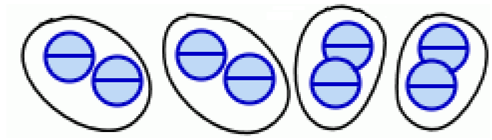
Multiplying Integers

Multiply a positive times a negative



The image illustrates 4 × (−2) as four groups of two negatives. We can solve it using repeated addition:

4 × (−2) = (−2) + (−2) + (−2) + (−2) = −8.

As a shortcut, just multiply the "plain numbers" 4 and 2, and write the answer as negative.

Example. 7 × (−8) = ?

This is illustrated by 7 groups of 8 negatives, which means the answer will be negative.
We multiply 7 × 8 = 56 to find how many negatives there are. The final answer is 7 × (−8) = −56.

1. Multiply.

a. 5 × (−4) = _______ 12 × (−2) = _______	**b.** 8 × (−1) = _______ 7 × (−6) = _______	**c.** 9 × (−9) = _______ 10 × (−7) = _______

2. Write each addition as a multiplication, and solve.

a. $^{-}4 + {}^{-}4 + {}^{-}4 + {}^{-}4$ = ______ × ______ = _______	**b.** $^{-}31 + {}^{-}31$ = ______ × ______ = _______	**c.** $^{-}200 + {}^{-}200 + {}^{-}200$ = ______ × ______ = _______

Multiply a negative times a positive

To solve (−8) × 4 or −5 × 6 or (a negative number times a positive number), we can "turn them around" because multiplication is commutative.

(−8) × 4 is the same as 4 × (−8) = −32.

−5 × 6 is the same as 6 × (−5) = −30.

BUT, −5 × 0 = 0. Zero is not written as −0, but as 0.

So, a negative times a positive gives a negative answer.

3. Multiply.

a. −5 × 7 = _______ 11 × (−3) = _______	**b.** (−9) × 1 = _______ −8 × 8 = _______	**c.** (−9) × 0 = _______ 8 × (−5) = _______

Multiply a negative times a negative What is $(-8) \times (-4)$ or $-5 \times (-6)$? This baffled real mathematicians in the past, too, so don't worry if the answer sounds confusing! <u>A negative times a negative number gives a **positive** result!</u> So, $(-8) \times (-4) = 32$ and $-5 \times (-6) = 30$.
Why? We will explore that in the exercise below.

4. Complete the patterns.

a.	**b.**	**c.**
$(-3) \times 3 =$ _______ $(-3) \times 2 =$ _______ $(-3) \times 1 =$ _______ $(-3) \times 0 =$ _______ $(-3) \times (-1) =$ _______ $(-3) \times (-2) =$ _______ $(-3) \times (-3) =$ _______ $(-3) \times (-4) =$ _______	$(-5) \times 3 =$ _______ $(-5) \times 2 =$ _______ $(-5) \times 1 =$ _______ $(-5) \times 0 =$ _______ $(-5) \times (-1) =$ _______ $(-5) \times (-2) =$ _______ $(-5) \times (-3) =$ _______ $(-5) \times (-4) =$ _______	$(-8) \times 3 =$ _______ $(-8) \times 2 =$ _______ $(-8) \times 1 =$ _______ $(-8) \times 0 =$ _______ $(-8) \times (-1) =$ _______ $(-8) \times (-2) =$ _______ $(-8) \times (-3) =$ _______ $(-8) \times (-4) =$ _______
In the above pattern, the products (answers) increase by 3 in each step!	In the above pattern, the products (answers) increase by ____ in each step!	In the above pattern, the products (answers) increase by _____ in each step!
It follows that the *negative times negative* products in the patterns must be <u>positive</u>.		

5. Multiply.

a. $-5 \times 4 =$ _______ $-5 \times (-4) =$ _______	**b.** $(-9) \times (-2) =$ _______ $2 \times (-11) =$ _______	**c.** $(-3) \times 30 =$ _______ $-7 \times (-80) =$ _______

6. Find the missing factors.

a. $4 \times$ _______ $= -32$	**b.** $-9 \times$ _______ $= 108$	**c.** $9 \times$ _______ $= -900$
d. $-4 \times$ _______ $= 32$	**e.** $-9 \times$ _______ $= -108$	**f.** $-9 \times$ _______ $= 900$

7. Multiply, and solve the riddle. *What is black when it is clean, and white when it is dirty?*

A. 5 × (−4) = _______	**D.** (−4) × 9 = _______	**C.** 6 × (−5) = _______
K. −7 × (−7) = _______	**O.** 7 × (−7) = _______	**R.** −3 × (−8) = _______
A. 2 × (−12) = _______	**L.** (−4) × (−5) = _______	**B.** (−3) × (−12) = _______
	A. −3 × (−10) = _______	**B.** (−2) × (−5) = _______

−20		10	20	−24	−30	49	36	−49	30	24	−36

8. The points (−2, 1), (0, 0), and (−1, 2) are vertices of a triangle.

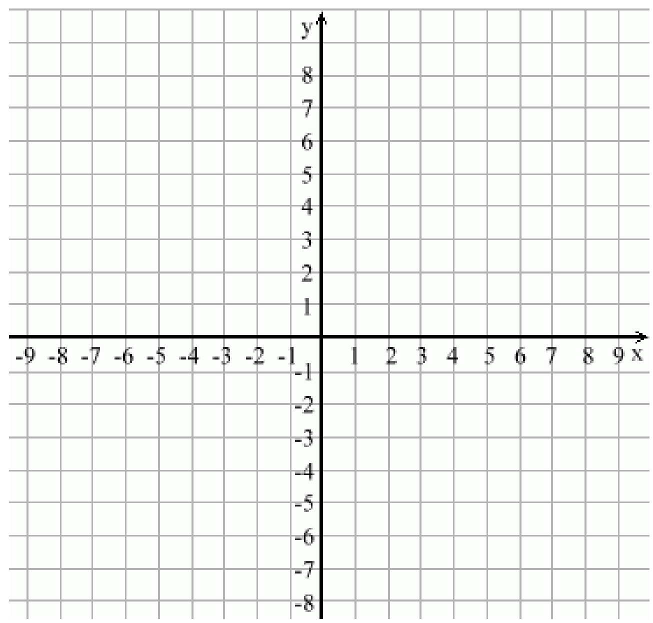
a. Draw the triangle.

b. Multiply each coordinate of each point by 2, to get three new points.
Write the coordinates of the new points:

c. Draw a new triangle using the three points from (b) as vertices.

d. Repeat this, multiplying the coordinates of the original three points by 3.

What you just did was *enlarge* the original triangle. The original and the two new triangles are *similar triangles*—they have the same shape.



(Optional) **Another justification for the rule *"Negative times negative makes positive"***

This justification can be seen using the distributive property. The distributive property of arithmetic states that $a(b + c) = ab + ac$. For example, 4 × (3 + 5) = 4 × 3 + 4 × 5.

Let's see what happens if $a = -1$, $b = 3$, and $c = -3$. We get (−1)[3 + (−3)] **=** (−1)(3) + (−1)(−3)

Now, since 3 + (−3) on the left side is zero, the whole left side is zero (−1 times zero equals zero). So the right side, (−1)(3) + (−1)(−3), must be zero as well!

On the right side, (−1)(3) is −3. It follows that (−1)(−3) has to be 3. That is the only way to make the right side equal zero. Therefore, (−1)(−3) is *positive 3*.

This same argument can be made using a, b, and $-b$ (variables instead of specific numbers). According to the distributive property: $a[b + (-b)] = ab + a(-b)$. The left side is always zero because $b + (-b) = 0$. Now, if a is negative, and b is positive, then on the right side ab is negative (positive times a negative). Then, $a(-b)$ MUST be positive so the right side can add up to zero.

So, if we made *"Negative times negative"* to be negative, then distributive property wouldn't hold for negative numbers. But mathematicians do want it to hold to keep mathematics a very consistent system. So, mathematicians have decided that negative times negative has to be positive.

The History of Negative Numbers: http://nrich.maths.org/public/viewer.php?obj_id=5961

Negative Numbers: http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2