Laws of Exponents, Part 1

1. **a.** In the expression $2^4\,2^3$, both powers have the same base of 2. See if you can find a way to write this expression in a shorter form, as a single power of 2 (using only one exponent).
Hint: Expand the powers as repeated multiplications..

b. Do the same with $3^2\,3^4$.

c. Do the same with a^3a^9.

2. Are the following statements true? If not, correct them.

a. $2^4\,2^2 = 2^8$

b. $2^3\,2^3 = 4^6$

c. $10^3\,10^2 = 10^5$

3. Expand the powers by writing out the repeated multiplications. Then simplify. Lastly, write the entire expression as a single power of 4.

$$\frac{4^7}{4^5} = \frac{\square \cdot \square \cdot \square \cdot \square \cdot \square \cdot \square \cdot \square}{\square \cdot \square \cdot \square \cdot \square \cdot \square} =$$

4. Simplify the expression, writing it as a single power of 5.

$$\frac{5^5}{5^2} = \frac{\qquad\qquad\qquad\qquad\qquad}{} =$$

5. Using the same technique as above, write the expression $\frac{x^6}{x^2}$ as a single power of x.

6. Sandra believes that $\frac{2^5}{2^4} \cdot 2 = 1$. Is she correct? If not, explain why not.

7. Are the following statements true? Use the table of powers of 3 to help.

Hint 1: Often estimation is sufficient to see that a statement is wrong.

Hint 2: To check the veracity of a division statement, you can also use multiplication.

a. $3^3 + 3^4 = 3^7$

b. $3^3 \cdot 3^4 = 3^7$

c. $\frac{3^6}{3^3} = 3^2$

d. $\frac{3^6}{3^3} = 3^3$

$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$
$3^6 = 729$
$3^7 = 2{,}187$
$3^8 = 6{,}561$

The first law of exponents: product of powers

When we multiply powers with the same base, we are dealing with a repeated multiplication of the same number.

For example, in $5^4 5^6$, we first multiply five by itself 4 times, and then 6 more times.

$$\underbrace{5 \cdot 5 \cdot 5 \cdot 5}_{\text{4 repetitions}} \cdot \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}_{\text{6 repetitions}} = 5^{10} \quad \text{(10 repetitions)}$$

We can simply add the exponents: $5^4 5^6 = 5^{10}$. Five is multiplied by itself ten times.

In general, $a^n a^m = a^{n+m}$ for any rational number a, and for any integer exponents n and m.

8. Write each expression as a single power (with one base and one exponent). You don't have to find the value of the expressions.

a. $4^3 4^{11} =$	**b.** $(-7)^{17}\,(-7)^2 =$	**c.** $\left(-\frac{2}{3}\right)^2\left(-\frac{2}{3}\right)^5 =$

d. $5 \cdot 5 \cdot 5 \cdot 5^8 =$	**e.** $0.3 \cdot 0.3^2 \cdot 0.3^4 =$	**f.** $a^2 \cdot a \cdot a \cdot a^{12} =$	**g.** $x^m x^7 =$

9. Write each expression as a single power. You don't have to find the value of the expressions.

a. $1000 \cdot 10^4 =$	**b.** $8 \cdot 2^5 =$	**c.** $3^6 \cdot 27 =$	**d.** $4^2 \cdot 64 =$

10. Find the value of x.

a. $6^2 6^x 6^9 = 6^{21}$	**b.** $b^x b^9 = b^5 b^6$	**c.** $9 \cdot 3^5 = 27 \cdot 3^x$	**d.** $8 \cdot 2^4 = 32 \cdot 2^x$

Example 1. Rewrite $2^9 7^4 7^2 2^5 \cdot 4$ as a product of powers, using as few exponents as possible.

Here, 2 is multiplied by itself nine times, and also five times, and then, 4 is actually $2 \cdot 2$. In total, 2 is multiplied by itself $9 + 5 + 2 = 16$ times.

And 7 is multiplied by itself, first of all four times, and then also two times — a total of 6 times.

In all, the simplified expression is $2^{16} 7^6$.

Example 2. Simplify $4x^9 y^4 x^2 y^5 x$.

The variable x is multiplied by itself $9 + 2 + 1 = 12$ times. The variable y is multiplied by itself $4 + 5 = 9$ times.

We cannot do anything about the coefficient 4.

In all, the simplified expression is $4x^{12} y^9$.

11. Rewrite each expression as a product of powers, using as few exponents as possible.

a. $10^2 8^3 10^5 8^6$	**b.** $3^3 5^6 3^3 4^3 5^2 \cdot 3$	**c.** $24 \cdot 2^3 3^6 2^5$
d. $25 \cdot 2^4 5^2 3^2 \cdot 9$	**e.** $64 \cdot 100 \cdot 7 \cdot 2^3$	**f.** $81 \cdot 36 \cdot 3 \cdot 2^3$

12. Simplify.

a. $x^7 y^3 x^3 y^9$	**b.** $2a^3 a^6 b^3 b^3 ab$	**c.** $5s \cdot r^4 \cdot s \cdot s^3 \cdot s \cdot 6 \cdot r \cdot t^3$