

# Division as Repeated Subtraction

Multiplication can be solved by repeated addition. Division is the opposite of multiplication. Division can be solved by **repeated subtraction**.

**Example 1.** Bag 771 apples so there are 3 apples in each bag. How many bags are needed?

You can start by putting 3 apples into one bag, which leaves you 768 apples. Then for each bag, subtract 3 apples, and keep counting the bags you use, until you have no apples.

771	- 3	- 3	- 3	- 3	- 3	- 3	... keep subtracting!
	1 bag	1 bag	1 bag	1 bag	1 bag	1 bag	... keep counting bags!

It just takes quite a long time, doesn't it? Instead you can take a 'shortcut' and initially subtract 300 apples (taking 100 bags) or some other big multiple of 3.

771	- 300	- 300	- 30	...
	100 bags	100 bags	10 bags	...

Let's figure it out and keep count of the bags as we subtract (put into bags) the apples.

Look at the two calculations on the right.

So, all total we needed  $200 + 50 + 7 = \underline{257 \text{ bags}}$  to bag all the apples. It went evenly; no apples were left over!

In other words,  $771 \div 3 = 257$ .

**Method 1 - slower**

<u>Apples</u>	<u>Bags</u>
771	
<u>- 300</u>	100 bags
471	
<u>- 300</u>	100 bags
171	
<u>- 30</u>	10 bags
141	
<u>- 30</u>	10 bags
111	
<u>- 30</u>	10 bags
81	
<u>- 30</u>	10 bags
51	
<u>- 30</u>	10 bags
21	
<u>- 21</u>	7 bags
0	

**Method 2 - quicker**

<u>Apples</u>	<u>Bags</u>
771	
<u>- 600</u>	200 bags
171	
<u>- 150</u>	50 bags
21	
<u>- 21</u>	7 bags
0	

**Example 2.** You have 567 apples. If you put 3 apples into one bag, how many bags will you need?

Look at the chart on the right.

The total needed is  $100 + 80 + 9 = \underline{189 \text{ bags}}$  to bag all of the apples with no apples left over. In other words,  $567 \div 3 = 189$ .

<u>Apples</u>	<u>Bags</u>
567	
<u>- 300</u>	100 bags
267	
<u>- 240</u>	80 bags
27	
<u>- 27</u>	9 bags
0	

**Example 3.** You have 646 apples. If you put 8 apples into one bag, how many bags will you need?

The total we need \_\_\_\_\_ + \_\_\_\_ = \_\_\_\_\_ **bags** for the apples and there are \_\_\_\_\_ apples left over.

In other words,  $686 \div 8 = \underline{\hspace{2cm}}$ , **R 6**.

<b>Apples</b>	<b>Bags</b>
686	
- _____	_____
46	_____
- _____	_____
6	

**Example 4.** It won't matter even if you do the subtracting in smaller steps. Compare the two ways to do the division  $795 \div 3$  by subtracting repeatedly.

Dividend	Quotient	Dividend	Quotient
795 - 300 <hr/> 495 - 300 <hr/> 195 - 30 <hr/> 165 - 30 <hr/> 135 - 30 <hr/> 105	100  100  10  10  10	795 - 600 <hr/> 195 - 180 <hr/> 15 - 15 <hr/> 0	200  60  5

The total of the quotient is

\_\_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_\_,

and the division is even.

In other words,  $795 \div 3 = \underline{\hspace{2cm}}$ .

Let's compare the repeated subtraction shown before and the conventional long division algorithm. The steps are the same, just written out differently.

For clarity's sake, we will initially write out the subtracted numbers with all the zeros included. Also, for clarity and for easy comparison, we will write the parts of the quotient above each other.

<p>Continued subtraction</p> <table border="0" style="width: 100%;"> <tr> <td style="text-align: right;"><b>Dividend</b> <small>(the apples)</small></td> <td style="text-align: right;"><b>Quotient</b> <small>(the bags)</small></td> </tr> <tr> <td style="text-align: right;">789</td> <td></td> </tr> <tr> <td style="text-align: right;">- 600</td> <td style="text-align: right;">200</td> </tr> <tr> <td style="text-align: right;"><hr/>189</td> <td></td> </tr> <tr> <td style="text-align: right;">- 180</td> <td style="text-align: right;">60</td> </tr> <tr> <td style="text-align: right;"><hr/>9</td> <td style="text-align: right;">3</td> </tr> <tr> <td style="text-align: right;">- 9</td> <td></td> </tr> <tr> <td style="text-align: right;"><hr/>0</td> <td style="text-align: right;">263</td> </tr> </table>	<b>Dividend</b> <small>(the apples)</small>	<b>Quotient</b> <small>(the bags)</small>	789		- 600	200	<hr/> 189		- 180	60	<hr/> 9	3	- 9		<hr/> 0	263	<table border="0" style="margin: auto;"> <tr> <td style="text-align: right;">200</td> </tr> <tr> <td style="text-align: right;">3 <math>\overline{)789}</math></td> </tr> <tr> <td style="text-align: right;">- 600</td> </tr> <tr> <td style="text-align: right;"><hr/>189</td> </tr> </table> <p><b>Hundreds.</b> 700 <math>\div</math> 3 = 200 R 100. Subtract to find the remainder. (100). Then add the 8 tens to it.</p>	200	3 $\overline{)789}$	- 600	<hr/> 189	<table border="0" style="margin: auto;"> <tr> <td style="text-align: right;">60</td> </tr> <tr> <td style="text-align: right;">200</td> </tr> <tr> <td style="text-align: right;">3 <math>\overline{)789}</math></td> </tr> <tr> <td style="text-align: right;">- 600</td> </tr> <tr> <td style="text-align: right;"><hr/>189</td> </tr> <tr> <td style="text-align: right;">- 180</td> </tr> <tr> <td style="text-align: right;"><hr/>09</td> </tr> </table> <p><b>Tens.</b> 180 <math>\div</math> 3 = 60 or 6 tens. This was exact; no remainder.</p>	60	200	3 $\overline{)789}$	- 600	<hr/> 189	- 180	<hr/> 09	<table border="0" style="margin: auto;"> <tr> <td style="text-align: right;">3</td> </tr> <tr> <td style="text-align: right;">60</td> </tr> <tr> <td style="text-align: right;">200</td> </tr> <tr> <td style="text-align: right;">3 <math>\overline{)789}</math></td> </tr> <tr> <td style="text-align: right;">- 600</td> </tr> <tr> <td style="text-align: right;"><hr/>189</td> </tr> <tr> <td style="text-align: right;">- 180</td> </tr> <tr> <td style="text-align: right;"><hr/>09</td> </tr> <tr> <td style="text-align: right;">- 9</td> </tr> <tr> <td style="text-align: right;"><hr/>0</td> </tr> </table> <p><b>Ones.</b> 9 <math>\div</math> 3 is 3. The final answer is 263.</p>	3	60	200	3 $\overline{)789}$	- 600	<hr/> 189	- 180	<hr/> 09	- 9	<hr/> 0
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One more example:

Continued subtraction		Hundreds	Tens	Ones
<b>Dividend</b> (the apples)	<b>Quotient</b> (the bags)			
635		<u>100</u>	<u>120</u>	<u>127</u>
- 500	100	5 ) 635	5 ) 635	5 ) 635
<u>135</u>		- 500	- 500	- 500
- 100	20	<u>135</u>	<u>135</u>	<u>135</u>
<u>35</u>	7		- 100	- 100
- 35	127		<u>35</u>	<u>35</u>
<u>0</u>				- 35
				<u>0</u>

1. Bag fruits. Also solve the problems using long division, and compare the methods.

<p><b>a. Bag 610 apples; 5 apples in each bag.</b></p> <table> <tr> <td><b>Apples</b></td> <td><b>Bags</b></td> <td></td> </tr> <tr> <td>610</td> <td></td> <td>5 ) 615</td> </tr> <tr> <td>- 500</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>110</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 100</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>10</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 10</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>0</u></td> <td></td> <td></td> </tr> </table>	<b>Apples</b>	<b>Bags</b>		610		5 ) 615	- 500	<input type="text"/>		<u>110</u>	<input type="text"/>		- 100	<input type="text"/>		<u>10</u>	<input type="text"/>		- 10	<input type="text"/>		<u>0</u>			<p><b>b. Bag 852 kiwis; 3 kiwis in each bag.</b></p> <table> <tr> <td><b>Kiwis</b></td> <td><b>Bags</b></td> <td></td> </tr> <tr> <td>852</td> <td></td> <td>3 ) _____</td> </tr> <tr> <td>- _____</td> <td>200</td> <td></td> </tr> <tr> <td><u>12</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 12</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>0</u></td> <td></td> <td></td> </tr> </table>	<b>Kiwis</b>	<b>Bags</b>		852		3 ) _____	- _____	200		<u>12</u>	<input type="text"/>		- 12	<input type="text"/>		<u>0</u>								
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<p><b>c. Bag 246 grapefruits; 2 grapefruits in each bag.</b></p> <table> <tr> <td><b>Grapefruits</b></td> <td><b>Bags</b></td> <td></td> </tr> <tr> <td>246</td> <td></td> <td>2 ) _____</td> </tr> <tr> <td>- _____</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>46</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 40</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>6</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 6</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>0</u></td> <td></td> <td></td> </tr> </table>	<b>Grapefruits</b>	<b>Bags</b>		246		2 ) _____	- _____	<input type="text"/>		<u>46</u>	<input type="text"/>		- 40	<input type="text"/>		<u>6</u>	<input type="text"/>		- 6	<input type="text"/>		<u>0</u>			<p><b>d. Bag 952 plums; 4 plums in each bag.</b></p> <table> <tr> <td><b>Plums</b></td> <td><b>Bags</b></td> <td></td> </tr> <tr> <td>952</td> <td></td> <td>4 ) _____</td> </tr> <tr> <td>- _____</td> <td>200</td> <td></td> </tr> <tr> <td><u>52</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 40</td> <td>30</td> <td></td> </tr> <tr> <td><u>12</u></td> <td><input type="text"/></td> <td></td> </tr> <tr> <td>- 12</td> <td><input type="text"/></td> <td></td> </tr> <tr> <td><u>0</u></td> <td></td> <td></td> </tr> </table>	<b>Plums</b>	<b>Bags</b>		952		4 ) _____	- _____	200		<u>52</u>	<input type="text"/>		- 40	30		<u>12</u>	<input type="text"/>		- 12	<input type="text"/>		<u>0</u>		
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