

Divisibility and Factors

Recall that in any multiplication, the numbers that are multiplied are called **factors** and the result is called a **product**.

(Even the multiplication “ 6×8 ” is called a product. You can call it the product written, whereas 48 is the product calculated or solved.)

We say that **6 is a factor of 48** -- because 6 times some whole number equals 48. Similarly, **8 is a factor of 48** (because 8 times some whole number equals 48).

$$6 \times 8 = 48$$

From the multiplication fact above we can make two division facts: $48 \div 6 = 8$ and $48 \div 8 = 6$.

These divisions are even divisions (no remainder). This means that both 6 and 8 are **divisors** of 48.

We also say that 48 is **divisible by** both 6 and 8.

$$48 \div 8 = 6$$

If a number is a factor of another number, it is also its divisor.

- How can you check whether a number is a factor of another number?
For example, how would you check whether 7 is a factor of 623?
- Answer the questions, and justify your answer.
 - Is 8 a factor of 100? (Yes/No), because
 - Is 9,896 divisible by 7? (Yes/No), because
 - Is 9 a divisor of 50? (Yes/No), because
- Mark said, “I know that 607 is divisible by 13, because its digits add up to 13 ($6 + 0 + 7 = 13$).”
Is Mark correct? If not, prove to him the truth of the matter.

4. a. How can you tell, without calculations, whether the number $3 \times 4 \times 87$ is divisible by 3?
- b. Is this number divisible by 12? Why or why not?
- c. Is $2 \times 758 \times 5$ divisible by 10?

Easy divisibility rules (You should already know these.)

A number is **divisible by 2** if it ends in 0, 2, 4, 6, or 8. Such a number is even.

A number is **divisible by 5** if it ends in 0 or 5. For example, 395 is divisible by 5.

A number is **divisible by 10** if it ends in 0. For example, 56,930 is divisible by 10.

A number is **divisible by 100** if it ends in “00”. For example, 450,000 is divisible by 100.

A number is **divisible by 1000** if it ends in “000”. For example, 450,000 is divisible by 1000.

Example 1. Is $2 \times 3 \times 17$ divisible by 10?

$2 \times 3 \times 17$ is 6×17 . Imagine doing this multiplication in columns. What will be the *last* digit of the answer?

The answer will end in 2, because $6 \times 7 = 42$ ends in 2. Therefore, the number is not divisible by 10.

5. Mark an “x” if the number is divisible by 2, 5, 10, 100, or 1,000.

Divisible by	2	5	10	100	1000
825					
400					
332					

Divisible by	2	5	10	100	1000
600,200					
56,000					
307,995					

6. Answer. In each case, explain why or why not it is so (justify your answer).
- a. Is 6×28 divisible by 5?
- b. Is 3×794 divisible by 10?
- c. Is $3 \times 3 \times 3 \times 3 \times 3$ divisible by 2?
- d. Is $2 \times 15 \times 2 \times 7$ divisible by 4?

Divisibility rules for 3, 6, and 9

A number is divisible by 3 if the sum of its digits is divisible by 3.

Example 2. To check if 93,025 is divisible by 3, add its digits: $9 + 3 + 0 + 2 + 5 = 19$. Since 19 is *not* divisible by 3, neither is 93,025. (In fact, $93,025 \div 3 = 31,008 \text{ R}1$.)

A number is divisible by 6 if it is divisible by both 2 and 3.

Example 3. The number 756 is divisible by 3, because the sum of its digits is 18, which *is* divisible by 3. Also, it is an even number so it's divisible by 2. Therefore, it is also divisible by 6. (In fact, $756 \div 6 = 126$.)

A number is divisible by 9 if the sum of its digits is divisible by 9.

Example 4. To check if 105,642 is divisible by 9, add its digits: $1 + 0 + 5 + 6 + 4 + 2 = 18$. Since 18 *is* divisible by 9, so is 105,642.

7. Tell whether these numbers are divisible by 3.
If yes, divide the number by 3 (long division).

- a. 539
- b. 43,719
- c. 9,032

Tip: In adding the digits, you can *omit* any digits that are divisible by 3 (namely 3, 6, and 9). For example, to check whether 99,378 is divisible by 3, just add $7 + 8 = 15$ and omit 9, 9, and 3. Since 15 is divisible by 3, so is 99,378.

8. Change one of the digits in the number 238,882 so that the number is divisible by 3, but *not* divisible by 2.

9. Tell whether these numbers are divisible by 9.
If yes, divide the number by 9 (long division).

- a. 888
- b. 576
- c. 44,082

10. Mark an “x” if the number is divisible by 2, 3, 5, 6, or 9.

Divisible by	2	3	5	6	9
589					
558					

Divisible by	2	3	5	6	9
495					
3,594					