

Scientific Notation: Small Numbers

When writing very small numbers in scientific notation, the exponent is negative. Recall that negative exponent does not signify a negative number, but a *fraction*: a reciprocal of the corresponding power of ten with a positive exponent. For example, $10^{-4} = \frac{1}{10^4} = \frac{1}{10,000}$. As a decimal, this is 0.0001.

Example 1. To write 0.00034 in scientific notation, we need to use 3.4 as the decimal that is multiplied by a power of ten. (Why?) Now note in which place the digit 3 is: it is in ten-thousandths place, which is the *fourth* digit after the decimal point. This means we use $\frac{1}{10,000} = \frac{1}{10^4} = 10^{-4}$ as the power of ten.

So, $0.00034 = 3.4 \cdot 10^{-4}$.

Example 2. To write $7.64 \cdot 10^{-6}$ in decimal notation, we note that the digit 7 has to be in the place indicated by the power of ten, which means 7 will be in the millionths place. The other digits will follow. The millionths place is the sixth decimal digit after the decimal point. So, $7.64 \cdot 10^{-6} = 0.00000764$.

1. Write the numbers given in scientific notation in decimal notation, and vice versa.

Scientific Notation	Decimal notation	Scientific Notation	Decimal notation
$3 \cdot 10^{-5}$			0.0000002388
	0.0008	$8.2 \cdot 10^{-4}$	
	0.00000203		0.0000000308
$6.108 \cdot 10^{-8}$		$4.539 \cdot 10^{-7}$	

2. Eric said that $7.61 \cdot 10^{-9}$ has a total of nine zeros, like this: 0.00000000761. Is he correct?

3. Compare the numbers, writing $<$ or $>$ in the box. How can you tell which of them is greater, without writing them in decimal notation?

a. $2 \cdot 10^{-7}$ $5 \cdot 10^{-8}$

b. $3 \cdot 10^{-9}$ $3 \cdot 10^{-7}$

c. $7.82 \cdot 10^{-5}$ 0.000075

d. $4 \cdot 10^{-4}$ 0.00046

e. $7 \cdot 10^{-4}$ 0.0065

f. 0.00000078 $2.8 \cdot 10^{-8}$

4. Write in order from smallest to greatest: $5.6 \cdot 10^7$ 10^{-6} 0.0003 10^8 $6 \cdot 10^7$ 0.00002 $9 \cdot 10^{-7}$

5. A 70-kg male body contains approximately 7,000,000,000,000,000,000,000 atoms. Of these, approximately $3.9 \cdot 10^{25}$ are nitrogen atoms, $1.61 \cdot 10^{27}$ are oxygen atoms, $8.03 \cdot 10^{26}$ are carbon atoms, and $4.22 \cdot 10^{27}$ are hydrogen atoms.
- Write the number of atoms in the male 70-kg body using scientific notation.
 - Are there more oxygen or carbon atoms in a human body?

Example 3. To write $77 \cdot 10^{-4}$ correctly in scientific notation, we need to use 7.7 instead of 77. Since changing 77 to 7.7 makes it *ten* times smaller, then 10^{-4} must become ten times larger, which means we multiply it by 10 and it becomes 10^{-3} . So, $77 \cdot 10^{-4} = 7.7 \cdot 10^{-3}$.

6. One gold atom weighs about $326.964 \cdot 10^{-24}$ grams. Write this number in scientific notation correctly.
7. Rewrite the numbers in scientific notation correctly.

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|--------------------------|-------------------------|
| a. $89 \cdot 10^{-5}$ | b. $479 \cdot 10^{-6}$ |
| c. $0.3 \cdot 10^{-4}$ | d. $208 \cdot 10^{-9}$ |
| e. $0.045 \cdot 10^{-8}$ | f. $0.02 \cdot 10^{-6}$ |

A *negative* number written in scientific notation is in the form of $a \cdot 10^n$, where n is an integer, and a is a decimal number so that $-10 < a \leq -1$. For example, $-7.909 \cdot 10^{10}$ becomes $-79,090,000,000$.

8. Fill in the table.

Scientific Notation	Decimal notation	Scientific Notation	Decimal notation
$-4 \cdot 10^5$		$-7 \cdot 10^{-3}$	
	$-59,000,000$	$-2.81 \cdot 10^{-7}$	
$-1.506 \cdot 10^6$			-0.0000098
	$-1,008,200,000$		-0.0000000503

9. In 2022, the public debt of the United States was around 26.82 trillion U.S. dollars. India's public debt in 2022 was about 1,261,000,000,000 USD. Write these numbers in scientific notation, as negative numbers. Which country has more debt?

10. Write the numbers in order from smallest to greatest: $4 \cdot 10^{-4}$ $-4 \cdot 10^4$ $-4 \cdot 10^{-4}$ 0.004 $4 \cdot 10^4$