Activity Library: Volume II

Activity Collections

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Toys, maps, and blueprints all use the idea of a scale. Based on the provided scale, we can “scale down” to make a smaller model of the object or distance. We can “scale up” to see how large the object is in reality. Scales are often provided in the form of a ratio. Based on this ratio, we can answer questions about the situation, as will be seen in this worksheet.

1. Hot Wheels offers a 5-pack of cars that were manufactured using the scale 1:64 (shop.mattel.com). Explain what this means.

2. For the Hot Wheel car manufactured using the scale 1:64, how tall would the driver be of this car? State any assumptions used.

3. We will continue using the context of the Hot Wheel car manufactured using the scale 1:64. Suppose they need to make a Ford Mustang tire that is a standard size of P235/50ZR18 (media.ford.com). This means that the diameter of the wheel is 18 inches. What will be the diameter of the scale model?
4. Google SketchUp is a free design tool for creating 3-D scale drawings of just about anything (sketchup.google.com). Once a SketchUp design project is completed, the project can be printed. In one case, the design project is printed using the scale 1 inch in the printout is equivalent to 33.84 inches in SketchUp. Suppose you are designing a 4-story building (about 40 feet in height). What will its height be in the printout? Will this design fit on a standard $8\frac{1}{2}$" by 11" piece of paper?

Using the scale 1:33.84, we say that 1 inch in the printout represents 33.84 inches in SketchUp. Therefore, 1 inch in SketchUp represents $\frac{1}{33.84}$ of an inch in the printout. So, 40 feet (480 inches) in SketchUp represents $\frac{480}{33.84} \approx 14.2$ inches in the printout. Note that we would have to use paper that is larger than the traditional $8\frac{1}{2}$" by 11" size.

5. The Burj Khalifa, located in downtown Dubai (United Arab Emirates) is the tallest skyscraper in the world at a height of 2,716.5 feet (www.burjkhalifa.ae). The Lego Company has created a Lego kit that models the Burj Khalifa. The Lego model is 10 inches tall (shop.lego.com). What scale was used to make the Lego model of the Burj Khalifa?

We say that 10 inches on the scale model represents 2,716.5 feet on the actual building. We make the units of the length the same for each. There are 12 inches in 1 foot, therefore 2,716.5 feet is equivalent to $\frac{2716.5}{12} = 226.375$ inches. So, the scale can be represented by 10:226.375 or 1:22.6375.

6. The Empire State Building, located in New York City, is one of the buildings that the Lego Company has available in their architecture collection (shop.lego.com). The height of the actual building is 1250 feet (www.pbs.org). Using the scale computed in Exercise 5, how tall should the Lego model of the Empire State Building be?

Using the scale 1:22.6375, we say that 1 inch on the scale model represents 22.6375 inches on the actual building. The Empire State Building is $\frac{1250}{22.6375} \approx 55.18$ inches tall. There are $\frac{1250}{22.6375} \approx 4.63$ copies of 22.6375 inches in 1250 so the scale model will need to be 4.6 inches in height using this scale.

7. The Lego model of the Empire State Building is actually 7.4 inches in height (shop.lego.com). If the model of the Empire State Building was placed next to the model of the Burj Khalifa, would it appear to be in the same proportion as the actual buildings if they could be placed next to one another?

No, it would not. If using the same scale as used for the Burj Khalifa, the Lego model of the Empire State Building should be 4.6 inches instead of 7.4 inches.
Scale Models #1
Working with Proportional Thinking

Toys, maps, and blueprints all use the idea of a scale. Based on the provided scale, we can “scale down” to make a smaller model of the object or distance. We can “scale up” to see how large the object is in reality. Scales are often provided in the form of a ratio. Based on this ratio, we can answer questions about the situation, as will be seen in this worksheet.

1. Hot Wheels offers a 5-pack of cars that were manufactured using the scale 1:64 (shop.mattel.com). Explain what this means.

This means that 1 unit of length (inch, meter, whatever) on the scale model represents 64 units of length (inch, meter, whatever) on the actual object.

2. For the Hot Wheel car manufactured using the scale 1:64, how tall would the driver be of this car? State any assumptions used.

Suppose we consider a driver of an actual car that is 6 feet tall. Using the scale 1:64, we say that 1 foot on the scale model represents 64 feet for the actual object. Therefore, 1 foot on the actual object represents \( \frac{1}{64} \) of a foot on the scale. So, 6 feet on the real object represents \( \frac{6}{64} = \frac{3}{32} \) of a foot on the scale. In inches, the driver of the Hot Wheels car is \( \frac{3}{32} \cdot 12 \) inches = \( \frac{9}{8} \) inches = \( 1 \frac{1}{8} \) inches.

3. We will continue using the context of the Hot Wheel car manufactured using the scale 1:64. Suppose they need to make a Ford Mustang tire that is a standard size of P235/50ZR18 (media.ford.com). This means that the diameter of the wheel is 18 inches. What will be the diameter of the scale model?

With a scale of 1:64, we know that the diameter of the wheel on the model must be \( \frac{1}{64} \) the diameter of the actual wheel.

\[
\frac{1}{64} \cdot 18 \text{ inches} = \frac{18}{64} \text{ inch} = \frac{9}{32} \text{ inch}.
\]

The scale model will have a wheel diameter of \( \frac{9}{32} \) inch.
4. Google SketchUp is a free design tool for creating 3-D scale drawings of just about anything (sketchup.google.com). Once a SketchUp design project is completed, the project can be printed. In one case, the design project is printed using the scale 1 inch in the printout is equivalent to 33.84 inches in SketchUp. Suppose you are designing a 4-story building (about 40 feet in height). What will its height be in the printout? Will this design fit on a standard $8\frac{1}{2}$ by 11" piece of paper?

Using the scale 1:33.84, we say that 1 inch in the printout represents 33.84 inches in SketchUp. Therefore, 1 inch in SketchUp represents $\frac{1}{33.84}$ of an inch in the printout. So, 40 feet (480 inches) in SketchUp represents $\frac{480}{33.84} \approx 14.2$ inches in the printout. Note that we would have to use paper that is larger than the traditional $8\frac{1}{2}$ by 11" size.

5. The Burj Khalifa, located in downtown Dubai (United Arab Emirates) is the tallest skyscraper in the world at a height of 2,716.5 feet (www.burjkhalifa.ae). The Lego Company has created a Lego kit that models the Burj Khalifa. The Lego model is 10 inches tall (shop.lego.com). What scale was used to make the Lego model of the Burj Khalifa?

We say that 10 inches on the scale model represents 2,716.5 feet on the actual building. We make the units of the length the same for each. There are 12 inches in 1 foot, therefore 2,716.5 feet is equivalent to $2,716.5 \cdot 12 = 32,598$ inches. So, the scale can be represented by 10:32,598 or 1:3,259.8.

6. The Empire State Building, located in New York City, is one of the buildings that the Lego Company has available in their architecture collection (shop.lego.com). The height of the actual building is 1250 feet (www.pbs.org). Using the scale computed in Exercise 5, how tall should the Lego model of the Empire State Building be?

Using the scale 1:3,259.8, we say that 1 inch on the scale model represents 3,259.8 inches on the actual building. The Empire State Building is $12 \cdot 1250 = 15,000$ inches tall. There are $\frac{15,000}{3,259.8} \approx 4.6$ copies of 3,259.8 in 15,000 so the scale model will need to be 4.6 inches in height using this scale.

7. The Lego model of the Empire State Building is actually 7.4 inches in height (shop.lego.com). If the model of the Empire State Building was placed next to the model of the Burj Khalifa, would it appear to be in the same proportion as the actual buildings if they could be placed next to one another?

No, it would not. If using the same scale as used for the Burj Khalifa, the Lego model of the Empire State Building should be 4.6 inches instead of 7.4 inches.
Outdoorsmen, like hikers or campers, often bring a map and a GPS device or compass to help make sure they don’t get lost in the wilderness. Compasses, like the one shown below, is based on bearings. A bearing tells you which way you’re heading, reported using the measure of the angle (rotated clockwise from North) generated by the red arrowheads. Two examples of bearings are shown below.

1. Draw the angle showing a bearing of $105^\circ$. In what general direction is this bearing?

2. Draw the angle showing a bearing of $300^\circ$. In what general direction is this bearing?
3. Draw the angle showing a bearing of 180°. In what general direction is this bearing?

Headings are also sometimes reported relative to other cardinal directions. For example, someone might say her heading is 15° degrees North of East.

4. Draw the angle showing a heading of 15° degrees North of East.

5. Draw the angle showing a heading of 40° East of South.

6. Draw the angle showing a heading of 35° South of West.
Where are We Heading? 
Working with Drawing Angles

Outdoorsmen, like hikers or campers, often bring a map and a GPS device or compass to help make sure they don’t get lost in the wilderness. Compasses, like the one shown below, is based on bearings. A bearing tells you which way you’re heading, reported using the measure of the angle (rotated clockwise from North) generated by the red arrowheads. Two examples of bearings are shown below.

1. Draw the angle showing a bearing of 105°. In what general direction is this bearing?

   This bearing is pointed in the general direction of East. (More specifically, a little South of East.)

2. Draw the angle showing a bearing of 300°. In what general direction is this bearing?

   This bearing is pointed in a Northwestern direction. (More specifically, a little North of West.) Note that drawing this bearing with a protractor can be difficult for students since a protractor usually only represents angle measures between 0° and 180°. The easiest method is to note that 300° is 60° short of 360°. So a bearing of 300° can be drawn by making an angle measuring 60° rotated counterclockwise from North.
3. Draw the angle showing a bearing of 180°. In what general direction is this bearing?

This bearing is heading due South.

Headings are also sometimes reported relative to other cardinal directions. For example, someone might say her heading is 15° degrees North of East.

4. Draw the angle showing a heading of 15° degrees North of East.

5. Draw the angle showing a heading of 40° East of South.

6. Draw the angle showing a heading of 35° South of West.
Savings for Retirement  
Using Finite Geometric Series

One of the best ways to save money for retirement is to make a savings deposit every month. Over time, even a small monthly contribution can increase in value dramatically. As of October 22, 2008, ING DIRECT offered a 5-year certificate of deposit paying 4.17% interest compounded monthly. The minimum investment required was one dollar (Source: www.bankrate.com).

1. Suppose that a working student deposits $100 per month on the first day of the month into an account paying 4.17% interest compounded monthly. What will be the value of the account after the fourth monthly deposit is made? (Assume interest on a monthly deposit is first paid one month after the deposit is made.)
2. The solution in Exercise 1 may be obtained by adding up the first four terms of a geometric sequence. Write the sum of the four terms.

3. The sum of the first n terms of a finite geometric sequence with first term a and common ratio r is

\[ S_n = \frac{a - ar^n}{1 - r} \].

Use this formula to calculate the solution to the problem given in Exercise 1. Explain any difference in the solution to Exercise 1 and the solution found by using this formula.

4. If the student continues to make $100 monthly contributions, what will be the value of the account after 480 monthly investments have been made?

5. How many monthly contributions will need to be made before the account balance first exceeds $100,000?
One of the best ways to save money for retirement is to make a savings deposit every month. Over time, even a small monthly contribution can increase in value dramatically. As of October 22, 2008, ING DIRECT offered a 5-year certificate of deposit paying 4.17% interest compounded monthly. The minimum investment required was one dollar (Source: www.bankrate.com).

1. Suppose that a working student deposits $100 per month on the first day of the month into an account paying 4.17% interest compounded monthly. What will be the value of the account after the fourth monthly deposit is made? (Assume interest on a monthly deposit is first paid one month after the deposit is made.)

Suppose that the first $100 deposit is made on January 1. Then the first interest payment will be made on February 1, the second interest payment will be made on March 1, and the third interest payment will be made on April 1.

\[ FV = 100 \left( 1 + \frac{0.0417}{12} \right)^3 \]
\[ = 100(1.0105) \]
\[ = 101.05 \]

The second $100 deposit will be made on February 1 and will receive two interest payments.

\[ FV = 100 \left( 1 + \frac{0.0417}{12} \right)^2 \]
\[ = 100(1.0070) \]
\[ = 100.70 \]

The third $100 deposit will be made on March 1 and will receive one interest payment.

\[ FV = 100 \left( 1 + \frac{0.0417}{12} \right) \]
\[ = 100(1.0035) \]
\[ = 100.35 \]

The fourth $100 deposit will be made on April 1 and will not receive an interest payment. The value of the entire investment is given by the following equation.

\[ FV = 101.05 + 100.70 + 100.35 + 100 \]
\[ = $402.10 \]
2. The solution in Exercise 1 may be obtained by adding up the first four terms of a geometric sequence. Write the sum of the four terms.

\[ r = \left( 1 + \frac{0.0417}{12} \right) \]

\[ = 1.003475 \]

\[ S_4 = 100 + 100(1.003475) + 100(1.003475)^2 + 100(1.003475)^3 \]

3. The sum of the first \( n \) terms of a finite geometric sequence with first term \( a \) and common ratio \( r \) is

\[ S_n = \frac{a - ar^n}{1 - r} \]. Use this formula to calculate the solution to the problem given in Exercise 1. Explain any difference in the solution to Exercise 1 and the solution found by using this formula.

\[ S_4 = \frac{100 - 100(1.003475)^4}{1 - (1.003475)} \]

\[ \approx 402.09 \]

*The $0.01 difference in the solutions is due to round off error. In Exercise 1, we rounded the intermediate values to the nearest cent whereas in this exercise we only rounded the final solution to the nearest cent.*

4. If the student continues to make $100 monthly contributions, what will be the value of the account after 480 monthly investments have been made?

\[ S_{480} = \frac{100 - 100(1.003475)^{480}}{1 - (1.003475)} \]

\[ \approx 123,345.24 \]

*The account value will be $123,345.24.*

5. How many monthly contributions will need to be made before the account balance first exceeds $100,000?

\[ 100,000 = \frac{100 - 100(1.003475)^n}{1 - (1.003475)} \]

\[ 100,000 = \frac{100(1 - 1.003475^n)}{-0.003475} \]

\[-347.5 = 100(1 - 1.003475^n) \]

\[-3.475 = 1 - 1.003475^n \]

\[-4.475 = -1.003475^n \]

\[ 4.475 = 1.003475^n \]

\[ \ln(4.475) = n \ln(1.003475) \]

\[ n \approx 432 \text{ monthly contributions} \]