

math

MAMMOTH

Grade 4-B Worktext

Division

Geometry

Fractions

Decimals



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Foreword

Math Mammoth Grade 4-A and *Grade 4-B* worktexts comprise a complete math curriculum for the fourth grade mathematics studies, which meets and exceeds the Common Core Standards for grade 4.

In fourth grade, our main focus is on multi-digit multiplication, long division, and areas related to those. Students also practice solving multi-step word problems, learn to use large numbers, and are introduced to fractions and decimals. This is accompanied by studies in geometry and measuring.

The year starts out with a review of addition and subtraction, patterns and graphs. We illustrate word problems with bar models and study how to solve simple equations, all of which teaches algebraic thinking. Students also learn terminology related to addition and subtraction, the order of operations, and statistical graphs.

Next come the concept of place value and large numbers—up to millions. First, we review four-digit numbers and do some mental math with them. Next students learn numbers up to one million and calculate with them based on the concepts of place value. In the end of the chapter we find out about millions and an introduction to multiples of 10, 100, and 1000.

The third chapter is all about multiplication. After briefly reviewing the concept of multiplication and the times tables, the lessons focus on multi-digit multiplication. First, we explore partial products (multiplying in parts) and students learn an “easy way to multiply,” where they multiply numbers part-by-part and add last. This is followed by the standard algorithm. You can omit the lesson about the “easy way” if you so wish; however, it shows students what multi-digit multiplication is based on (the “why”) better than the standard algorithm, and that is why I’ve included it. We also study some related concepts, such as the order of operations and proportional reasoning, and problem solving in context of money and change.

The last chapter in part A is about time, temperature, length, weight, and volume. Students will learn to solve more complex problems using various measuring units and to convert between measuring units.

In part B, we first study division. The focus is on learning long division, using division in word problems, and studying factors and divisibility.

In the chapter on geometry, we first review area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple unknown angle problems, and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last. These two chapters teach and practice only some of the operations with fractions and decimals. The focus is still on building conceptual understanding of these concepts as a foundation towards 5th grade, when fraction and decimal arithmetic will be in focus.

Math Mammoth curriculum provides you everything you need for a full year’s mathematics studies. Yet you still have liberty and can choose in which order some topics are studied. Chapters 1, 2, and 3 should be studied in order, and Chapter 3 (Multiplication) needs to come before Chapter 5 (long division). However, you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry) and schedule them earlier or later. Also, most lessons from chapters 7 and 8 (Fractions and Decimals) can be studied earlier; however the topic of finding parts with division should naturally be studied only after mastering division.

I wish you success in teaching math!

Maria Miller, the author

Chapter 5: Division

Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, average, and problem solving. It is a long chapter, because division and long division are “in focus” in fourth grade.

We start out by reviewing basic division facts by single-digit numbers (such as $24 \div 4$ or $56 \div 7$). After that, we study division terminology and dividing numbers by whole tens and hundreds (such as $400 \div 20$).

The lesson *Finding Fractional Parts with Division* shows an important relationship between fractions and division. For example, we can find $\frac{3}{4}$ of a number by first finding $\frac{1}{4}$ (dividing by 4) and then multiplying the result by 3.

Next students practice the order of operations again—this time with division as one of the operations.

Then we study the concept of remainder, preparing students to study long division right after that. The concept of remainder is presented visually at first. Soon, students encounter simple division problems with remainder written with the long division symbol or “corner”, as I like to call it.

Next comes a set of lessons intended to teach long division in several small steps. We start with divisions where each of the digits in the dividend (thousands, hundreds, tens, and ones) can be divided evenly by the divisor (for example, $3096 \div 3$). As the next step, there is a remainder in the ones. Then, the divisions have a remainder in the tens. Finally, there is a remainder in the hundreds and in the thousands, and this completes the step-by-step learning process for long division. The lessons also include lots of word problems to solve.

After long division, we study the concept of average, which is a nice application of division, and problems that involve a fractional part of a quantity (such as $\frac{3}{4}$ of \$600). Students get help from visual bar models to solve the problems.

The last section deals with elementary number theory. We study basic divisibility rules (though not all of them), prime numbers, and finding all factors of a given two-digit number.

The Lessons in Chapter 5

	page	span
Review of Division	10	3 pages
Division Terms and Division with Zero	13	2 pages
Dividing with Whole Tens and Hundreds	15	2 pages
Finding Fractional Parts with Division	17	3 pages
Order of Operations and Division.....	20	2 pages
The Remainder, Part 1	22	3 pages
The Remainder, Part 2	25	3 pages
Long Division 1	28	4 pages
Long Division 2	32	3 pages
Long Division 3	35	4 pages

Long Division with 4-Digit Numbers	39	4 pages
More Long Division	43	3 pages
Remainder Problems	46	4 pages
Long Division with Money	50	2 pages
Long Division Crossword Puzzle	52	1 page
Average	53	3 pages
Problems with Fractional Parts	56	2 pages
Problems to Solve	58	3 pages
Divisibility	61	4 pages
Prime Numbers	65	3 pages
Finding Factors	68	2 pages
Mixed Review	70	2 pages
Review	72	2 pages

Helpful Resources on the Internet

DIVISION CONCEPT AND DIVISION FACTS

The Forty Frogs Game

Learn to find fractions of sets by splitting a set of baby frogs into equal groups.

<http://www.mathactivities.net/frogs.htm>

Patty's Paints Division

Help Patty paint cars by solving basic division questions. Lastly you can drive your newly painted car in a fun race!

<http://www.multiplication.com/games/play/pattys-paints-division>

Flying High Division

Fly your plane safely through the storm clouds by answering the division facts correctly.

<http://www.multiplication.com/games/play/flying-high-division>

Times or Divide Bingo

A useful class teaching resource on division and multiplication by 10 and 100. It includes decimals and is suitable for use on an interactive whiteboard.

<http://www.topmarks.co.uk/Flash.aspx?f=bingotimesordivide>

Multiply & Divide by 10, 100, 1000

Help the log jumper jump across the river by clicking on the logs that have the correct answers. Choose "Level 3".

http://kids.britannica.com/lm/games/GM_5_5/GM_5_5.htm

Fraction of a Number

Practice finding a fraction of a given number.

http://www.mathplayground.com/fractions_fractionof.html

Order of Ops

Save seven members of a Royal Family from prison by using your order of operation skills. Choose the expression to be solved in each step, and solve it. The program uses a visual representation of a stairway to show how the expression gets shorter at each step.

<http://mrnussbaum.com/orderops/>

Free worksheets for order of operations

Generate printable & customizable worksheets for the order of operations. Choose from five operations and parentheses. You can choose the number range used, number of problems, and more.

http://www.homeschoolmath.net/worksheets/order_of_operations.php

ITP Remainders

This ITP sets up an empty grid into which you can place counters. Removing or highlighting extra counters will change the calculation displayed.

http://mathsframe.co.uk/en/resources/resource/67/itp_remainders

Leftovers — game with beads

Practice division with this fun dice game!

<http://www.learn-with-math-games.com/long-division-games-for-the-classroom.html>

Grab the Remainder

Solve the given division equation and identify the correct remainder and gobble it before the time lapses.

<http://www.turtlediary.com/game/interpret-the-remainder.html>

LONG DIVISION

MathFrog Dividerama!

Interactive long division practice. Guided help available.

<http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml>

Mr. Martini's Classroom: Long Division

An interactive long division tool.

<http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html>

Drag-and-Drop Math

Practice division interactively. Choose “Division”, 2-digit dividend, and 1-digit divisor.

<http://mrnussbaum.com/drag-and-drop-math/>

Long Division Millionaire Game

Learn to divide large numbers up to thousands. Can you answer all 15 questions?

<http://www.kidsmathtv.com/free/long-division-game-for-6th-grade-millionaire-game/>

Division Jump — board game

Practice division of one-digit numbers into two, three, and four-digit numbers.

<http://www.learn-with-math-games.com/division-activities.html>

Bike Racing Math Average

Race your motorcycle against others while answering questions about average. Speed up with each correct answer!

<http://www.mathnook.com/math/bike-racing-math-average.html>

Long Division Quiz

Practice dividing four-digit numbers by single-digit numbers in this online quiz.

<http://i4c.xyz/nmenbdv>

Double-Division.org

Double-division is a form of the long division algorithm that takes away the guesswork of finding how many times the divisor goes into the number to be divided. Also called 1-2-4-8 division.

<http://www.doubledivision.org/>

Short Division

This is a web page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in your head and not written down.

<http://www.themathpage.com/ARITH/divide-whole-numbers.htm>

FACTORS AND PRIMES

Arrays and factors

Drag rectangles to show the factorizations of a given number on a grid.

<http://www.shodor.org/interactivate/activities/FactorizeTwo/>

Factor Game

Choose a number from the game board, and your opponent gets all the numbers that are its proper factors. If a player chooses a number with no proper factors remaining, that player loses a turn. You can adjust the number of rows and columns on the game board to get a more challenging (and interesting) game. This game can easily be adapted to be played offline, with paper and colored pencils.

<http://illuminations.nctm.org/Activity.aspx?id=4134>

Sliding Tile Factorization Game

Slide a number over another to capture it, but you can only do this if the number you slide is a factor of the other. Number 1 is only supposed to be used to capture prime numbers.

http://www.visualmathlearning.com/Games/sliding_factors.html

Octopus Factors

Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.

<http://www.counton.org/games/map-numbers/octopus/>

Not a Factor

Choose a number that is *not* a factor of the given number.

http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

Product Game

The players choose factors and the product of those gets colored in on the game board. The player who gets four products in row wins. You can play against the computer or with a friend. This game can easily be adapted to be played offline, with paper and colored pencils.

<http://illuminations.nctm.org/Activity.aspx?id=4213>

Primes, Factors and Divisibility—Explorer at CountOn.org

Lessons explaining divisibility tests, primes, and factors.

<http://www.counton.org/explorer/primes>

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Long Division 1

Divide hundreds, tens, and ones separately.

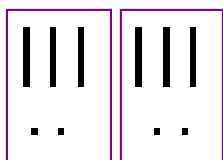
Write the dividend inside the long division “corner”, and the quotient on top.

$$\underline{64 \div 2 = ?}$$

Divide tens and ones separately:

$$6 \text{ tens} \div 2 = 3 \text{ tens (t)}$$

$$4 \text{ ones} \div 2 = 2 \text{ ones (o)}$$



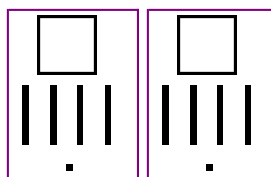
$$\begin{array}{r} \text{t o} \\ 32 \\ 2 \overline{) 64} \end{array}$$

$$\underline{282 \div 2 = ?}$$

$$2 \text{ hundreds} \div 2 = 1 \text{ hundred (h)}$$

$$8 \text{ tens} \div 2 = 4 \text{ tens (t)}$$

$$2 \div 2 = 1. \text{ (o)}$$



$$\begin{array}{r} \text{h t o} \\ 141 \\ 2 \overline{) 282} \end{array}$$

1. Make groups. Divide. Write the dividend inside the “corner” if it is missing.

a. Make 2 groups



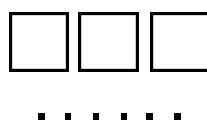
$$2 \overline{) 62}$$

b. Make 3 groups



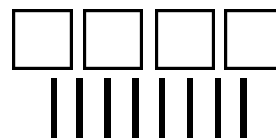
$$3 \overline{) \quad}$$

c. Make 3 groups



$$3 \overline{) \quad}$$

d. Make 4 groups



$$4 \overline{) \quad}$$

2. Divide thousands, hundreds, tens, and ones separately.

a. $4 \overline{) 84}$

b. $3 \overline{) 393}$

c. $3 \overline{) 660}$

d. $4 \overline{) 8040}$

e. $3 \overline{) 66}$

f. $6 \overline{) 6036}$

g. $3 \overline{) 330}$

h. $4 \overline{) 4804}$

<div style="text-align: center;"> h t o 0 4 $\overline{) 248}$ </div> <p>4 does not go into 2. You can put zero in the quotient in the hundreds place or omit it. 4 does go into 24, six times. Put 6 in the quotient.</p>	<div style="text-align: center;"> h t o 0 6 2 4 $\overline{) 248}$ </div>	<div style="text-align: center;"> th h t o 0 5 $\overline{) 3505}$ </div>	<div style="text-align: center;"> th h t o 0 7 0 1 5 $\overline{) 3505}$ </div> <p>5 does not go into 3. You can put zero in the quotient. 5 does go into 35, seven times.</p>
<p style="text-align: center;"><u>Explanation:</u></p> <p>The 2 of 248 is of course 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient will not have any whole hundreds.</p> <p>Then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result, 6 tens goes as part of the quotient.</p> <p>Check the final answer: $4 \times 62 = 248$.</p>		<p style="text-align: center;"><u>Explanation:</u></p> <p>$3,000 \div 5$ will not give any whole thousands to the quotient because the answer is less than 1,000.</p> <p>But 3 thousands and 5 hundreds make 35 hundreds together. You can divide $3,500 \div 5 = 700$, and place 7 as part of the quotient in the hundreds place.</p> <p>Check the final answer: $5 \times 701 = 3,505$.</p>	
<p>If the divisor does not “go into” the first digit of the dividend, look at the <u>first two digits</u> of the dividend.</p>			

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. $3 \overline{) 123}$

b. $4 \overline{) 284}$

c. $6 \overline{) 360}$

d. $8 \overline{) 248}$

e. $2 \overline{) 184}$

f. $7 \overline{) 427}$

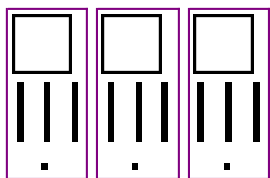
g. $3 \overline{) 1833}$

h. $4 \overline{) 2404}$

i. $7 \overline{) 4970}$

j. $5 \overline{) 4505}$

Ones division is not even. There is a remainder.



$$395 \div 3 = 131 \text{ R}2$$

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 1 \quad 3 \\ 3 \overline{) 395} \end{array}$$

3 goes into 3 one time.
3 goes into 9 three times.

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 1 \quad 3 \quad 1 \text{ R}2 \\ 3 \overline{) 395} \end{array}$$

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 1 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th} \quad \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 0 \quad 0 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

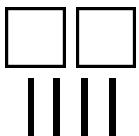
8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

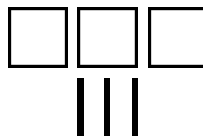
4. Divide into groups. Find the remainder.



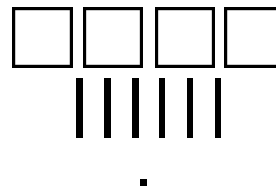
a. $2 \overline{) 63}$



b. $2 \overline{) \quad}$



c. $3 \overline{) \quad}$



d. $2 \overline{) \quad}$

5. Divide. Indicate the remainder if any.

a. $4 \overline{) 847}$

b. $2 \overline{) 69}$

c. $3 \overline{) 367}$

d. $4 \overline{) 89}$

e. $2 \overline{) 121}$

f. $6 \overline{) 1805}$

g. $7 \overline{) 215}$

h. $8 \overline{) 2482}$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

$$\begin{array}{r} \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 6 \quad 1 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th} \quad \text{h} \quad \text{t} \quad \text{o} \\ 0 \quad 4 \quad 0 \quad 2 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

6. Practice some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3 \overline{) 128}$

b. $3 \overline{) 95}$

c. $6 \overline{) 4267}$

d. $4 \overline{) 2845}$

e. $5 \overline{) 5507}$

f. $2 \overline{) 8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. $440 \div 4 =$

$820 \div 2 =$

b. $3600 \div 400 =$

$369 \div 3 =$

c. $824 \div 2 =$

$560 \div 90 =$

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Divisibility

A number n is **divisible** by another number m if the division $n \div m$ is exact (no remainder).

For example, $18 \div 3 = 6$, so 18 is divisible by 3.

Also, 18 is divisible by 6, because we can write the other division $18 \div 6 = 3$.

So, 18 is divisible by *both* 6 and 3. We say 6 and 3 are **divisors** of 18.

You can use long division to check if a number is divisible by another.

For example, $67 \div 4 = 16$, R3. There is a remainder, so 67 is *not* divisible by 4.

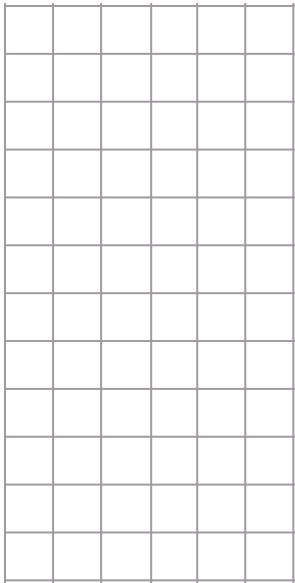
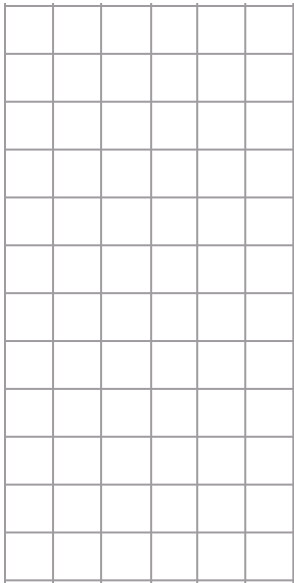
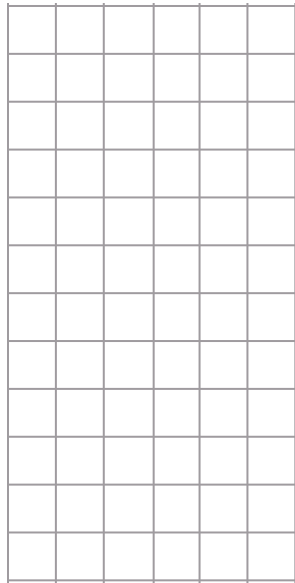
Also, from this we learn that neither 4 nor 16 is a divisor of 67.

$$\begin{array}{r} 16 \\ 4 \overline{) 67} \\ \underline{-4} \\ 27 \\ \underline{-24} \\ 3 \end{array}$$

1. Divide and determine if the number is divisible by the other number.

<p>a. $21 \div 3 = \underline{\hspace{2cm}}$ Is 21 divisible by 3?</p>	<p>b. $40 \div 6 = \underline{\hspace{2cm}}$ Is 40 divisible by 6?</p>	<p>c. $17 \div 5 = \underline{\hspace{2cm}}$ Is 5 a divisor of 17?</p>	<p>d. $84 \div 7 = \underline{\hspace{2cm}}$ Is 7 a factor of 84?</p>
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2. Answer the questions. You may need long division.

<p>a. Is 98 divisible by 4?</p> 	<p>b. Is 603 divisible by 7?</p> 	<p>c. Is 3 a factor of 1,256?</p> 
---	--	---

In any multiplication, the numbers that are multiplied are called **factors** and the result is called a **product**.

factor		factor		product
7	\times	6	$=$	42

For example, since $6 \times 7 = 42$, 6 and 7 are **factors** of 42.

From this multiplication fact we can write two divisions: $42 \div 6 = 7$ and $42 \div 7 = 6$.
So, this means that 6 and 7 are also divisors of 42.

From this we can notice the following:

If a number is a factor of another number, it is also its divisor.

There is yet one more new word to learn that ties in with all of this: **multiple**.

We say **42 is a multiple of 6**, because 42 is some number times 6 (namely 7×6).

And of course 42 is also a multiple of 7, because 42 is some number times 7 (namely, 6×7)!

3. Fill in.

We know that $8 \times 9 = 72$. So, 8 is a _____ of 72, and so is 9.

Also, 72 is a _____ of 8, and 72 is a _____ of 9.

And, 72 is _____ by 8 and by 9.

4. Fill in.

<p>a. Is 5 a factor of 55?</p> <p>Yes, because $___ \times ___ = ______$.</p>	<p>b. Is 8 a divisor of 45?</p> <p>No, because $___ \div ___ = ______$.</p>
<p>c. Is 36 a multiple of 6?</p> <p>_____, because $___ \times ___ = ______$.</p>	<p>d. Is 34 a multiple of 7?</p> <p>_____, because $___ \div ___ = ______$.</p>
<p>e. Is 7 a factor of 46?</p> <p>_____, because _____.</p>	<p>f. Is 63 a multiple of 9?</p> <p>_____, because _____.</p>

Multiples of 6 are all those numbers we get when we multiply 6 by other numbers. For example, we can multiply 0×6 , 7×6 , 11×6 , 109×6 , and so on. The resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, and so on.

5. **a.** Make a list of multiples of 11, starting at 0 and going at least till 154.

b. Make a list of multiples of 111, starting at 0. Make it as long as you can in this space!

Divisibility by 2

Numbers that are divisible by 2 are called **even** numbers.
Numbers that are NOT divisible by 2 are called **odd** numbers.

Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

Divisibility by 5

Numbers that end in 0 and 5 are divisible by 5.

For example, 10, 35, 720, and 3,675 are such numbers.

6. Mark an “x” if the number is divisible by 2 or by 5.

number	divisible		number	divisible		number	divisible		number	divisible	
	by 2	by 5		by 2	by 5		by 2	by 5		by 2	by 5
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

Divisibility by 10

Numbers that end in 0 are divisible by 10.

For example, 10, 60, 340, and 2,570 are such numbers.

7. Mark an “x” if the number is divisible by 2, by 5, or by 10.

number	divisible			number	divisible			number	divisible		
	by 2	by 5	by 10		by 2	by 5	by 10		by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in a zero, so it is ALSO divisible by ____ and ____.

8. **a.** Write a list of numbers that are divisible by 2, from 0 to 60.

This is also a list of _____ of 2.

- b.** In the list above, *underline* those numbers that are divisible by 4.
What do you notice?
- c.** In the list above, *color* those numbers that are divisible by 6.
What do you notice?
- d.** Which numbers are divisible by both 4 and by 6?
9. **a.** Write a list of numbers that are divisible by 3, from 0 to 60.
-
- This is also a list of _____ of 3.
- b.** In the list above, *underline* those numbers that are divisible by 6.
What do you notice?
- c.** In the list above, *color* those numbers that are divisible by 9.
What do you notice?
10. Use the lists you made in (8) and (9). Find numbers that are divisible by *both* 2 and 9.
11. What number is a factor of every number?
12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers.
Which ones?

Who am I?

(Hint: I am less than 50.)

Mystery Number

38 25 199
47 101

Divided by 9, I leave a remainder of 6.
Divided by 4, I leave a remainder of 1.
Divided by 10, I leave a remainder of 3.

Who am I?

(Hint: I am less than 100.)

Mystery Number

38 25 199
47 101

I am a multiple of 3, 4, 5, and 6.
I am a factor of 120.
Divided by 7, I leave a remainder of 4.

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Chapter 6: Geometry

Introduction

We start our study of geometry by reviewing the concepts of area and the perimeter of rectangles (from third grade). Students also apply these concepts in various problems, including problems where they write simple equations and a problem where they explore all possible perimeters for a given area.

The focus of this chapter is angles. Students learn about lines, rays, and angles, and about acute, right, obtuse, and straight angles. Next they learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations and estimate some common angles.

The lesson *Parallel and Perpendicular Lines* also ties in with the topic of angles, because two lines are perpendicular if they form a right angle. After that, we study parallelograms and other quadrilaterals in more detail, paying attention to their angles and lengths of sides.

We also study triangles and classify them according to their angles (acute, obtuse, or right triangles). Classifying triangles according to their sides (equilateral, isosceles, or scalene) will be studied in 5th grade. The last topic in this chapter, and an easy one, is line symmetry.

The study of geometry is full of strange-sounding words. I suggest that you have your student(s) start a geometry notebook, where they will draw picture(s) and text to explain every new concept or term, which will help them to learn and remember those terms better. They can also do the drawing exercises from this chapter in the notebook. Encourage the students to make this book to be as good as they possibly can and be creative so that it becomes their own special work. You can even give them credit for it.

The Lessons in Chapter 6

	page	span
Review: Area of Rectangles	79	5 pages
Review: Area and Perimeter	84	4 pages
Lines, Rays, and Angles	88	5 pages
Measuring Angles	93	5 pages
Drawing Angles	98	2 pages
Angle Problems	100	5 pages
Estimating Angles	105	5 pages
Parallel and Perpendicular Lines	110	5 pages
Parallelograms	115	3 pages
Triangles	118	4 pages
Line Symmetry	122	3 pages
Mixed Review	125	2 pages
Review	127	4 pages

Helpful Resources on the Internet

AREA AND PERIMETER

Free Worksheets for Area and Perimeter

Create worksheets for the area and the perimeter of rectangles/squares with images, word problems, or problems where the student writes an expression for the area using the distributive property.

http://www.homeschoolmath.net/worksheets/area_perimeter_rectangles.php

Area Blocks

Cover your grid with shapes before your opponent does!

http://www.mathplayground.com/area_blocks.html

Shape explorer

Find the perimeter and area of odd shapes on a rectangular grid.

<http://www.shodor.org/interactivate/activities/ShapeExplorer/>

Math Playground: Party Designer

You need to design areas for the party, such as a crafts table, food table, seesaw, and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

Area and Perimeter Builder

Create your own rectangular shapes using colorful blocks and explore the relationship between perimeter and area. You can choose to show the side lengths to understand how a perimeter works. You can also use two work areas (grids) to compare the area and perimeter of two shapes side-by-side. Lastly, challenge yourself in the game screen to build shapes or find the area of various figures.

http://phet.colorado.edu/sims/html/area-builder/latest/area-builder_en.html

Geometry Area/Perimeter Quiz from ThatQuiz.org

An online quiz, about the area and perimeter of rectangles, triangles, and trapezoids. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when the perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1200b-lc-p0>

Area: Missing Side Length Quiz

Practice finding the value of the unknown side in this 10-question quiz.

<http://www.thatquiz.org/tq-4/?-j8001-lc-p0>

Area and Perimeter of Rectangles

A 10-question quiz with varying questions concerning the area and perimeter of rectangles.

<https://www.ck12.org/assessment/ui/views/test.view.new.html?practice/Area-and-Perimeter-of-Rectangles-Practice?type=practice>

ANGLES

Online Protractor

Investigate angles and the use of protractors.

<http://www.amblesideprimary.com/ambleweb/mentalmaths/protractor.html>

Measuring Angles

Practice measuring angles with a protractor.

<http://www.mathplayground.com/measuringangles.html>

Draw Angles - Khan Academy

Use a protractor to construct angles.

<https://www.khanacademy.org/math/on-sixth-grade-math/on-geometry-spatial-sense/on-angles-polygons/e/drawing-angles>

Topmarks - Angles activity

Squirt the dog with water by dragging the correct angle onto the screen.

<http://www.topmarks.co.uk/Flash.aspx?b=maths/angles>

Fruit Picker

Try to pick 6 apples in 6 shots by turning at the correct angle.

<http://www.fruitpicker.co.uk/activity/>

Estimating Angles

Estimate the target angle. The closer you get to the target angle the more points you will score.

<https://nrich.maths.org/1235>

Draw Perpendicular and Parallel lines - interactive

Learn about perpendicular and parallel lines and practice drawing them.

<https://www.mathsisfun.com/perpendicular-parallel.html>

Parallel and Perpendicular Lines in Shapes Quiz

Classify the lines as parallel, perpendicular, or neither.

<https://www.studyladder.com/games/activity/parallel-and-perpendicular-lines-in-shapes-28459>

Turtle Pond

Guide a turtle to a pond using commands, which include turning him in certain angles or moving him a specific distance.

<http://illuminations.nctm.org/Activity.aspx?id=3534>

SHAPES / POLYGONS

Polygon Matching Game

Learn all the common polygons by playing this fun, timed matching game.

http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary

A matching game.

<http://www.quia.com/cc/2758.html>

Interactive Quadrilaterals

See all the different kinds of quadrilaterals “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.

<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

Interactive Parallelogram

Drag the parallelogram and learn about its properties, angles, and sides.

<http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php>

Sample worksheet from
www.mathmammoth.com

Interactive Triangles Tool

Read about triangles, and then play with them to become familiar with them from all angles.

<https://www.mathsisfun.com/geometry/triangles-interactive.html>

Triangles Splat

“Shoot” the triangles as their names appear on the screen. Choose “Right, Acute, Obtuse”.

http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/triangles_shoot.htm

Classifying Triangles Drag-and-Drop Game

Drag each triangle in the correct basket before time runs out!

<http://www.math-play.com/classifying-triangles/Triangles-Drag-and-Drop-Game.html>

Lines of Symmetry

Match the corresponding lines of symmetry.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/SymmetryLinesShapesShoot.htm>

Symmetry Shapes Shoot

Practice identifying symmetrical shapes by clicking on them.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/SymmetryShapesShoot.htm>

Line Shoot

Learn about lines of symmetry the fun way in this line shoot geometry math game.

http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/line_shoot.htm

General

Interactivate! Tessellate

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>

Patch Tool

An online activity where the student designs a pattern using geometric shapes.

<http://illuminations.nctm.org/Activity.aspx?id=3577>

Polygon Playground

Drag various colorful polygons to the work area to make your own creations!

<http://www.mathcats.com/explore/polygons.html>

Geometry Worksheets

Worksheets about complementary and supplementary angles, parallel, perpendicular, and intersecting lines, types of angles, basic shapes, area & perimeter of rectangles, and parts of a circle.

<http://www.dadsworksheets.com/worksheets/basic-geometry.html>

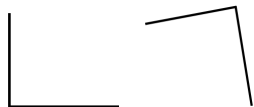
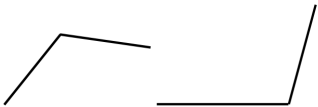

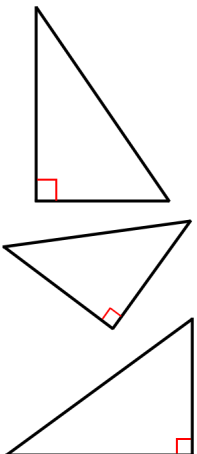
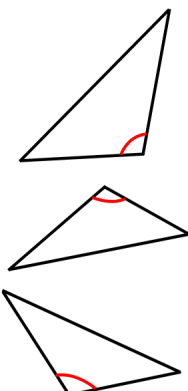
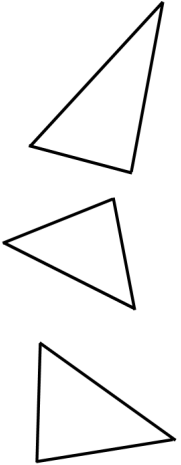
Space Logic

Write a program that will guide the space rover to its target. Watch out for the boulders!

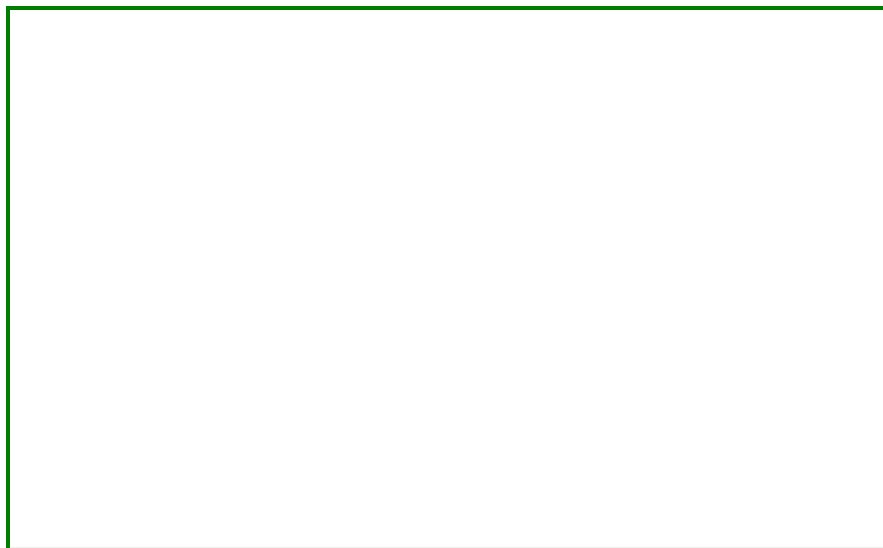
http://www.mathplayground.com/space_logic.html

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Triangles

 <p>right angles (exactly 90°)</p>	 <p>obtuse angles (more than 90°, less than 180°)</p>	 <p>acute angles (less than 90°)</p>
 <p>Right triangles have exactly one right angle.</p>	 <p>Obtuse triangles have exactly one obtuse angle.</p>	 <p>Acute triangles have three acute angles. In other words, <i>all</i> the angles are acute.</p>

1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.
- b.** Draw another, different right triangle.
- c.** A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?



A right triangle has one right angle. The other two angles are _____.

2. **a.** Draw an obtuse angle.
Then make it into an obtuse triangle by drawing in the third side.



- b.** Draw another, different obtuse triangle.
- c.** An obtuse triangle has one obtuse angle. Are the other two angles in a obtuse triangle acute, right, or obtuse?



An obtuse triangle has one obtuse angle. The other two angles are _____.

3. **a.** Draw an acute triangle.
The lengths of sides can be any.

- b.** Measure its angles.

They measure _____ $^{\circ}$,
_____ $^{\circ}$, and _____ $^{\circ}$.



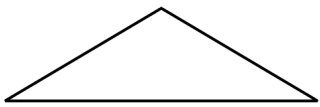
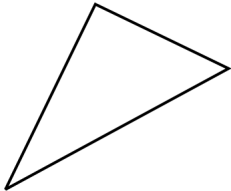
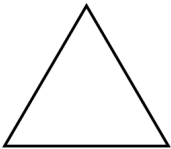
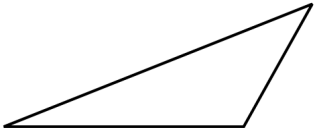
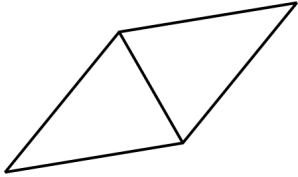
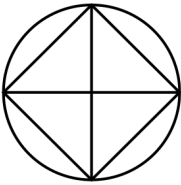
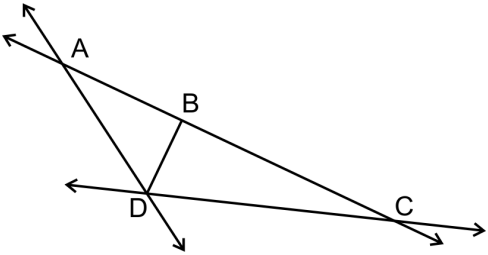
4. Observe all that you have done in this lesson thus far, and fill in.

Right triangles have exactly 1 _____, _____,
and the other two angles are _____.

Obtuse triangles have exactly 1 _____, _____,
and the other two angles are _____.

Acute triangles have _____ angles.

5. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p> 	<p>b.</p> 	<p>c.</p> 	<p>d.</p> 
<p>e.</p> 	<p>f.</p> 	<p>g.</p> 	

6. Label the triangles in the pictures as right, acute, or obtuse.

<p>a.</p>  <p>National Museum of Marine Corps Photo by Ron Cogswell</p> <p>_____</p>	<p>b.</p>  <p>The City Center of Woburn, MA Photo by Randy Robertson</p> <p>_____</p>	<p>c.</p>  <p>The White House Photo by Glyn Lower</p> <p>_____</p>
<p>d.</p>  <p>Waterlily Photo by Francis Chung</p> <p>_____</p>	<p>e.</p>  <p>Warning of a steep hill ahead Photo by www.flickr.com/photos/shirokazan/</p> <p>The black triangle is _____</p> <p>The red triangle is _____</p>	

Find acute, obtuse, and right triangles outside in nature, in buildings, in signs, and so on!

7. **a.** Draw a triangle with 85° and 40° angles.

Hint: First draw a 85° angle. Then, mark a point anywhere on one side of that angle to be the second vertex of the triangle. Use that point as a vertex for the 40° angle, and draw the 40° angle.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



8. **a.** Draw a triangle with 125° and 40° angles.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



9. **a.** Draw a triangle with 55° and 35° angles.

- b.** Measure the third angle.
It is _____ degrees.
- c.** What kind of triangle is it?
(acute, right, obtuse)
- d.** What is the angle sum?



New Terms

- *an acute triangle*
- *a right triangle*
- *an obtuse triangle*

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Chapter 7: Fractions

Introduction

In third grade, students have studied equivalent fractions and comparing some easy fractions. In fourth grade, it is time to expand their knowledge of fraction topics. We study:

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (sums where the denominators are the same, such as $\frac{5}{6} + \frac{3}{6}$ or $1\frac{2}{3} + 2\frac{1}{3}$)
- equivalent fractions (for example, $\frac{2}{3} = \frac{8}{12}$)
- comparing fractions
- multiplying a fraction by a whole number (for example $5 \times \frac{1}{2}$)

Then in fifth grade, students tackle *all* four operations with fractions. Our studies here are laying groundwork for that by emphasizing conceptual understanding and relying a lot on visual models.

The lessons in this chapter are also important because they are the basis for understanding decimal numbers, which is the topic of the next chapter. Decimals are just another way of writing fractions with denominators 10, 100, 1,000, *etc.* For example, 0.4 is $\frac{4}{10}$, and 3.29 is $\frac{329}{100}$.

In this grade, we continue studying fractions and their operations with the help of visual models. This is done in order to build a strong conceptual understanding of these operations and to avoid presenting fraction arithmetic as a list of computational rules to be learned by rote memory. If students learn to calculate with fractions only by using memorized rules, they will also easily confuse those rules because there are so many of them. There is a place for the rules: they become *shortcuts* for ideas that are already understood, but we don't want to start with them. The goal is to let the ideas and concepts "sink in" first, and the shortcuts (or rules) will follow in 5th grade.

The Lessons in Chapter 7

	page	span
One Whole and its Fractional Parts	135	3 pages
Mixed Numbers	138	4 pages
Adding Fractions and Mixed Numbers 1	142	4 pages
Adding Fractions and Mixed Numbers 2	146	3 pages
Equivalent Fractions	149	5 pages
Subtracting Fractions and Mixed Numbers	154	3 pages
Comparing Fractions	157	4 pages
Multiplying Fractions by Whole Numbers	161	3 pages
Practicing With Fractions	164	2 pages
Mixed Review	166	2 pages
Review	168	2 pages

Helpful Resources and Games on the Internet

FRACTIONS AND MIXED NUMBERS

Identifying Fractions at Conceptua Fractions

A tool that shows fractions or mixed numbers using a pie, a bar, dots, and a number line.

<https://www.conceptuamath.com/app/tool/identifying-fractions>

Number Bonds - Fractions

Combine balls whose fractions add to one.

http://www.mathplayground.com/number_bonds_fractions.html

Puzzle Pics Fractions

Reveal the mystery picture by dragging each puzzle piece to the number line that illustrates the fraction.

http://www.mathplayground.com/puzzle_pics_fractions.html

Animal Rescue: Fractions Number Line Game

Find and free the trapped animals by moving the arrow to the correct place on the number line.

<http://www.sheppardsoftware.com/mathgames/fractions/AnimalRescueFractionsNumberLineGame.htm>

Clara Fraction's Ice Cream Shop

Convert improper fractions into mixed numbers, and scoop the right amount of ice cream flavors onto the cone for your customers.

<http://mrnussbaum.com/icecream/>

Mixed Numbers and Improper Fractions

Practice converting mixed numbers to improper fractions.

http://www.mathplayground.com/fractions_mixed.html

ADDITION AND SUBTRACTION

Adding of Like Fractions with Circle Models

Practice adding fractions with the help of a visual model.

<http://www.visualfractions.com/AddEasyCircle/addcircles.html>

Fractions Workshop

Choose “Add mixed fractions with like denominators” in order to practice adding mixed numbers.

<http://mrnussbaum.com/fractions-workshop-ipad.html>

Action Fraction

A racing game with several levels where you add and subtract fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.

http://www.solveymath.com/math_games/arithmetic_games/action_fraction/

Add Mixed Numbers: 10-Question Quiz

Practice adding mixed numbers. Express the answer as a mixed fraction in lowest terms.

<http://www.thatquiz.org/tq-3/?-j301-l1-p0>

Fraction Game

Move the markers on the fraction number line from left to right according to the given fraction cards.

<http://illuminations.nctm.org/Activity.aspx?id=4148>

Sample worksheet from

www.mathmammoth.com

Four-Sum Fractions Board Game

Practice adding and simplifying fractions with common denominators. Be the first player to score four in a row!

<http://www.learn-with-math-games.com/fraction-math-games.html>

Subtracting Mixed Numbers with Borrowing

Perform subtraction calculations using borrowing with mixed number fractions.

<https://www.wisc-online.com/learn/formal-science/mathematics/abm701/subtracting-mixed-number-fractions-with-borro>

Subtracting Mixed Fractions Quiz

Practice subtracting mixed fractions in this multiple-choice quiz. Drag and drop corresponding answers.

<http://www.fractions4kids.com/subtracting-mixed-fractions-quiz/>

Fruit Shoot Fractions Addition

Click the fruit with the correct answer. To match the topics students learn in this section, choose adding 2 or 3 fractions with like denominators. You can also choose your mode (untimed or timed) and speed (slow vs. fast fruit).

<http://www.sheppardsoftware.com/mathgames/fractions/FruitShootFractionsAddition.htm>

Fraction Worksheets: Addition and Subtraction

Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose “Like Fractions” for this level.

<http://www.homeschoolmath.net/worksheets/fraction.php>

EQUIVALENT FRACTIONS

Equivalent Fractions

Draw two equivalent fractions for the given fraction. Choose either a square or a circle for the shape.

<http://illuminations.nctm.org/Activity.aspx?id=3510>

Fresh Baked Fractions

Practice equivalent fractions by clicking on a fraction that is not equal to the others.

<http://www.funbrain.com/fract/index.html>

Fraction Dolphins

Click on the dolphin with the correct equivalent fraction to the fraction on the bucket.

<http://mrnussbaum.com/fraction-dolphins-ipad.html>

Free Equivalent Fractions Worksheets

Create custom-made worksheets for equivalent fractions that can either include pie images or not.

http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

COMPARING FRACTIONS

Comparison Shoot Out

Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.

<http://www.fuelthebrain.com/games/comparison-shootout/>

Comparing Fractions—XP Math

Simple timed practice for comparing two fractions.

<http://xpmath.com/forums/arcade.php?do=play&gameid=8>

Sample worksheet from

www.mathmammoth.com

Ordering Fractions at Conceptua Fractions

An interactive tool where students place numbers, visual models, and decimals on a number line.

<https://www.conceptuamath.com/app/tool/comparing-fractions>

Ordering Fractions

Drag the fractions into the right order, from lowest to highest.

<http://www.topmarks.co.uk/Flash.aspx?b=maths/fractions>

Number Climb

Click the balls in ascending order of numbers.

http://www.mathplayground.com/number_climb.html

MULTIPLYING FRACTIONS

Multiplying Fractions with Circle Models

This page illustrates fraction multiplication with circle models.

<http://www.visualfractions.com/MultiplyCircle/>

Multiply Fractions by Whole Numbers

Practice multiplying fractions by whole numbers in this simple online exercise.

<http://www.mathgames.com/skill/4.67-multiply-fractions-by-whole-numbers>

Multiplying Fractions by Whole Numbers Quiz

This is a multiple-choice quiz that practices multiplying fractions by whole numbers.

<http://www.turtlediary.com/quiz/multiplying-fractions-by-whole-numbers.html>

GENERAL

Visual Fractions

Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated with a visual model. Also includes a couple of games.

<http://www.visualfractions.com/>

Conceptua Math Fraction Tools

Interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, finding common denominators and more. Each activity uses several visual models, such as fraction circles, horizontal and vertical bars, number lines, etc.

<https://www.conceptuamath.com/app/tool-library>

Who Wants Pizza?

This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.

<http://math.rice.edu/~lanius/fractions/index.html>

Fractioncity

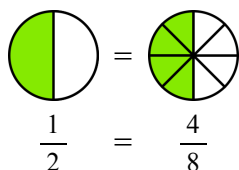
Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.

<http://www.teachnet.com/lesson/math/fractioncity.htm>

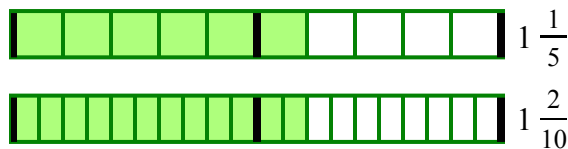
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Equivalent Fractions

If you eat half of a pizza, or if you eat $\frac{4}{8}$ of a pizza, you have eaten the same amount.



$\frac{1}{2}$ and $\frac{4}{8}$ are **equivalent fractions**.



The two fraction strips show an equal amount. So, we can write an equal sign between the

two mixed numbers: $1 \frac{1}{5} = 1 \frac{2}{10}$.

1. Color the first fraction. Shade the same amount of pie in the second picture. Write the second fraction.

<p>a. $\frac{1}{2} =$</p>	<p>b. $\frac{3}{4} =$</p>	<p>c. $\frac{6}{10} =$</p>	<p>d. $\frac{8}{12} =$</p>
<p>e. $\frac{1}{3} =$</p>	<p>f. $1 \frac{2}{3} =$</p>	<p>g. $1 \frac{10}{12} =$</p>	

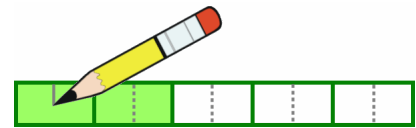
2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.

	a. $\frac{3}{3} =$	b. $\frac{4}{3} =$	c. $\frac{7}{3} =$
	d. $2 \frac{1}{3} =$	e. $1 \frac{2}{3} =$	f. $2 \frac{2}{3} =$

3. Mark the equivalent fractions on the number lines.

<p>a. $\frac{4}{5} =$</p>	<p>b. $\frac{3}{9} =$</p>
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Example 1. The fraction strip illustrates $\frac{2}{5}$. If you split each piece (both the colored and white pieces) into *two* new pieces, what fraction do you get?



You get $\frac{4}{10}$: four colored pieces, and ten pieces total.

You have *two* times as many colored pieces, and *two* times as many total pieces as before.

4. Split both the colored and white pieces as instructed. Write the fraction after you change it.

a. Split all the pieces into two new ones.



$$\frac{1}{2} = \frac{\text{yellow square}}{\text{yellow square}}$$

b. Split all the pieces into four new ones.



$$\frac{1}{2} = \frac{\text{yellow square}}{\text{yellow square}}$$

c. Split all the pieces into three new ones.



$$\frac{1}{3} = \frac{\text{yellow square}}{\text{yellow square}}$$

d. Split all the pieces into three new ones.



$$\frac{1}{3} = \frac{\text{yellow square}}{\text{yellow square}}$$

e. Split all the pieces into two new ones.



$$\frac{5}{6} = \frac{\text{yellow square}}{\text{yellow square}}$$

f. Split all the pieces into three new ones.



$$\frac{2}{5} = \frac{\text{yellow square}}{\text{yellow square}}$$

Can you notice a *shortcut* for finding the second fraction?

g. Split all the pieces into four new ones.



$$\frac{\text{yellow square}}{\text{yellow square}} = \frac{\text{yellow square}}{\text{yellow square}}$$

h. Split all the pieces into two new ones.



$$\frac{\text{yellow square}}{\text{yellow square}} = \frac{\text{yellow square}}{\text{yellow square}}$$

i. Split all the pieces into three new ones.



$$\frac{\text{yellow square}}{\text{yellow square}} = \frac{\text{yellow square}}{\text{yellow square}}$$

If you found the shortcut, explain how it works in these problems:

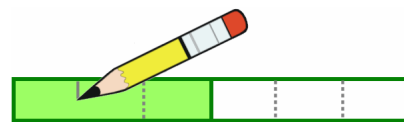
Split all the pieces into three new ones.

$$\frac{1}{3} = \frac{\text{yellow square}}{\text{yellow square}}$$

Split all the pieces into two new ones.

$$\frac{3}{5} = \frac{\text{yellow square}}{\text{yellow square}}$$

Example 2. The fraction strip illustrates $\frac{1}{2}$. If we split each piece into *three* new pieces, we get $\frac{3}{6}$.



We now have *three* times as many colored pieces, and *three* times as many total pieces as before. We can show this in writing this way →

We multiply both the top and bottom number in a fraction by 3. We get an equivalent fraction—it is the same amount, just cut into more pieces. *This does not mean we multiply the whole fraction by 3.*

$$\frac{1}{2} = \frac{3}{6}$$

$\times 3$
 $\times 3$

5. Split the pieces. Fill in the missing parts.



a. This is $\frac{3}{4}$. Make it $\frac{9}{12}$.

Each piece is split into ____ new ones.

$$\frac{3}{4} = \frac{9}{12}$$

\times
 \times



b. This is $\frac{1}{3}$. Make it $\frac{4}{12}$.

Each piece is split into ____ new ones.

$$\frac{1}{3} = \frac{4}{12}$$

\times
 \times



c. This is $\frac{1}{2}$. Make it $\frac{5}{10}$.

Each piece is split into ____ new ones.

$$\frac{1}{2} = \frac{5}{10}$$

\times
 \times



d. This is $\frac{1}{4}$. Make it $\frac{4}{16}$.

$$\frac{1}{4} = \frac{4}{16}$$

\times
 \times



e. This is $\frac{2}{3}$. Make it $\frac{6}{9}$.

$$\frac{2}{3} = \frac{6}{9}$$

\times
 \times



f. This is $\frac{2}{3}$. Make it $\frac{8}{12}$.

$$\frac{2}{3} = \frac{8}{12}$$

\times
 \times



$$\frac{4}{5} = \frac{\quad}{10}$$

\times
 \times



$$\frac{2}{3} = \frac{\quad}{15}$$

\times
 \times



$$\frac{3}{5} = \frac{\quad}{15}$$

\times
 \times

6. Write the equivalent fraction. Use multiplication.

a. Split all the pieces into three new ones. $\frac{5}{6} = \frac{\text{yellow box}}{\text{yellow box}}$	b. Split all the pieces into five new ones. $\frac{3}{4} = \frac{\text{yellow box}}{\text{yellow box}}$	c. Split all the pieces into four new ones. $\frac{2}{5} = \frac{\text{yellow box}}{\text{yellow box}}$	d. Split all the pieces into ten new ones. $\frac{9}{10} = \frac{\text{yellow box}}{\text{yellow box}}$
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7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.

a. Pieces were split into ____ new ones. $\frac{1}{2} = \frac{\text{yellow box}}{6}$	b. Pieces were split into ____ new ones. $\frac{3}{10} = \frac{30}{\text{yellow box}}$	c. Pieces were split into ____ new ones. $\frac{2}{5} = \frac{\text{yellow box}}{30}$	d. Pieces were split into ____ new ones. $\frac{7}{8} = \frac{35}{\text{yellow box}}$
e. $\frac{2}{3} = \frac{\text{yellow box}}{6}$	f. $\frac{3}{5} = \frac{9}{\text{yellow box}}$	g. $\frac{5}{6} = \frac{\text{yellow box}}{12}$	h. $\frac{1}{3} = \frac{\text{yellow box}}{9}$

8. Write the fractions that have tenths with hundredths instead.

a. $\frac{1}{10} = \frac{\text{yellow box}}{100}$	b. $\frac{3}{10} =$	c. $\frac{6}{10} =$	d. $\frac{4}{10} =$	e. $\frac{13}{10} =$
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9. Connect the equivalent fractions with a line.

a.	$\frac{2}{3}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{10}$	$\frac{2}{8}$
	$\frac{2}{6}$	$\frac{6}{9}$

b.	$\frac{1}{2}$	$\frac{2}{10}$
	$\frac{3}{4}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{6}{12}$
	$\frac{4}{12}$	$\frac{9}{12}$

c.	$\frac{3}{6}$	$\frac{3}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$

10. Write chains of equivalent fractions!




a. $\frac{1}{2} = \frac{\text{yellow box}}{4} = \frac{\text{yellow box}}{6} = \frac{\text{yellow box}}{8} = \frac{\text{yellow box}}{\text{yellow box}} = \frac{\text{yellow box}}{\text{yellow box}} = \frac{\text{yellow box}}{\text{yellow box}}$	b. $\frac{1}{3} = \frac{\text{yellow box}}{6} = \frac{\text{yellow box}}{9} = \frac{\text{yellow box}}{12} = \frac{\text{yellow box}}{\text{yellow box}}$
---	--

We can use equivalent fractions to add fractions that have different denominators.

Example 3. Add $\frac{2}{10} + \frac{17}{100}$. First, write $\frac{2}{10}$ as $\frac{20}{100}$ (an equivalent fraction).

Then you can add, because the fractions now have the same denominator: $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$.

11. Add.

a. $\frac{1}{10} + \frac{8}{100}$ $\downarrow \quad \downarrow$  $+$ $\frac{8}{100} =$	b. $\frac{7}{10} + \frac{3}{100}$ $\downarrow \quad \downarrow$  $+$  $=$	c. $\frac{45}{100} + \frac{3}{10}$
d. $\frac{9}{10} + \frac{9}{100}$	e. $\frac{7}{10} + \frac{23}{100}$	f. $\frac{24}{100} + \frac{9}{10}$
g. $\frac{7}{100} + 1\frac{4}{10}$	h. $2\frac{28}{100} + 1\frac{5}{10}$	i. $\frac{6}{10} + \frac{35}{100} + \frac{7}{100}$

12. Draw a picture showing that $\frac{1}{3}$ and $\frac{4}{12}$ are equivalent fractions.

Puzzle Corner

Add. This is challenging. *Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.*

a. $\frac{3}{4} + \frac{1}{2}$	b. $\frac{1}{5} + \frac{3}{10}$	c. $\frac{2}{3} + \frac{2}{9}$
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Chapter 8: Decimals

Introduction

In fourth grade, students learn about decimal numbers that have one or two decimal digits, and they learn to add and subtract them. It is important to grasp these simple topics well because we are laying a groundwork for fifth and sixth grade where decimal operations take a “center stage.”

The focus is first of all, on understanding that decimals are simply fractions with a denominator of 10 or 100. Then with that in mind, we study comparing, adding, and subtracting them.

Take note of this common student misconception. Many students add $0.5 + 0.9 = 0.14$. The correct way to view $0.5 + 0.9$ is as 5 tenths plus 9 tenths, which is 14 tenths = 1.4.

An example of another misconception is when a student adds $0.5 + 0.11 = 0.16$. This student is thinking of the decimal parts as if they were “whole numbers” and adding $5 + 11 = 16$. To solve $0.5 + 0.11$ correctly, students can rewrite 0.5 as 0.50, and then the problem becomes $0.50 + 0.11 = 0.61$.

In the lesson *Using Decimals with Measuring Units*, students encounter decimals in connection with metric units, such as 0.1 km or 2.4 kg, and they also convert between the units, such as writing 0.5 km as 500 m. This topic will be studied further in 5th grade.

The Lessons in Chapter 8

	page	span
Decimal Numbers—Tenths.....	173	2 pages
Adding and Subtracting with Tenths	175	2 pages
Two Decimal Digits—Hundredths	177	4 pages
Adding and Subtracting Hundredths	181	4 pages
Adding and Subtracting Decimals in Columns	185	3 pages
Using Decimals with Measuring Units	188	2 pages
Mixed Review	190	2 pages
Review	192	2 pages

Helpful Resources on the Internet

Decimal Detective

Identify the decimal number with the help of given hints.

<http://www.turtlediary.com/game/decimal-place-value.html>

Modeling Decimals (Area and Grid Models)

An interactive “gizmo” for modeling decimals in a grid or on a number line. It is by subscription, but you can try the gizmo for 5 minutes for free.

<http://www.explorelarning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007>

Decimals on a Number Line

This multiple-choice quiz asks questions about the position of letters on the number line.

http://www.henryanker.com/Math/Fractions/Number_Line_Fractions_Decimals/Decimals_on_Number_Line_1.swf

Number Conundrum with Decimals

Complete the number puzzle using decimal numbers. The number in each block is the sum of the two numbers directly underneath.

http://www.mathplayground.com/number_conundrum_decimals.html

Decimal Challenge

Guess the decimal number between 0 and 10. Each time you get a response that tells whether your guess was too high or too low.

<http://www.interactivestuff.org/sums4fun/decchall.html>

Switch

Switch the decimals around until they are in ascending order. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/switch.html>

Scales

Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.

<http://www.interactivestuff.org/sums4fun/scales.html>

Fraction/Decimal Worksheets

Change fractions to decimal numbers or decimal numbers to fractions.

<http://www.homeschoolmath.net/worksheets/fraction-decimal.php>

Rock Hopper

Help the frog reach the other side of the pond by clicking on the rocks that add up to the correct answer.

http://www.eduplace.com/kids/mw/swfs/rockhopper_grade4.html

Bubble Burst

Burst the bubble that has the correct answer to the decimal addition.

<http://www.mathnook.com/math/bubbleburst.html>

Decimals Quiz

Practice adding and subtracting decimals in this 10-question quiz.

<https://www.thatquiz.org/tq-3/?-j163-l5-p0>

Decimal Subtraction - Matching

Match each decimal subtraction with the correct answer.

<http://www.sheppardsoftware.com/mathgames/decimals/matchingDecimalsMinus.htm>

Decimal Mania - Addition and subtraction

Practice decimal addition and subtraction with this interactive exercise.

<http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/addsubdec.shtml>

Get to the (Decimal) Point Addition and Subtraction card games (pp. 60-63 of the PDF file)

This is a card game with four different variations that practice decimal addition and subtraction.

http://www.pepnonprofit.org/uploads/2/7/7/2/2772238/acing_math.pdf

Adding and Subtracting Decimals Quiz

This 10-question multiple-choice quiz focuses on adding and subtracting decimals.

<http://www.mrmaisonet.com/index.php?/Decimal-Quizzes/Adding-and-Subtracting-Decimals.html>

Decimals Magic Square

Add and subtract decimals to make a "magic square". Add to find a magic sum and subtract to find the missing addends.

<http://www.hbschool.com/activity/elab2004/gr4/14.html>

Taking Measures Capacity Game

A quiz with varying questions concerning the concept of volume and how to measure volume.

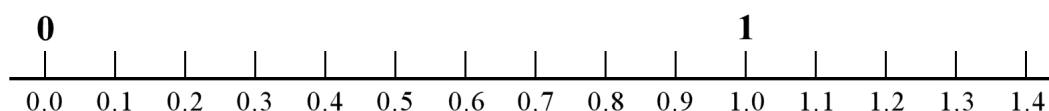
<http://www.bbc.co.uk/skillswise/game/ma23capa-game-taking-measures-capacity>

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Adding and Subtracting with Tenths

<p>You already know how to add or subtract decimals that have tenths, such as $0.8 + 0.5$. They are just fractions with a denominator of 10.</p> <p>Compare the two additions in each box. One of them is written with decimals and the other with fractions.</p>	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
<p>There is one tricky thing: $0.6 + 0.7$ is NOT 0.13 !!</p> <p>To see why, add the corresponding fractions. Notice that six tenths and seven tenths makes thirteen tenths, which is more than one!</p>	$0.6 + 0.7 = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each “number line jump.”



- a. You're at 0.7, and you jump *five tenths* to the right. _____
- b. You're at 0.6, and you jump *eight tenths* to the right. _____
- c. You're at 1.1, and you jump *eight tenths* to the left. _____
- d. You're at 1.3, and you jump *four tenths* to the left. _____
- e. You're at 0.2, and you jump *eleven tenths* to the right. _____

2. Solve the fraction additions, and then write them using decimals.

<p>a. $\frac{2}{10} + \frac{7}{10} =$</p> <p>$0.2 +$</p>	<p>b. $\frac{5}{10} + \frac{6}{10} =$</p>	<p>c. $\frac{9}{10} + \frac{8}{10} =$</p>
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3. Add or subtract.

a.	b.	c.	d.
$0.9 + 0.2 =$ _____	$0.5 + 0.7 =$ _____	$0.8 + 0.7 =$ _____	$1.8 - 0.9 =$ _____
$1.9 + 0.2 =$ _____	$3.5 + 0.7 =$ _____	$0.8 + 2.7 =$ _____	$5.8 - 0.9 =$ _____

4. Fill in the missing parts.

a. $2.3 + 0.9 = \underline{\hspace{2cm}}$	b. $1.5 + 0.7 = \underline{\hspace{2cm}}$	c. $6.6 - 0.5 = \underline{\hspace{2cm}}$	d. $4.7 - 1.7 = \underline{\hspace{2cm}}$
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5. Write the numbers.

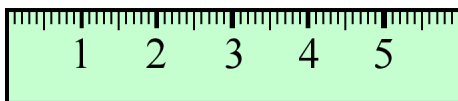
a. 3 tenths, 5 ones	d. Write the numbers in order. 9 8.9 9.1 9.0 9.9 1.9
b. 7 tens, 8 ones, 4 tenths	
c. 4 tenths, 3 ones, 6 tens	

6. Continue the patterns by adding or subtracting the same number repeatedly.

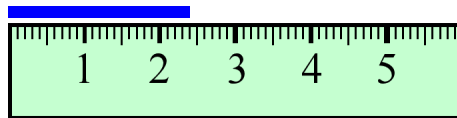
a. 0.1 $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$ $+ 0.2 = \underline{\hspace{2cm}}$	b. 1.1 $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$ $+ 0.5 = \underline{\hspace{2cm}}$	c. 2.5 $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$ $+ 0.3 = \underline{\hspace{2cm}}$	d. 3.6 $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$ $- 0.4 = \underline{\hspace{2cm}}$
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7. Remember? **1 millimeter is one-tenth of a centimeter.** Or, $1 \text{ mm} = 0.1 \text{ cm}$.

a. Draw a line that is 4.7 cm long.



b. Measure the line in centimeters.
Use a decimal.



8. In (a) and (b), convert. In (c), add and give your answer in centimeters.

a. $0.5 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

b. $7 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$

c. $5 \text{ mm} + 0.9 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$

$1.2 \text{ cm} = \underline{\hspace{2cm}} \text{ mm}$

$35 \text{ mm} = \underline{\hspace{2cm}} \text{ cm}$

$4 \text{ cm} + 3.4 \text{ cm} = \underline{\hspace{2cm}} \text{ cm}$

9. The two sides of a rectangle measure 6.5 cm and 3.6 cm.
Draw the rectangle on blank paper. What is its perimeter?