

Make It Real Learning

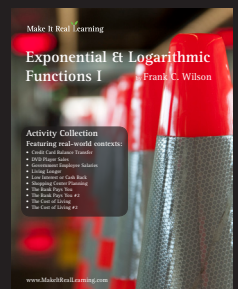
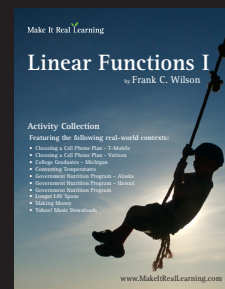
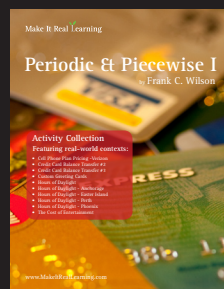
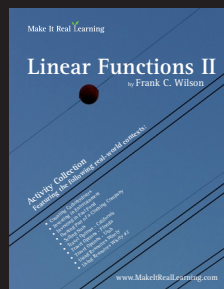
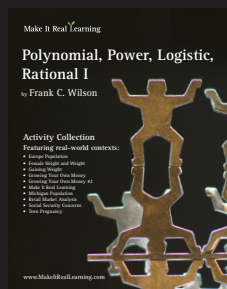
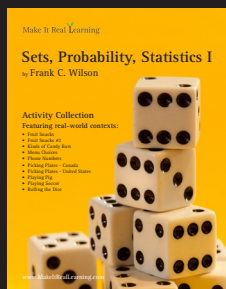
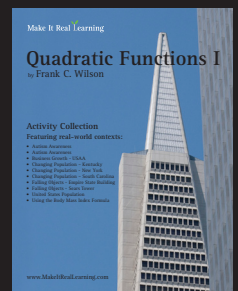
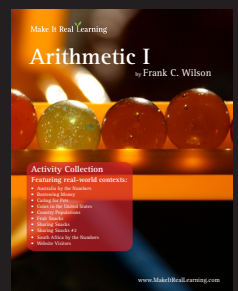
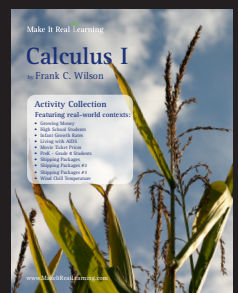
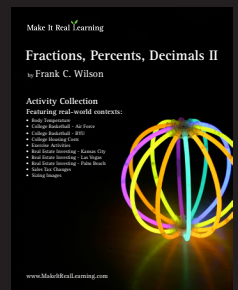
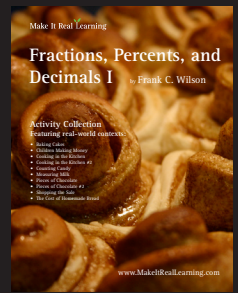
# Volume I Activity Sampler

by Frank C. Wilson

## Activity Collection

### Featuring real-world contexts:

- Choosing a Cell Phone Plan - Verizon (Linear I)
- Cooking in the Kitchen (Fractions, Percents, Decimals I)
- High School Students (Calculus I)
- Rolling the Dice (Sets, Probability, Statistics I)
- Fruit Snacks (Arithmetic I)
- Teen Pregnancy (Polynomial, Power, Logistic, Rational I)
- Shopping Center Planning (Exponential and Logarithmic I)
- Hours of Daylight - Anchorage (Periodic and Piecewise I)
- United States Population (Quadratic I)
- Travel Options - Florida (Linear II)



[www.MakeItRealLearning.com](http://www.MakeItRealLearning.com)

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## Introduction

“When am I ever going to use this?” It is a question that has plagued teachers and learners for decades. Now, with the help of the Make It Real Learning workbook series, you can answer the question.

The *Volume I Activity Sampler* workbook features ten activities from the *Make It Real Learning Activity Library, Volume I*. The library contains the following workbooks:

- Arithmetic I
- Calculus I
- Exponential and Logarithmic Functions I
- Fractions, Percents, Decimals I
- Linear Functions I
- Linear Functions II
- Periodic and Piecewise Functions I
- Polynomial, Power, Logistic, Rational Functions I
- Quadratic Functions I
- Sets, Probability, Statistics I

Each workbook contains ten activities. From determining which cell phone plan is the best value to forecasting the population of the United States, learners get to use mathematics in meaningful ways. Rest assured that each activity integrates real world information not just “realistic” data. These are real companies (e.g. Aeropostale, Disneyland) and real world issues (e.g. Social Security, teen pregnancy).

The mathematical objectives of each activity are clearly specified on the Activity Objectives page following introduction in each workbook. (No objectives are shown in this sampler.) Through the workbook series, we have consistently sought to address the content and process standards of the National Council of Teachers of Mathematics.

There are multiple ways to use the activities in a teaching environment. Many teachers find that the activities are an excellent tool for stimulating mathematical discussions in a small group setting. Due to the challenging nature of each activity, group members are motivated to brainstorm problem solving strategies together. The interesting real world contexts motivate them to want to solve the problems. The activities may also be used for individual projects and class-wide discussions.

As a ready-resource for teachers, the workbook also includes completely worked out solutions for each activity. To make it easier for teachers to assess student work, the solutions are included on a duplicate copy of each activity.

We hope you enjoy the activities! We continue to increase the number of workbooks in the Make It Real Learning workbook series. Please visit [www.MakeItRealLearning.com](http://www.MakeItRealLearning.com) for the most current list of activities. Thanks!

Frank C. Wilson  
Author

# Choosing a Cell Phone Plan–Verizon

## Investigating Linear Equations



**When am I ever going to use this?** ”  
Using the concepts in this worksheet, you will be able to determine which cell phone plan is the best value.

In 2008, Verizon offered the following cell phone plans to consumers. (Source: [www.verizon.com](http://www.verizon.com))

Verizon: Nationwide Basic		
Monthly Anytime Minutes	Monthly Fee	Charge for Extra Minutes
450 minutes	\$39.99	\$0.45 per minute
900 minutes	\$59.99	\$0.40 per minute
1350 minutes	\$79.99	\$0.30 per minute

1. If a customer expects to use 500 minutes per month, which plan is the best value?
2. A cell phone plan consists of a fixed cost (the monthly fee) and a variable cost (charge for extra minutes). Find a linear function that gives the total cost of each plan when  $x$  extra minutes are used.
3. Using the cost functions from (2) above, determine the cost of using 600 total minutes and 1000 total minutes with each plan. Then determine which plan is the best deal for each level.

4. The number of minutes a person uses each month often varies. Suppose that a Verizon customer uses the total number of minutes shown below.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
423	441	404	387	417	628	448	402	356	402	436	639

Which plan would cost the consumer the least amount money over the year?

5. At what number of extra minutes does the annual cost of the 900 Plan become less expensive than the 450 Plan?
6. Which plan would you recommend to a consumer who expects to use between 400 and 600 minutes each month? Explain your reasoning.

# Choosing a Cell Phone Plan–Verizon

## Investigating Linear Equations

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1350 minutes	\$79.99	\$0.30 per minute

1. If a customer expects to use 500 minutes per month, which plan is the best value?

450 Plan:  $\$39.99 + (50 \text{ extra minutes})(\$0.45 \text{ per minute}) = \$62.49$

900 Plan: \$59.99

For this customer, the 900 minute plan costs less than the 450 minute plan.

2. A cell phone plan consists of a fixed cost (the monthly fee) and a variable cost (charge for extra minutes). Find a linear function that gives the total cost of each plan when  $x$  extra minutes are used.

*Total Cost = Fixed Cost + Variable Cost*

*= Monthly Fee + (Cost per Extra Minute)(Extra Minute)*

*= Monthly Fee + (Cost per Extra Minute)( $x$ )*

450 Plan:  $C = 39.99 + 0.45x$

900 Plan:  $C = 59.99 + 0.40x$

1350 Plan:  $C = 79.99 + 0.30x$

3. Using the cost functions from (2) above, determine the cost of using 600 total minutes and 1000 total minutes with each plan. Then determine which plan is the best deal for each level.

For the 450 Plan, 600 minutes requires 150 minutes more than the included minutes and 1000 minutes requires 550 minutes more than the included minutes. For the 900 Plan, 0 and 100 extra minutes are needed, respectively. For the 1350 Plan, no extra minutes are needed.

Plan	600 minutes	1000 minutes
450	$C = 39.99 + 0.45(150) = \$107.49$	$C = 39.99 + 0.45(550) = \$287.49$
900	$C = 59.99 + 0.40(0) = \$59.99$ *	$C = 59.99 + 0.40(100) = \$99.99$
1350	$C = 79.99 + 0.30(0) = \$79.99$	$C = 79.99 + 0.30(0) = \$79.99$ *



4. The number of minutes a person uses each month often varies. Suppose that a Verizon customer uses the total number of minutes shown below.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
423	441	404	387	417	628	448	402	356	402	436	639

Which plan would cost the consumer the least amount money over the year?

*This consumer used less than 450 minutes a month every month except June and December. In June, they used 178 extra minutes and in December they used 189 extra minutes. In total, they used 367 extra minutes throughout the year. The basic fee for the 450 Plan is \$39.99 per month so their annual cost is*

$$\begin{aligned}
 \text{Annual Cost} &= (\text{Monthly fee})(\text{months}) + (\text{cost per minute})(\text{extra minutes}) \\
 &= (39.99)(12) + (0.45)(367) \\
 &= 479.88 + 165.15 \\
 &= 645.03
 \end{aligned}$$

*The annual cost for the 450 Plan is \$645.03.*

*The 900 Plan costs \$59.99 per month for all 12 months for a total cost of \$719.88. Even though the consumer had to pay extra during June and December, the 450 Plan was still less expensive than the 900 Plan.*

5. At what number of extra minutes does the annual cost of the 900 Plan become less expensive than the 450 Plan?

*The annual cost of the 450 Plan if no extra minutes are used is  $(39.99)(12) = \$479.88$ . Extra minutes are \$0.45 each. The annual cost of the 900 Plan if no extra minutes are used is  $(59.99)(12) = \$719.88$ . We need to solve the following equation.*

$$\begin{aligned}
 &450 \text{ Plan Cost} = \text{Lowest Cost for 900 Plan} \\
 &479.88 + 0.45x = 719.88 \\
 &0.45x = 240 \\
 &x \approx 533
 \end{aligned}$$

*As long as the total number of extra minutes per year remains at 533 minutes or below, the 450 Plan is the least expensive of the two plans.*

6. Which plan would you recommend to a consumer who expects to use between 400 and 600 minutes each month? Explain your reasoning.

*If the consumer uses 450 minutes or less each month, the 450 Plan is clearly the better plan. In (5) above, we saw that after 533 extra minutes annually (average of 44 minutes monthly), the 900 Plan becomes the better value. In other words, if the consumer consistently uses more than 494 minutes monthly, the 900 Plan is the better deal. If the consumer is on the 450 Plan and talks 600 minutes each month, their annual cost is \$1289.88! In contrast, the 900 Plan annual cost is \$719.88. We choose to play it safe and pick the 900 Plan.*

# Cooking in the Kitchen

## Working with Fractions

“When am I ever going to use this?”  
Using the concepts in this worksheet, you will be able to adjust recipe ingredients to make a larger or smaller batch of cookies.

A favorite chocolate chip cookie recipe of our family is shown below.

### Oatmeal Chocolate Chip Cookies

1½ cups sugar	2 cups wheat flour
2 cups brown sugar	2 cups oatmeal
4 cubes of butter	½ cup oat bran
4 eggs beaten	1 teaspoon salt
2 Tablespoons vanilla	1½ teaspoon soda
2½ cups flour	4 cups chocolate chips

Mix ingredients. Spoon onto cookie sheet.  
Bake at 350 degrees for 9 minutes.

As shown in the recipe, various units of measure (teaspoons, tablespoons, cups) are used in making the cookies.

1. If we cut the recipe in half, how many cups of chocolate chips and how many cups of flour will be needed?

2. If a  $\frac{1}{2}$ -cup measure is used to measure the sugar and brown sugar for the original recipe, how many times will the  $\frac{1}{2}$ -cup measure need to be filled for each type of sugar?
3. One way to compare two different quantities is to calculate their ratio. That is, divide one quantity by the other. Calculate the ratio of sugar to brown sugar in the original recipe. Then explain what the ratio means. (Hint: You may find it helpful to use the answer from (2).)
4. We want to make as many cookies as possible but only have 3 cups of oatmeal available. If we plan to use all of the oatmeal, how much sugar and how much oat bran will we need?

# Cooking in the Kitchen

## Working with Fractions

“When am I ever going to use this?”  
Using the concepts in this worksheet, you will be able to adjust recipe ingredients to make a larger or smaller batch of cookies.

A favorite chocolate chip cookie recipe of our family is shown below.

### Oatmeal Chocolate Chip Cookies

1½ cups sugar

2 cups brown sugar

4 cubes of butter

4 eggs beaten

2 Tablespoons vanilla

2½ cups flour

2 cups wheat flour

2 cups oatmeal

½ cup oat bran

1 teaspoon salt

1½ teaspoon soda

4 cups chocolate chips

Mix ingredients. Spoon onto cookie sheet.

Bake at 350 degrees for 9 minutes.

As shown in the recipe, various units of measure (teaspoons, tablespoons, cups) are used in making the cookies.

1. If we cut the recipe in half, how many cups of chocolate chips and how many cups of flour will be needed?

To cut the recipe in half, we may divide the quantities by 2 or multiply them by  $\frac{1}{2}$ . We will use both methods.

$$\begin{aligned}\frac{(4 \text{ cups chocolate chips})}{2} &= \frac{4}{2} \text{ cups chocolate chips} \\ &= 2 \text{ cups of chocolate chips}\end{aligned}$$

$$\begin{aligned}\frac{1}{2} \times (2\frac{1}{2} \text{ cups flour}) &= \frac{1}{2} \times (\frac{5}{2} \text{ cups flour}) \\ &= \frac{1 \times 5}{2 \times 2} \text{ cups flour} \\ &= \frac{5}{4} \text{ cups flour} \\ &= 1\frac{1}{4} \text{ cups flour}\end{aligned}$$

Two cups of chocolate chips and  $1\frac{1}{4}$  cups flour will be needed.

2. If a  $\frac{1}{2}$ -cup measure is used to measure the sugar and brown sugar for the original recipe, how many times will the  $\frac{1}{2}$ -cup measure need to be filled for each type of sugar?

*Since two  $\frac{1}{2}$ -cup measures make up a whole cup, three  $\frac{1}{2}$ -cup measures are needed for the  $1\frac{1}{2}$  cups sugar and four  $\frac{1}{2}$ -cup measures are needed for the 2 cups of brown sugar.*

3. One way to compare two different quantities is to calculate their ratio. That is, divide one quantity by the other. Calculate the ratio of sugar to brown sugar in the original recipe. Then explain what the ratio means. (Hint: You may find it helpful to use the answer from (2).)

$$\begin{aligned}\frac{3\frac{1}{2}\text{-cups of sugar}}{4\frac{1}{2}\text{-cups of brown sugar}} &= \frac{3}{4} \times \frac{\frac{1}{2}\text{-cups of sugar}}{\frac{1}{2}\text{-cups of brown sugar}} \\ &= \frac{3}{4} \times \frac{\cancel{\frac{1}{2}\text{-cups of sugar}}}{\cancel{\frac{1}{2}\text{-cups of}} \text{ brown sugar}} \\ &= \frac{3 \text{ sugar}}{4 \text{ brown sugar}}\end{aligned}$$

*This means that for every 4 measures of brown sugar in the recipe, 3 measures of sugar should be added. This will be true whether cups,  $\frac{1}{2}$ -cups, tablespoons, or other type of measure is used.*

4. We want to make as many cookies as possible but only have 3 cups of oatmeal available. If we plan to use all of the oatmeal, how much sugar and how much oat bran will we need?

*We must determine by what proportion we are increasing the recipe. To do this, we divide the amount of oatmeal available by the amount shown in the recipe.*

$$\frac{3 \text{ cups oatmeal}}{2 \text{ cups oatmeal}} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

*We need  $1\frac{1}{2}$  times of each ingredient. The original recipe calls for  $1\frac{1}{2}$  cups sugar and  $\frac{1}{2}$  -cup of oat bran.*

$$\begin{aligned}1\frac{1}{2} \times (1\frac{1}{2} \text{ cups sugar}) &= \frac{3}{2} \times \left(\frac{3}{2} \text{ cups sugar}\right) & 1\frac{1}{2} \times \left(\frac{1}{2} \text{ cup oat bran}\right) &= \frac{3}{2} \times \left(\frac{1}{2} \text{ cup oat bran}\right) \\ &= \frac{3 \times 3}{2 \times 2} \text{ cups sugar} & &= \frac{3 \times 1}{2 \times 2} \text{ cup oat bran} \\ &= \frac{9}{4} \text{ cups sugar} & &= \frac{3}{4} \text{ cup oat bran} \\ &= 2\frac{1}{4} \text{ cups sugar} & &\end{aligned}$$

*We need  $2\frac{1}{4}$  cups of sugar and  $\frac{3}{4}$  cup of oat bran.*

# High School Students

## Working with Rates of Change

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

**B**ased on projections from 2006–07 to 2016–17, the number of students in grades 9 through 12 in the United States may be modeled by

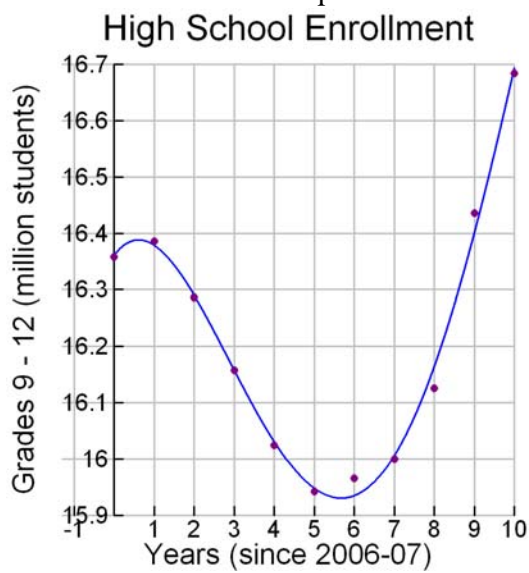
$$s(t) = -0.4936t^4 + 13.27t^3 - 89.50t^2 + 95.10t + 16,360 \text{ thousand students}$$

where  $t$  is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 -10 or in 2014 -15? Use calculus to find your answer and show the work that leads to your conclusion.

2. Use technology and the derivative of  $s$  to find the absolute extrema of  $s$  on the interval  $[0,10]$ . Then interpret the real world meaning of each extreme value.

3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of  $s$  on the interval  $[0, 10]$  and interpret what it represents in terms of the rate of change of  $s$ . (Note: The units on the graph are million students and the units of the equation of  $s$  are thousand students.)



4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

# High School Students

## Working with Rates of Change

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Using the concepts in this worksheet, you will be able to use calculus to find and interpret rates of change in the real world.

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$$s(t) = -0.4936t^4 + 13.27t^3 - 89.50t^2 + 95.10t + 16,360 \text{ thousand students}$$

where  $t$  is the number of years since the 2006 – 07 school year (Source: Modeled from the Statistical Abstract of the United States 2008, Table 209).

1. According to the model, will the number of students be changing more rapidly in 2009 –10 or in 2014 –15? Use calculus to find your answer and show the work that leads to your conclusion.

$$s'(t) = -1.9744t^3 + 39.81t^2 - 179.0t + 95.10$$

$$\begin{aligned} s'(3) &= -1.9744(3)^3 + 39.81(3)^2 - 179.0(3) + 95.10 \\ &\approx -136.9 \text{ thousand students per year} \end{aligned}$$

$$\begin{aligned} s'(8) &= -1.9744(8)^3 + 39.81(8)^2 - 179.0(8) + 95.10 \\ &\approx 200.0 \text{ thousand students per year} \end{aligned}$$

According to the model, the number of students will be decreasing at a rate of 136.9 thousand students per year in 2009 – 10 and increasing at a rate of 200.0 thousand students per year in 2014 – 15. The number of students is changing more rapidly in 2014 – 15.

2. Use technology and the derivative of  $s$  to find the absolute extrema of  $s$  on the interval  $[0,10]$ . Then interpret the real world meaning of each extreme value.

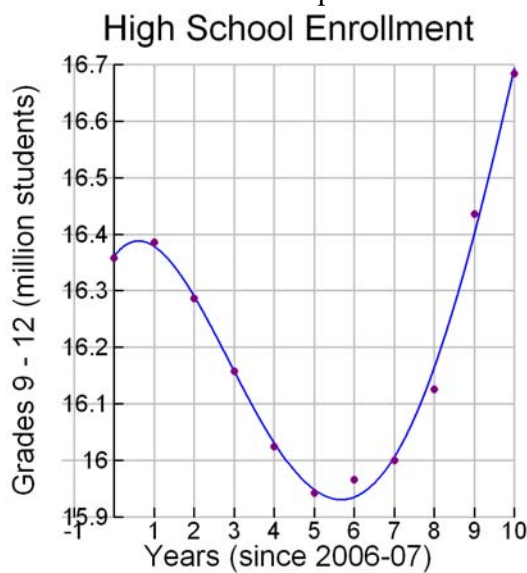
Relative extrema of  $s$  occur wherever  $s'$  changes sign. We graph  $s'(t)$  and  $y = 0$  simultaneously and find the points of intersection to determine the critical values of  $s$ . The critical values of  $s$  are  $s \approx 0.61$  and  $s = 5.67$ .

$t$	$s$
0	16,360
0.61	16,390
5.67	15,930
10	16,700

Rounded to the nearest multiple of 10 thousand, the absolute maximum number of students enrolled will be 16,700 thousand and the absolute minimum number of students enrolled will be 15,930 thousand.



3. Referring to the graph in the figure below, estimate the coordinates of the inflection point of  $s$  on the interval  $[0,10]$  and interpret what it represents in terms of the rate of change of  $s$ . (Note: The units on the graph are million students and the units of the equation of  $s$  are thousand students.)



We are looking for where the graph changes concavity. It appears that the graph changes from concave down to concave up at around  $(3, 16.15)$ . So the inflection point of  $s$  is approximately  $(3, 16150)$ . That is, in 2009 – 2010 the number of students is about 16,150,000. In terms of the rate of change of  $s$ , this is the year when the number of students is decreasing most rapidly.

4. According to the model, what was the predicted average rate of change in the number of high school students between 2006 – 07 and 2016 – 17?

$$\begin{aligned} s(0) &= -0.4936(0)^4 + 13.27(0)^3 - 89.50(0)^2 + 95.10(0) + 16,360 \\ &= 16,360 \end{aligned}$$

$$\begin{aligned} s(10) &= -0.4936(10)^4 + 13.27(10)^3 - 89.50(10)^2 + 95.10(10) + 16,360 \\ &= 16,695 \end{aligned}$$

$$\begin{aligned} \frac{s(10) - s(0)}{10 - 0} &= \frac{16,695 - 16,360}{10 - 0} \frac{\text{thousand students}}{\text{year}} \\ &= 33.5 \text{ thousand students per year} \end{aligned}$$

Between 2006 – 07 and 2016 – 17, the number of students in grades 9 – 12 was expected to increase by an average of 33,500 students per year. That is, although the number of students increased and decreased over the ten year period, the average of the rates of change was 33,500 students per year.

# Rolling the Dice

## Using Probability

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to predict how likely it is you will score points when playing a game of dice.

**D**ice games have been around for centuries. The game of Pig is a dice game with simple rules. Each turn a player rolls a die repeatedly until a 1 is rolled or the player decides to end his turn. After each roll, the player may choose to roll again or to end his turn. If the player decides to end his turn, he adds up the face values of each roll and adds the result to his total score. If the player rolls a 1, his turn ends and he scores 0 points for that turn.

For example, Marisol gets a 3 on her first roll, a 6 on her second roll, and a 5 on her third roll. She decides to end her turn and adds up her points. Since  $3 + 6 + 5 = 14$ , she scores 14 points on that turn.

Alex goes next and rolls a 4, 5, 3, and 1, in that order. Since he rolled a 1, his turn ends and he scores 0 points on that turn.

The first player to 100 points wins the game.

1. What is the probability of scoring 0 points on the first dice roll of a turn?
2. A player rolls the die twice and then chooses to end her turn. Given that she didn't roll a 1 on either turn, what is the probability that she scored exactly 7 points on the turn?

3. What is the probability that a player will roll the die six times on a turn without rolling a 1?
4. A player decides that she will roll the die three times on each turn, unless she rolls a 1 on the first or second roll of the turn. What is the probability that she will score more than 0 points on the turn?
5. Is the strategy of the player in (4) a winning strategy? Explain.

# Rolling the Dice

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For example, Marisol gets a 3 on her first roll, a 6 on her second roll, and a 5 on her third roll. She decides to end her turn and adds up her points. Since  $3 + 6 + 5 = 14$ , she scores 14 points on that turn.

Alex goes next and rolls a 4, 5, 3, and 1, in that order. Since he rolled a 1, his turn ends and he scores 0 points on that turn.

The first player to 100 points wins the game.

1. What is the probability of scoring 0 points on the first dice roll of a turn?

*There are 6 possible outcomes:  $\{1, 2, 3, 4, 5, 6\}$ . The only way to earn 0 points is to roll a 1.*

$$P(1) = \frac{1}{6}$$
$$\approx 16.7\%$$

*There is a 16.7% probability that the player scores 0 points on his first turn.*

2. A player rolls the die twice and then chooses to end her turn. Given that she didn't roll a 1 on either turn, what is the probability that she scored exactly 7 points on the turn?

*We list the possible outcomes as two digit numbers with the first digit being the value of the first roll and the second digit being the value of the second roll.*

$$\left\{ \begin{array}{l} 22 \ 23 \ 24 \ 25 \ 26 \\ 32 \ 33 \ 34 \ 35 \ 36 \\ 42 \ 43 \ 44 \ 45 \ 46 \\ 52 \ 53 \ 54 \ 55 \ 56 \\ 62 \ 63 \ 64 \ 65 \ 66 \end{array} \right\}$$

*There are 25 outcomes. To score 7 points, she must roll 25, 34, 43, or 52.*

$$P(\text{scores 7 points}) = \frac{4}{25}$$
$$= 16\%$$

*There is a 16% probability that she scored 7 points on the turn.*

3. What is the probability that a player will roll the die six times on a turn without rolling a 1?

$$\begin{aligned}
 P(\text{no 1 for six rolls}) &= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) \\
 &= \frac{15,625}{46,656} \\
 &\approx 33.5\%
 \end{aligned}$$

*There is a 33.5% probability that a player will roll the die six times in a row without getting a 1.*

4. A player decides that she will roll the die three times on each turn, unless she rolls a 1 on the first or second roll of the turn. What is the probability that she will score more than 0 points on the turn?

*The probability she will score 0 points on the first roll is  $\frac{1}{6} \approx 16.7\%$ .*

*To have a second roll, she has to roll something other than 1 on the first roll. To score 0 on the second roll, she has to roll a 1. So the probability that she will score 0 on the second roll is*

$$\begin{aligned}
 P(1 \text{ on second roll}) &= P(\text{not 1 on first roll}) \cdot P(1 \text{ on second roll}) \\
 &= \frac{5}{6} \left(\frac{1}{6}\right) \\
 &= \frac{5}{36} \\
 &\approx 13.9\%
 \end{aligned}$$

*To have a third roll, she has to roll something other than a 1 on the first two rolls. To score a 0 on the third roll, she has to roll a 1. So the probability she will score a 0 on the third roll is*

$$\begin{aligned}
 P(1 \text{ on third roll}) &= P(\text{not 1 on first roll}) \cdot P(\text{not 1 on second roll}) \cdot P(1 \text{ on third roll}) \\
 &= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) \\
 &= \frac{25}{216} \\
 &\approx 11.6\%
 \end{aligned}$$

*We add up the individual probabilities*

$$\begin{aligned}
 \frac{1}{6} + \frac{5}{36} + \frac{25}{216} &= \frac{91}{216} \\
 &\approx 42.1\%
 \end{aligned}$$

*The probability of the person scoring 0 is 42.1%. Therefore, the probability of scoring more than 0 is 57.9%.*

5. Is the strategy of the player in (4) a winning strategy? Explain.

*Yes. The player will score more than 0 points 57.9% of the time. In the long run, the player will score points more often than not.*

# Fruit Snacks

## Working with Averages

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to find the average number of items in a group.

Many people enjoy eating fruit snacks. Kirkland Signature Fruit Snacks are made from seven different fruit juices (Source: package labeling). A father opened five bags of the Fruit Snacks, counted the number of pieces in each bag, and recorded the results in the table.

Bag #1	14 pieces	4 red, 5 green, 4 orange, 1 yellow
Bag #2	13 pieces	4 red, 3 green, 4 orange, 2 yellow
Bag #3	12 pieces	5 red, 3 green, 4 orange, 0 yellow
Bag #4	14 pieces	2 red, 6 green, 4 orange, 2 yellow
Bag #5	13 pieces	2 red, 6 green, 4 orange, 1 yellow

1. What was the average number of pieces in each bag?
2. Which is greater: the average number of green pieces in each bag or the average number of orange pieces in each bag?
3. The number of yellow pieces in each bag was less than the number of pieces of every other color. What was the average number of yellow pieces in each bag?

4. Is it likely that there is exactly 1 red piece in a bag of Fruit Snacks?
5. Using the results from Exercises 1 through 4, predict the number of pieces of each color in an unopened bag of Fruit Snacks.
6. After predicting the number of Fruit Snacks in an unopened bag, the father opened up a sixth bag of Fruit Snacks. It contained 1 yellow piece, 5 green pieces, 4 orange pieces, and 4 red pieces. How accurate was the prediction in Exercise 5?
7. When the sixth bag is included with the first five bags, what is the average number of pieces in each bag?

# Fruit Snacks

## Working with Averages

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Bag #5	13 pieces	2 red, 6 green, 4 orange, 1 yellow

1. What was the average number of pieces in each bag?

$$\frac{14 + 13 + 12 + 14 + 13}{5} = \frac{66}{5} \\ = 13.2 \text{ pieces}$$

*The average number of pieces in each bag is 13.2 pieces.*

2. Which is greater: the average number of green pieces in each bag or the average number of orange pieces in each bag?

$$\frac{5 + 3 + 3 + 6 + 6}{5} = \frac{23}{5} \\ = 4.6 \text{ green pieces}$$

$$\frac{4 + 4 + 4 + 4 + 4}{5} = \frac{20}{5} \\ = 4 \text{ orange pieces}$$

*The average number of green pieces in each bag is greater than the average number of orange pieces?*

3. The number of yellow pieces in each bag was less than the number of pieces of every other color. What was the average number of yellow pieces in each bag?

$$\frac{1 + 2 + 0 + 2 + 1}{5} = \frac{6}{5} \\ = 1.2 \text{ yellow pieces}$$



4. Is it likely that there is exactly 1 red piece in a bag of Fruit Snacks?

*To answer the question we first find the average number of red pieces in each bag.*

$$\frac{4 + 4 + 5 + 2 + 2}{5} = \frac{17}{5} \\ = 3.4 \text{ red pieces}$$

*Since the average number of red pieces is 3.4 pieces, it is unlikely that there is exactly 1 red piece in a bag of Fruit Snacks.*

5. Using the results from Exercises 1 through 4, predict the number of pieces of each color in an unopened bag of Fruit Snacks.

*On average there were 1.2 yellow pieces, 4.6 green pieces, 4 orange pieces, and 3.4 red pieces. Since it doesn't make sense to talk about less than a whole piece, we round each of the amounts. We predict that there is 1 yellow piece, 5 green pieces, 4 orange pieces, and 3 red pieces in an unopened bag. This adds up to 13 pieces in the bag which is close to the average of 13.2 pieces.*

6. After predicting the number of Fruit Snacks in an unopened bag, the father opened up a sixth bag of Fruit Snacks. It contained 1 yellow piece, 5 green pieces, 4 orange pieces, and 4 red pieces. How accurate was the prediction in Exercise 5?

*The sixth bag contained the exact number of yellow, green and orange pieces predicted and contained one red piece more than predicted. There were 14 pieces in the bag.*

7. When the sixth bag is included with the first five bags, what is the average number of pieces in each bag?

$$\frac{14 + 13 + 12 + 14 + 13 + 14}{6} = \frac{80}{6} \\ = 13\frac{1}{3} \text{ pieces}$$

*The average number of pieces is  $13\frac{1}{3}$  pieces.*

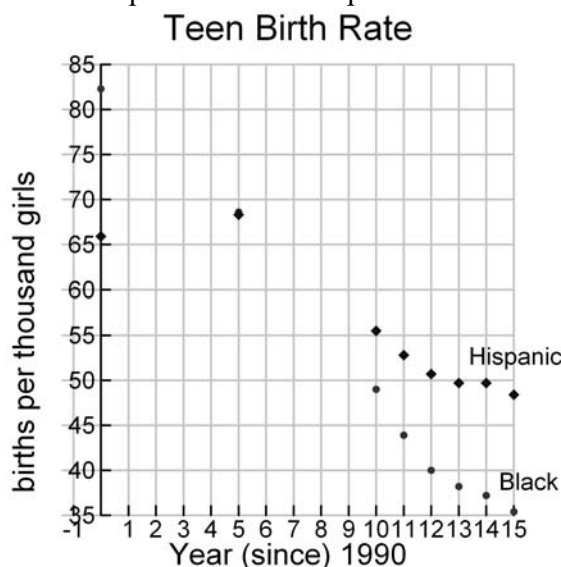
# Teen Pregnancy

## Investigating Cubic Functions

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to use cubic function models to predict unknown results.

In any society, the choices of one person often impact the lives of others. One of the most dramatic life changes a teenage girl can experience is an unexpected pregnancy. This event can dramatically alter the young woman's educational and employment opportunities. Despite this fact, many young women have overcome the challenges of being a single mother and have achieved success in their personal and professional lives.

The scatter plots below show the pregnancy rate of 15 – 17 year old young women racially classified as Black and Hispanic. The birth rate is reported as births per 1000 women.



1. Based on the shape of the scatter plots, what type of function do you think will best fit each of the data sets? Explain.
2. The cubic function model for each of the data sets is given below together with the coefficient of determination ( $r^2$ ). Use technology to find the point of intersection of the two functions.  
Hispanic birth rate:  $h(x) = 0.0280x^3 - 0.718x^2 + 3.34x + 65.9$ ;  $r^2 = 0.997$   
Black birth rate:  $b(x) = 0.0215x^3 - 0.475x^2 - 0.846x + 82.3$ ;  $r^2 = 0.999$

3. What is the real-life meaning of the point of intersection in (3)?
  
  
  
  
  
  
  
  
  
  
4. From the scatter plots, does it appear that efforts to educate youth about abstinence and safe sex are having an impact? Explain.
  
  
  
  
  
  
  
  
  
  
5. Use each model to predict the birth rate in 2010.
  
  
  
  
  
  
  
  
  
  
6. Do the pregnancy rates in (5) seem reasonable? Explain.
  
  
  
  
  
  
  
  
  
  
7. Over what domain do you expect the models will give reasonable results? Explain.

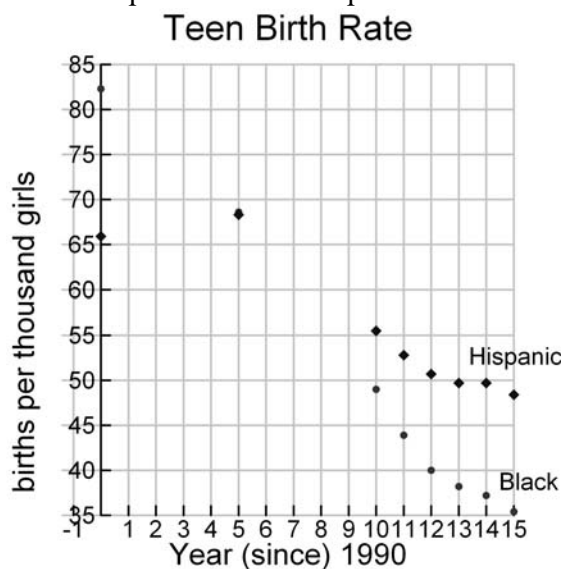
# Teen Pregnancy

## Investigating Cubic Functions

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The scatter plots below show the pregnancy rate of 15 – 17 year old young women racially classified as Black and Hispanic. The birth rate is reported as births per 1000 women.



1. Based on the shape of the scatter plots, what type of function do you think will best fit each of the data sets? Explain.

*Both data sets start out concave down but appear to change to concave up part way through the data set. Since there is a change in concavity, a cubic function model may fit each data set.*

2. The cubic function model for each of the data sets is given below together with the coefficient of determination ( $r^2$ ). Use technology to find the point of intersection of the two functions.

Hispanic birth rate:  $h(x) = 0.0280x^3 - 0.718x^2 + 3.34x + 65.9$ ;  $r^2 = 0.997$

Black birth rate:  $b(x) = 0.0215x^3 - 0.475x^2 - 0.846x + 82.3$ ;  $r^2 = 0.999$

*Using a graphing calculator, we find the point of intersection is (5.33, 67.5).*

3. What is the real-life meaning of the point of intersection in (3)?

*(5.33, 67.5) tells us that Hispanic teens and Black teens had the same birth rate about 4 months into 1996. ( $t = 5$  is the end of 1995. The extra 0.33 puts us about 4 months into 1996.) At that time, the birth rate was 67.5 births per thousand teens aged 15 – 17 years.*

4. From the scatter plots, does it appear that efforts to educate youth about abstinence and safe sex are having an impact? Explain.

*Yes. Both of the data sets are decreasing. For Black teens, the birth rate dropped from approximately 82 to 36 between 1990 and 2005. For Hispanic teens, the birth rate dropped from 66 to 49 over the same time period.*

5. Use each model to predict the birth rate in 2010.

*Hispanic birth rate:  $h(20) = 0.0280(20)^3 - 0.718(20)^2 + 3.34(20) + 65.9 = 69.5$*

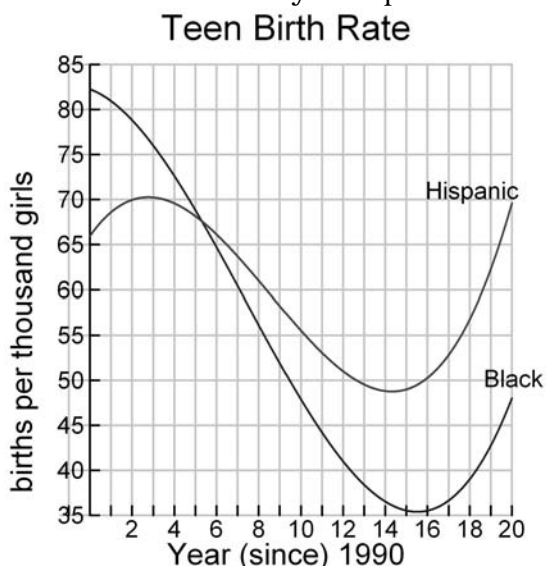
*Black birth rate:  $b(20) = 0.0215(20)^3 - 0.475(20)^2 - 0.846(20) + 82.3 = 47.38$*

*The models predict the Hispanic birth rate to be 69.5 births per thousand and the Black birth rate to be 47.38 births per thousand.*

6. Do the pregnancy rates in (5) seem reasonable? Explain.

*Between 1995 and 2005, both data sets are decreasing. The projected birth rates for 2010 represent an increase not a decrease in the birth rate of both groups. Consequently, we don't think that these predictions are reasonable.*

7. Over what domain do you expect the models will give reasonable results? Explain.



*We expect the birth rate to decrease or remain flat. Both models begin to increase around  $t = 15$ . Therefore, we pick the interval  $[0, 15]$  to be a reasonable domain for the model.*

# Shopping Center Planning

## Looking at Exponential and Linear Models

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to create and use linear and exponential models to forecast growth.

Queen Creek Marketplace is a 1.1 million square foot commercial center built in Queen Creek, Arizona. It is anchored by stores such as Kohls, Lowes, Super Target, and others. In its report for the city of Queen Creek, Vestar Development projected the following for its marketing area.

	2006	2008	2010
population	100,945	148,138	194,207
households	30,776	45,441	49,756
average income	\$65,136	\$69,775	\$74,024

1. Which of the above demographic factors (population, households, average income) is closest to being an exponential function of time? Explain.
2. For the factor identified in (1), what is the growth factor and what does it represent in this context?
3. For the factor identified in (1), find an exponential model and forecast the value of the factor in 2012.

4. Which of the demographic factors (population, households, average income) is closest to being a linear function of time? Explain.
5. Use regression to find a linear function model for the factor identified in (4) and predict the value of the factor in 2012.
6. Referring to the model in (5), what is the real world meaning of the slope of the model?

# Shopping Center Planning

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average income	\$65,136	\$69,775	\$74,024

- Which of the above demographic factors (population, households, average income) is closest to being an exponential function of time? Explain.

We calculate the ratio of consecutive output values. We looking to see which factor has a near-constant ratio.

$$\begin{array}{ll}
 \text{Population:} & \frac{148,138}{100,945} \approx 1.468 \quad \frac{194,207}{148,138} \approx 1.311 \\
 \text{Households:} & \frac{45,441}{30,776} \approx 1.477 \quad \frac{49,756}{45,441} \approx 1.095 \\
 \text{Income:} & \frac{69,775}{65,136} \approx 1.071 \quad \frac{74,024}{69,775} \approx 1.061
 \end{array}$$

Of the three factors, the ratios for income are closest to being constant. Therefore, average income is closest to being an exponential function of time.

- For the factor identified in (1), what is the growth factor and what does it represent in this context?

$$b = \left( \frac{74,024}{65,136} \right)^{\frac{1}{4-0}} \approx 1.032$$

Each year, the average household income increases by about 3.2%.

- For the factor identified in (1), find an exponential model and forecast the value of the factor in 2012.

$$I(t) = 65,136(1.032)^t$$

$$\begin{aligned}
 I(6) &= 65,136(1.032)^6 \\
 &\approx 78,686
 \end{aligned}$$

In 2012, the average household income is projected to be \$78,686.



4. Which of the demographic factors (population, households, average income) is closest to being a linear function of time? Explain.

*We calculate the difference of consecutive output values. We are looking to see which factor has a near-constant difference.*

$$\text{Population: } 148,138 - 100,945 = 47,193 \qquad 194,207 - 148,138 = 46,069$$

$$\text{Households: } 45,441 - 30,776 = 14,665 \qquad 49,756 - 45,441 = 4,315$$

$$\text{Income: } 69,775 - 65,136 = 4,639 \qquad 74,024 - 69,775 = 4,249$$

*The differences for population and for income are close to being constant. By calculating the ratios of the differences, we can further quantify how close the values are to being constant.*

$$\frac{47,193}{46,069} \approx 1.024$$

*The population differences are within 2.4% of being constant.*

$$\frac{4639}{4249} \approx 1.092$$

*The income differences are within 9.2% of being constant. Therefore, population is the demographic factor closest to being a linear function of time.*

5. Use regression to find a linear function model for the factor identified in (4) and predict the value of the factor in 2012.

$$p(t) = 23,316t + 101,132 \text{ people}$$

$$\begin{aligned} p(6) &= 23,316(6) + 101,132 \text{ people} \\ &= 241,028 \text{ people} \end{aligned}$$

*We predict there will be 241,028 people in the market area in 2012.*

6. Referring to the model in (5), what is the real world meaning of the slope of the model?

*The slope represents the annual rate of change in the population. In this case, the population is increasing at a rate of 23,316 people per year.*

# Hours of Daylight – Anchorage

## Working with Sinusoidal Models

“When am I ever going to use this?”  
Using the concepts in this worksheet, you will be able to create a mathematical model for the hours of daylight where you live.

The hours of daylight in a given part of the world changes from day to day. In the northern hemisphere, the Summer Solstice (around June 21) is the longest day of the year and the Winter Solstice (around December 21) is the shortest day of the year. The United States Naval Observatory allows website visitors to download the hours of daylight for any day of the year and any location in the world (Source: aa.usno.navy.mil).

The hours of daylight on the 21<sup>st</sup> day of the month for selected cities are shown in the tables below.

Hours of Daylight on 21 <sup>st</sup> Day of the Month – Anchorage, Alaska											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
06:52	09:39	12:20	15:18	17:59	19:22	18:00	15:16	12:21	09:31	06:47	05:27

Hours of Daylight on 21 <sup>st</sup> Day of the Month – Phoenix, Arizona											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
10:19	11:13	12:10	13:13	14:02	14:22	14:02	13:12	12:11	11:10	10:18	09:56

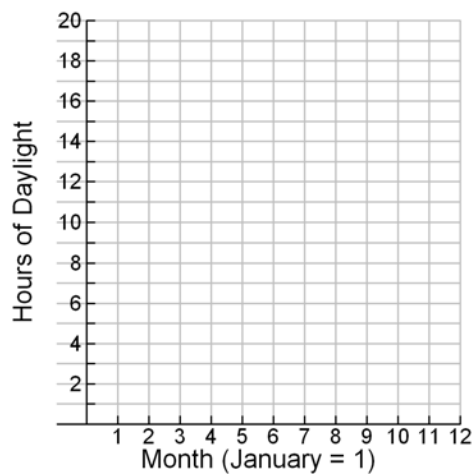
Hours of Daylight on 21 <sup>st</sup> Day of the Month – Easter Island, Chile											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
13:34	12:52	12:07	11:18	10:41	10:26	10:40	11:17	12:05	12:52	13:34	13:51

Hours of Daylight on 21 <sup>st</sup> Day of the Month – Perth, Australia											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
14:39	13:25	12:07	10:43	09:37	09:09	09:36	10:42	12:05	13:27	14:40	15:11

1. What is the significance of June 21 for each of the cities?

2. What is the significance of December 21 for each of the cities?

3. On the axes below, draw the scatter plot for the hours of daylight in Anchorage, Alaska. Be sure to convert the times from hour-minute format (e.g. 12:20) to hour format (e.g. 12.33).



4. Which type of sinusoidal function will best fit the data without requiring a horizontal shift:  $\sin(m)$ ,  $-\sin(m)$ ,  $\cos(m)$ , or  $-\cos(m)$ ? Explain.

5. Find a sinusoidal model of the type identified in Exercise 4 that fits the data well.

# Hours of Daylight – Anchorage

## Working with Sinusoidal Models

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Using the concepts in this worksheet, you will be able to create a mathematical model for the hours of daylight where you live.

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The hours of daylight on the 21<sup>st</sup> day of the month for selected cities are shown in the tables below.

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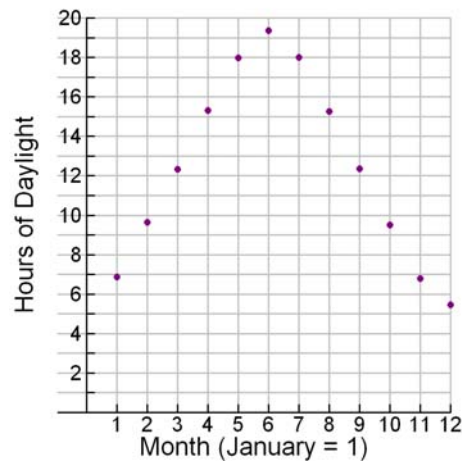
1. What is the significance of June 21 for each of the cities?

*That day is the longest day of the year for Anchorage and Phoenix and is the shortest day of the year for Easter Island and Perth.*

2. What is the significance of December 21 for each of the cities?

*That day is the shortest day of the year for Anchorage and Phoenix and is the longest day of the year for Easter Island and Perth.*

3. On the axes below, draw the scatter plot for the hours of daylight in Anchorage, Alaska. Be sure to convert the times from hour-minute format (e.g. 12:20) to hour format (e.g. 12.33).



4. Which type of sinusoidal function will best fit the data without requiring a horizontal shift:  $\sin(m)$ ,  $-\sin(m)$ ,  $\cos(m)$ , or  $-\cos(m)$ ? Explain.

We first observe that month 12 (December) is the same as month 0 (December of the previous year). Consequently, we predict that the vertical intercept will be 5.45 hours as well. The data appears to start at its minimum value (0, 5.45), climb to its maximum value (6, 19.37), and then descend to its minimum value (12, 5.45). The function  $h(m) = -\cos(m)$  exhibits this same basic behavior.

5. Find a sinusoidal model of the type identified in Exercise 4 that fits the data well.

The maximum value is 19.37 and the minimum value is 5.45. We use these values to find the amplitude and midline.

$$\begin{aligned} \text{midline} &= \frac{\text{max} + \text{min}}{2} \\ &= \frac{19.37 + 5.45}{2} \\ &= 12.41 \end{aligned}$$

$$\begin{aligned} \text{amplitude} &= \frac{\text{max} - \text{min}}{2} \\ &= \frac{19.37 - 5.45}{2} \\ &= 6.96 \end{aligned}$$

Since there are 12 months in a year, we expect that the period of the function is 12 months. We find  $B$ .

$$\begin{aligned} B &= \frac{2\pi}{\text{period}} \\ &= \frac{2\pi}{12} \\ &= \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} h(m) &= -\text{amplitude}(\cos(B \cdot m)) + \text{midline} \\ &= -6.96 \cos\left(\frac{\pi}{6} m\right) + 12.41 \end{aligned}$$

The hours of daylight on the 21<sup>st</sup> day of month  $m$  are given by  $h(m) = -6.96 \cos\left(\frac{\pi}{6} m\right) + 12.41$ .

# United States Population

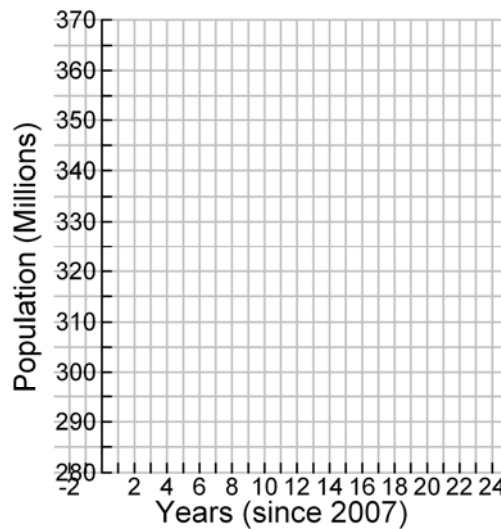
## Using Quadratic Models

**“When am I ever going to use this?”**  
Using the concepts in this worksheet, you will be able to use quadratic models of real world data sets to forecast unknown values.

Based on Census Bureau projections for 2007 – 2050, the population of the United States may be modeled by  $P(t) = 0.0022675t^2 + 2.6804t + 300.760$  where  $P$  is the population (in millions) and  $t$  is the number of years since 2007 (Source: Modeled from Statistical Abstract of the United States 2008, Table 3).

1. Use the Quadratic Formula to solve  $P(t) = 350$ . Then explain what the solution means.
2. According to the population model, what will be the population of the United States in 2010? Round your answer to the nearest hundred thousand.

3. Graph  $P(t) = 0.0022675t^2 + 2.6804t + 300.760$  and  $y = 320$  on the axes below.



4. Referring to the graph in the solution of Exercise 3, estimate the coordinates of the point of intersection of the two graphs. Then interpret the practical meaning of the point of intersection.
5. The graph of  $P(t) = 0.0022675t^2 + 2.6804t + 300.760$  in Exercise 3 appears to be linear not quadratic. Referring to the equation of  $P$ , explain why this is.
6. Give three examples of groups who benefit from population models such as the one above and explain why the model would be of value to them.

# United States Population

## Using Quadratic Models

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1. Use the Quadratic Formula to solve  $P(t) = 350$ . Then explain what the solution means.

$$350 = 0.0022675t^2 + 2.6804t + 300.760$$

$$0 = 0.0022675t^2 + 2.6804t - 49.240$$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2.6804 \pm \sqrt{(2.6804)^2 - 4(0.0022675)(-49.240)}}{2(0.0022675)} \\ &\approx -1200 \text{ and } 18.09 \end{aligned}$$

Since the domain of the original data set used to create the model is  $0 \leq t \leq 43$ , the value of  $t = -1200$  is not reasonable. Since  $t = 18$  is 2025, we predict that near 2025 the population of the United States will reach 350 million (350,000 thousand).

2. According to the population model, what will be the population of the United States in 2010? Round your answer to the nearest hundred thousand.

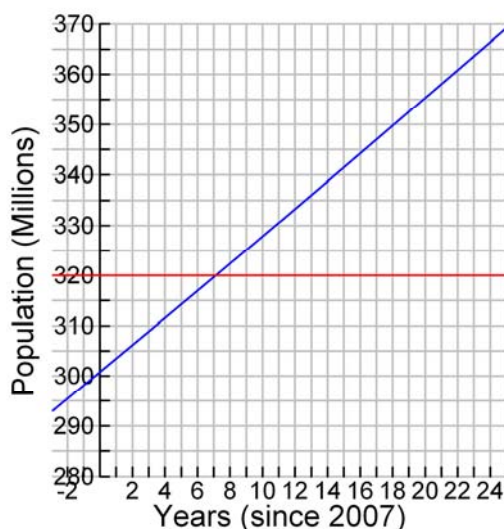
Since  $t = 3$  corresponds with 2010, we evaluate the function at  $t = 3$ .

$$\begin{aligned} P(3) &= 0.0022675(3)^2 + 2.6804(3) + 300.760 \\ &= 308.822 \end{aligned}$$

We estimate that the population in the United States will be 308.8 million in 2010.



3. Graph  $P(t) = 0.0022675t^2 + 2.6804t + 300.760$  and  $y = 320$  on the axes below.



4. Referring to the graph in the solution of Exercise 3, estimate the coordinates of the point of intersection of the two graphs. Then interpret the practical meaning of the point of intersection.

*The point of intersection is approximately (7, 320). In 2014, the population of the United States is predicted to be 320 million.*

5. The graph of  $P(t) = 0.0022675t^2 + 2.6804t + 300.760$  in Exercise 3 appears to be linear not quadratic. Referring to the equation of  $P$ , explain why this is.

*Since  $a = 0.0022675$  is close to 0, the term  $0.0022675t^2$  will have little effect on the value of  $P$  for small values of  $t$ . For example,  $0.0022675(25)^2 \approx 1.42$ . In contrast, the term  $2.6804t$  changes the value of  $P$  much more dramatically. For example,  $2.6804(25) \approx 67.01$*

6. Give three examples of groups who benefit from population models such as the one above and explain why the model would be of value to them.

*Internal Revenue Service – Based on the population model, the IRS can forecast tax revenue.*

*Congressional Budget Planners – Based on the model, budget planners can forecast spending requirements for government programs such as Medicare, Medicaid, and similar programs.*

*Military Planners – Based on the model, military planners can predict the number of citizens who are eligible to defend their country in time of war.*

# Travel Options – Florida

## Working with Linear Systems



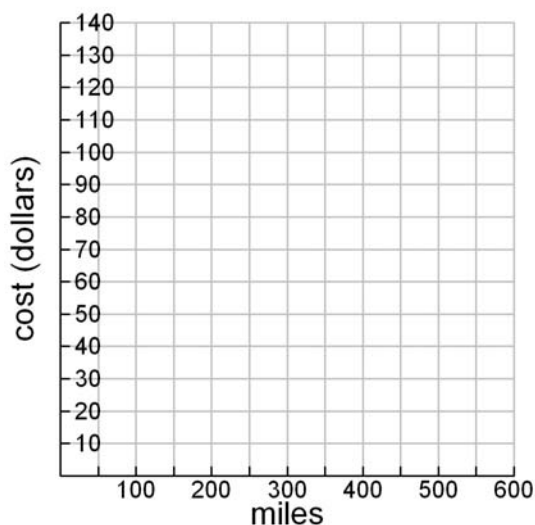
**When am I ever going to use this?**

Using the concepts in this worksheet, you will be able to determine if renting a car for a day trip will save you money.

On May 25, 2008, the average price for unleaded gasoline in Florida was \$3.939 per gallon (Source: [www.floridastategasprices.com](http://www.floridastategasprices.com)). A driver in Tallahassee, Florida, plans to drive to Orlando, Florida, a distance of roughly 515 miles round trip. The driver owns an SUV that gets 18 miles per gallon. Dollar Rent A Car, Inc., offers a Ford Focus car rental in Tallahassee or \$42.12 per day. It is estimated the car gets 35 miles per gallon on the highway. (Sources: [www.hertz.com](http://www.hertz.com), [www.ford.com](http://www.ford.com)).

1. What is the gasoline cost per mile for the SUV and for the Ford Focus?
2. Create a function for the fuel cost of driving the SUV  $m$  miles.
3. Create a function for the fuel plus rental cost of driving the Ford Focus  $m$  miles.

4. Graph each of the functions in (2) and (3) on the axes below.



5. Estimate the point of intersection of the graphs in (4). Then explain the real-world meaning for the point of intersection.

6. Set up a system of equations using the equations from (2) and (3). Then solve the system algebraically.

7. Explain how the solution to (5) and the solution to (6) are related.

8. Will it cost the driver more money to rent a Ford Focus to drive to Orlando or to drive the SUV?

# Travel Options – Florida

## Working with Linear Systems

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1. What is the gasoline cost per mile for the SUV and for the Ford Focus?

$$\begin{array}{c} \text{SUV} \\ \frac{3.939 \frac{\text{dollars}}{\text{gallon}}}{18 \frac{\text{miles}}{\text{gallon}}} = \left( 3.939 \frac{\text{dollars}}{\text{gallon}} \right) \left( \frac{1 \cancel{\text{gallon}}}{18 \text{ mile}} \right) \\ \approx 0.219 \text{ dollars per mile} \end{array}$$

$$\begin{array}{c} \text{Ford Focus} \\ \frac{3.939 \frac{\text{dollars}}{\text{gallon}}}{35 \frac{\text{miles}}{\text{gallon}}} = \left( 3.939 \frac{\text{dollars}}{\text{gallon}} \right) \left( \frac{1 \cancel{\text{gallon}}}{35 \text{ mile}} \right) \\ \approx 0.113 \text{ dollars per mile} \end{array}$$

2. Create a function for the fuel cost of driving the SUV  $m$  miles.

Let  $V$  be the fuel cost for driving the SUV  $m$  miles. We have  $V = 0.219m$ .

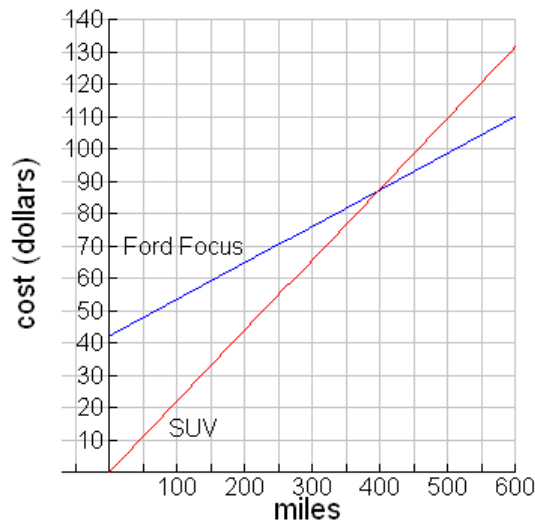
3. Create a function for the fuel plus rental cost of driving the Ford Focus  $m$  miles.

Let  $F$  be the fuel and rental cost for driving the Ford Focus  $m$  miles. We have

$$F = \text{fuel cost} + \text{rental cost}$$

$$F = 0.113m + 42.12$$

4. Graph each of the functions in (2) and (3) on the axes below.



5. Estimate the point of intersection of the graphs in (4). Then explain the real-world meaning for the point of intersection.  
*It appears that the graphs intersect at about (400,88). This means that when 400 miles are driven the cost of driving each vehicle will be the same: \$88.*

6. Set up a system of equations using the equations from (2) and (3). Then solve the system algebraically.  
*Since we are interested in when the costs will be equal, we use the same variable,  $C$ , to represent the cost in the system of equations.*

$$C = 0.219m$$

$$C = 0.113m + 42.12$$

*Since both equations equal  $C$ , we set the right hand sides of the equations equal to each other and solve.*

$$0.219m = 0.113m + 42.12$$

$$0.106m = 42.12$$

$$m \approx 397$$

$$C = 0.219m$$

$$C = 0.219(397.4)$$

$$C \approx 87.0$$

*The solution to the system is  $m = 397$  and  $C = 87$ .*

7. Explain how the solution to (5) and the solution to (6) are related.

*The point of intersection of the graphs is the solution to the linear system of equations. Although we can estimate the solution from the graph, solving the system algebraically allows us to calculate the solution with more precision*

8. Will it cost the driver more money to rent a Ford Focus to drive to Orlando or to drive the SUV?  
 Explain.

*Since the trip is more than 397 miles, it will cost the driver more money to drive the SUV than the Ford Focus. From the graph in (5), it appears that the driver will save about \$12 by renting the Ford Focus.*

## About the Author

Frank Wilson earned his B.S. and M.S. degrees in mathematics from Brigham Young University. He spent six years serving as an officer in the United States Air Force before returning to civilian life. He has taught students math at the United States Air Force Academy, Park College, Green River Community College, and Chandler-Gilbert Community College. In addition to teaching, Frank is a popular author and workshop presenter. His college mathematics textbooks (Finite Mathematics, Finite Mathematics and Applied Calculus, Brief Applied Calculus, and Applied Calculus) are used at colleges and universities across the United States. Finite Mathematics and Applied Calculus was selected by the Textbook and Academic Author's Association as the winner of the 2007 TEXTY Textbook Excellence award for mathematics. Frank's picture book on measurement, *Measure Up! A Bug Contest*, is popular among teachers and children alike.

Frank lives with his wife and five living children in Queen Creek, Arizona.

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