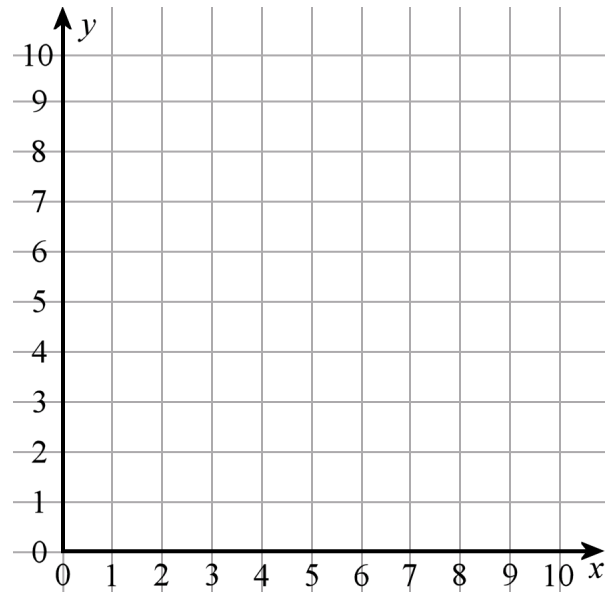


2. Plot and label the following points on the coordinate grid.

A $(2, 8)$ B $(0, 5)$ C $(4, 0)$

D $(9, 10)$ E $(8, 5)$ F $(1, 4)$

G $(1, 0)$ H $(0, 8)$ I $(3, 7)$



3. The coordinate grid is very useful for many things. For example, computer drawing programs use it frequently. Let's say "LINE $(5, 6) - (2, 7)$ " means a straight line segment that is drawn from the point $(5, 6)$ to the point $(2, 7)$.

Draw the following line segments.
What figure is formed?

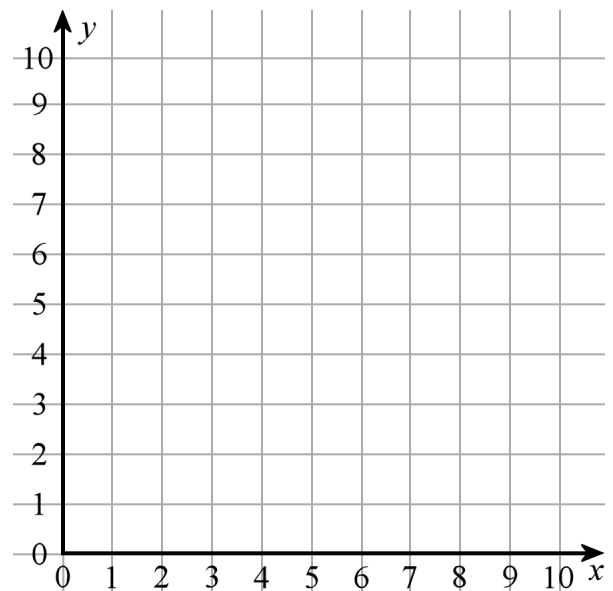
LINE $(1, 0) - (7, 0)$ LINE $(7, 0) - (7, 5)$

LINE $(1, 0) - (1, 5)$ LINE $(1, 5) - (0, 5)$

LINE $(0, 5) - (4, 7)$ LINE $(4, 7) - (8, 5)$

LINE $(8, 5) - (7, 5)$ LINE $(3, 0) - (3, 3)$

LINE $(5, 0) - (5, 3)$ LINE $(3, 3) - (5, 3)$



This example shows point A moving four units down and then two units to the right. The new location is called point A' (read "A prime").

Originally A's coordinates were (1, 6).

After the movement, the coordinates are (3, 2)

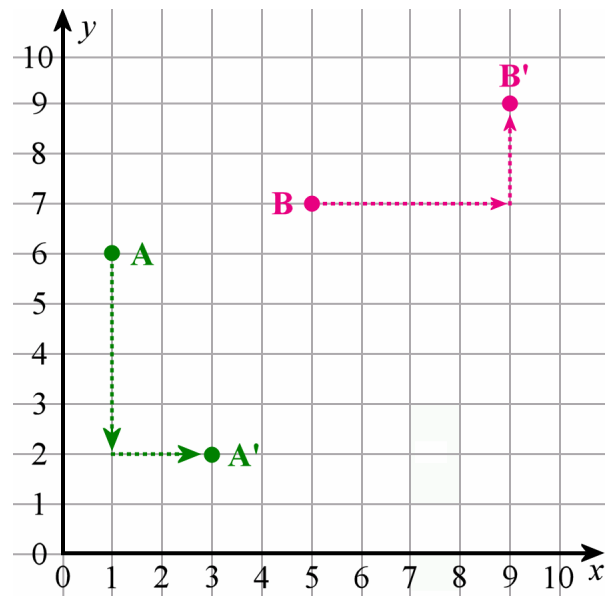
Notice how you can just subtract four units from the y -coordinate (the movement four units straight down) and add two units to the x -coordinate (movement two units to the right).

Point B is originally at (5, 7). It moves four units to the right and two up. You add four to the x -coordinate, and two to the y -coordinate. Its new coordinates are (9, 9).

Movement up or down affects the y -coordinate.

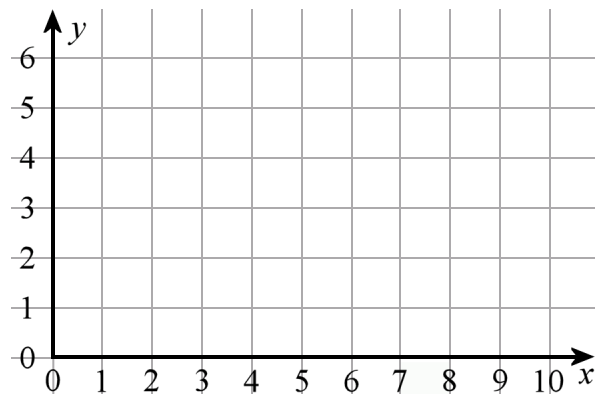
Movement right or left affects the x -coordinate.

In other words, movement *parallel* to an axis affects that same coordinate.



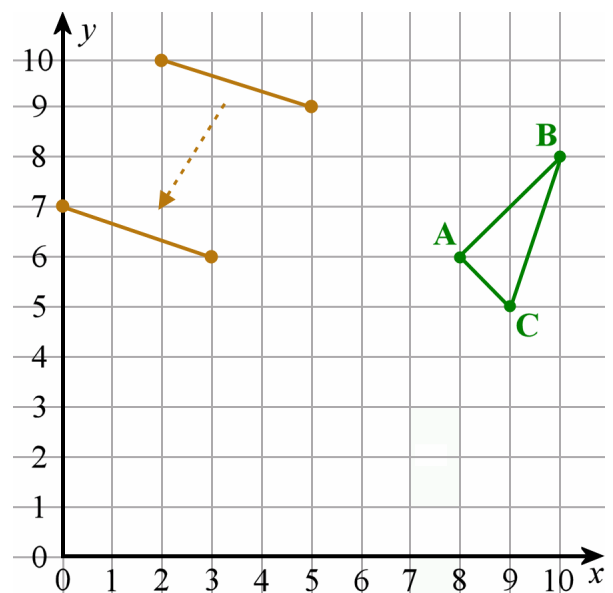
4. The three vertices of a triangle are (2, 0), (5, 1) and (3, 4). The triangle is moved three units to the right and two up.

- Plot the vertices of the triangle before and after the movement.
- Write the coordinates of the vertices after the movement.



5. a. Determine how the line segment has been moved, and move the triangle ABC the same way. Let's call the new triangle A'B'C'. Write the coordinates of the vertices of the triangle A'B'C' after the movement.

- Let's say the point (3, 5) moves to (2, 7). Move the triangle ABC in a similar way. Write the coordinates of the triangle's vertices after the movement.



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Mean, Mode, and Bar Graphs

Do you think you could calculate the average of the data shown in the bar graph? After all, there are numbers involved.

Actually, we cannot. To see why, think *what kind of data* produced this graph originally. What were the people in the study asked? What did they respond?

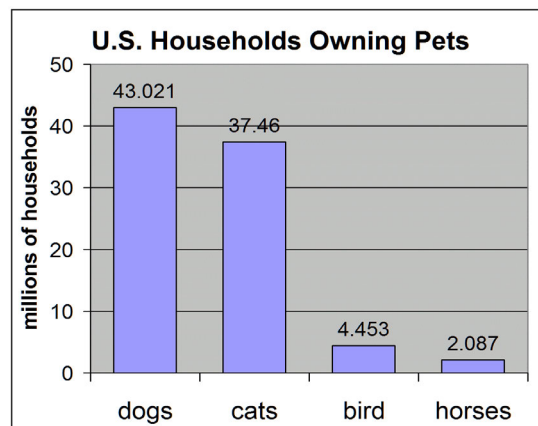
The people were asked something like, “What pets do you have?” The people would have answered, “cat,” “dog,” “fish,” and similar.

The original data set consists simply of the words “cat,” “dog,” “bird,” and “horse”—each one listed many times, because each word corresponds to the answer of one particular household.

cat, cat, dog, dog, dog, bird, dog, dog, bird, cat, dog, horse, dog, cat, dog....

We cannot calculate anything from this kind of data set because it is **not numerical data**. The only thing we can do is to determine the most commonly occurring item—the **mode**.

In this case, the mode is *dog*. You can see that from the graph: the tallest bar is for dogs.



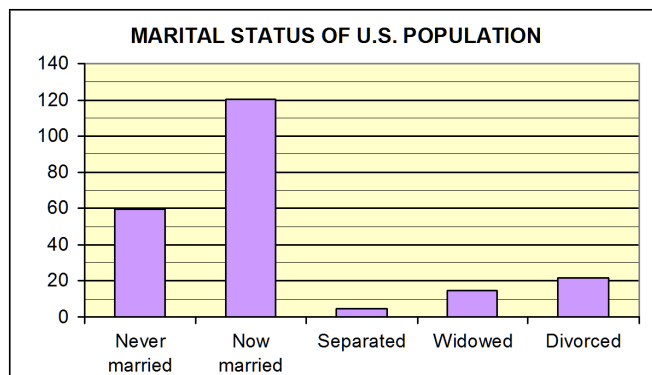
Mode is the most commonly occurring item in a data set.

- Sometimes a set of data has two or more modes. For example, the data set *green, green, blue, blue, black, brown, hazel* has two modes: both green and blue are equally common.
- If none of the items occurs twice or more, there is no mode. For example, this data *green, blue, pink, red, black, brown, purple* has no mode.

1. Find the mode of the data set shown in the bar graph on the right.

2. a. Find the mode of this data:

water, pop, juice, pop, juice, water,
milk, water, pop, pop, juice, pop

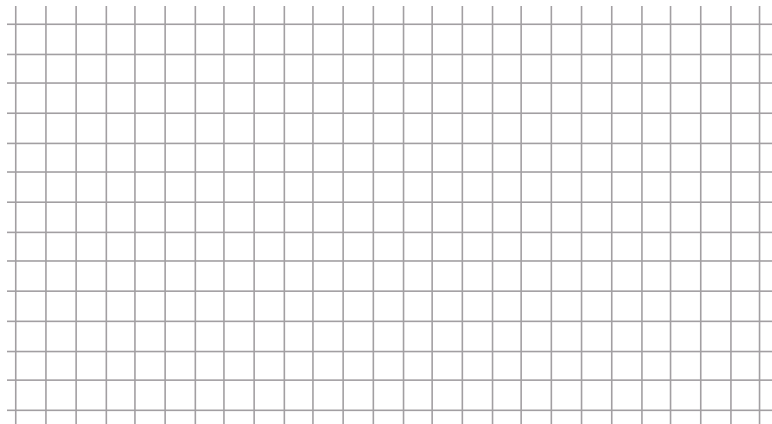


b. If the above words are the answers of 12 people to some question, what could have been the question?

3. Nineteen children were asked about their favorite ice cream flavor. Here are their responses:

strawberry, vanilla, chocolate, vanilla, chocolate chip, chocolate, pecan, pecan, vanilla, vanilla, strawberry, chocolate chip, vanilla, chocolate, chocolate, vanilla, strawberry, chocolate chip, vanilla.

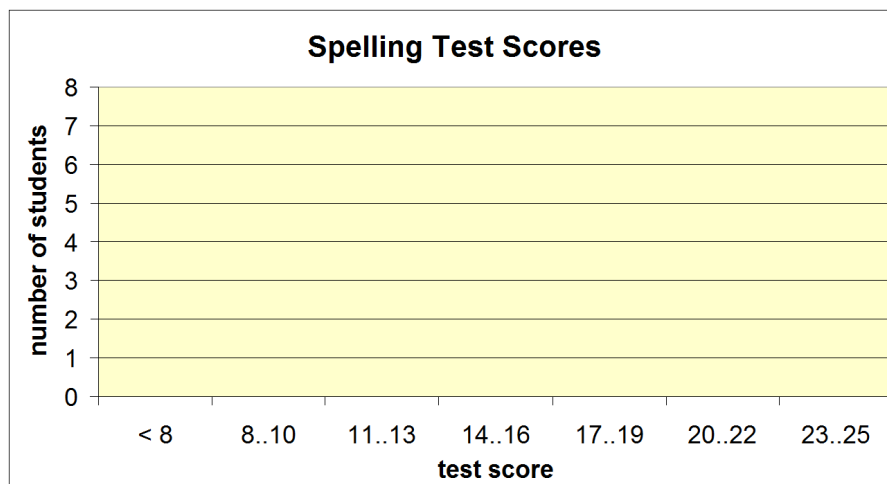
- Find the mode.
- Draw a bar graph.
- If possible, calculate the mean.



4. These are the spelling test scores of a fifth grade class:
4 5 7 9 9 10 10 11 11 12 12 12 13 14 17 18 18 18 19 19 19 20 24 25

- Find the mode.
- Draw a bar graph.
- If possible, calculate the mean.

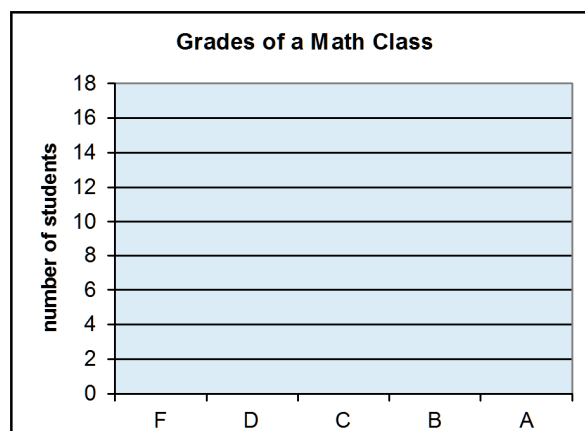
Test Score	Frequency
< 8	
8..10	
11..13	
14..16	
17..19	
20..22	
23..25	



5. a. Find the mode.
- b. Draw a bar graph.
- c. If possible, calculate the average.
- d. There were ____ students in all. What *fraction* of the students got grade B?

Grades of a math class

Grade	Frequency
F	3
D	8
C	12
B	17
A	10



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Chapter 6: Fractions: Add and Subtract

Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade. I hope that students have already built a solid conceptual understanding in their minds in previous years, so we can build on that foundation.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a “stumbling block.” Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to “tackle” the more complex situation of *unlike* fractions.

First, we review how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a “brute force” method based on converting fractions to equivalent fractions. This chapter ends with a lesson on measuring in inches, using units as small as $1/16$ of an inch.

The Lessons in Chapter 6

	<i>page</i>	<i>span</i>
Fraction Terminology	50	1 page
Review: Mixed Numbers	51	4 pages
Adding Mixed Numbers	55	3 pages
Subtracting Mixed Numbers 1	58	4 pages
Subtracting Mixed Numbers 2	62	2 pages
Equivalent Fractions 1	64	3 pages
Equivalent Fractions 2	67	2 pages
Adding and Subtracting Unlike Fractions	69	3 pages
Finding the (Least) Common Denominator	72	3 pages
Add and Subtract: More Practice	75	3 pages
Adding and Subtracting Mixed Numbers	78	5 pages
Comparing Fractions	83	4 pages
Measuring in Inches	87	5 pages
Mixed Review	92	3 pages
Review	95	2 pages

Helpful Resources on the Internet

MIXED NUMBERS

Clara Fraction Ice Cream Shop

Convert improper fractions to mixed numbers, while scooping various ice cream flavors onto the cone.

<http://mrnussbaum.com/clarafraction/>

Fraction Models

Explore improper fractions, mixed numbers, decimals, and percentages using several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the models.

<http://illuminations.nctm.org/Activity.aspx?id=3519>

Fractions Workshop

Choose “Add mixed fractions with like denominators” and the number of problems you would like to do.

<http://mrnussbaum.com/fractions-workshop-ipad.html>

Subtracting Mixed Fractions Quiz (Like Denominators)

Drag and drop each answer to the corresponding subtraction problem.

<http://www.fractions4kids.com/subtracting-mixed-fractions-quiz/>

Subtracting Mixed Numbers with Borrowing

Learn how to borrow mixed fractions with this animation.

<https://www.wisc-online.com/learn/formal-science/mathematics/abm701/subtracting-mixed-number-fractions-with-borro>

EQUIVALENT FRACTIONS

Equivalent Fractions

You are given a fraction that is shown with a visual model and on a number line, and you need to construct two *other* fractions that are equivalent to the given fraction. Drag two sliders to choose the denominators for your fractions and then click pieces to color them.

<http://illuminations.nctm.org/Activity.aspx?id=3510>

Fresh Baked Fractions

Practice equivalent fractions by clicking on a fraction that is not equal to others.

<http://www.funbrain.com/fract/>

Triplets: Equivalent Fractions

Sort the space teams by equivalent fractions to make sure all the athletes get to the correct starting place before the games begin.

<http://www.mathplayground.com/Triplets/Triplets.html>

Fraction Dolphins

Click on the dolphin with the correct equivalent fraction to the fraction on the bucket of fish.

<http://mrnussbaum.com/fraction-dolphins-ipad.html>

Fraction Worksheets: Equivalent Fractions with Visual Models

Create custom-made worksheets for equivalent fractions. Choose to include pie images or not.

http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers

Create custom-made worksheets for these fraction operations.

<http://www.homeschoolmath.net/worksheets/fraction-b.php>

Sample worksheet from

www.mathmammoth.com

ADDITION AND SUBTRACTION

Fraction Videos 1: Addition and Subtraction

A set of videos by the author that cover topics in this chapter.

http://www.mathmammoth.com/videos/fractions_1.php

Adding Fractions with Uncommon Denominators Tool at Conceptua Fractions

A tool that links a visual model to the procedure of adding two unlike fractions.

<https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators>

Add Unlike Fractions with Number Line Models

Practice adding unlike fractions. Click “EXPLAIN” to see a visual illustration and the answer.

<http://www.visualfractions.com/AddUnlike/>

Drop Zone

Practice making a sum of one using fractions in this interactive online activity.

<https://www.brainpop.com/games/dropzone/>

Add Mixed Numbers with Unlike Denominators - Quiz

Use this simple online quiz for extra practice.

<http://www.mathgames.com/skill/5.72-add-mixed-numbers-with-unlike-denominators>

Fruit Shoot Fractions

This game practices addition of fractions. There are several different levels to choose from.

<http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.htm>

Fruit Splat

Practice finding the least common denominator. This game has three different levels to choose from.

<http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonDenominator.htm>

Fraction Word Problems

Practice adding and subtracting fractions with these interactive word problems.

<http://mrnussbaum.com/grade5standards/568-2/>

Math Balloons: Fractions

Answer whether the fraction additions are true or false in this timed activity.

<http://www.mathnook.com/math/math-balloons-fractions.html>

Fraction Bars Blackjack

The computer gives you two fraction cards. You have the option of getting more or “holding”. The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.

http://fractionbars.com/Fraction_Bars_Black_Jack/

Old Egyptian Fractions

Puzzles to solve: add fractions like a true Old Egyptian Math Cat!

<http://www.mathcats.com/explore/oldegyptianfractions.html>

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division

Create custom-made worksheets for the four operations with fractions and mixed numbers.

<http://www.homeschoolmath.net/worksheets/fraction.php>

ORDERING AND COMPARING

Comparing Fractions Tool at Conceptua Fractions

An interactive tool where students place numbers, visual models, and decimals on a number line.

<http://www.conceptuamath.com/app/tool/comparing-fractions>

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.

<http://www.fuelthebrain.com/games/comparison-shootout/>

Comparing Fractions—XP Math

Simple timed practice with comparing two fractions.

<http://xpmath.com/forums/arcade.php?do=play&gameid=8>

Visual Fractions Game

Find a Fraction Between Two Given Fractions with the help of this visual tool.

http://www.mathplayground.com/visual_fractions.html

Fractional Hi Lo

The computer has selected a fraction. You make guesses and it tells if your guess was too high or low.

<http://www.theproblemsite.com/games/hilo.asp>

My Closest Neighbor

A neat card game where you need to make a fraction that is as close as possible to the given fraction.

<https://denisegaskins.com/2014/08/06/fraction-game-my-closest-neighbor/>

Comparing/Ordering Fractions Worksheets

Create customizable worksheets for comparing or ordering fractions. You can include pie images.

http://www.homeschoolmath.net/worksheets/comparing_fractions.php

MEASURING & GENERAL

Measure It!—Practice measuring lines in inches.

<http://onlineintervention.funbrain.com/measure/>

Measuring—Practice measuring with a virtual ruler. Choose the category “Inches, Sixteenths”.

<http://www.abcya.com/measuring.htm>

Sal’s Sub Shop—Cut the subs to the given measurements.

<http://mrnussbaum.com/sal/>

Fraction Word Problems

This is a set of 10 interactive word problems with multiple-choice answers involving mixed numbers.

<http://mrnussbaum.com/grade5standards/572-2>

Who Wants Pizza?—A tutorial and interactive exercises about fraction addition and multiplication.

<http://math.rice.edu/~lanius/fractions/>

Fraction Lessons—Tutorials, examples, and videos explaining all the basic fraction topics.

<http://www.mathexpression.com/learning-fractions.html>

Online Fraction Calculator

http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Sample worksheet from
www.mathmammoth.com

Fraction Terminology

As we study fractions and their operations, it is important that you understand the terms, or words, that we use. This page is for reference. You can even post it on your wall or make your own fraction poster based on it.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.
The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\frac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

Like fractions have the same denominator.
They have the same kind of parts.

It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$

and



$\frac{7}{9}$

are like fractions.

Unlike fractions have a different denominator.
They have different kinds of parts.

It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.



$\frac{2}{9}$

and



$\frac{3}{4}$

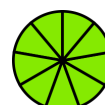
are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). $\frac{2}{9}$ is a proper fraction.



$\frac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). It is a *fraction*, so it is written as a fraction and *not* as a mixed number.



$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value.

If you think in terms of pies, they have the same amount of “pie to eat,” but they are written using different denominators, or are “cut into different kinds of slices.”



$\frac{3}{9}$

and



$\frac{1}{3}$

are equivalent fractions.

Simplifying or reducing a fraction means that, for a given fraction, you find an equivalent fraction that has a “simpler,” or smaller, numerator and denominator. (It has fewer but bigger slices.)



$\frac{9}{12}$

simplifies to

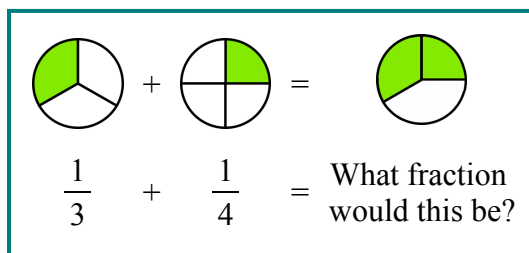
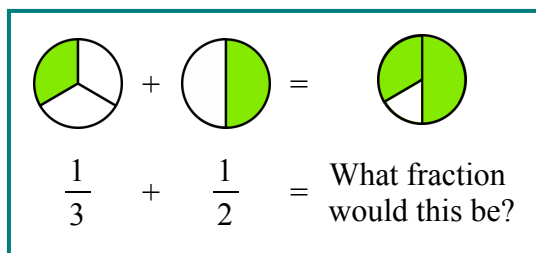


$\frac{3}{4}$

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Adding and Subtracting Unlike Fractions

Cover the page below the black line. Then try to figure out the addition problems below.






























































$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$	$\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$	<p>Did you solve the problems above?</p> <p>The solution is this:</p> <p>We convert the fractions so that they become <i>like</i> fractions (with a same denominator), using equivalent fractions.</p> <p>Then we can add or subtract.</p>
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- Write the fractions shown by the pie images. Convert them into equivalent fractions with the same denominator (like fractions), and then add them. Color the missing parts.

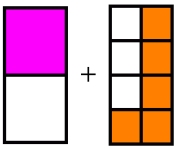
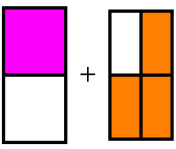
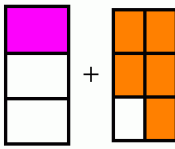
<p>a.</p> $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$	<p>b.</p> $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$	<p>c.</p> $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$
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2. Convert the fractions to like fractions first, then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!

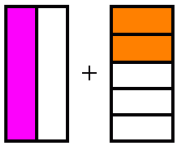
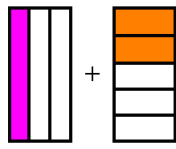
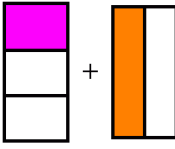
<p>a.  + </p> $\frac{1}{2} + \frac{1}{6}$ <p style="text-align: center;">↓ ↓</p>  +  =   + $\frac{1}{6}$ = 	<p>b.  + </p> $\frac{1}{8} + \frac{1}{4}$ <p style="text-align: center;">↓ ↓</p>  +  =  $\frac{1}{8} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$	<p>c.  + </p> $\frac{1}{6} + \frac{1}{4}$ <p style="text-align: center;">↓ ↓</p>  +  =   +  = 
<p>d. $\frac{5}{6} - \frac{1}{2}$</p> <p style="text-align: center;">↓ ↓</p> $\frac{5}{6} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$	<p>e. $\frac{5}{8} - \frac{1}{4}$</p> <p style="text-align: center;">↓ ↓</p>  -  = 	<p>f. $\frac{5}{6} - \frac{1}{4}$</p> <p style="text-align: center;">↓ ↓</p>  -  = 

<p>g.  + </p> $\frac{1}{2} + \frac{1}{8}$ <p style="text-align: center;">↓ ↓</p>  +  =   +  = 	<p>h.  + </p> $\frac{3}{10} + \frac{1}{5}$ <p style="text-align: center;">↓ ↓</p>  +  =   +  = 	<p>i.  + </p> $\frac{2}{5} + \frac{1}{2}$ <p style="text-align: center;">↓ ↓</p>  +  =   +  = 
<p>j. $\frac{1}{2} + \frac{3}{8}$</p> <p style="text-align: center;">↓ ↓</p>  +  = 	<p>k. $\frac{9}{10} - \frac{2}{5}$</p> <p style="text-align: center;">↓ ↓</p>  -  = 	<p>l. $\frac{4}{5} - \frac{1}{2}$</p> <p style="text-align: center;">↓ ↓</p>  -  = 

3. Split the parts only in the *first* fraction so that both fractions will have the same kind of parts. Add.

 <p>a. $\frac{\quad}{8} + \frac{5}{8} =$</p>	 <p>b. $\frac{\quad}{4} + \frac{3}{4} =$</p>	 <p>c. $\frac{\quad}{6} + \frac{5}{6} =$</p>
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Now split the parts in *both* fractions so that they will have the same kind of parts. Add.

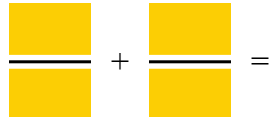

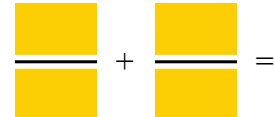
 <p>d. $\frac{\quad}{10} + \frac{\quad}{10} =$</p>	 <p>e. $\frac{\quad}{15} + \frac{\quad}{15} =$</p>	 <p>f. $\frac{\quad}{10} + \frac{\quad}{10} =$</p>
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4. Fill in the table based on the problems above. What kind of parts did the two fractions have at first? What kind of parts did you use in the final addition?

Types of parts:	Converted to:	Types of parts:	Converted to:
a. 2nd parts and 8th parts	<u>8th</u> parts	d. 2nd parts and 5th parts	_____ parts
b. 2nd parts and 4th parts	_____ parts	e. 3rd parts and 5th parts	_____ parts
c. 3rd parts and 6th parts	_____ parts	f. 3rd parts and 2nd parts	_____ parts

5. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table above?

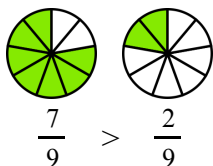
6. Challenge: If you think you know what kind of parts to convert these fractions into, then try these problems. Do not worry if you don't know how to do them—we will study this in the next lesson.

<p>a. $\frac{1}{2} + \frac{2}{3}$</p> <p style="text-align: center;">↓ ↓</p>  <p>$\frac{\quad}{6} + \frac{\quad}{6} =$</p>	<p>b. $\frac{2}{3} - \frac{2}{5}$</p> <p style="text-align: center;">↓ ↓</p>  <p>$\frac{\quad}{15} - \frac{\quad}{15} =$</p>	<p>c. $\frac{1}{3} + \frac{3}{4}$</p> <p style="text-align: center;">↓ ↓</p>  <p>$\frac{\quad}{12} + \frac{\quad}{12} =$</p>
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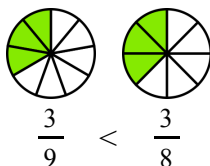
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Comparing Fractions

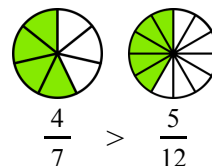
Sometimes it is easy to know which fraction is the greater of the two. Study the examples below!



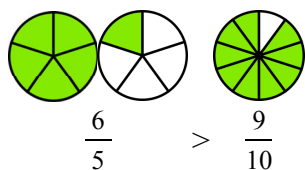
With **like fractions**, all you need to do is to check **which fraction has more “slices,”** and that fraction is greater.



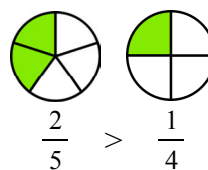
If both fractions have the **same number of pieces**, then the one with bigger pieces is greater.



Sometimes you can **compare to 1/2**. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.



Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here, 6/5 is more than 1, and 9/10 is less than 1.



If you can imagine the pie pictures in your mind, you can sometimes “see” which fraction is bigger. For example, it is easy to see that 2/5 is more than 1/4.

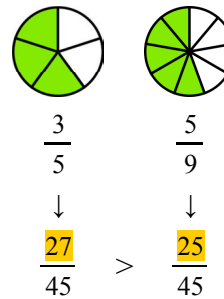
1. Compare the fractions, and write $>$, $<$, or $=$.

a. $\frac{1}{8}$ $\frac{1}{10}$	b. $\frac{4}{9}$ $\frac{1}{2}$	c. $\frac{6}{10}$ $\frac{1}{2}$	d. $\frac{3}{9}$ $\frac{3}{7}$
e. $\frac{8}{11}$ $\frac{4}{11}$	f. $\frac{7}{4}$ $\frac{7}{6}$	g. $\frac{5}{14}$ $\frac{5}{9}$	h. $\frac{4}{20}$ $\frac{2}{20}$
i. $\frac{2}{11}$ $\frac{2}{5}$	j. $\frac{1}{2}$ $\frac{5}{8}$	k. $\frac{3}{6}$ $\frac{1}{2}$	l. $\frac{1}{20}$ $\frac{1}{8}$
m. $\frac{1}{2}$ $\frac{3}{4}$	n. $\frac{8}{7}$ $\frac{3}{3}$	o. $\frac{49}{100}$ $\frac{61}{100}$	p. $\frac{7}{8}$ $\frac{8}{7}$
q. $\frac{9}{10}$ $\frac{3}{4}$	r. $\frac{6}{5}$ $\frac{3}{4}$	s. $\frac{4}{4}$ $\frac{9}{11}$	t. $\frac{1}{3}$ $\frac{3}{9}$









Sometimes none of the “tricks” explained in the previous page work, but we do have one more up our sleeve!

Convert both fractions into like fractions. Then compare.

In the picture on the right, it is hard to be sure if $\frac{3}{5}$ is really more than $\frac{5}{9}$. Convert both into 45th parts, and then it is easy to see that $\frac{27}{45}$ is more than $\frac{25}{45}$. Not by much, though!



2. Convert the fractions into like fractions, and then compare them.

<p>a.  $\frac{2}{3}$</p> <p> $\frac{5}{8}$</p> <p>↓ ↓</p>	<p>b.  $\frac{5}{6}$</p> <p> $\frac{7}{8}$</p> <p>↓ ↓</p>	<p>c.  $\frac{1}{3}$</p> <p> $\frac{3}{10}$</p> <p>↓ ↓</p>	<p>d.  $\frac{8}{12}$</p> <p> $\frac{7}{10}$</p> <p>↓ ↓</p>
<p>e. $\frac{5}{8}$</p> <p>↓</p>	<p>f. $\frac{11}{8}$</p> <p>↓</p>	<p>g. $\frac{6}{10}$</p> <p>↓</p>	<p>h. $\frac{6}{5}$</p> <p>↓</p>
<p>i. $\frac{7}{10}$</p> <p>↓</p>	<p>j. $\frac{43}{100}$</p> <p>↓</p>	<p>k. $\frac{9}{8}$</p> <p>↓</p>	<p>l. $\frac{7}{10}$</p> <p>↓</p>

3. One cookie recipe calls for $\frac{1}{2}$ cup of sugar. Another one calls for $\frac{2}{3}$ cup of sugar.
Which uses more sugar, a triple batch of the first recipe, or a double batch of the second?

How much more?

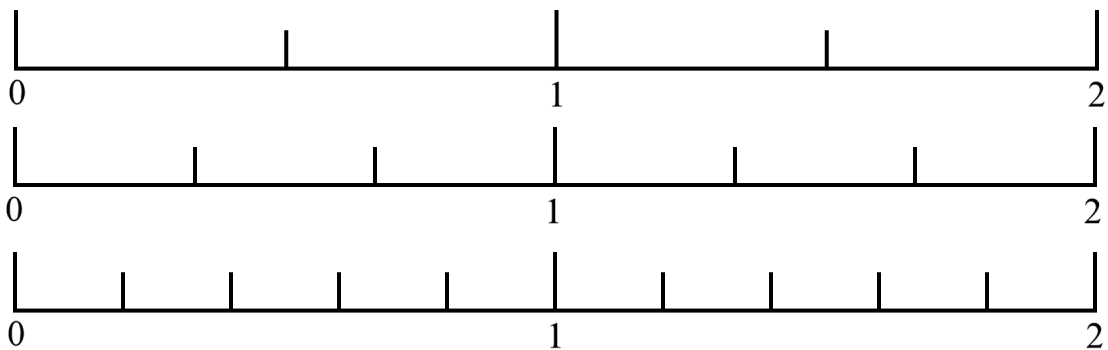
4. Compare the fractions using any method.

a. $\frac{5}{12}$ $\frac{3}{8}$	b. $\frac{5}{12}$ $\frac{4}{11}$	c. $\frac{3}{10}$ $\frac{1}{5}$	d. $\frac{3}{8}$ $\frac{4}{7}$
e. $\frac{4}{15}$ $\frac{1}{3}$	f. $\frac{5}{6}$ $\frac{11}{16}$	g. $\frac{7}{6}$ $\frac{10}{8}$	h. $\frac{5}{12}$ $\frac{5}{8}$
i. $\frac{3}{4}$ $\frac{4}{11}$	j. $\frac{13}{10}$ $\frac{9}{8}$	k. $\frac{2}{13}$ $\frac{1}{5}$	l. $\frac{1}{10}$ $\frac{1}{11}$

5. A coat costs \$40. Which is a bigger discount:
 $\frac{1}{4}$ off the normal price, or $\frac{3}{10}$ off the normal price?

Does your answer change if the original price
of the coat was \$60 instead? Why or why not?

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



a. $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

b. $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

_____ < _____ < _____ < _____ < _____

_____ < _____ < _____ < _____ < _____

7. Write the three fractions in order.

a. $\frac{7}{8}, \frac{9}{10}, \frac{7}{9}$

___ < ___ < ___

b. $\frac{1}{3}, \frac{4}{10}, \frac{2}{9}$

___ < ___ < ___

8. Rebecca made a survey of a group of 600 women. She found that $\frac{1}{3}$ of them never exercised, that $\frac{22}{100}$ of them swam regularly, $\frac{1}{5}$ of them jogged regularly, and the rest were involved in other sports.

a. Which was a bigger group, the women who jogged or the women who swam?

b. What fraction of this group of women exercise?

c. *How many women* in this group exercise?

d. How many women in this group swim?

The seven dwarfs could not divide a pizza into seven equal slices. The oldest suggested, "Let's cut it into eight slices, let each dwarf have one piece, and give the last piece to the dog."

Then another dwarf said, "No! Let's cut it into 12 slices instead, and give each of us $1\frac{1}{2}$ of those pieces, and the dog gets the $1\frac{1}{2}$ pieces left over."

Puzzle Corner



Which suggestion would give more pizza to the dog?

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Chapter 7: Fractions: Multiply and Divide

Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 8 on geometry in between chapters 6 and 7.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 6, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication $(2/3) \times 18$ km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that $5/3$ is the same as $5 \div 3$. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, $20 \div 6 = 3 \frac{2}{6}$).

Then students encounter sharing divisions with fractions. For example, if two people share equally $4/5$ of a pizza, how much will each person get? This is represented by the division $(4/5) \div 2 = 2/5$. Another case we study is dividing unit fractions by whole numbers (such as $(1/2) \div 4$).

We also divide whole numbers by unit fractions, such as $6 \div (1/3)$. Students will solve these thinking how many times the divisor “fits into” the dividend.

The last lesson is an introduction to ratios, and is optional. Ratios will be studied a lot in 6th and 7th grades, including in connection with proportions. We are laying the groundwork for that here.

The Lessons in Chapter 7

	<i>page</i>	<i>span</i>
Simplifying Fractions 1	102	4 pages
Simplifying Fractions 2	106	4 pages
Multiply Fractions by Whole Numbers	110	4 pages
Multiplying Fractions by Fractions	114	5 pages
Fraction Multiplication and Area	119	6 pages
Simplifying Before Multiplying	125	4 pages
Multiplying Mixed Numbers	129	5 pages
Multiplication as Scaling/Resizing	134	4 pages
Fractions Are Divisions	138	4 pages
Dividing Fractions 1: Sharing Divisions	142	4 pages
Dividing Fractions 2: Fitting the Divisor	147	4 pages
Introduction to Ratios	151	4 pages
Mixed Review	155	3 pages
Review	158	4 pages

Helpful Resources on the Internet

Fraction Videos 2: Multiplication and Division

My own videos that cover multiplying and dividing fractions.

http://www.mathmammoth.com/videos/fractions_2.php

REDUCING/SIMPLIFYING FRACTIONS

Canceling Demonstration

Watch a movie that uses circles to demonstrate how to rename to lowest terms with canceling.

<http://www.visualfractions.com/cancel/>

Reduce Fractions Shoot

Reduce the fraction on the screen to the lowest terms by clicking the correct answer.

http://www.sheppardsoftware.com/mathgames/fractions/reduce_fractions_shoot.htm

Fraction Worksheets: Simplifying and Equivalent Fractions

Create custom-made worksheets for fraction simplification and equivalent fractions.

<http://www.homeschoolmath.net/worksheets/fraction.php>

Reducing Fractions to Lowest Terms

This is a simple online exercise that you can use for extra practice.

<http://www.mathgames.com/skill/3.46-reducing-fractions-to-lowest-terms>

Fractions Booster

How much pizza is left? Be sure to reduce the answer down to lowest terms!

http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/fractions/level5.htm

Add and Subtract Fractions Game

Solve the given equation and gobble the correct answer as fast as possible, or you will lose to the computer!

<http://www.turtlediary.com/game/add-and-subtract-fractions.html>

Frosty Fractions

Add together the two fractions given. If the answer is available on the board, place a snowflake token over it. The winner is the first player to get a straight line of three snowflakes, either horizontally, vertically, or diagonally (this game is for two players).

<http://www.counton.org/games/map-fractions/frosty/>

FRACTION MULTIPLICATION

Multiply Fractions by Whole Numbers

Use this simple online exercise for additional practice as needed.

<http://www.mathgames.com/skill/4.67-multiply-fractions-by-whole-numbers>

Multiply Fractions and Whole Numbers

Practice multiplying a whole number times a fraction in this online exercise.

https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/multiplying-fractions-pre-alg/e/multiplying_fractions_by_integers

Interactive Model for the Multiplication of Fractions

In this interactive activity, you will see how to use area models to multiply fractions.

https://www.learner.org/courses/learningmath/number/session9/part_a/try.html

Fraction Multiplication TeacherTool

Students multiply two fractions together and use an area model to represent the product. Scroll down to “Fifth Grade Multiplication and Division” and click on “Fraction Multiplication 2”.

www.dreambox.com/teachertools

Soccer Math - Multiplying Fractions

Answer the multiple-choice fraction multiplication problems and play soccer in between the questions.

<http://www.math-play.com/soccer-math-multiplying-fractions-game/multiplying-fractions-game.html>

Snow Sprint Fractions

Practice fraction multiplication while participating in a snowmobile race!

http://www.mathplayground.com/ASB_SnowSprint.html

Multiply Fractions with Models

Use this simple online exercise for additional practice as necessary.

<http://www.mathgames.com/skill/5.109-multiply-fractions-with-models>

Product Fractions Card Activity (p. 36 of the PDF)

In this activity, players work in pairs to multiply fractions. This is not a “game”, as such, but rather an opportunity for students to work collaboratively and manipulate the problems.

http://www.pepnonprofit.org/uploads/2/7/7/2/2772238/acing_math.pdf

Fraction Multiplication Quiz

Practice multiplying like and unlike fractions in this 10-question interactive quiz.

<http://www.thegreatmartinicompany.com/Math-Quick-Quiz/fraction-multiply-quiz.html>

Multiply Mixed Numbers Quiz

Practice multiplying mixed numbers. Express the answers as mixed numbers and in lowest terms.

<https://www.thatquiz.org/tq-3/?-j304-la-p0>

Multiplying Fractions Word Problems

Solve and interpret fraction multiplication word problems in this interactive exercise from Khan Academy.

<https://www.khanacademy.org/math/arithmetic/fractions/multiplying-fractions-word-probl/e/multiplying-fractions-by-fractions-word-problems>

Fraction Multiplication as Scaling

Interpret how multiplying by a fraction greater or less than 1 affects the product in this interactive online exercise.

<https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/multiplying-fractions-pre-alg/e/fraction-multiplication-as-scaling>

Multiplying Fractions Word Problems

Practice multiplying mixed numbers with these interactive word problems.

<http://mrnussbaum.com/grade5standards/577-2/>

Who Wants Pizza?

A tutorial that explains fraction multiplication using a pizza, followed by some interactive exercises.

<http://math.rice.edu/~lanius/fractions/>

Multiply Fractions Jeopardy

Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.

<http://www.quia.com/cb/95583.html>

Multiplying Fractions

Multiply the fractions shown and reduce to the answer to the lowest terms.

http://www.mathplayground.com/fractions_mult.html

Fractions Mystery Picture Game

Solve problems where you find a fractional part of a quantity, and uncover a picture.

<http://www.dositey.com/2008/math/mistery2.html>

FRACTION DIVISION

Divide Fractions by Whole Numbers - Word Problems

These are simple word problems which can be used to reinforce the topic of division with fractions.

<http://www.mathgames.com/skill/5.94-divide-fractions-by-whole-numbers>

Dividing Unit Fractions by Whole Numbers

Use this simple interactive exercise to reinforce fraction division skills.

https://www.khanacademy.org/math/arithmetic/fractions/dividing-fractions-tutorial/e/dividing_fractions_0.5

Dividing Whole Numbers by Unit Fractions

Practice dividing a whole number by a unit fraction in this interactive exercise.

https://www.khanacademy.org/math/arithmetic/fractions/dividing-fractions-tutorial/e/dividing_fractions

Partitive Division of Fractions Tool

This tool provides you with a connection between a story problem (context), paraphrase, model, and procedure where a fractional value is shared equally. Students complete phrases like: $5\frac{4}{5}$ shared equally among 12. They use a double number line or an area model to visualize the answer, and they connect this understanding to the numeric procedure.

<https://www.conceptuamath.com/app/tool/divide-partitive>

Fractions as Divisions

In this educational video, Sal shows how a/b and $a\div b$ are equivalent. That is, the fraction bar and the division symbol mean the same thing.

<https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic/tcc-5th-fractions-as-division/v/fractions-as-division>

Fractions as Divisions Word Problems

Practice word problems that involve using the fraction bar as division in this interactive exercise.

<https://www.khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic/tcc-5th-fractions-as-division/e/understanding-fractions-as-division--word-problems>

Dividing Fractions: Word Problems

Solve word problems by dividing fractions by fractions in this interactive exercise.

<https://www.khanacademy.org/math/arithmetic/fractions/div-fractions-fractions/e/dividing-fractions-by-fractions-word-problems>

Seven Cookies for Grampy

Make seven whole cookies for Grampy by rearranging the fractional parts of other whole cookies.

<http://www.visualfractions.com/sevencookies/>

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division

Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.

<http://www.homeschoolmath.net/worksheets/fraction.php>

Thinking Blocks: Ratio Word Problems

Model and solve word problems with ratios using this interactive bar model tool.

http://www.mathplayground.com/NewThinkingBlocks/thinking_blocks_ratios.html

GENERAL

Visual Fractions

A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division.

<http://www.visualfractions.com/>

Conceptua Math Fractions Tools

Free and interactive fraction tools. Each activity uses several models, such as circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions.

<http://www.conceptuamath.com/app/tool-library>

Fraction Lessons at MathExpression.com

Tutorials, examples, and videos explaining all the basic fraction topics.

<http://www.mathexpression.com/learning-fractions.html>

Online Fraction Calculator

Add, subtract, multiply, or divide fractions and mixed numbers.

http://www.homeschoolmath.net/worksheets/fraction_calculator.php

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Multiplying Fractions by Fractions

We have studied how to find a fractional part of a whole number using multiplication.



For example, $\frac{3}{5}$ of 80 is written as the multiplication $\frac{3}{5} \times 80 = \frac{240}{5} = 48$.

REMEMBER: The word “of” translates here into **multiplication**.

We can use the same idea to find a fractional part of a fraction.















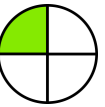

Example 1. One-half of  is .

As a multiplication, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Example 2. One-fourth of  is .

As a multiplication, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

<p>a. Find $\frac{1}{2}$ of </p> <p>$\frac{1}{2} \times \frac{1}{4} =$</p>	<p>b. Find $\frac{1}{2}$ of </p> <p> \times  $=$ </p>	<p>c. Find $\frac{1}{2}$ of </p> <p> \times  $=$ </p>
<p>d. Find $\frac{1}{3}$ of </p>	<p>e. Find $\frac{1}{3}$ of </p>	<p>f. Find $\frac{1}{3}$ of </p>
<p>g. Find $\frac{1}{4}$ of </p>	<p>h. Find $\frac{1}{4}$ of </p>	<p>i. Find $\frac{1}{4}$ of </p>
<p>Did you notice a shortcut? If so, calculate $\frac{1}{5} \times \frac{1}{6} =$ </p>		

Shortcut: multiplying fractions of the type $1/n$

To multiply fractions of the form $1/n$ where n is a whole number, simply multiply the denominators to get the new denominator.

$$\rightarrow \frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \quad \text{or} \quad \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

2. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$

b. $\frac{1}{13} \times \frac{1}{3}$

c. $\frac{1}{5} \times \frac{1}{20}$

We have now studied how to find $1/2$ or $1/3$ or $1/5$ of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

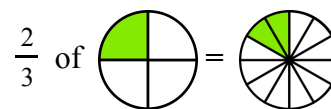
Review: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part of another fraction.

Example 3. Find $\frac{2}{3}$ of $\frac{1}{4}$. First, we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$.

Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.



Example 4. Find $\frac{4}{5}$ of $\frac{1}{7}$.

First, we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part of the fraction.
It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Color in a picture to show the answer.

a. $\frac{3}{4} \times \frac{1}{2} =$

b. $\frac{2}{3} \times \frac{1}{2} =$

c. $\frac{3}{4} \times \frac{1}{3} =$

d. $\frac{2}{3} \times \frac{1}{3} =$

e. $\frac{2}{5} \times \frac{1}{2} =$

f. $\frac{4}{5} \times \frac{1}{2} =$

4. Solve the multiplications by using two helping multiplications. Lastly, simplify if possible.

<p>a. $\frac{2}{3} \times \frac{1}{8} =$</p> <p>First find $\frac{1}{3}$ of $\frac{1}{8}$, then multiply the result by 2.</p> <p>$\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$ and $\frac{1}{24} \times 2 = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$</p>	<p>b. $\frac{3}{4} \times \frac{1}{10} =$</p> <p>First find $\frac{1}{4}$ of $\frac{1}{10}$, then multiply the result by 3.</p> <p>$\frac{1}{4} \times \frac{1}{10} = \frac{\boxed{}}{\boxed{}}$ and $\frac{\boxed{}}{\boxed{}} \times 3 = \frac{\boxed{}}{\boxed{}}$</p>
<p>c. $\frac{3}{5} \times \frac{1}{6} =$</p> <p>First find $\frac{1}{5}$ of $\frac{1}{6}$, then multiply the result by 3.</p> <p>$\frac{1}{5} \times \frac{1}{6} = \frac{\boxed{}}{\boxed{}}$ and $\frac{\boxed{}}{\boxed{}} \times 3 = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$</p>	<p>d. $\frac{5}{6} \times \frac{1}{9} =$</p> <p>First find $\frac{1}{6}$ of $\frac{1}{9}$, then multiply the result by 5.</p> <p>$\frac{1}{6} \times \frac{1}{9} = \frac{\boxed{}}{\boxed{}}$ and $\frac{\boxed{}}{\boxed{}} \times 5 = \frac{\boxed{}}{\boxed{}}$</p>
<p>e. $\frac{2}{3} \times \frac{1}{7} =$</p>	<p>f. $\frac{3}{8} \times \frac{1}{4} =$</p>

A shortcut for multiplying fractions

Multiply the numerators to get the numerator for the answer.

Multiply the denominators to get the denominator for the answer.

Study the examples on the right.

Remember always to give your final answer as a mixed number and in lowest terms (simplified).

$$\frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$$

$$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1 \frac{1}{10}$$

5. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$	b. $\frac{11}{12} \times \frac{1}{6}$
c. $\frac{1}{3} \times \frac{3}{13}$	d. $9 \times \frac{2}{3}$
e. $\frac{2}{9} \times \frac{6}{7}$	f. $10 \times \frac{5}{7}$

COMPARE	
The roundabout way	The shortcut
$\frac{5}{6} \times \frac{1}{2} = ?$ First find $\frac{1}{6}$ of $\frac{1}{2}$, then multiply the result by 5. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ Find $\frac{1}{8}$ of $\frac{3}{5}$, then multiply that result by 2. And to find $\frac{1}{8}$ of $\frac{3}{5}$, first find $\frac{1}{8}$ of $\frac{1}{5}$, and then multiply that by 3. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$. That multiplied by 3 is $\frac{1}{40} \times 3 = \frac{3}{40}$. Then, that multiplied by 2 is $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$.	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the “roundabout way,” we do each multiplication separately. In the shortcut, we can just do them all at once.	

6. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8} =$	b. $\frac{7}{10} \times \frac{8}{5} =$
c. $\frac{9}{20} \times \frac{4}{5} =$	d. $\frac{2}{5} \times \frac{1}{3} =$
e. $\frac{1}{4} \times \frac{2}{7} =$	f. $\frac{9}{4} \times \frac{1}{3} =$
g. $\frac{2}{3} \times \frac{11}{8} =$	h. $\frac{2}{9} \times \frac{3}{10} =$

7. There was $\frac{1}{4}$ of the pizza left. Marie ate $\frac{2}{3}$ of that.

- What part of the *original* pizza did she eat?
- What part of the *original* pizza is left now?

8. Theresa has painted $\frac{5}{8}$ of the room.

- a. What part is still left to paint?
- b. Now, Theresa has painted half of what was still left.
Draw a bar model of the situation.
What part of the room is still left to paint?

9. Ted has completed $\frac{2}{3}$ of a job that his boss gave him.

- a. What part is still left to do?
- b. Now Ted has completed a third of what was still left to do.
Draw a bar model of the situation.
What (fractional) part of the original job is still left undone?

What part is completed?

10. Sally wants to make $\frac{1}{3}$ of the recipe on the right.
How much does she need of each ingredient?

Carob Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
 $\frac{1}{2}$ cup honey
1 teaspoon vanilla
 $\frac{3}{4}$ cup whole wheat flour
 $\frac{3}{4}$ teaspoon baking powder
1 cup walnuts or other nuts

11. For an upcoming get-together, Alison needs to multiply the coffee recipe. Assume that half of the guests drink one serving, and the other half drink two servings. Find how much coffee she will need, if she has:

- a. 30 guests
- b. 50 guests
- c. 80 guests

Coffee (5 servings)

$3\frac{1}{2}$ cups water
 $\frac{1}{4}$ cup coffee

Puzzle Corner

Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$

b. $\times \frac{1}{4} = \frac{5}{16}$

c. $\times \frac{3}{8} = \frac{1}{16}$


d. $\times \frac{2}{5} = \frac{3}{10}$

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Chapter 8: Geometry

Introduction

The problems in this chapter involve lots of drawing. Geometry is a hands-on subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol “” are meant to be done in a notebook or on blank paper.

This chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and how to draw a perpendicular line through a given point on a line. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of a rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

The Lessons in Chapter 8

	<i>page</i>	<i>span</i>
Review: Angles	166	2 pages
Review: Drawing Polygons	168	4 pages
Circles	172	3 pages
Quadrilaterals	175	4 pages
Equilateral, Isosceles, and Scalene Triangles	179	5 pages
Area and Perimeter Problems	184	3 pages
Volume	187	5 pages
Volume of Rectangular Prisms (Cuboids)	192	4 pages
A Little Bit of Problem Solving	196	2 pages
Mixed Review.....	198	3 pages
Review.....	201	3 pages

Helpful Resources on the Internet

FOR REVIEW OF ANGLES AND POLYGONS

Measuring Angles

Rotate the protractor into position and give your measurement to the nearest whole number.

<http://www.mathplayground.com/measuringangles.html>

Turtle Pond

Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.

<http://illuminations.nctm.org/Activity.aspx?id=3534>

Interactive Polygon Crossword Puzzle

Use the clues to help you guess the words that go in the puzzle, and fill it in.

<http://www.mathgoodies.com/puzzles/crosswords/ipolygon3.html>

Types of Polygons Vocabulary Quiz

In this interactive quiz you have to quickly name different types of polygons based on given clues. For each question you will have only 30 seconds to write your answer!

<http://www.math-play.com/types-of-polygons.html>

Polygon Matching Game

Many of the polygons included are quadrilaterals.

http://www.mathplayground.com/matching_shapes.html

Free Worksheets for Area and Perimeter

Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property. Options also include area and perimeter problems for irregular rectangular areas, and more.

http://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php

Areas of Rectangular Shapes Quiz

Practice finding the area of rectangular compound shapes with this interactive quiz.

<https://www.studyladder.com/games/activity/area-of-irregular-shapes-13136>

Circle

This page includes a detailed lesson about circles, as well as interactive exercises to practice the topic.

<http://www.mathgoodies.com/lessons/vol2/geometry.html>

QUADRILATERALS

Interactive Quadrilaterals

See all the different kinds of quadrilateral “in action.” You can drag the corners, see how the angles change, and observe what properties do not change.

<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

Quadrilateral Properties

Investigate the properties of square, rectangle, rhombus, an isosceles trapezoid, and a non-isosceles trapezoid in this dynamic, online activity.

http://www.glencoe.com/sites/texas/student/mathematics/assets/interactive_lab/geometry/G_08/G_08_dev_100.html

Sample worksheet from
www.mathmammoth.com

Complete the Quadrilateral

This is a hands-on activity (printable worksheets) where students join the dots to complete quadrilaterals, which helps students learn about the different types of quadrilaterals.

<http://fawnnguyen.com/don-stewards-complete-quadrilateral/>

Types of Quadrilaterals Quiz

Identify the quadrilaterals that are shown in the pictures in this interactive multiple-choice quiz.

http://www.softschools.com/math/geometry/quadrilaterals/types_of_quadrilaterals/

Quadrilateral Types Practice at Khan Academy

Identify quadrilaterals based on pictures or attributes in this interactive quiz.

https://www.khanacademy.org/math/basic-geo/basic-geo-shapes/basic-geo-classifying-shapes/e/quadrilateral_types

Classify Quadrilaterals Worksheets

Make free printable worksheets for classifying (identifying, naming) quadrilaterals.

http://www.homeschoolmath.net/worksheets/classify_quadrilaterals.php

TRIANGLES

Triangle Shoot

Practice classifying triangles by their angles or by their sides, or identifying types of angles, with this “math splat” game.

http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/triangles_shoot.htm

Rags to Riches: Classify Triangles by Sides and Angles

Answer multiple-choice questions about classifying triangles by their angles and sides and about angle measures of a triangle in a quest for fame and fortune.

<http://www.quia.com/rr/457498.html>

Identify Triangles Quiz

A simple multiple-choice quiz about identifying (classifying) triangles either by their sides or angles. You can modify some of the quiz parameters, such as the number of problems in it.

<http://www.thatquiz.org/tq-A/?-j1-l34-p0>

Interactive Triangles Activity

Play with different kinds of triangles (scalene, isosceles, equilateral, right, acute, obtuse). Drag the vertices and see how the triangle's angles and sides change.

<https://www.mathsisfun.com/geometry/triangles-interactive.html>

Classify Triangles Worksheets

Make free printable worksheets for classifying triangles by their sides, angles, or both.

http://www.homeschoolmath.net/worksheets/classify_triangles.php

VOLUME

Geometric Solids

Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.

<http://illuminations.nctm.org/Activity.aspx?id=3521>

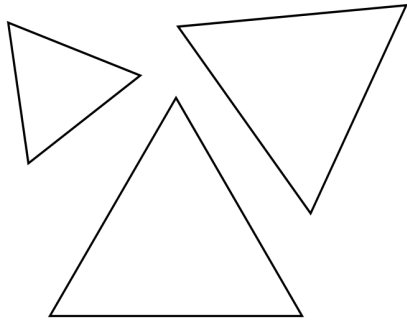
Sample worksheet from
www.mathmammoth.com

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Equilateral, Isosceles, and Scalene Triangles

If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

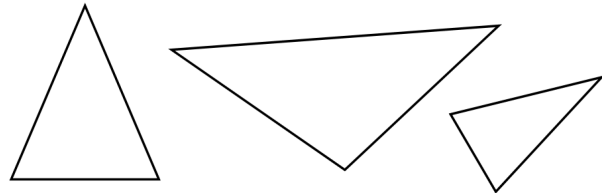
“*Equi-*” refers to things that are the same or equal, and “*lateral*” refers to sides. Think of it as a “same-sided” triangle.



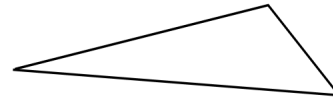
If only *two* of a triangle’s sides are congruent, then it is called an **isosceles triangle**.

Think of it as a “same-legged” triangle, the “legs” being the two sides that are the same length.

Mark the two congruent sides of each isosceles triangle:

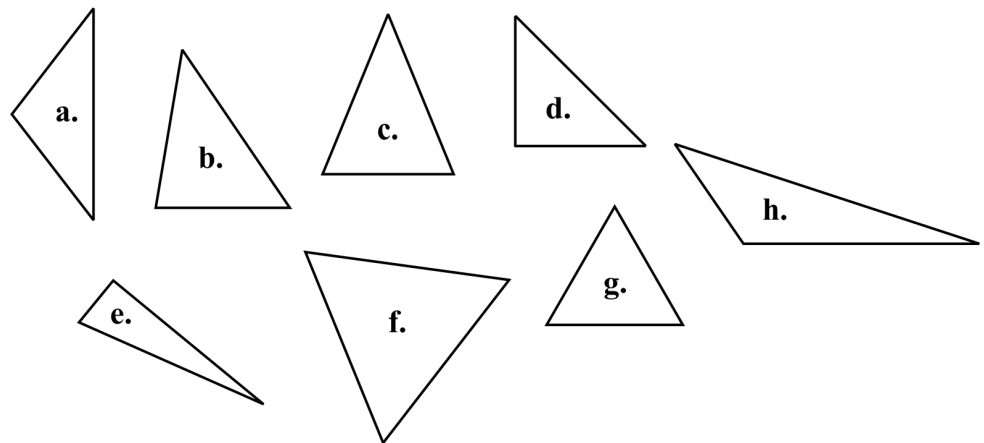


Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.



- Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

You can mark each triangle with an “e,” “i,” or “s” correspondingly.



- Fill in the table by classifying the triangles labeled as (a), (d), (e), and (g) above as “acute,” “right,” or “obtuse” (by their angles), and also as “equilateral,” “isosceles,” or “scalene” (by their sides).

Triangle	Classification by the sides	Classification by the angles
a		
d		
e		
g		

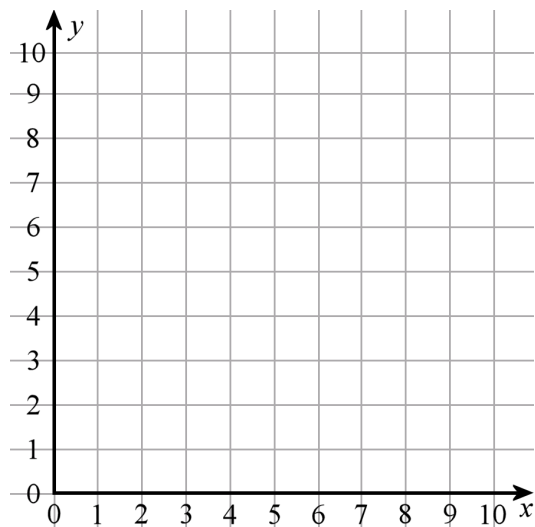
3. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles and sides.

Triangle 1: (0, 0), (4, 0), (0, 4)

_____ and

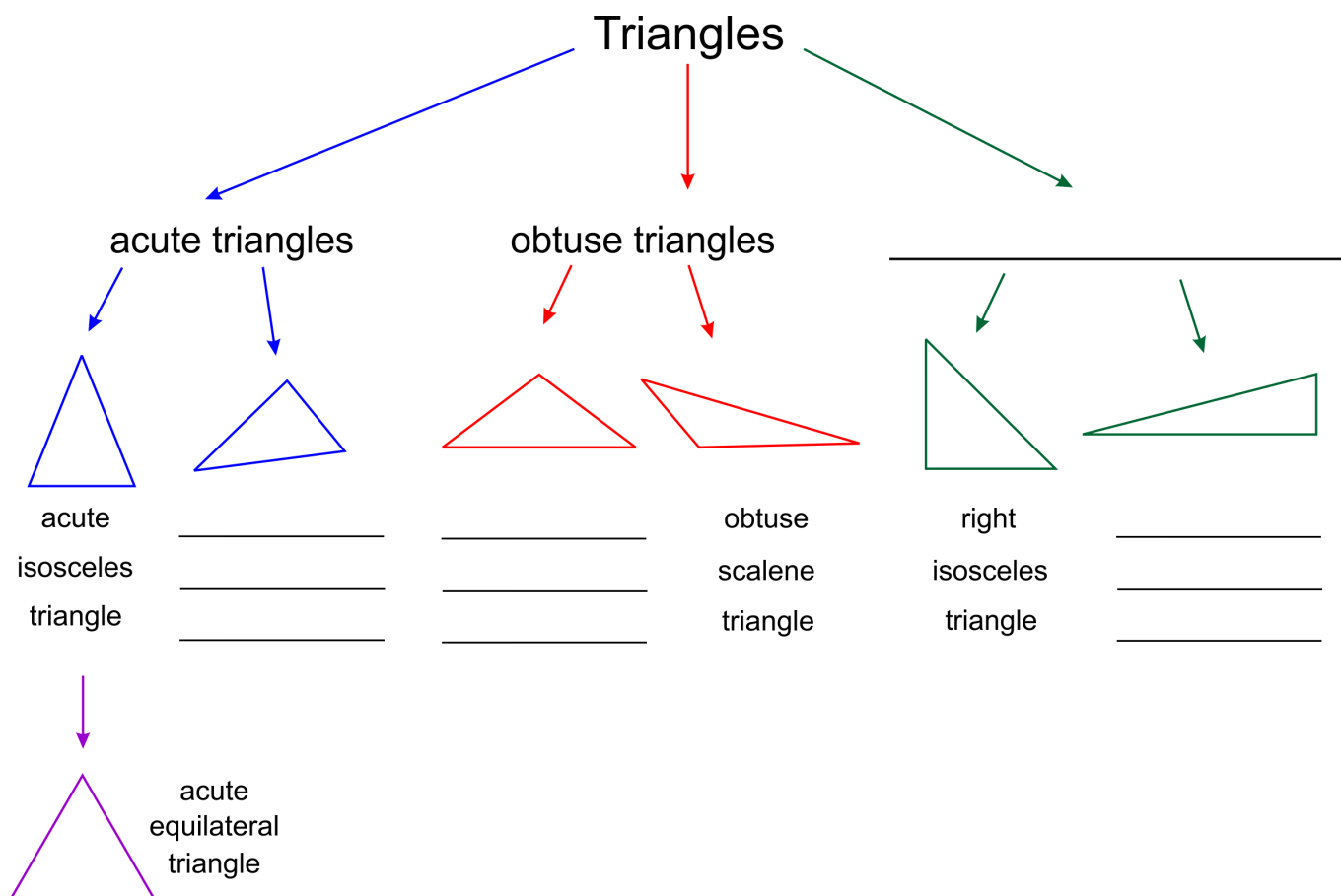
Triangle 2: (5, 5), (1, 8), (9, 4)

_____ and

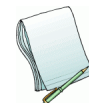


4. Plot in the coordinate grid an acute scalene triangle.

5. Fill in the missing parts in this tree diagram classification for triangles.

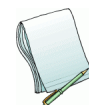


6. **a.** Draw a scalene obtuse triangle where one side is 3 cm and another is 7 cm.
Hint: Draw the 7-cm side first, then the 3-cm side forming any obtuse angle with the first side.



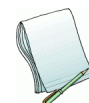
- b.** Measure the third side.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

7. **a.** Draw an isosceles right triangle whose two sides measure 5 cm.
Hint: Draw a right angle first. Then, measure off the 5-cm sides. Then draw in the last side.



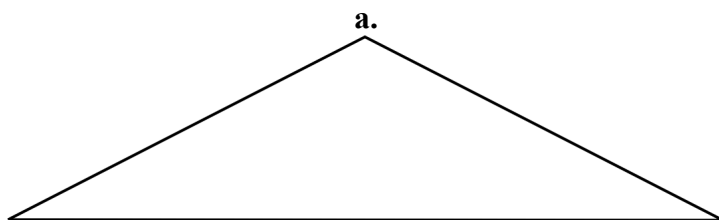
- b.** Measure the third side. It is _____ cm.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

8. **a.** Draw any isosceles triangle.
*Hint: Draw any angle. Then, measure off the two congruent sides, making sure they have the same length.
Then draw the last side.*



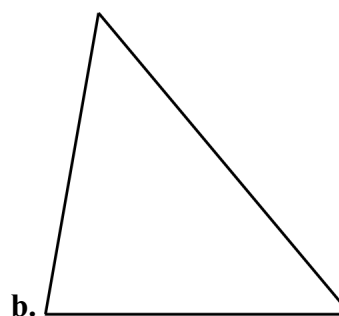
- b.** Measure the angles of your triangle. They measure _____ $^{\circ}$, _____ $^{\circ}$, and _____ $^{\circ}$.
The angle sum is _____ $^{\circ}$.

9. Measure all the angles in the isosceles triangles (a) and (b).
Continue their sides, if necessary.



_____ $^{\circ}$, _____ $^{\circ}$, and _____ $^{\circ}$.

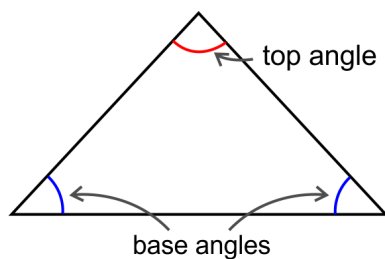
The angle sum is _____ $^{\circ}$.



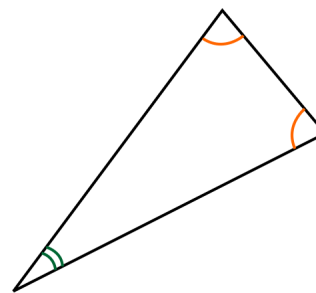
_____ $^{\circ}$, _____ $^{\circ}$, and _____ $^{\circ}$.

The angle sum is _____ $^{\circ}$.

What do you notice?

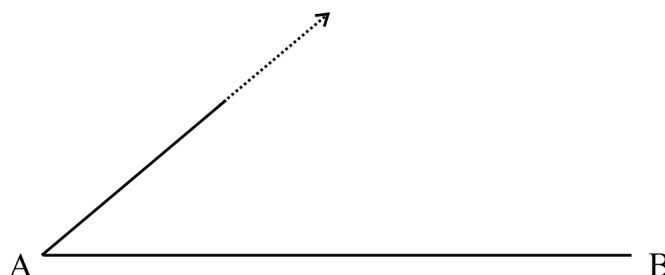


There are two angles in an isosceles triangle that have the same angle measure. They are called the **base angles**. The remaining angle is called the **top angle**.



Can you find the top angle and the base angles in this isosceles triangle?

10. The angle at A measures 40° . Draw another angle of 40° at B, and then continue its side so that you get an isosceles triangle with 40° base angles.



Measure the top angle. It is _____ $^\circ$. The three angle measures add up to _____ $^\circ$.

11. a. Draw an isosceles triangle with 75° base angles. (The length of the sides can be anything.)

Hint: start by drawing the base side (of any length). Then, draw the 75° angles.



b. Measure the top angle. It is _____ $^\circ$. The three angle measures add up to _____ $^\circ$.

c. Compare your triangle to those of your classmates, or draw another one yourself.

Can you draw several different-looking triangles with this information, or are they all identical?

12. a. Draw an isosceles triangle with a 50° top angle.

Hint: start by drawing a 50° angle. The two sides of the angle you drew are the two congruent sides of the triangle, so choose how long those sides should be, measure, and mark them. Then draw in the third side.

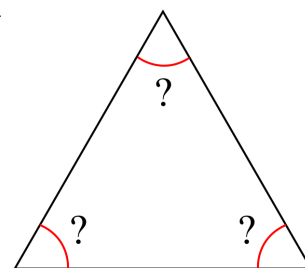


b. The base angles are _____ $^\circ$ each. The three angle measures add up to _____ $^\circ$.

c. Compare your triangle to those of your classmates, or draw another one yourself.

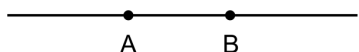
Can you draw several different-looking triangles with this information, or are they all identical?

13. Make a guess about the angle measures in an equilateral triangle: _____°
Measure to check.

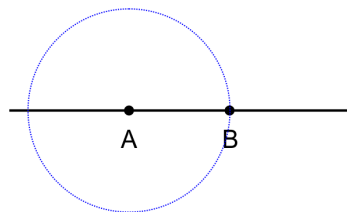


14. a. Could an equilateral triangle be a right triangle?
If yes, sketch an example. If not, explain why not.
- b. Could a scalene triangle be obtuse?
If yes, sketch an example. If not, explain why not.
- c. Could an acute triangle be scalene?
If yes, sketch an example. If not, explain why not.
- d. Could a right triangle be scalene?
If yes, sketch an example. If not, explain why not.
- e. Could an obtuse triangle be equilateral?
If yes, sketch an example. If not, explain why not.

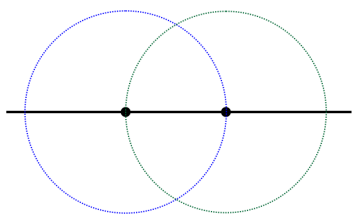
Draw an equilateral triangle with a compass and a ruler (optional)



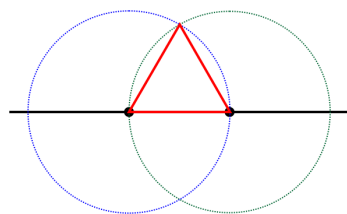
Draw a line segment and mark two points on it. These points mark one of the sides of the triangle.



Draw a circle using point A as the center point and the length AB as the radius. The third vertex of the triangle MUST lie on this circle. Can you see why?

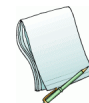


Can you see what was done in this picture?



The triangle is finished!

15. Draw at least three different equilateral triangles in your notebook using the method above.
Can you explain why this method works?



Important Terms

- *equilateral triangle*
- *isosceles triangle*
- *scalene triangle*

Area and Perimeter Problems

Example 1. Find the area of the shaded figure.

The easiest way to do this is:

- (1) Find the area of the larger outer rectangle.
- (2) Find the area of the white inner rectangle.
- (3) Subtract the two.

1. The area of the large rectangle is $7 \text{ cm} \times 10 \text{ cm} = 70 \text{ cm}^2$.

2. We find the *sides* of the white rectangle by subtracting.

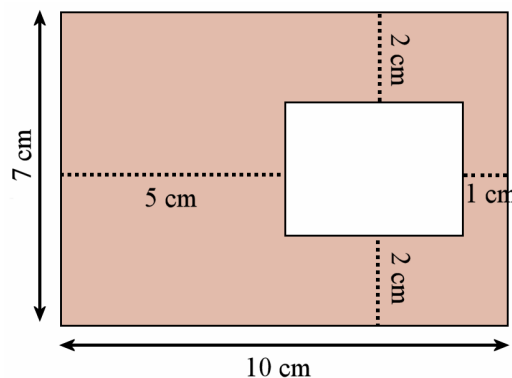
The longer side of the white rectangle is

$$10 \text{ cm} - 5 \text{ cm} - 1 \text{ cm} = 4 \text{ cm}.$$

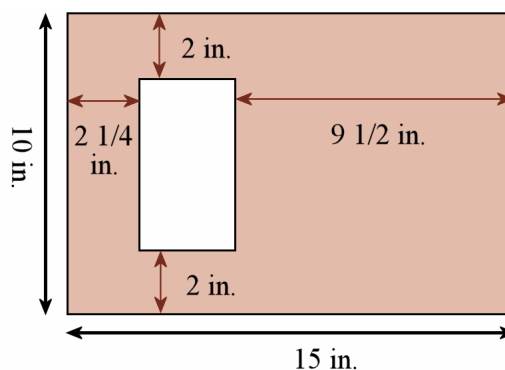
$$\text{The shorter side is } 7 \text{ cm} - 2 \text{ cm} - 2 \text{ cm} = 3 \text{ cm}.$$

So, the area of the white rectangle is $4 \text{ cm} \times 3 \text{ cm} = 12 \text{ cm}^2$.

3. Now we subtract to find the shaded area: $70 \text{ cm}^2 - 12 \text{ cm}^2 = 58 \text{ cm}^2$.



1. **a.** Find the area of the white rectangle.
All lines meet at right angles.



- b.** Find the area of the shaded figure.

2. The image on the right shows a picture frame.
Find the area of the actual frame (that is, of the shaded part).
All lines meet at right angles.

