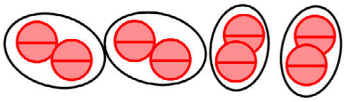
Dividing Integers

1. What division equation is illustrated by the image?



2. Complete the patterns in a logical way. Think of multiplication.

a.	b.	c.
$12 \div 4 =$ ______	______ $\div (-7) = -3$	$60 \div$ ______ $= 2$
$8 \div 4 =$ ______	______ $\div (-7) = -2$	$40 \div$ ______ $= 2$
$4 \div 4 =$ ______	______ $\div (-7) = -1$	$20 \div$ ______ $= 2$
$0 \div 4 =$ ______	______ $\div (-7) = 0$	$-20 \div$ ______ $= 2$
$(-4) \div 4 =$ ______	______ $\div (-7) = 1$	$-40 \div$ ______ $= 2$
$(-8) \div 4 =$ ______	______ $\div (-7) = 2$	$-60 \div$ ______ $= 2$
$(-12) \div 4 =$ ______	______ $\div (-7) = 3$	$-80 \div$ ______ $= 2$
$(-16) \div 4 =$ ______	______ $\div (-7) = 4$	$-100 \div$ ______ $= 2$

What do the patterns suggest for the <u>sign</u> of these types of quotients?

A negative divided by a positive	A positive divided by a negative	A negative divided by a negative
____________________	____________________	____________________

In mathematics, **we want the properties of the operations to be preserved** as much as possible when venturing into “new realms”, and integers are such a “new realm” compared to the counting numbers.

For example, what is the value of $14 \div (-2)$?

Let s be the value of $14 \div (-2)$. For multiplication to still be the opposite operation to division, it follows that $(-2)s = 14$. The only number that fulfills the latter equation is $s = -7$. Therefore, $14 \div (-2) = -7$.

Using variables instead of specific numbers, we could prove that each time a positive number is divided by a negative, the quotient is <u>negative</u>. Using similar reasoning, we can show that each time you divide a positive integer by a negative integer, the quotient is <u>negative</u>.

3. Multiply. Then use the same numbers to write *two* equivalent division equations.

a.	b.	c.
a. $-5 \cdot (-5) =$ ______	**b.** $9 \cdot (-6) =$ ______	**c.** $-80 \cdot 8 =$ ______
______ $\div$ _____ $=$ ______	______ $\div$ _____ $=$ ______	______ $\div$ _____ $=$ ______
______ $\div$ _____ $=$ ______	______ $\div$ _____ $=$ ______	______ $\div$ _____ $=$ ______

Summary. The symbols below show whether you get a positive or negative answer when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

Multiplication	**Examples**	**Division**	**Examples**
⊕ · ⊖ = ⊖	$4 \cdot (-5) = -20$	⊕ ÷ ⊖ = ⊖	$20 \div (-5) = -4$
⊖ · ⊕ = ⊖	$-4 \cdot 5 = -20$	⊖ ÷ ⊕ = ⊖	$-20 \div 5 = -4$
⊖ · ⊖ = ⊕	$-4 \cdot (-5) = 20$	⊖ ÷ ⊖ = ⊕	$-20 \div (-5) = 4$
⊕ · ⊕ = ⊕	$4 \cdot 5 = 20$	⊕ ÷ ⊕ = ⊕	$20 \div 5 = 4$

Here are the shortcuts for *multiplication* and *division* with signed numbers (NOT for addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- If the numbers have different signs, the answer is negative.

4. Divide.

a. $24 \div (-8) =$ _______	**b.** $-24 \div 8 =$ _______	**c.** $(-24) \div (-8) =$ _______
d. $-40 \div 8 =$ ________	**e.** $(-8) \div (-1) =$ ________	**f.** $72 \div (-9) =$ ________

5. Four people shared a debt of $280 equally. How much did each owe? Write a quotient to represent this.

6. In a math game, you get a negative point for every wrong answer and a positive point for every correct answer. Additionally, if you answer in 1 second, your negative points from the past get slashed in half!

Angie had accumulated 14 negative and 25 positive points in the game. Then she answered a question correctly in 1 second. Write an equation for her current "point balance."

7. Here's a funny riddle. Solve the math problems to uncover the answer.

E ____ $\div (-8) = 2$	**N** $-12 \cdot (-10) = 2 \cdot$ ______	**E** $(-144) \div 12 =$ _____
E $3 \cdot (-12) =$ _____	**H** ____ $\div 12 = -5$	**T** $-4 \cdot (-9) =$ ______
N $-5 = -15 \div$ ____	**E** $0 =$ ____ $\cdot (-6)$	**V** $-45 \div$ ______ $= 30 \div 6$
G $-3 \cdot (-9) =$ ______ $\cdot 3$	**I** $-27 \div 9 = 9 \div$ ______	**I** $-7 \cdot$ ____ $= 4 \cdot (-21)$
S $-42 \div 6 =$ ______	**N** $3 \cdot$ _____ $= -2 \cdot 12$	

Why is six afraid of seven? Because…

−7	−12	−9	−36	60		0	12	9	−60	36		3	−3	−8	−16

Example 1. $-5 \cdot 8 - \dfrac{20}{-4}$

Here we have a multiplication, a subtraction, and a division. The multiplication and division are done first, before the subtraction. Notice carefully how the simplification is done, on the right.

$$\underbrace{-5 \cdot 8}_{\downarrow\,-40} - \underbrace{\frac{20}{-4}}_{\downarrow\,(-5)} = -40 - (-5) = -40 + 5 = -35$$

8. Find the value of each expression.

a. $8 \cdot \dfrac{(-8)}{4}$	**b.** $1 - \dfrac{2}{(-5)}$	**c.** $-8 + \dfrac{21}{4-7}$
d. $2 + \dfrac{(-6)}{5} \cdot 10$	**e.** $1 + \dfrac{(-2) \cdot 15}{-3}$	**f.** $30 - \dfrac{24}{(-4)} - 11$

9. A submarine was located 300 ft below the surface. Then it rose so that its depth was only one-third of what it had been. Write an equation to represent this, using negative integers.

10. Give a real-world context for each quotient.

a. $-100 \div 2$

b. $-60 \div 3$

11. Write two different division equations for each quotient.

a. _______ ÷ ______ = −7 _______ ÷ _______= −7	**b.** _______ ÷ ______ = −1 _______ ÷ _______= −1	**c.** _______ ÷ ______ = −5 _______ ÷ _______= −5

Puzzle Corner

If you multiply both coordinates of the point $(-2, 3)$ by -1, it becomes $(2, -3)$. Investigate what happens to a geometric figure if you multiply each of its coordinates by -1.