The Sieve of Eratosthenes and Prime Factorization

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in grades 4 and 5... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________	**b.** 60 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________
c. 84 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________	**d.** 97 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________

For your reference, here are some of the common divisibility tests for whole numbers.

A number is...	A number is...
divisible by 2 if it ends in 0, 2, 4, 6, or 8.	**divisible by 3** if the sum of its digits is divisible by 3.
divisible by 5 if it ends in 0 or 5.	**divisible by 4** if the number formed from its last two digits is divisible by 4.
divisible by 10 if it ends in 0.	**divisible by 6** if it is divisible by both 2 and 3.
divisible by 100 if it ends in "00".	**divisible by 9** if the sum of its digits is divisible by 9.

Use the various divisibility tests when building a factor tree for a composite number.

```
  135            27           135
  / \       5 ) 1 3 5        /   \
 5 · ?        - 1 0         5 · 27
                3 5             / \
              - 3 5            3 · 9
                  0                / \
                                  3 · 3
```

We start out by noticing that 135 is **divisible by 5.** From long division, we get 135 = 5 · 27. The final factorization is 135 = 3 · 3 · 3 · 5 or $3^3 \cdot 5$.

```
  441            4 9           441
  / \       9 ) 4 4 1         /   \
 9 · ?        - 3 6          9  ·  49
                8 1         / \   / \
              - 8 1        3 · 3 · 7 · 7
                  0
```

Adding the digits of 441, we get 9, so it is **divisible by 9.** We divide to get 441 = 9 · 49. The end result is 441 = 3 · 3 · 7 · 7 or $3^2 \cdot 7^2$.

2. Find the prime factorization of these composite numbers. Use a notebook for long divisions. Give each factorization below the factor tree.

a. 124 / \\ 2 · ___ / \\ 2	**b.** 260 / \\ 10 · ___ / \\ / \\	**c.** 96 / \\ 3 · ___ / \\
124 =	**260 =**	**96 =**
d. 90	**e.** 165	**f.** 95
90 =	**165 =**	**95 =**