A Variable on Both Sides

Example 1. Solve $2x + 8 = -5x$.

Notice that the unknown appears on both sides of the equation. To isolate it, we need to

- either subtract $2x$ from both sides—because that makes $2x$ disappear from the left side
- or add $5x$ to both sides—because that makes $-5x$ disappear from the right side.

$$\begin{aligned} 2x + 8 &= -5x && \mid + 5x \\ 2x + 8 + 5x &= 0 && \text{(now add } 2x \text{ and } 5x \text{ on the left side)} \\ 7x + 8 &= 0 && \mid - 8 \\ 7x &= -8 && \mid \div 7 \\ x &= -8/7 \end{aligned}$$

Check:

$$\begin{aligned} 2 \cdot (-8/7) + 8 &\overset{?}{=} -5 \cdot (-8/7) \\ -16/7 + 8 &\overset{?}{=} 40/7 \\ -2\ 2/7 + 8 &= 5\ 5/7 \ \checkmark \end{aligned}$$

Example 2. Solve $10 - 2s = 4s + 9$.

To isolate s, we need to

- either add $2s$ to both sides
- or subtract $4s$ from both sides.

The choice is yours. Personally, I like to keep the unknown on the left side and eliminate it from the right.

$$\begin{aligned} 10 - 2s &= 4s + 9 && \mid - 4s \\ 10 - 2s - 4s &= 9 && \text{(now simplify } -2s - 4s \text{ on the left side)} \\ 10 - 6s &= 9 && \mid - 10 \\ -6s &= -1 && \mid \div (-6) \\ s &= 1/6 \end{aligned}$$

Check:

$$\begin{aligned} 10 - 2 \cdot (1/6) &\overset{?}{=} 4 \cdot (1/6) + 9 \\ 10 - 2/6 &\overset{?}{=} 4/6 + 9 \\ 9\ 4/6 &= 9\ 4/6 \ \checkmark \end{aligned}$$

1. Solve. Check your solutions (as always!).

a. $3x + 2 = 2x - 7$	**b.** $9y - 2 = 7y + 5$

2. Solve. Check your solutions (as always!).

a. $11 - 2q = 7 - 5q$	**b.** $6z - 5 = 9 - 2z$
c. $8x - 12 = -1 - 3x$	**d.** $-2\text{y} - 6 = 20 + 6y$
e. $6w - 6.5 = 2w - 1$	**f.** $5g - 5 = -20 - 2g$

Combining like terms

Remember, in algebra, a *term* is an expression that consists of numbers, fractions, and/or variables that are multiplied. This means that the expression $-2y + 7 + 8y$ has three terms, separated by the plus signs.

In the expression $-2y + 7 + 8y$, the terms $-2y$ and $8y$ are called **like terms** because they have the same variable part (in this case a single y). We can **combine** (add or subtract) like terms.

To do that, it helps to organize the terms in the expression in alphabetical order according to the variable part and write the constant terms last. We get $-2y + 8y + 7$ ($8y - 2y + 7$ is correct, too).

Next, we add $-2y + 8y$ and get $6y$. So the expression $-2y + 7 + 8y$ simplifies to $6y + 7$.

Example 3. Simplify $6y - 8 - 9y + 2 - 7y$.

First, we organize the expression so that the terms with y are written first, followed by the constant terms.

For that purpose, we **view each operation symbol** (+ or −) **in front of the term as the *sign* of each term.** In a sense, you can imagine each plus or minus symbol as being "glued" to the term that follows it. Of course the first term, $6y$, gets a "+" sign.

$$+6y \; -8 \; -9y \; +2 \; -7y$$

Why can we do it this way?

Because subtracting a term is the same as adding its opposite. In symbols,

$$6y \; - 8 \; - 9y \; + 2 \; - 7y$$
$$= 6y + (-8) + (-9y) + 2 + (-7y).$$

In other words, the expression $6y - 8 - 9y + 2 - 7y$ is the SUM of the terms $6y$, -8, $-9y$, 2, and $-7y$.

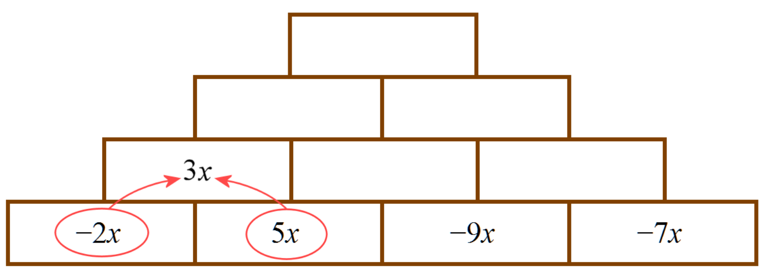
After reordering the terms, the expression becomes $6y - 9y - 7y - 8 + 2$.

Now we need to combine the like terms $6y$, $-9y$, and $-7y$. We do that by finding the sum of their coefficients 6, −9, and −7. Since $6 - 9 - 7 = -10$, we know that $6y - 9y - 7y = -10y$.

Similarly, we combine the two constant terms: $-8 + 2 = -6$.

Our expression therefore simplifies to $-10y - 6$.

3. Fill in the pyramid! Add each pair of terms in neighboring blocks and write its sum in the block above it.



4. Organize the expressions so that the variable terms are written first, followed by constant terms.

a. $6 + 2x - 3x - 7 + 11$

b. $-s - 12 + 15s + 9 - 7s$

c. $-8 + 5t - 2 - 6t$

5. Simplify the expressions in the previous exercise.