

Angles in Polygons

The angle sum in a quadrilateral is 360° .

See if you can understand and fill in this proof about the angle sum in a quadrilateral!

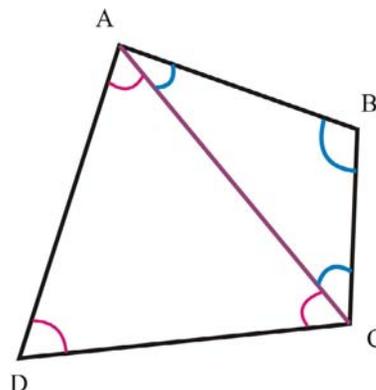
PROOF. Look at the quadrilateral ABCD. We draw a diagonal into it. The diagonal divides the quadrilateral into two triangles, triangle ABC and triangle ACD.

The angle B from triangle ABC is also an angle of the quadrilateral. The angle D from triangle ACD is also an angle of the quadrilateral.

Angle BCA and angle ACD are angles in the two triangles (the two angles with vertex C), but they also form together one angle of the quadrilateral. Similarly, angle CAB and angle DAC are angles in the two triangles (the two marked angles with vertex A), but they also form together another angle of the quadrilateral.

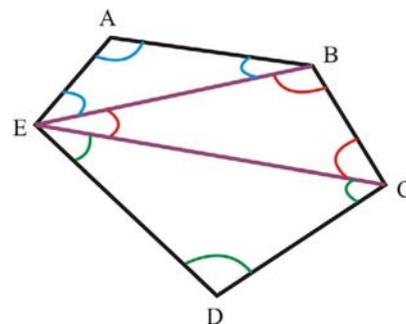
The angle sum of triangle ADC is _____ degrees, and the angle sum of triangle ABC is also _____ degrees,

It follows that the four angles in the quadrilaterals ABCD are formed of the angles of the two triangles. Thus, the angle sum of a quadrilateral is twice _____ $^\circ$, or _____ $^\circ$.



1. The figure illustrates how we can find the angle sum in a pentagon. Use the reasoning above to find the angle sum of a pentagon.

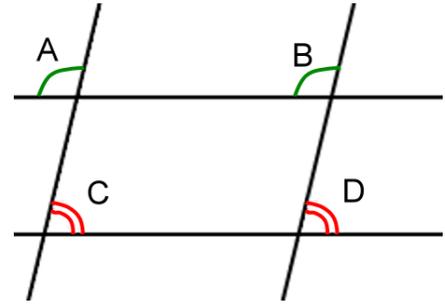
The angle sum of a pentagon is _____ $^\circ$.



2. Draw six points randomly. Connect them with a ruler so you get a hexagon. Then, divide it into triangles using diagonals. Use the reasoning above to find the angle sum of a hexagon.

It is _____ $^\circ$.

3. You have seen this picture earlier. It has two sets of parallel lines. We see lots of *vertical angles* and *corresponding angles*.



a. Angle A is 102° . Mark in the picture (using a single arc) all the other angles that are also 102° .

b. Mark in the picture (using a double arc) all the other angles that measure the same as angle C.

c. How many degrees is angle C? _____ $^\circ$

d. What quadrilateral is enclosed by the two sets of parallel lines? _____

From this figure we can learn something special about the angles in a parallelogram:

In a parallelogram, the opposite angles are congruent.

Also, two "neighboring" angles have the angle sum of 180 degrees.

In total, the four angles of course add up to 360° , just like in any quadrilateral.

4. One angle in a parallelogram is 74° . What are the measures of its other angles?

_____ $^\circ$, _____ $^\circ$ and _____ $^\circ$.

Now draw one such parallelogram.

You can choose the side lengths.

5. One angle of a rhombus is 115° . What are the measures of its other angles?

_____ $^\circ$, _____ $^\circ$ and _____ $^\circ$.

Now draw one such rhombus. You can choose the side length. Just remember, in a rhombus, all sides are congruent.

Puzzle Corner

a. This is a regular pentagon. The angle marked with "?" is called an *exterior angle* of the pentagon. Figure out its angle measure.

b. This is a regular hexagon. Figure out the measure of the exterior angle (marked with "?").

c. How many degrees is the exterior angle of a regular nonagon?

