

Multiplying Fractions by Fractions

We have studied how to find a fractional part of a whole number using multiplication.

For example, $\frac{3}{5}$ of 80 is written as a multiplication: $\frac{3}{5} \times 80 = \frac{240}{5} = 48$.

NOTE: The word *OF* translates here into **MULTIPLICATION**.

We can use the same idea to find a *fractional part of a fraction*!




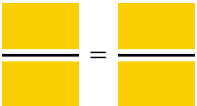



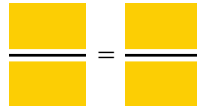




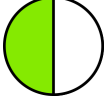

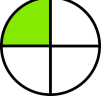

One-half of  is .

As a multiplication, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

One-fourth of  is .

As a multiplication, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

<p>a. Find $\frac{1}{2}$ of </p> <p>$\frac{1}{2} \times \frac{1}{4} =$</p>	<p>b. Find $\frac{1}{2}$ of </p> <p> \times  $=$ </p>	<p>c. Find $\frac{1}{2}$ of </p> <p> \times  $=$ </p>
<p>d. Find $\frac{1}{3}$ of </p>	<p>e. Find $\frac{1}{3}$ of </p>	<p>f. Find $\frac{1}{3}$ of </p>
<p>g. Find $\frac{1}{4}$ of </p>	<p>h. Find $\frac{1}{4}$ of </p>	<p>i. Find $\frac{1}{4}$ of </p>
<p>Did you notice a shortcut? If so, calculate $\frac{1}{5} \times \frac{1}{6} =$ </p>		

Shortcut: multiplying fractions of the type $1/n$

To multiply fractions of the form $1/n$ where n is a whole number, simply multiply the denominators to get the new denominator \rightarrow

$$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \quad \text{or} \quad \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

2. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$	b. $\frac{1}{13} \times \frac{1}{3}$	c. $\frac{1}{5} \times \frac{1}{20}$
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We have now studied how to find $1/2$ or $1/3$ or $1/5$ of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

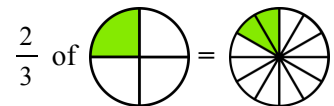
Review: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part of another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First, we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$.

Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.


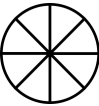

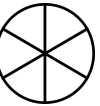

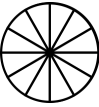

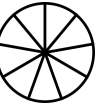

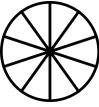

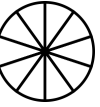


Example. Find $\frac{4}{5}$ of $\frac{1}{7}$.

First, we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part of the fraction. It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Color in a picture to show the answer.

a. $\frac{3}{4} \times$  $=$ 	b. $\frac{2}{3} \times$  $=$ 
c. $\frac{3}{4} \times$  $=$ 	d. $\frac{2}{3} \times$  $=$ 
e. $\frac{2}{5} \times$  $=$ 	f. $\frac{4}{5} \times$  $=$ 

4. Solve the multiplications by using two helping multiplications. Lastly, simplify if possible.

<p>a. $\frac{2}{3} \times \frac{1}{8} =$</p> <p>First find $\frac{1}{3}$ of $\frac{1}{8}$, then multiply the result by 2.</p> <p>$\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$ and $\frac{1}{24} \times 2 = \frac{\square}{\square} = \frac{\square}{\square}$</p>	<p>b. $\frac{3}{4} \times \frac{1}{10} =$</p> <p>First find $\frac{1}{4}$ of $\frac{1}{10}$, then multiply the result by 3.</p> <p>$\frac{1}{4} \times \frac{1}{10} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 3 = \frac{\square}{\square}$</p>
<p>c. $\frac{3}{5} \times \frac{1}{6} =$</p> <p>First find $\frac{1}{5}$ of $\frac{1}{6}$, then multiply the result by 3.</p> <p>$\frac{1}{5} \times \frac{1}{6} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 3 = \frac{\square}{\square} = \frac{\square}{\square}$</p>	<p>d. $\frac{5}{6} \times \frac{1}{9} =$</p> <p>First find $\frac{1}{6}$ of $\frac{1}{9}$, then multiply the result by 5.</p> <p>$\frac{1}{6} \times \frac{1}{9} = \frac{\square}{\square}$ and $\frac{\square}{\square} \times 5 = \frac{\square}{\square}$</p>
<p>e. $\frac{2}{3} \times \frac{1}{7} =$</p>	<p>f. $\frac{3}{8} \times \frac{1}{4} =$</p>

A shortcut for multiplying fractions

Multiply the numerators to get the numerator for the answer.
 Multiply the denominators to get the denominator for the answer.

Study the examples on the right.

Remember always to give your final answer as a mixed number and in lowest terms (simplified).

$$\frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$$

$$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1 \frac{1}{10}$$

5. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

<p>a. $\frac{3}{9} \times \frac{2}{9}$</p>	<p>b. $\frac{11}{12} \times \frac{1}{6}$</p>
<p>c. $\frac{1}{3} \times \frac{3}{13}$</p>	<p>d. $9 \times \frac{2}{3}$</p>
<p>e. $\frac{2}{9} \times \frac{6}{7}$</p>	<p>f. $10 \times \frac{5}{7}$</p>

COMPARE	
The roundabout way	The shortcut
$\frac{5}{6} \times \frac{1}{2} = ?$ First find $\frac{1}{6}$ of $\frac{1}{2}$, then multiply the result by 5. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ Find $\frac{1}{8}$ of $\frac{3}{5}$, then multiply that result by 2. And to find $\frac{1}{8}$ of $\frac{3}{5}$, first find $\frac{1}{8}$ of $\frac{1}{5}$, and then multiply that by 3. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$. That multiplied by 3 is $\frac{1}{40} \times 3 = \frac{3}{40}$. Then, that multiplied by 2 is $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$.	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the “roundabout way,” we do each multiplication separately. In the shortcut, we can just do them all at once.	

6. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8} =$	b. $\frac{7}{10} \times \frac{8}{5} =$
c. $\frac{9}{20} \times \frac{4}{5} =$	d. $\frac{2}{5} \times \frac{1}{3} =$
e. $\frac{1}{4} \times \frac{2}{7} =$	f. $\frac{9}{4} \times \frac{1}{3} =$
g. $\frac{2}{3} \times \frac{11}{8} =$	h. $\frac{2}{9} \times \frac{3}{10} =$

7. There was $\frac{1}{4}$ of the pizza left. Marie ate $\frac{2}{3}$ of that.

- What part of the *original* pizza did she eat?
- What part of the *original* pizza is left now?

8. Theresa has painted $\frac{5}{8}$ of the room.

a. What part is still left to paint?

b. Now, Theresa has painted half of what was still left.

Draw a bar model of the situation.

What part of the room is still left to paint?

9. Ted has completed $\frac{2}{3}$ of a job that his boss gave him.

a. What part is still left to do?

b. Now Ted has completed a third of what was still left to do.

Draw a bar model of the situation.

What (fractional) part of the original job is still left undone?

What part is completed?

10. Sally wants to make $\frac{1}{3}$ of the recipe on the right.

How much does she need of each ingredient?

Carob Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
 $\frac{1}{2}$ cup honey
1 teaspoon vanilla
 $\frac{3}{4}$ cup whole wheat flour
 $\frac{3}{4}$ teaspoon baking powder
1 cup walnuts or other nuts

11. For an upcoming get-together, Alison needs to multiply the coffee recipe. Assume that half of the guests drink one serving, and the other half drink two servings. Find how much coffee she will need, if she has:

a. 30 guests

b. 50 guests

c. 80 guests.

Coffee (5 servings)

$3\frac{1}{2}$ cups water
 $\frac{1}{4}$ cup coffee

Puzzle Corner

Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$

b. $\times \frac{1}{4} = \frac{5}{16}$

c. $\times \frac{3}{8} = \frac{1}{16}$

d. $\times \frac{2}{5} = \frac{3}{10}$