# Contents

Geometry 3 (Blue Series)

# Angle Relationships 1

A **ray** has a starting point and continues indefinitely in one direction (indicated by one arrowhead).

In contrast to a ray, a **line** continues indefinitely in *two* directions (indicated by two arrowheads).

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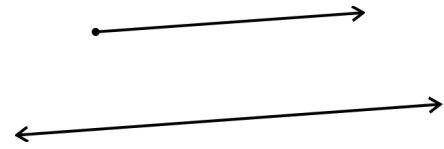
An **angle** consists of **two rays that start at the same point**, called the **vertex**. Each ray is called a **side** of the angle.

We can denote the angle on the right as angle BAC, or using the symbol "∠" for "angle," as ∠BAC.

Note that we list the vertex point in the middle: it is ∠B**A**C, not ∠ABC. We could also name it ∠CAB.
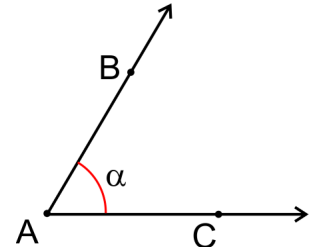
In mathematics, we also often denote angles with the beginning letters of the Greek alphabet: α (alpha), β (beta), γ (gamma), and δ (delta). So ∠BAC can also be called "angle α."

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Two angles are **adjacent** if they have a **common vertex and share one side.**

In the image on the right, ∠α and ∠β are adjacent (side-by-side) angles.

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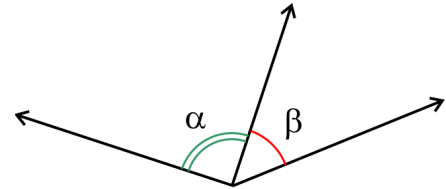
1. B is a point on line AD. Find the measures of the three angles, and also the angle sums. Do you notice any special numbers?

    ∠ABC = _____ °

    ∠CBD = _____ °

    ∠ABD = _____ °

    sum of ∠ABC and ∠CBD : _____ °

    sum of all three angles: _____ °    (This should be 360°.)
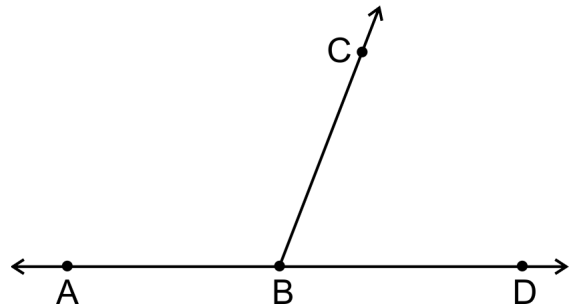
2. Several rays start at point A. Measure the angles. Calculate their sum.

    ∠1 = _____ °    ∠2 = _____ °

    ∠5 = _____ °    ∠4 = _____ °
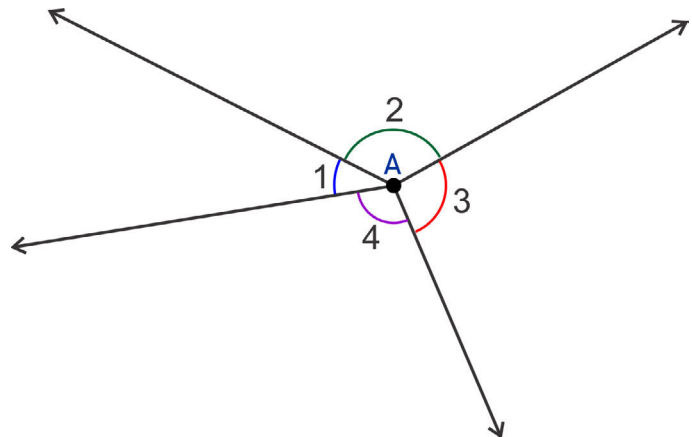
    Sum of the angles = _____ °

P is a point on line *l*. The angles ∠α and ∠β in this image are adjacent, and they form a straight angle (an angle of 180 degrees). They are called **supplementary angles**.

Two angles are supplementary if their **sum is 180 degrees:**

$$∠α + ∠β = 180°$$

We also say that α supplements β.

Rays *r* and *s* start at point P and form a right angle. The adjacent angles ∠α and ∠β form a right angle. They are called **complementary angles**.

Two angles are complementary if their **sum is 90 degrees:**

$$∠α + ∠β = 90°$$
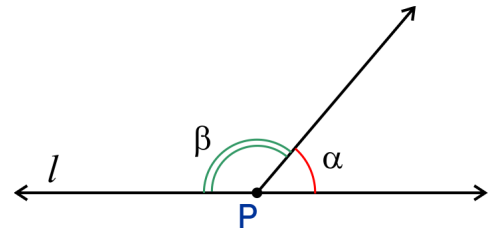
We also say that α complements β.

Here's a mnemonic to help you remember the difference: **S**upplementary angles form a **S**traight line, and **C**omplementary angles form a **C**orner (a right angle).
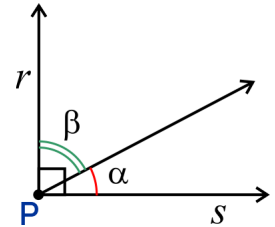
Supplementary angles don't have to be adjacent, and neither do complementary angles.
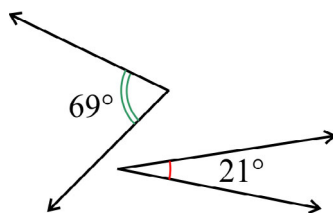
These are still complementary angles, because 21° + 69° = 90°.
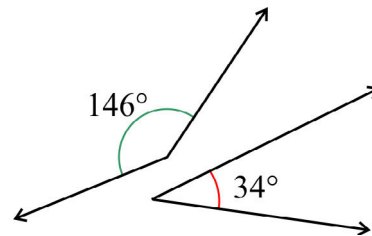
These are still supplementary angles, because 146° + 34° = 180°.

3. **a.** Draw a 38° angle. Then draw an adjacent angle that complements it.

   **b.** Draw an 82° angle. Then draw an adjacent angle that supplements it.

4. Write an equation for the unknown and solve it. Do not measure any angles.

**a.** $x$, $78°$

**b.** $61°$, $\alpha$, $14°$

**c.** $19°$, $47°$, $98°$, $65°$, $y$

**d.** $85°$, $x$, $x$, $79°$, $131°$
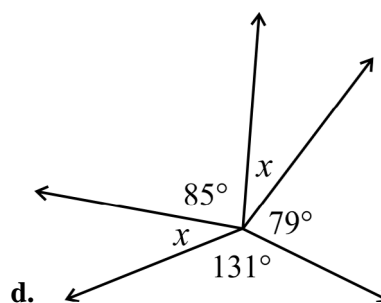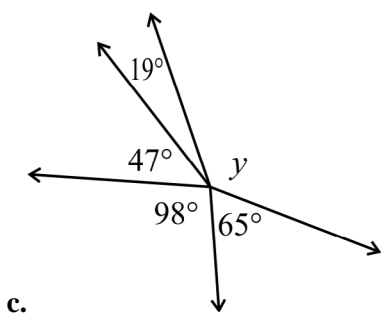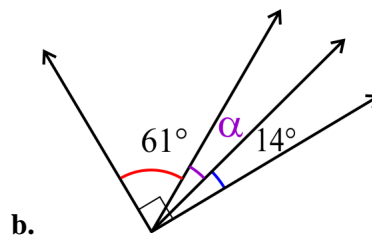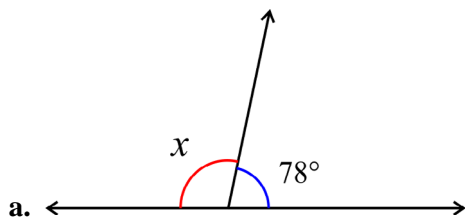
5. Figure out the missing entries in the tables without measuring any angles.

**a.**

| Angle | Degrees | Fraction | Percentage |
|-------|---------|----------|------------|
| α     |         | 1/4      |            |
| β     | 120°    |          |            |
| γ     |         |          |            |
| δ     | 75°     |          |            |

**b.**

| Angle | Degrees | Fraction | Percentage |
|-------|---------|----------|------------|
| α     | 50°     |          |            |
| β     |         |          |            |
| γ     |         | 1/6      |            |
| δ     |         |          |            |

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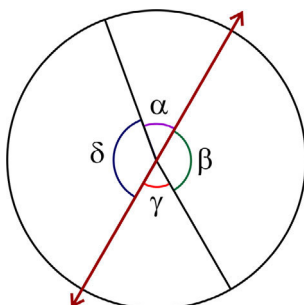
**Example.** Point B is on line AD. Write an equation to solve for the unknown. What is the measure of angle ABC?

Since angle ABD is a straight angle (180°), the equation is:

$$2x + 78 \;=\; 180$$
$$2x \;=\; 102$$
$$x \;=\; 51$$

So, $x$ is 51°. However, angle ABC does not measure 51° because its measure is $2x$, not $x$. So, we double the value of $x$ to get that $\angle ABC = 102°$.
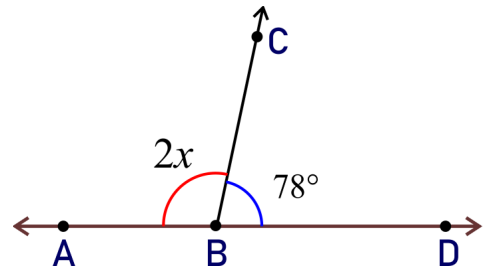
6. Angle ADC measures 95°.

   **a.** Write an equation for the unknown and solve it.

   **b.** Find the measure of $\angle BDC$.
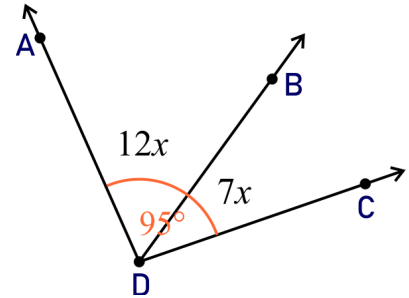
7. **a.** Write an equation for the unknown and solve it.

   **b.** Find the measure of $\angle ABC$.
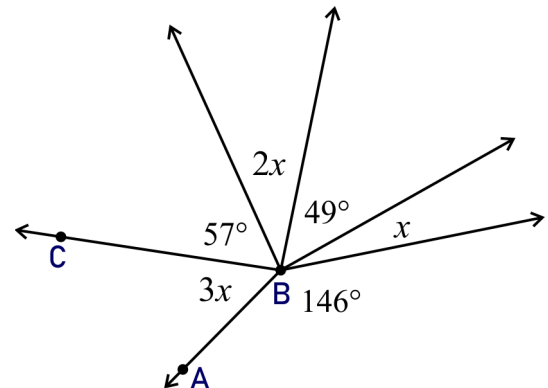
8. **a.** Write an equation for the unknown and solve it.

   **b.** Find the measure of each angle in the image, excluding those whose angle measure is given.
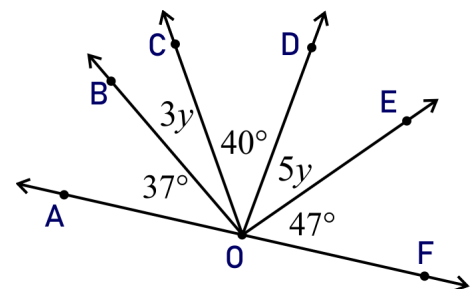
# Area of a Circle

The area of a circle is given by this formula: $A = \pi r^2$, where $r$ is the radius of the circle.

Read the formula as: "Area equals pi $r$ squared." It means that you first multiply the radius by itself and then multiply the result by $\pi$.

You can remember the formula by thinking, "Pie are square." Of course pies are usually round, not square! It is bad English, as well, but the purpose of this silly mnemonic is just to help you remember the formula.

**Example 1.** The radius of a circle measures 8 cm. What is its area?

We use the formula: $A = \pi r^2 = \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm} \approx 3.14 \cdot 64 \text{ cm}^2 = 200.96 \text{ cm}^2$, or about 200 cm$^2$.

**Remember to always give your answer for an area in <u>square</u> units**, be it square inches, square centimeters, square meters, square feet, *etc*. If no measuring unit is given, use "square units."

*You can use a calculator for all the problems in this lesson.*

1. Estimate the area of the circles by counting squares and parts of squares. After that, calculate the area to the nearest tenth of a square unit.
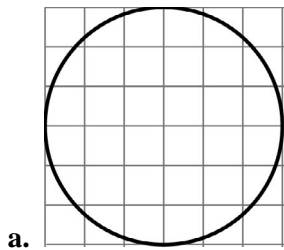
**a.**

Estimation: _____ square units

Calculation: _____ square units

**b.**
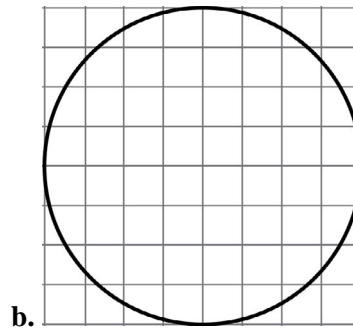
Estimation: _____ square units

Calculation: _____ square units

2. Find the areas of these circles.

| | |
|---|---|
| **a.** A circle with a radius of 7.0 cm.<br><br>(Round the answer to the nearest 10 cm$^2$.)<br><br>Area = | **b.** A circle with a radius of 10 1/4 in.<br><br>(Round the answer to the nearest square inch.)<br><br>Area = |
| **c.** A circle with a *diameter* of 75.0 cm.<br><br>(Round the answer to the nearest 10 cm$^2$.)<br><br>Area = | **d.** A circle with a radius of 17 ft 4 in.<br><br>(Round the answer to the nearest 1000 sq. in.)<br><br>Area = |

3. The radius of a certain circle is 8 units. Which expression gives you the circumference of that circle?

**a.** $16\pi$  **b.** $8\pi$  **c.** $\dfrac{8}{\pi}$  **d.** $\dfrac{\pi}{8}$

4. The circumference of a certain circle is 11 units. Which expression gives you the diameter of that circle?

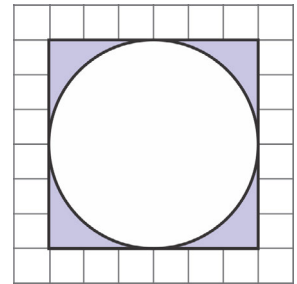**a.** $121\pi$  **b.** $11\pi$  **c.** $\dfrac{11}{\pi}$  **d.** $\dfrac{\pi}{11}$

5. The diameter of a certain circle is 6 units. Which expression gives you the area of that circle?

**a.** $36\pi$  **b.** $6\pi$  **c.** $12\pi$  **d.** $9\pi$

6. A circle is drawn inside a square. Find the area of the *shaded figure* to the nearest tenth of a square unit.

7. A square is drawn inside a circle. The diameter of the circle is 15.0 cm, and the side of the square is 10.6 cm. What percentage of the area of the circle is the area of the square? Give it to the nearest tenth of a percent.

*Note: You will need a more exact value for π than 3.14, so use 3.1416 or the π-button on your calculator.*
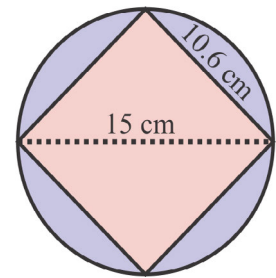
8. A water fountain is surrounded by a flower bed as shown by the image. The two circles in the image are concentric (they share the same center point).

**a.** Find the area of the portion that is planted with flowers, to the nearest ten square feet.

**b.** If fertilizer is to be applied at the rate of 2 lb per 50 square feet, then how much fertilizer is needed for the flower bed?
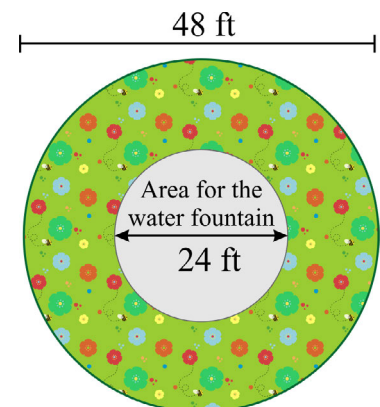
9. Joe's pizzeria offers the following pizzas. Let's find out which is the best buy.

| Pizza diameter | Cost | Area (sq. in.) | Cost per square inch ($) |
|---|---|---|---|
| 10″ | $10.99 | | |
| 12″ | $12.99 | | |
| 14″ | $14.99 | | |
| 18″ | $21.99 | | |

**a.** Find the area of each pizza to the nearest hundredth of a square inch. (Use $\pi$ = 3.1416 or the $\pi$-button.)

**b.** Find the cost per square inch to the tenth of a cent (three decimals).

**c.** Which pizza is the cheapest per square inch?

**d.** Which gives you more to eat, two 10″ pizzas or one 14″ pizza?

10. This shape consists of two half-circles and a rectangle. The side of each little square in the grid is one unit.
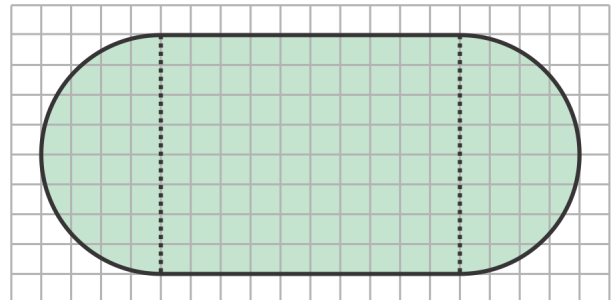
    **a.** Find the perimeter of the shape to the nearest unit.



    **b.** Find the area of the shape to the nearest square unit.

    **c.** If each unit in the grid represents 0.5 m in reality, what is the area of the shape in reality?

    **a.** Daniel calculated the area of a certain circle, using $\pi$ = 3.14. He got 153.86 cm². What was the radius of that circle?

    **b.** Another time, Daniel got $256\pi$ for the area of a certain circle. What is the circumference of that circle?

# Drawing Triangles, Part 2

The angles of any triangle always add up to 180°. We're not proving it right now, but this fact will be handy in the exercises of this lesson.

In the triangle on the right, the angle measures add to 180°:

$$46° + 52° + 82° = 180°$$
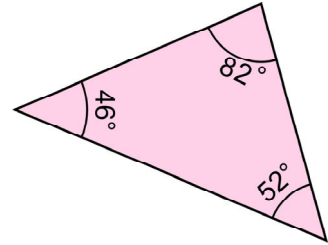
1. The two angles are angles in a triangle. Find the third angle measure.

   **a.** 100°, 30°, and _____    **b.** 19°, 58°, and _____    **c.** 87°, 25°, and _____

2. Why does a triangle with 90° and 100° angles not exist?
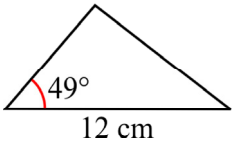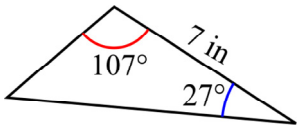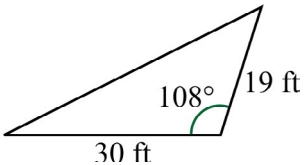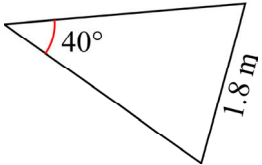
3. **a.** Draw a triangle with 50°, 40°, and 90° angles.

   **b.** Is the triangle you drew unique? Or can you draw other, noncongruent triangles that also have angles with these measures?
   You can compare drawings with a friend or a classmate.

   **c.** What do you think will happen in other cases where three angles are given but no information about the side lengths?

   Test your idea with a triangle with 70°, 70°, and 40° angles and/or with a triangle with 36°, 112° and 32° angles.

| **Terminology** | | | |
|---|---|---|---|
| The 12-cm side is **adjacent** to the 49° angle. |  | The 7-inch side is **between** the 107° and the 27° angles. |  |
| The 108° angle is **formed by** the 30-ft and 19-ft sides. |  | The 40° angle is **opposite** to the 1.8-meter side. |  |

4. Match the pictures and the descriptions. The same picture can match several descriptions and vice versa.

This triangle has one 50° angle and 9 cm and 5 cm sides.

A triangle with a 9-cm side adjacent to a 50° angle.

A triangle with 9 cm and 5 cm sides, and a 65° angle opposite the 9-cm side.

A triangle with a 9-cm side between 50° and 65° angles.

This triangle has a 5-cm side and 50° and 65° angles.
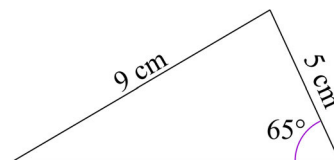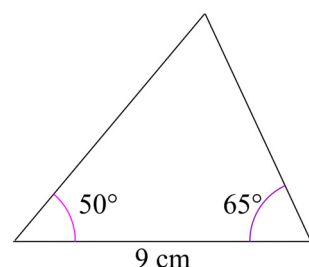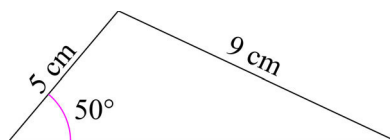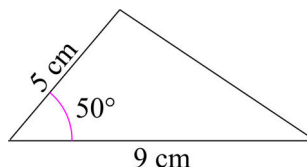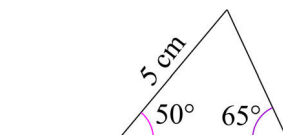


5. Are any of the triangles in the exercise above congruent with one another? If so, which?

6. Students were given the task of drawing a triangle with 3.5 cm and 5 cm sides, and a 55° angle formed by those sides. Elijah says, "Zoe's triangle does not fit what was asked." Is he correct in saying that? Explain.
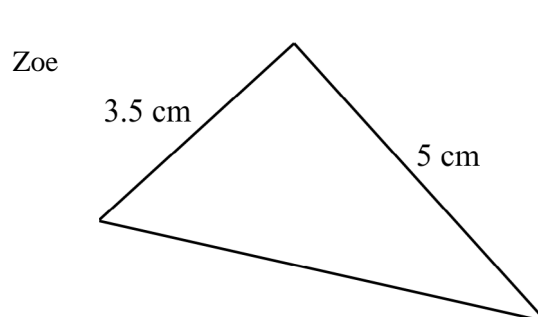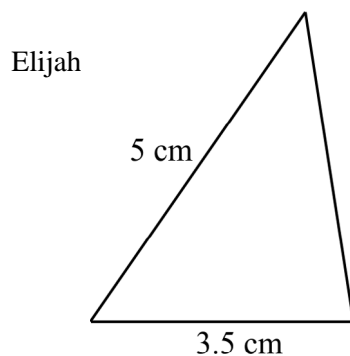
7. For each of the information cards below, determine whether the information given defines a unique triangle.
   If so, draw the triangle. If not, sketch at least two non-congruent triangles that satisfy the given conditions.
   Use blank paper as necessary. You may work with a partner.

**(1)**
A triangle with a 70° angle and
with 10 cm and 8 cm sides,

**(2)**
An isosceles triangle
with a 90° top angle.

**(3)**
A triangle with 30° and 90° angles
and a 6-cm side between them.

**(4)**
A triangle with 10 cm and 8 cm
sides that form a 70° angle.

**(5)**
An isosceles triangle with a
90° top angle and a 12-cm base.

**(6)**
A triangle with 30° and
90° angles plus a 6-cm side.

**Sample worksheet from**
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8. Sarah has drawn a secret triangle. She is giving you information about it, one clue at a time.
   After which clue can you draw Sarah's triangle, and be certain it is congruent to her triangle?

   1. My triangle has a 45° angle.
   2. It also has a 95° angle.
   3. It also has a side that measures 4 inches.
   4. That side is opposite the 95° angle.
   5. It also has a side that measures 2 9/16 in.
   6. It also has a 40° angle.

9. (optional) We haven't studied all the possible combinations of pieces of information (angles and sides) that
   determine a triangle, but we have looked at several. Based on the exercises in this and the previous lesson,
   and based on your further investigations, fill in the table. You will study this topic more in high school.

| Givens | Determines a unique triangle? (yes/no) |
|---|---|
| Three sides | |
| Two sides and a given angle formed by those sides | |
| Two sides and a given angle (location of angle not specified) | |
| Three angles | |
| Two angles and a side between them | |
| Two angles and a side (location of side not specified) | |
| One side and one angle | |

Anita says, "One of the angles of my isosceles triangle measures
104 degrees, and one of its sides measures 5 inches."

**Puzzle Corner**

How many different noncongruent triangles are there that satisfy the given information?

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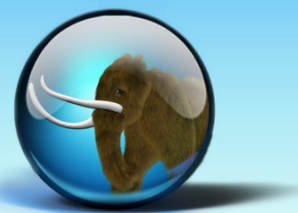
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