

The Sieve of Eratosthenes and Prime Factorization

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes*.

Here is an online interactive version: <https://www.mathmammoth.com/practice/sieve-of-eratosthenes>

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, _____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in grades 4 and 5... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54

Check 1 2 3 4 5 6 7 8 9 10

factors: _____

b. 60

Check 1 2 3 4 5 6 7 8 9 10

factors: _____

c. 84

Check 1 2 3 4 5 6 7 8 9 10

factors: _____

d. 97

Check 1 2 3 4 5 6 7 8 9 10

factors: _____

For your reference, here are some of the common divisibility tests for whole numbers.

A number is...

divisible by 2 if it ends in 0, 2, 4, 6, or 8.

divisible by 5 if it ends in 0 or 5.

divisible by 10 if it ends in 0.

divisible by 100 if it ends in "00".

A number is...

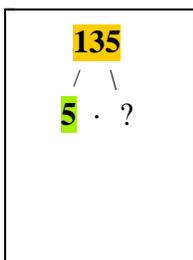
divisible by 3 if the sum of its digits is divisible by 3.

divisible by 4 if the number formed from its last two digits is divisible by 4.

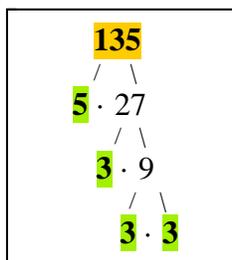
divisible by 6 if it is divisible by both 2 and 3.

divisible by 9 if the sum of its digits is divisible by 9.

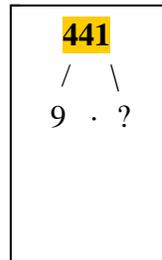
Use the various divisibility tests when building a factor tree for a composite number.



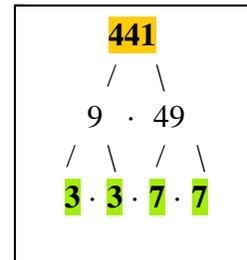
$$\begin{array}{r} 27 \\ 5 \overline{) 135} \\ \underline{-10} \\ 35 \\ \underline{-35} \\ 0 \end{array}$$



We start out by noticing that 135 is **divisible by 5**. From long division, we get $135 = 5 \cdot 27$. The final factorization is $135 = 3 \cdot 3 \cdot 3 \cdot 5$ or $3^3 \cdot 5$.



$$\begin{array}{r} 49 \\ 9 \overline{) 441} \\ \underline{-36} \\ 81 \\ \underline{-81} \\ 0 \end{array}$$



Adding the digits of 441, we get 9, so it is **divisible by 9**. We divide to get $441 = 9 \cdot 49$. The end result is $441 = 3 \cdot 3 \cdot 7 \cdot 7$ or $3^2 \cdot 7^2$.

2. Find the prime factorization of these composite numbers. Use a notebook for long divisions. Give each factorization below the factor tree.

<p>a. 124</p> <p>2</p>	<p>b. 260</p>	<p>c. 96</p>
124 =	260 =	96 =
<p>d. 90</p>	<p>e. 165</p>	<p>f. 95</p>
90 =	165 =	95 =