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# Introduction

*Math Mammoth Geometry 2* continues the study of geometry after *Math Mammoth Geometry 1*, and is suitable for grades 6 - 7. It concentrates on two broad and important topics: **area and volume** of all common shapes.

In the first section of the book, which is also the longest, students learn to calculate the area of all common shapes: triangles, parallelograms, other polygons, and circles. They encounter Pi and how it relates to the circumference of a circle. We also briefly study the proof for the formula for the area of a circle. I feel it is important that students encounter justifications for mathematical formulas and procedures and even read some proofs before high school. We don't want students to think that mathematics is only a bag of magic tricks or formulas to memorize that seemingly came out of nowhere. Proofs and logical thinking are foundations to mathematics and school mathematics should not be left without them.

Next, we study nets and the surface area of common solids. Naturally the student needs to know how to calculate the area of two-dimensional shapes by this point (specifically, the area of rectangles, triangles, and circles).

There is one more section about area, in which we learn how to convert between various units of area, both metric and customary.

In the lesson *Slicing Three-Dimensional Shapes*, we slice three-dimensional solids with a plane, and learn that the result is always a two-dimensional shape. Students see that in a concrete way by slicing cubes and pyramids made of modeling clay. Some Internet links (provided in the lesson) will also help students to visualize what happens when a solid is cut with a plane.

Lastly, the book teaches about volume of common solids. I assume the students already know how to find the volume of a right rectangular prism (a box). First we expand this topic by calculating volumes of rectangular prisms with edges of fractional length. Then we go on to calculate volumes of other solids: prisms, cylinders, pyramids, and cones.

Besides simple calculation exercises, the lessons contain many real-life applications, word problems, and mathematical problems concerning area and volume. I have tried to create a variety of problems to encourage students' problem-solving skills.

These topics (area and volume) involve lots of calculations, and the calculator is allowed in the problems that are marked with a little calculator image. Middle school students also need to learn other geometry topics that are more "geometric" in nature: congruent transformations, angle relationships, and compass-and-ruler constructions (covered in *Math Mammoth Geometry 3*). Besides, they also need to study the Pythagorean Theorem (covered in *Math Mammoth Geometry 3*).

*I wish you success in teaching math!*  
*Maria Miller, the author*

## Helpful Resources on the Internet

Use the online resources as you see fit to supplement the main text.

### *Area and perimeter*

#### **Free worksheets for the area of triangles, quadrilaterals, and polygons**

Generate printable and customizable worksheets to practice finding the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include choosing either the first or all quadrants, scaling, image size, workspace, and border.

[http://www.homeschoolmath.net/worksheets/area\\_triangles\\_polygons.php](http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php)

#### **BBC Bitesize - Area**

Revision “bites” (brief reviews), including a few interactive questions, about the area of triangles, of parallelograms, and of compound shapes. Includes an activity and a test.

<http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/>

#### **Geometry Area/Perimeter Quiz from ThatQuiz.org**

An online quiz that asks either the area or the perimeter of rectangles, triangles, parallelograms, and trapezoids. You can also modify the quiz parameters to your liking, for example to omit a shape, or instead of solving for the area, solve for an unknown side when perimeter/area is given.

<http://www.thatquiz.org/tq-4/?-j1i00f-lc-p0>

#### **Area Tool**

Use this tool to determine how the length of the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapezoids, parallelograms, and triangles?

<http://illuminations.nctm.org/Activity.aspx?id=3567>

#### **Area of a Triangle**

In this activity, you try to make a parallelogram using a copy of the given triangle.

<http://illuminations.nctm.org/Activity.aspx?id=4160>

#### **Area of a Parallelogram**

In this activity, you rearrange the pieces of a parallelogram to make a rectangle.

<http://illuminations.nctm.org/Activity.aspx?id=4158>

#### **Math Playground: Party Designer**

You need to design areas for the party, such as crafts table, food table, seesaw, and so on, so that they have the given perimeters and areas.

<http://www.mathplayground.com/PartyDesigner/PartyDesigner.html>

### *Polygons*

#### **Triangle Explorer**

Practice calculating the area of a triangle using this interactive tool.

<http://www.shodor.org/interactivate/activities/TriangleExplorer/>

#### **Properties of Kites**

Investigate the interior angles and diagonals of kites with these interactive tools.

<http://math.kendallhunt.com/x19428.html>

### **Interactive Quadrilaterals**

See all the different kinds of quadrilaterals “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.

<http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html>

### **Looking at Polygons**

An interactive lesson with explanations and a quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.

<http://www.absorblearning.com/mathematics/demo/units/KCA007.html>

### **Interactive Tangram Puzzle**

Place the tangram pieces so they form the given shape.

[http://nlvm.usu.edu/en/nav/frames\\_asid\\_112\\_g\\_2\\_t\\_1.html](http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html)

### **Tangram set**

Cut out your Tangram set by folding paper

<http://tangrams.ca/inner/foldtan.htm>

### *Circle /Pi*

### **Interactive Area of a circle**

Explore and discover the relationship between the area, radius, and graph of a circle. Just click and drag the points.

<http://www.mathwarehouse.com/geometry/circle/interactive-area.php>

### **Circle tool from Illuminations**

How do the area and circumference of a circle compare to its radius and diameter? This activity allows you to investigate these relationships in the Intro and Investigation sections and then hone your skills in the Problems section.

<http://illuminations.nctm.org/Activity.aspx?id=3547>

### **Area of a Circle, Formula & Illustrated Lesson**

The interactive tool shows you the area of the circle as the radius increases. The page also includes a short quiz.

<http://www.mathwarehouse.com/geometry/circle/area-of-circle.php>

### **Circle Tool**

An applet that allows you to investigate how the area and circumference of a circle compare to its radius and diameter in the Intro and Investigation sections and then hone your skills in the Problems section.

You can drag the radius to various lengths, and then click the “Add to Table” button to record the data in the table.

You can also examine the ratios of any two measures, and make a graph of the data

<http://illuminations.nctm.org/ActivityDetail.aspx?ID=116>

### **Radius, Diameter, and Circumference**

A simple lesson with an interactive quiz about radius, diameter, and circumference of a circle.

<http://www.mathgoodies.com/lessons/vol2/circumference.html>

### **The Area of a Circle as a Limit**

An animation that illustrates how the area of a circle is a limit of the sum of the areas of interior triangles as the number of triangles goes to infinity. This is the idea I explain in this book (Math Mammoth Geometry 2).

<http://www.learnerstv.com/animation/animation.php?ani=96&cat=physics>

### **Area of Circle**

An activity where you measure a circle and the resulting figure when you cut it into wedges and tape them together to form a crude parallelogram.

[http://www.learner.org/courses/learningmath/measurement/session7/part\\_b/index.html](http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html)

### **Area & Perimeter Practice**

A 10-question quiz that will let you practice finding the area and circumference of a circle.

<http://www.thatquiz.org/tq-4/?-j201g-la-p1ug>

### **Area and circumference of a circle - Test from BBC Bitesize**

A 10-question multiple-choice quiz about the area and circumference of a circle where the questions increase with difficulty.

<http://www.bbc.co.uk/bitesize/quiz/q90581037>

### **Area and Circumference of a Circle**

A 15-question multiple-choice quiz.

<http://www.proprofs.com/quiz-school/story.php?title=area-circumference--circle>

### **Mangahigh.com - Shape**

Questions on the area of a circle, including the area of a semi-circle and simple compound shapes.

[https://www.mangahigh.com/en/maths\\_games/shape/circles\\_and\\_cylinders/area\\_of\\_a\\_circle](https://www.mangahigh.com/en/maths_games/shape/circles_and_cylinders/area_of_a_circle)

### **Area of Circle**

An interactive calculator that calculates the circumference, radius, diameter, and area of a circle when any one of those is entered.

<http://www.mathsisfun.com/geometry/circle-area.html>

### **Area of Circles**

An interactive applet that allows students to explore the formula for the area of a circle by cutting it into sectors and rearranging the sectors to form a figure close to a parallelogram. By increasing the number of sectors, students can see that the figure gets closer and closer to a perfect parallelogram.

<http://www.geogebra.org/student/m279>

### **Amazing History of Pi**

A short and simple introduction to the history of pi.

<http://ualr.edu/lasmoller/pi.html>

### **Approximating Pi**

How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.

<http://www.pbs.org/wgbh/nova/archimedes/pi.html>

### **Pi Day Activities & Links**

Two pi-related activities to do at home, plus a lot of fun links about pi. Originally meant for Pi Day (celebrated 3/14 each year).

[http://www.exploratorium.edu/pi/pi\\_activities/index.html](http://www.exploratorium.edu/pi/pi_activities/index.html)

## 5 Trillion Digits of Pi

As of August 2010, the world record for computing digits of pi was 5 trillion digits. This will keep changing, of course.

<http://www.numberworld.org/digits/Pi/>

**A Rolling Circle Illustrating Pi** This is a short animation where a circle with diameter 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at 3.14 or Pi.

<http://i.imgur.com/dsCw0.gif>

## *Volume and Surface Area*

### 2-D and 3-D Shapes

Learn about different solids: rotate them and see their nets.

[http://www.bgfl.org/bgfl/custom/resources\\_ftp/client\\_ftp/ks2/maths/3d](http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/3d)

### Geometric Solids

Manipulate (rotate) various geometric solids by dragging with the mouse, and see their nets. Count the number of faces, edges, and vertices.

<http://illuminations.nctm.org/Activity.aspx?id=3521>

### Cuboid Exploder and Isometric Shape Exploder

These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes and then “explode” them into their unit cubes, thus illustrating volume.

<http://www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html> and

<http://www.teacherled.com/resources/isoexplode/isoexplodeload.html>

### Volume Shoot Game

Shoot (select) the shapes with a given volume in cubic units.

<http://www.sheppardsoftware.com/mathgames/geometry/shapeshoot/VolumeShapesShoot.htm>

### Interactivate: Surface Area and Volume

Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.

<http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/>

### Surface area practice

A 10-question quiz where you can practice finding the surface area of prisms, pyramids, and cylinders. Note: you need to input the surface area of cylinders as a multiple of  $\pi$  · cm. For example, a cylinder with height 7 cm and bottom radius of 2 cm has the surface area of  $(4 \text{ cm} \cdot \pi \cdot 7 \text{ cm}) + (2 \cdot (2 \text{ cm})^2 \cdot \pi) = 28\pi \text{ cm} + 8\pi \text{ cm} = 36\pi \text{ cm}$ .

<http://www.thatquiz.org/tq-4/?-j824a0-l6-p1ug>

### Geometry Volume/Surface Area Quiz from ThatQuiz.org

An online quiz that asks for either the volume or surface area of cubes, prisms, spheres, cylinders, or cones. You can modify the quiz parameters to your liking, for example to omit some shapes, solve only for volume, only for surface area, or even for the length of an unknown side with the volume or surface area given.

<http://www.thatquiz.org/tq-4/?-j3vu0-lc-m2kc0-na-p0>

### **Making Cuboids**

An interactive activity to explore the surface area and volume of a cuboid, calculate them, or find the volume when the areas of the faces are known.

<http://www.mrbartonmaths.com/resources/keystage3/shape/Volume%20and%20Surface%20Area%20of%20Cuboids.swf>

### **Volume of Right Rectangular Prisms with Fractional Edges**

Word problems from OpusMath: Choose the ones you want and then build a Word document from them. Answer keys available. Free registration required.

<http://www.opusmath.com/common-core-standards/6.g.2-find-the-volume-of-a-right-rectangular-prism-with-fractional-edge-lengths>

### **Interactives - Surface area - Cylinders**

A lesson that includes an interactive portion where you calculate the surface area of a cylinder step-by-step.

[http://www.learner.org/interactives/geometry/area\\_surface2.html](http://www.learner.org/interactives/geometry/area_surface2.html)

### **Volume of a Cylinder**

Discussion about the volume of a cylinder, including a regular calculator and an interactive calculator where you can drag the orange dot to resize the cylinder, and the volume is calculated as you drag. By allowing oblique cylinders and then dragging the top orange dot, you can clearly see that the volume of an oblique cylinder is equal to the volume of a right cylinder.

<http://www.mathopenref.com/cylindervolume.html>

### **Surface Area and Volume of 3-D Shapes**

This page contains three worked out examples about the volume and surface area of cylinders and prisms, followed by over a dozen exercises and word problems. You can self-check your answers by a clicking the "Check" buttons.

[http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i9/bk9\\_9i4.html](http://www.cimt.plymouth.ac.uk/projects/mepres/book9/bk9i9/bk9_9i4.html)

### **Volume of a cylinder**

Interactive and guided questions about the volume of a cylinder. Students also find the height or the radius of a cylinder when given the volume. Some problems leave the answer in terms of pi.

[https://www.mangahigh.com/en/math\\_games/shape/circles\\_and\\_cylinders/volume\\_of\\_a\\_cylinder](https://www.mangahigh.com/en/math_games/shape/circles_and_cylinders/volume_of_a_cylinder)

### **Geometry Volume Quiz**

A 10-question quiz where you can practice finding the volume of prisms and cylinders. Note: you need to input the volume of cylinders as a multiple of  $\pi \cdot \text{cm}$ . For example, a cylinder with height 7 cm and bottom radius of 2 cm has the volume of  $(2 \text{ cm})^2 \cdot \pi \cdot 7 \text{ cm} = 28\pi \text{ cm}$ .

<http://www.thatquiz.org/tq-4/?-j28g0-la-p1ug>

### **MathGuide's Volume of Cylinders Quizmaster**

Interactive questions where you calculate the area of the bottom and the volume of a cylinder, given its radius and height.

<http://www.mathguide.com/cgi-bin/quizmasters/CylindersV.cgi>

*Cross-sections of solids*

### **Cross sections**

An interactive activity that allows you to slice a cube with a plane and find its different cross sections.

[http://www.learner.org/courses/learningmath/geometry/session9/part\\_c/index.html](http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html)

### **Cross-Section Flyer from Shodor**

Examine cross-sections of a cone, cylinder, pyramid, prism, and a double-cone. You can rotate and move the cutting plane, plus decide the number of faces for the pyramid and prism. An excellent tool!

<http://www.shodor.org/interactivate/activities/CrossSectionFlyer/>

### **Shapes – 3D Geometry Learning**

An app for iPad and iPhone

Explore various properties of 3D shapes, such as edges, vertices, and faces. You can rotate the solids and unfold them into their nets. Choose from 27 different solids.

<https://itunes.apple.com/au/app/solids-elementary-hd/id501650786?mt=8>

**Sample worksheet from**  
[www.mathmammoth.com](http://www.mathmammoth.com)



## **General**

### **Geometry - Math Warehouse**

Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.

<http://www.mathwarehouse.com/geometry/>

### **Geometry Course from Learning Math**

This online geometry course includes readings, problems, videos, interactive activities, homework problems and solutions. It is meant for K-8 teachers but will work well for middle school students as well.

<http://www.learner.org/courses/learningmath/geometry>

### **Geometry Reference Sheet**

Both online and printable versions; includes area and volume formulas for common shapes plus Pythagorean Theorem.

<http://www.ecalc.com/math-help/worksheet/geometry>

### **Geometry Tutorials**

Simple tutorials on triangles and their properties, polygons, symmetry, angles and much more. Also includes lots of solved geometry questions and some interactive applets.

<http://www.analyzmath.com/geometry.html#tutorials>

### **Online Kaleidoscope**

Create your own kaleidoscope pattern with this interactive tool.

[http://www.zefrank.com/dtoy\\_vs\\_byokal/](http://www.zefrank.com/dtoy_vs_byokal/)

### **Make Your Own Mandala**

A Mandala is a circular symmetrical design based on eights. Make your own and experiment with symmetry.

[http://www.girlsgotech.org/world\\_around\\_us.html](http://www.girlsgotech.org/world_around_us.html)

### **National Library of Virtual Manipulatives for Interactive Mathematics: Geometry**

A collection of interactive geometry activities: Congruent triangles, fractals, geoboard, golden rectangle, ladybug leaf, ladybug mazes, platonic solids, tangrams, tessellations, transformations and more.

[http://nlvm.usu.edu/en/nav/category\\_g\\_3\\_t\\_3.html](http://nlvm.usu.edu/en/nav/category_g_3_t_3.html)

### **Interactivate! Tessellate**

An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!

<http://www.shodor.org/interactivate/activities/Tessellate>