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Introduction

Math Mammoth Early Geometry is a worktext that covers geometry topics for grade 3. It focuses on these three topics: classifying shapes, area, and perimeter.

In the first three lessons of the book, students sort shapes into categories based on their attributes. They are learning, in particular, about quadrilaterals as a category. Students observe attributes such as the number of sides, presence of right angles, and whether the shape has equal sides or not. They learn about rhombuses, rectangles, and squares, and how a square is *also* a rectangle and a rhombus.

Their work here prepares students to classify quadrilaterals in great detail in grade 5. Students classify figures in grade 4 also, based on angles, and focusing on triangles in particular.

Next, we focus on the area of rectangles and shapes composed of rectangular areas. (How to calculate the area of other common shapes, such as triangles, polygons, and circles, is studied in 6th and 7th grades.)

We start the topic with the lesson *Getting Started with Area*, where students tile shapes with unit squares cut out of paper, thus learning the foundation of how area is measured. From there, the lessons gradually lead to the thought that area of a rectangle can be found by multiplying its side lengths. Students learn about different units for area (both metric and customary) and get hands-on experience on finding the area of rectangles by measuring the sides in inches or centimeters.

Then the lessons deal with the area of compound (rectilinear) shapes, whose area can be found by decomposing the shape into (non-overlapping) rectangles.

In this context, we also study the distributive property “in disguise”, in the lesson *Area of Decomposed Rectangles*. A rectangle is divided into two parts, and its area is found in two different ways: either as the area of the entire rectangle, or by adding the areas of the two parts.

From this area model, we can see that $a \times (b + c)$ is equal to $a \times b + a \times c$. The expression $a \times (b + c)$ is the area of the entire rectangle, with side lengths a and $(b + c)$, and this is equal to the sum of the areas of two rectangles, one with sides a and b , and the other with sides a and c . (The images in the lesson make it more clear.)

The distributive property is studied in its algebraic form in 6th grade, but students will use it again in 4th grade in the context of finding common factors of two whole numbers. For example, they will express the sum $36 + 8$ as $4(9 + 2)$.

In the lesson *Multiplying by Whole Tens* students learn to multiply a single-digit number by a multiple of ten, for example 3×40 or 90×7 . After that, students use this skill to calculate areas of bigger rectangles in the rest of the lessons in the book.

The last major topic in the book is perimeter. Students learn perimeter as the “go-around measure”: the total distance when you go or travel around the shape. Then, several lessons focus on both area and perimeter, to make sure students learn to distinguish between these two concepts. We investigate rectangles that have the same area but different perimeters, and also rectangles that have the same perimeter but different areas.

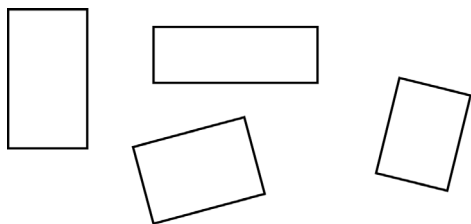
Good Mathematical Practices

- Mathematics has a lot to do with structures and patterns. In the beginning lessons of the book, students begin classifying (sorting) shapes based on their properties. This task of analyzing and sorting shapes continues all through the elementary school, and it is typical of the work that mathematicians do: look for patterns or structures.
- Encourage and require students to include the proper unit for area and for perimeter in their answers. Area is measured in square units of some kind (e.g. square meters, square inches), whereas perimeter is measured in linear units (e.g. meters, inches). Giving a numerical answer only (e.g. “24”) is not totally accurate. Remembering to include the proper unit is part of learning to convey mathematical results in a precise manner.
- While the geometric word problems in this book are still very simple, it is a good practice for students to learn to draw a sketch of the figure in question. With more complex problems in later grade levels, drawing a picture of the shape(s) becomes an essential tool in one’s “mathematical toolbox”.
- The area of a rectangle is found by *multiplying* the side lengths. The area model becomes an important tool in subsequent grade levels in the context of multiplying multi-digit whole numbers, multiplying mixed numbers, and even in multiplying polynomials in algebra.
- The lessons *Polygons* and *Multiplying by Whole Tens* give students opportunities to look for structure in sorting shapes and for numerical patterns. Mathematics has been called the “science of patterns”, so these types of tasks have to do with the essence of mathematics.
- Note that when a problem asks for the area or a perimeter of a figure in centimeters, millimeters, or inches, and the figure does not indicate the side lengths, the student is supposed to use a ruler to measure. By not including that instruction in the problem, we are giving the student opportunity to reason out the need for a ruler as a tool.

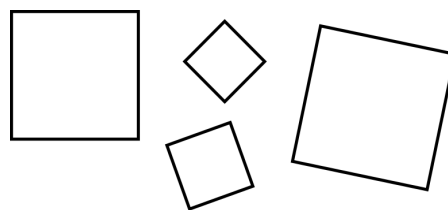
I wish you success in teaching math!

Maria Miller

Some Special Quadrilaterals



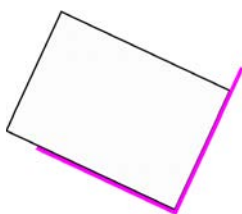
Rectangles are quadrilaterals (they have four sides) with four **right angles**. A right angle is a “straight” corner — like a corner of a book.



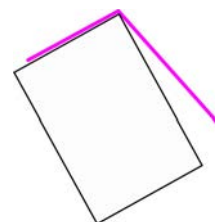
Squares are quadrilaterals with four equal sides (each side has the same length) and four right angles.

You can use the corner of a book to check if an angle is right (if the corner is “straight”). Or, fold a piece of paper twice in half, to get a right-angle checking tool.

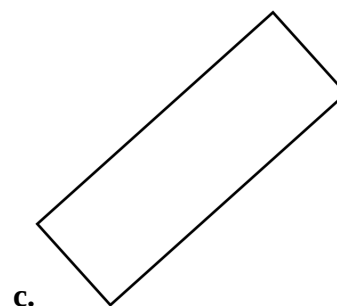
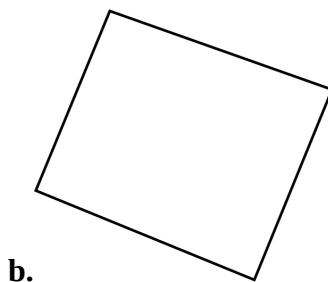
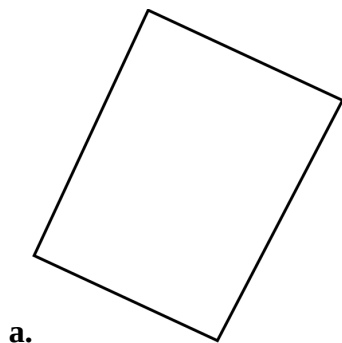
This is a right angle!



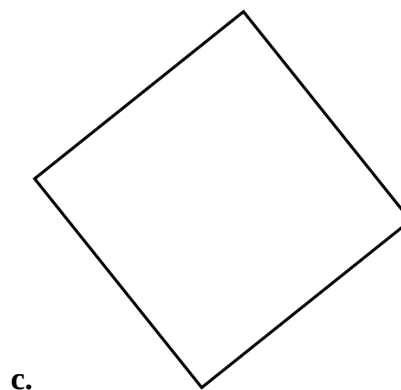
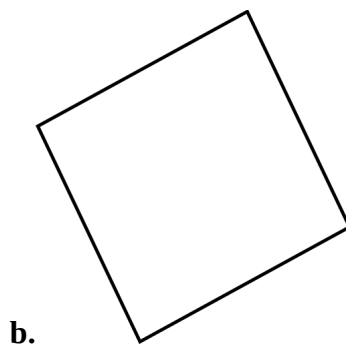
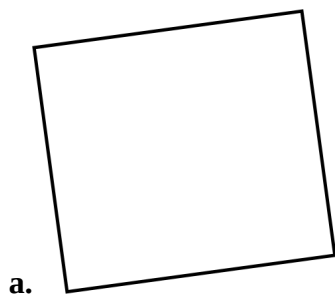
This is *not* a right angle!



1. Are these shapes rectangles or not? Check if the angles are right angles.

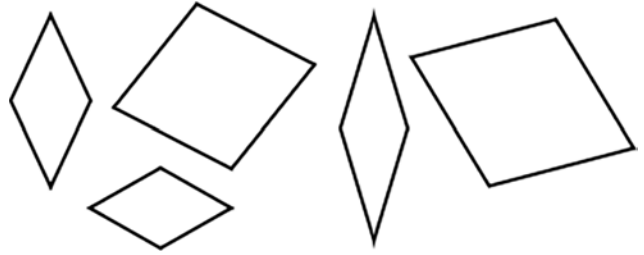


2. Are these shapes squares? Check if the angles are right angles *and* if the sides have the same length (use a ruler).



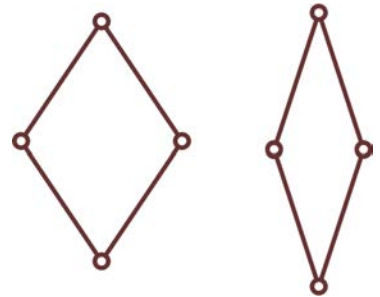
A **rhombus** is a quadrilateral with four equal sides. A rhombus is also called a diamond-shape or a diamond in common language.

The plural of rhombus is **rhombi**.



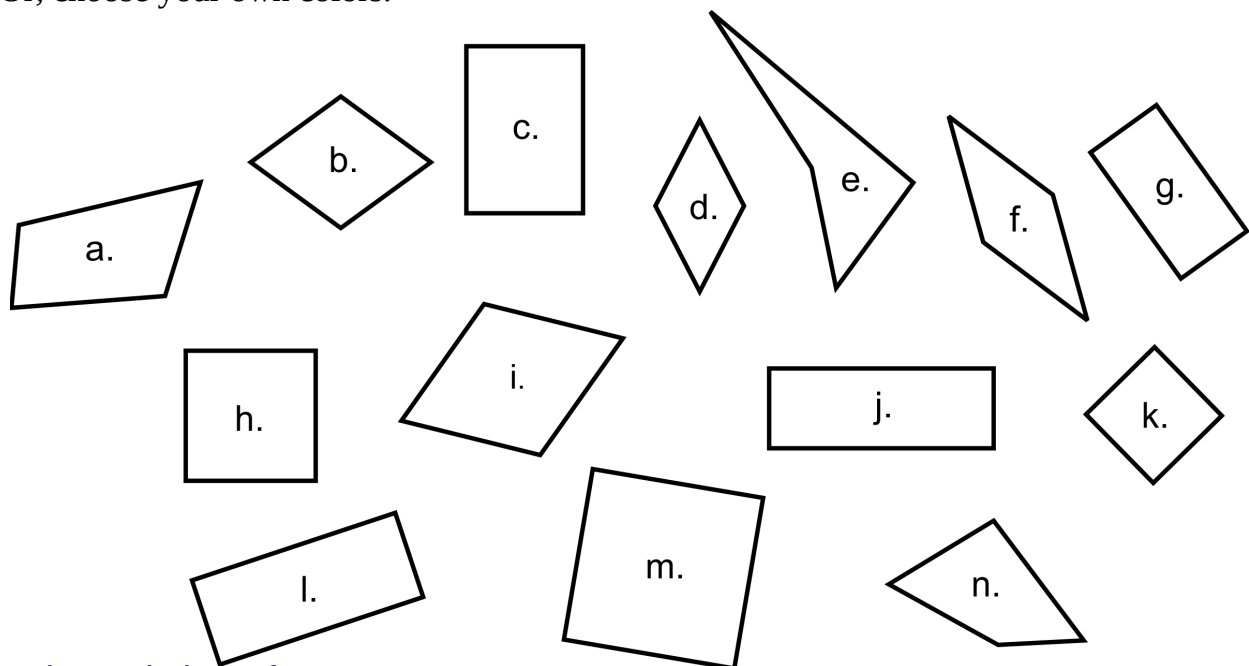
3. You can make a rhombus by taking four popsicle sticks, pencils, matches, or other sticks of the same length.

Arrange the four sticks into a diamond shape. Now, change it slightly to get another rhombus. Make a skinny one, a less skinny one, and so on. You can even make a square!



4. Can the angles of a rhombus be right angles?
If they are, what do we get? Sketch an example.

5. Color all the rectangles green, squares blue, rhombi red, and other quadrilaterals yellow.
Or, choose your own colors.

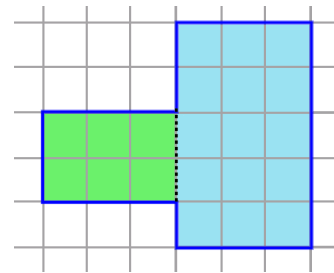


Area of Compound Shapes 1

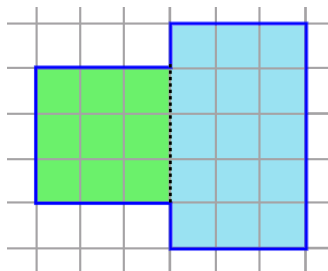
Example: To find the area of this figure, we can divide the shape into two rectangles. We then use two multiplications, and add their results:

$$3 \times 2 + 3 \times 5 = 6 + 15 = 21$$

The area is 21 square units.



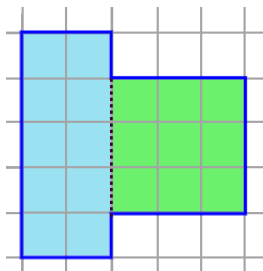
1. Write two multiplications to find the total area.



a.

$$\underline{\quad} \times \underline{\quad} + \underline{\quad} \times \underline{\quad} = \underline{\quad}$$

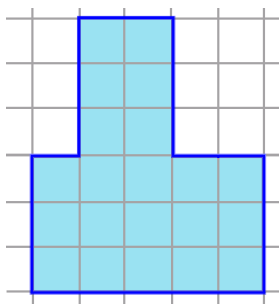
Area = square units



b.

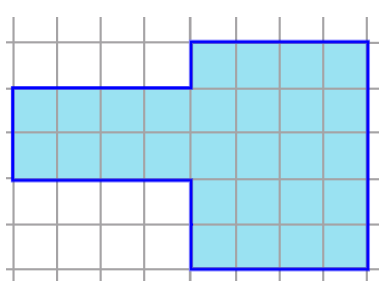
$$\underline{\quad} \times \underline{\quad} + \underline{\quad} \times \underline{\quad} = \underline{\quad}$$

Area = square units



c.

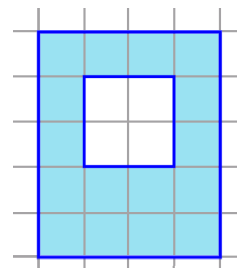
Area = square units



d.

Area = square units

2. Write a number sentence for the shaded area that uses multiplication and *subtraction*.



Area of Decomposed Rectangles

1. This rectangle is divided into two parts.

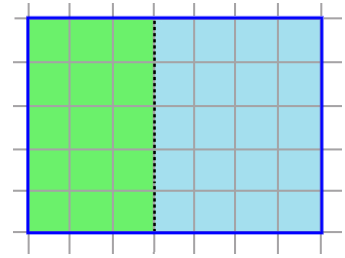
a. What is the area of the smaller part? Write a multiplication.

b. What is the area of the larger part? Write a multiplication.

c. What is the area of the entire rectangle? Write a multiplication.

d. You've written three multiplications. What is same about each one?

e. If we write the length of this rectangle as $3 + 4$, then what would $5 \times (3 + 4)$ signify?

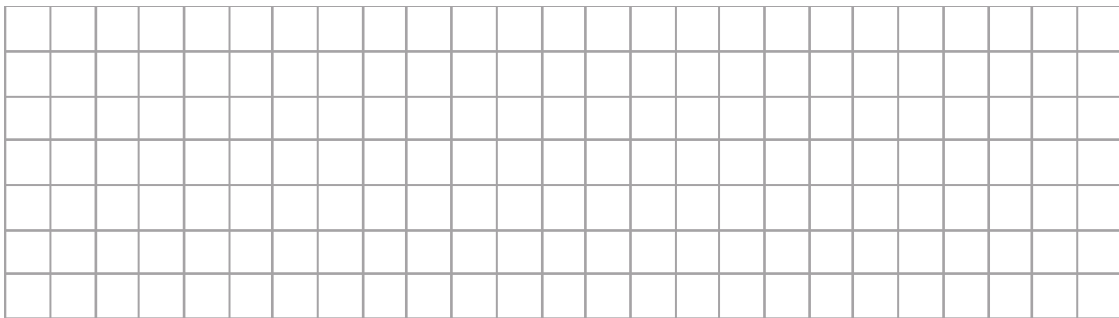


2. a. Draw here a rectangle that is divided into two smaller rectangles in this manner:

The entire rectangle is 4 units high by 12 units long.

The first part will be 4 units high by 3 units long.

The second part is 4 units high by 9 units long.



b. Write a multiplication for the area of the smaller part.

c. Write a multiplication for the area of the larger part.

d. Write a single multiplication for the area of the entire rectangle.

e. What would $4 \times (3 + 9)$ signify in this context?

The **height** of this rectangle is **4** units. Since the rectangle is divided into two parts, we can write its **length** as the sum $2 + 5$.

All in all, the area of the entire rectangle is $4 \times (2 + 5)$
 $= 4 \times 7 = 28$ square units.

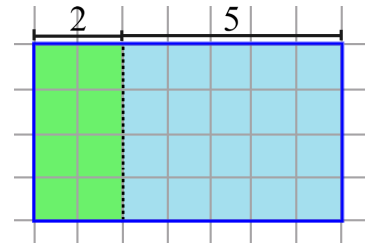
But we can also think of it this way. The area of the first part is 4×2 , or 8, and the area of the larger part is 4×5 , or 20.

Adding those together, the area of the entire rectangle can be written as $4 \times 2 + 4 \times 5$.

Both of those calculations give us the area of the entire rectangle — so they are equal:

$$4 \times (2 + 5) = 4 \times 2 + 4 \times 5$$

area of the whole rectangle
area of the first part
area of the second part

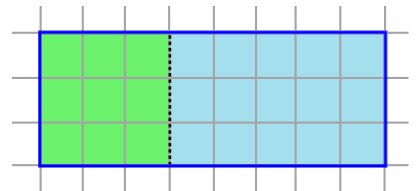


3. Fill in the missing parts to match the expressions (number sentences) for the area.

a.

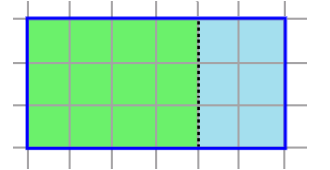
$$\underline{\quad} \times (\underline{\quad} + \underline{\quad}) = \underline{\quad} \times \underline{\quad} + \underline{\quad} \times \underline{\quad}$$

area of the whole rectangle
area of the first part
area of the second part



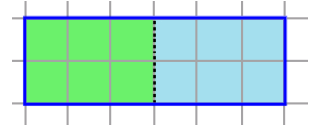
b.

$$\underline{\quad} \times (\underline{\quad} + \underline{\quad}) = \underline{\quad} \times \underline{\quad} + \underline{\quad} \times \underline{\quad}$$



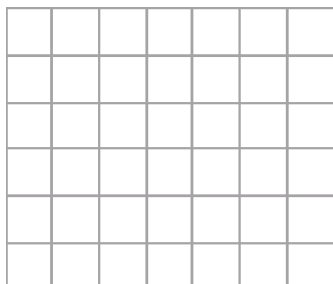
c.

$$\underline{\quad} \times (\underline{\quad} + \underline{\quad}) = \underline{\quad} \times \underline{\quad} + \underline{\quad} \times \underline{\quad}$$

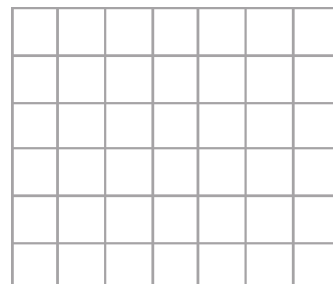


4. Draw a rectangle that matches the given expression (number sentence). Also find its area.

a. $3 \times (2 + 4) =$



b. $5 \times (1 + 4) =$



Same Area, Different Perimeter

1. Draw in the grid rectangles that have an area of 24 square units. Draw as many different ones as you can. Write the side lengths in the table. Lastly, calculate the perimeter of each. One is already given.

Rectangle	Side 1	Side 2	Area	Perimeter
Rectangle 1	2 units	12 units	24 square units	48 units
Rectangle 2			24 square units	
Rectangle 3			24 square units	



2. Jane is making a chicken coop. She was told that there should be 4 square feet of area for each bird.
- If her flock is 12 birds, what should be the area of her coop?
 - What could be the side lengths of the coop? Give at least two possibilities.