Contents

Chapter 9: Geometry

Chapter 10: Statistics

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The Sieve of Eratosthenes and Prime Factorisation

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is 1 · 13. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as 3 · 5. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: 2 · 11, 3 · 11, 4 · 11, 5 · 11, 6 · 11, 7 · 11, 8 · 11 and 9 · 11. The multiples of 11 that have not been crossed out are 10 · 11 and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are 2 · 13, 3 · 13, ..., 7 · 13, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in 4th and 5th grades... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________	**b.** 60 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________
c. 84 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________	**d.** 97 Check 1 2 3 4 5 6 7 8 9 10 factors: ______________________________

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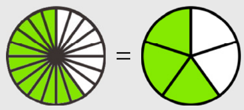
Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the "same amount of pie" as it was before. It is just cut differently.



Every four slices have been joined together.

$$\frac{12}{20} \overset{\div 4}{=} \frac{3}{5}$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →

Notice that here, the 4's that were cancelled out do ***not*** get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5's, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Leah simplified various fractions like you see below. She did not get them right though. Explain to her what she is doing wrong.

$\frac{24}{84} = \frac{20}{80} = \frac{1}{4}$ $\quad$ $\frac{27}{60} = \frac{7}{40}$ $\quad$ $\frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$

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The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write 36 = 1 · 36, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- 36 is also divisible by 2. We write 36 = 2 · 18, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write 36 = 3 · 12, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write 36 = 4 · 9, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write 36 = 6 · 6, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

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Revision: Multiplying Fractions 1

The **shortcut for multiplying fractions** is:

- Multiply the numerators.
- Multiply the denominators.

$$\frac{6}{7} \cdot \frac{5}{2} \cdot \frac{1}{3} = \frac{6 \cdot 5 \cdot 1}{7 \cdot 2 \cdot 3} = \frac{30}{42} = \frac{5}{7}$$

To **multiply mixed numbers**, *first* write them as fractions, then multiply.

$$2\,\frac{1}{3} \cdot 1\,\frac{1}{10} = \frac{7}{3} \cdot \frac{11}{10} = \frac{7 \cdot 11}{3 \cdot 10} = \frac{77}{30} = 2\,\frac{17}{30}$$

If one of the factors is a whole number, write it as a fraction with a denominator of 1.

$$6 \cdot \frac{11}{12} = \frac{6}{1} \cdot \frac{11}{12} = \frac{66}{12} = \frac{11}{2} = 5\,\frac{1}{2}$$

1. Multiply. Give your answer in lowest terms, and as a mixed number, if applicable.

a. $5 \cdot \frac{7}{8}$	**b.** $\frac{2}{7} \cdot \frac{5}{6}$
c. $\frac{9}{10} \cdot \frac{6}{7} \cdot \frac{1}{2}$	**d.** $1\,\frac{1}{3} \cdot 2\,\frac{2}{3}$
e. $\frac{1}{10} \cdot 3\,\frac{1}{5}$	**f.** $2\,\frac{5}{6} \cdot 10 \cdot \frac{1}{2}$

2. Find the area of a square with sides 1 ¼ units. Use fractions.

3. A biscuit recipe calls for 1½ cups of buttermilk, and Mary plans to make the recipe one and a half times, three times a week, in order to sell biscuits. How much buttermilk does she need in a week?

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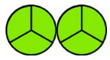
Dividing Fractions: Reciprocal Numbers

One interpretation of division is **measurement division**, where we think: *How many times does one number go into another?* For example, to solve how many times 11 fits into 189, we divide 187 ÷ 11 = 17.

(The other interpretation is equal sharing; we will come to that later.)

Let's apply that to fractions. How many times does  go into  ?

We can solve this just by looking at the pictures: three times. We can write the division: $2 \div \frac{2}{3} = 3$.

To check the division, we multiply: $3 \cdot \frac{2}{3} = \frac{6}{3} = 2$. Since we got the original dividend, it checks.

We can use measurement division to check whether an answer to a division is reasonable.

For example, if I told you that $7 \div 1\frac{2}{3}$ equals $14\frac{1}{3}$, you can immediately see it doesn't make sense:

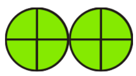
1 2/3 surely does not fit into 7 that many times. Maybe three to four times, but not 14!

You could also multiply to see that: 14-*and-something* times 1-*and-something* is way more than 14, and closer to 28 than to 14, instead of 7.

1. Find the answers that are unreasonable without actually dividing.

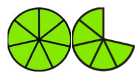
a. $\frac{4}{5} \div 6 = \frac{2}{15}$ **b.** $2\frac{3}{4} \div \frac{1}{4} = \frac{7}{12}$ **c.** $\frac{7}{9} \div 2 = \frac{7}{18}$ **d.** $8 \div 2\frac{1}{3} = 18\frac{1}{3}$ **e.** $5\frac{1}{4} \div 6\frac{1}{2} = 3\frac{1}{8}$

2. Solve with the help of the visual model, checking how many times the given fraction fits into the other number. Then write a division. Lastly, write a multiplication that checks your division.

a. How many times does  go into  ?

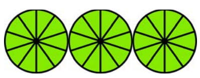
$2 \div \frac{3}{4} =$

Check: _____ $\cdot \frac{3}{4} =$

b. How many times does  go into  ?

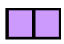
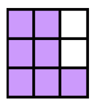
____ $\frac{\_\_}{\_\_} \div \frac{\_\_}{\_\_} =$

Check:

c. How many times does  go into  ?

$3 \div \frac{\_\_}{\_\_} =$

Check:

d. How many times does  go into  ?

$\frac{\_\_}{\_\_} \div \frac{\_\_}{\_\_} =$

Check:

3. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$5 \div \frac{1}{4}$ ↓ ↓ $5 \cdot 4 = 20$

$3 \div \frac{1}{8}$ ↓ ↓ $3 \cdot 8 = 24$

$9 \div \frac{1}{7}$ ↓ ↓ $9 \cdot 7 = 63$

Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem 5 ÷ (1/4). Since 1/4 goes into 1 exactly four times, it must go into 5 exactly 5 · 4 = 20 times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \cdot 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \cdot \frac{n}{m} = \frac{n \cdot m}{m \cdot n} = \frac{m \cdot n}{m \cdot n} = 1$.

To find the reciprocal of a mixed number or a whole number, first write it as a fraction, then “flip” it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$. And since $28 = \frac{28}{1}$, its reciprocal number is $\frac{1}{28}$.

4. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\frac{7}{8}$	**d.** 32	**e.** $2\frac{1}{8}$
$\frac{5}{8} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$32 \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$

5. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**c.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**d.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$	**e.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$

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Coordinate Grid

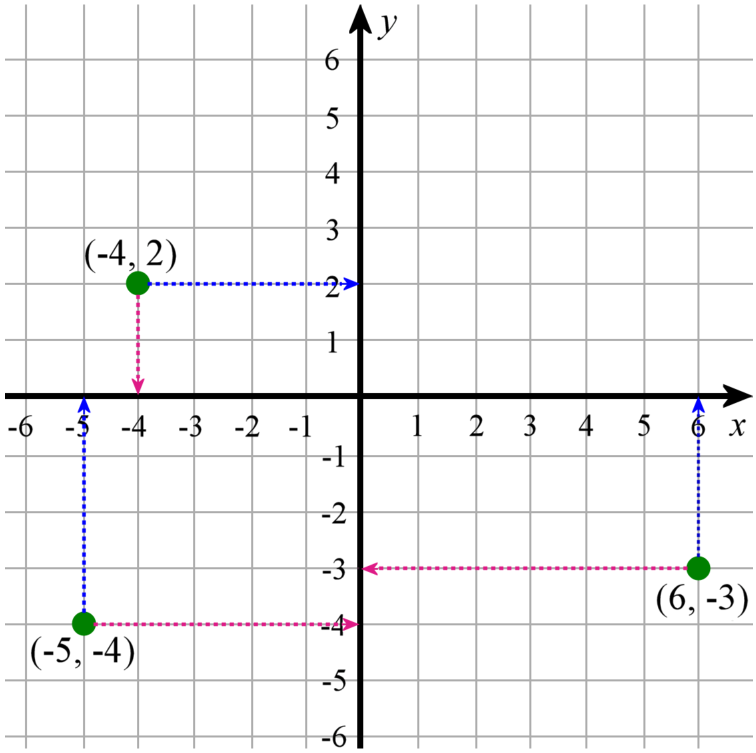
This is the *coordinate grid* or *coordinate plane*. We have extended the *x*-axis and the *y*-axis to include negative numbers now. The axes cross each other at the *origin*, or the point (0, 0).

The axes divide the coordinate plane into four parts, called *quadrants*. Previously, you have worked in only the so-called first quadrant, but now we will use all four quadrants.

The coordinates of a point are found in the same manner as before. Draw a vertical line (either up or down) from the point towards the *x*-axis. Where this line crosses the *x*-axis tells you the point's *x*-coordinate.

Similarly, draw a horizontal line (either right or left) from the point towards the *y*-axis. Where this line crosses the y-axis tells you the point's *y*-coordinate.

We list first the point's *x*-coordinate and then the *y*-coordinate. Look at the examples in the picture.



1. Write the *x*- and *y*-coordinates of the points.

A (____ , ____)

B (____ , ____)

C (____ , ____)

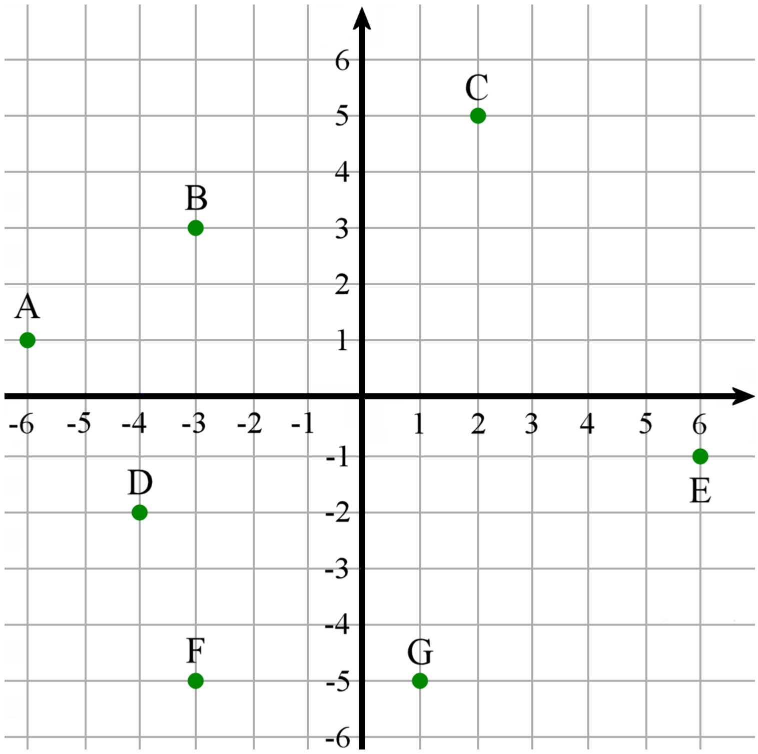
D (____ , ____)

E (____ , ____)

F (____ , ____)

G (____ , ____)

Self-check: Add the *x*-coordinates of all points. You should get −7.



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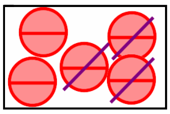
Subtracting a Negative Integer

We have already looked at such subtractions as 3 − 5 or −2 − 8, which you can think of as number line jumps. But what about **subtracting a negative integer?** What is 5 − (−4)? Or (−5) − (−3)?

Let's look at this kind of expression with a "double negative" in several different ways.

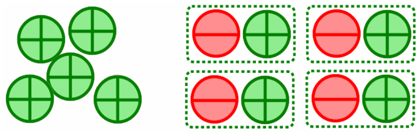
1. Subtraction as "taking away":

We can model subtracting a negative number using counters. (−5) − (−3) means we start with 5 negative counters, and then we *take away* 3 negative counters. That leaves 2 negatives, or −2.



5 − (−4) cannot easily be modelled that way, because it is hard to take away 4 negative counters when we do not have any negative counters to start with. But you *could* do it this way:

Start out with 5 positives. Then *add* four positive-negative pairs, which is just adding zero! Now you can take away four negatives. You are left with nine positives.



Start out with 5.

Add four positive-negative pairs, which amount to zero.

Lastly, cross out four negatives. You are left with nine positives.

2. Subtracting a negative number as a number line jump:

5 − (−4) is like standing at 5 on the number line, and getting ready to subtract, or go to the left. But, since there is a minus sign in front of the 4, it "turns you around" to face the positive direction (to the right), and you take 4 steps to the right instead. So, 5 − (−4) = 5 + 4 = 9.

(−5) − (−3) is like standing at −5, ready to go to the left, but the minus sign in front of 3 turns you "about face," and you take 3 steps to the right instead. You end up at −2.

3. Subtraction as a difference/distance:

To find the difference or distance between 76 and 329, subtract 329 − 76 = 253 (the smaller-valued number from the bigger-valued one). If you subtract the numbers the other way, 76 − 329, the answer is −253.

By the same analogy, we can think of 5 − (−4) as meaning the difference or distance between 5 and −4. From the number line we can see the distance is **9**.

(−5) − (−3) *could* be the distance between −5 and −3, except it has the larger number, −3, subtracted from the smaller number, −5.

If we turn them around, (−3) − (−5) would give us the distance between those two numbers, which is 2. Then, (−5) − (−3) would be the opposite of that, or −2.

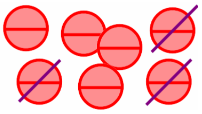
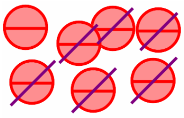
Two negatives make a positive!

You have probably already noticed that, any way you look at it, we can, in effect, replace those two minuses in the middle with a + sign. In other words, 5 − (−4) has the same answer as 5 + 4. And (−5) − (−3) has the same answer as −5 + 3. It may look a bit strange, but it works out really well.

5 − (−4)
5 + 4 = 9

(−5) − (−3)
(−5) + 3 = −2

1. Write a subtraction sentence to match the pictures.

a. 	**b.** 

2. Write an addition or subtraction sentence to match the number line movements.

a. You are at −2. You jump 6 steps to the left.

b. You are at −2. You get ready to jump 6 steps to the left,
but turn around at the last minute and jump 6 steps to the right instead.

3. Find the distance between the two numbers. Then, write a matching subtraction sentence. To get a positive distance, remember to *subtract the* <u>*smaller*</u> *number from the* <u>*bigger*</u> *number*.

a. The distance between 3 and −7 is _______. Subtraction: _______ − _______ = _______	**b.** The distance between −3 and −9 is _______. Subtraction: _______ − _______ = _______
c. The distance between −2 and 10 is _______. Subtraction: _______ − _______ = _______	**d.** The distance between −11 and −20 is _______. Subtraction: _______ − _______ = _______

4. Solve. Remember the shortcut: you can change each double minus "− −" into a plus sign.

a. −8 − (−4) =	**b.** −1 − (−5) =	**c.** 12 − (−15) =
8 − (−4) =	1 − (−5) =	−12 + 15 =
−8 + (−4) =	−1 − 5 =	−12 − 15 =
8 + (−4) =	1 − 5 =	12 + (−15) =

5. Connect with a line the expressions that are equal (have the same value).

a.		**b.**	
10 − (−3)	10 − 3	−9 + 2	−9 + (−2)
10 + (−3)	10 + 3	−9 − 2	−9 − (−2)

6. Write an integer addition or subtraction to describe the situations.

a. A roller coaster begins at 27 m above ground level.
Then it descends 32 metres.

b. Matt has $25. He wants to buy a bicycle from his friend that costs $40.
How much will he owe his friend?

Solve −1 + (−2) − (−3) − 4.

Puzzle Corner

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Graphing

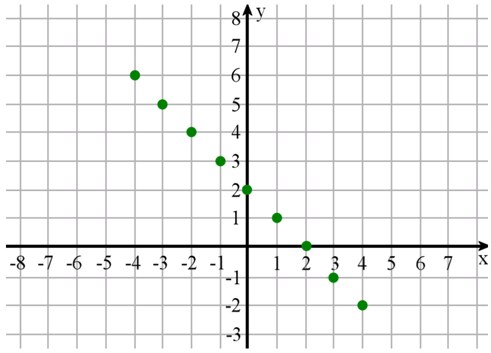
Remember? When an equation has two variables, there are many values of x and y that make that equation true.

Example. Note the equation $y = 2 - x$. If $x = 0$, then we can <u>calculate</u> the value of y using the equation: $y = 2 - 0 = 2$.

So, when $x = 0$ and $y = 2$, that equation is true. We can plot the number pair (0, 2) on the coordinate grid.

Some of the other (x, y) values that make the equation true are listed below, and they are plotted on the right.

x	−4	−3	−2	−1	0	1	2	3	4
y	6	5	4	3	2	1	0	−1	−2

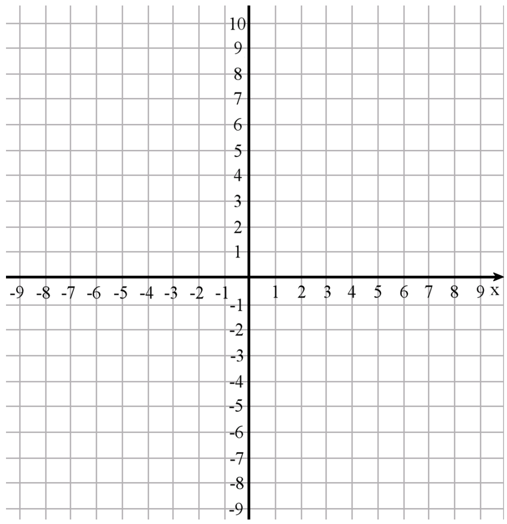


1. Plot the points from the equations. Graph both (b) and (c) in the same grid.

a. $y = x + 4$

x	−9	−8	−7	−6	−5	−4	−3	−2
y								

x	−1	0	1	2	3	4	5	6
y								



b. $y = 6 - x$

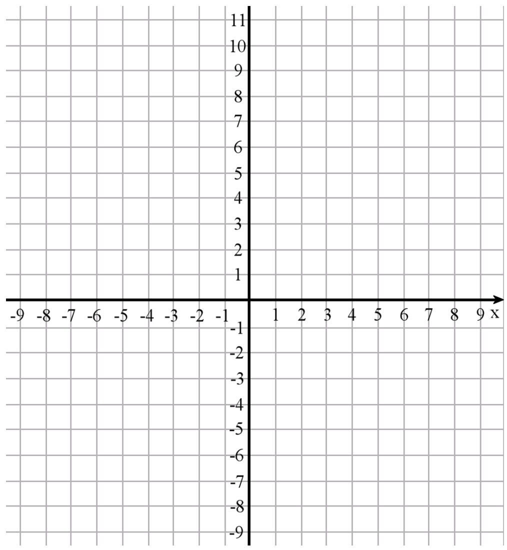
x	−3	−2	−1	0	1	2	3
y							

x	4	5	6	7	8	9
y						

c. $y = x - 2$

x	−5	−4	−3	−2	−1	0	1	2
y								

x	3	4	5	6	7	8	9
y							



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Polygons in the Coordinate Grid

Here is a neat way to **find the area of any polygon whose vertices are points in a grid.**

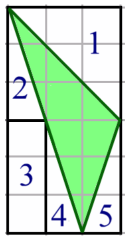
(1) Draw a rectangle around the polygon.

(2) Divide the area between the polygon and the rectangle into triangles and rectangles.

(3) Calculate those areas.

(4) <u>Subtract</u> the calculated areas from the total area of the large rectangle to find the area of the polygon.

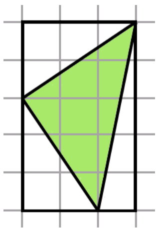
Example. To find the area of the coloured triangle, we draw a rectangle around it that is 3 units by 6 units. Then we find the areas marked with 1, 2, 3, 4, and 5:



1: a triangle; $3 \cdot 3 \div 2 = 4.5$ square units
2: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
3: a rectangle; $1 \cdot 3 = 3$ square units
4: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
5: a triangle; $1 \cdot 3 \div 2 = 1.5$ square units
The total for the shapes 1, 2, 3, 4, and 5 is 12 square units.

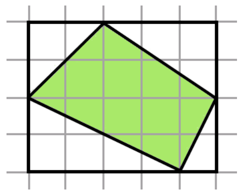
Therefore, the area of the coloured triangle is 18 square units − 12 square units = 6 square units.

1. Find the areas of the shaded figures.



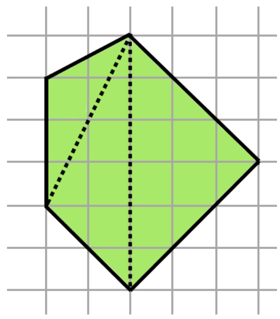
a.

b.



2. This figure is called a ________________________.

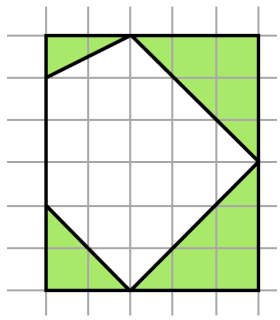
Calculate its area using the three triangles.
For each triangle, use the *vertical* side as the base.



3. Let’s use another way of calculating the area of the same figure.

1. Calculate the area of the rectangle that encloses the figure.
2. Calculate the areas of the four shaded triangles.
3. Subtract.

Verify that you get the same result as in exercise #1.



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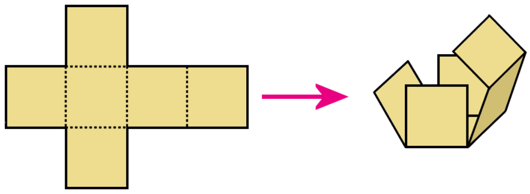
Nets and Surface Area 1

This picture shows a flat figure, called a **net**, that can be folded up to form a solid, in this case a cube.

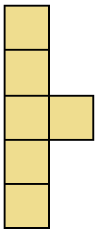
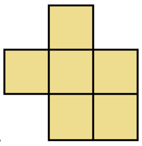
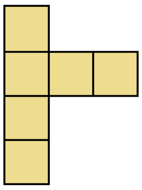
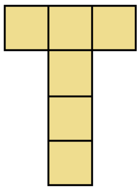
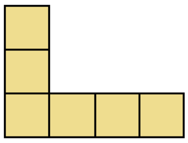
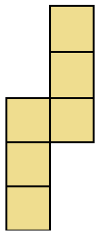


Each face of a cube is a square. If we find the total area of its faces, we will have found the **surface area** of the cube.

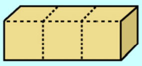
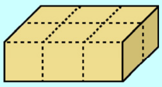
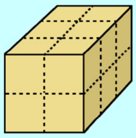
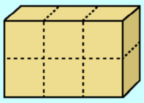
Let’s say that each edge of this cube measures 2 cm. Then one face would have an area of $2 \text{ cm} \cdot 2 \text{ cm} = 4 \text{ cm}^2$, and the total surface area of the six faces of the cube would be $6 \cdot 4 \text{ cm}^2 = 24 \text{ cm}^2$.

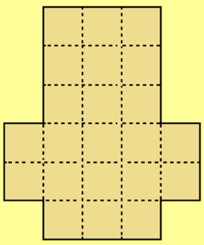
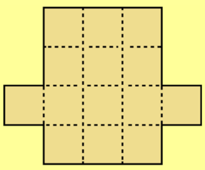
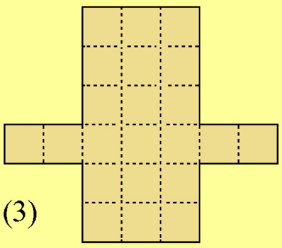
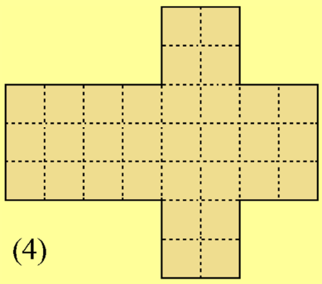
What is its volume? Remember, **volume** has to do with how much space a figure takes up, and not with “flat” area. Volume is measured in *cubic* units, whereas area is measured in *square* units. The volume of this cube is $2 \text{ cm} \cdot 2 \text{ cm} \cdot 2 \text{ cm} = (2 \text{ cm})^3 = 8 \text{ cm}^3$.

1. Which of these patterns are nets of a cube? In other words, which ones can be folded into a cube? You can copy the patterns on paper, cut them out and fold them.

a. 
b. 
c. 
d. 
e. 
f. 

2. Match each rectangular prism (a), (b), (c) and (d) with the correct net (1), (2), (3) and (4). Again, if you would like, you can copy the nets onto paper, cut them out, and fold them.

a. 
b. 
c. 
d. 

(1) 
(2) 
(3) 
(4) 

3. Find the surface area (A) and volume (V) of each rectangular prism in problem #2 if the edges of the little cubes are 1 cm long.

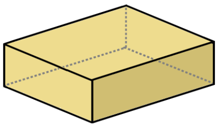
a. A = ________ cm^2 **b.** A =________ cm^2 **c.** A =________ cm^2 **d.** A =________ cm^2

V = ________ cm^3 V = ________ cm^3 V = ________ cm^3 V = ________ cm^3

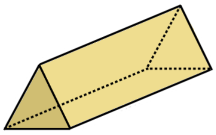
A **prism** has two identical polygons as its top and bottom faces. These polygons are called the *bases* of the prism. The bases are connected with faces that are parallelograms (and often rectangles).

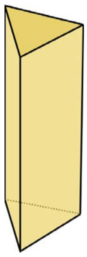
Prisms are named <u>after the polygon used as the bases</u>.

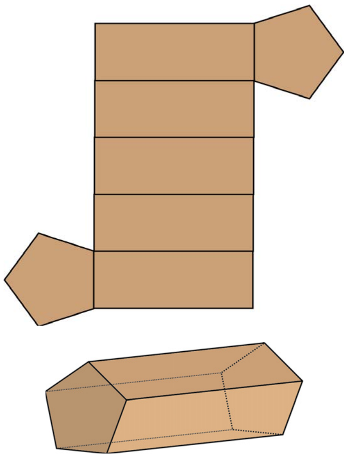
A rectangular prism.
The bases are rectangles.



Two **triangular prisms**.
One is lying down, where the base is facing you.
The other is "standing up".



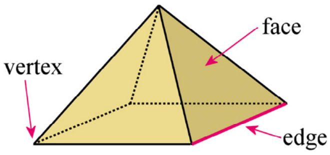


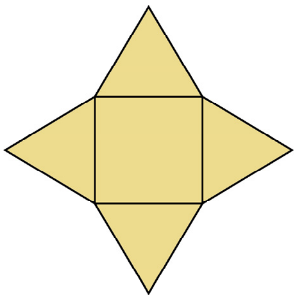


A pentagonal prism and its net. The bases are pentagons. Again, the base is not "on the bottom" but facing you.

A **pyramid** has a polygon as its bottom face (the base), and triangles as other faces.

Pyramids are named after the polygon at the base: a triangular pyramid, square pyramid, rectangular pyramid, pentagonal pyramid, and so on.

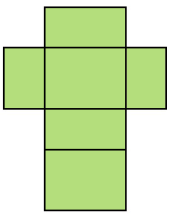


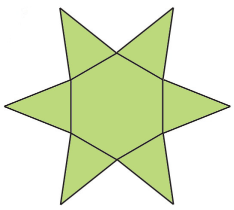


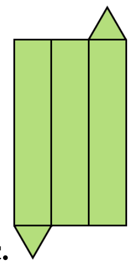
A **square pyramid** and its net.

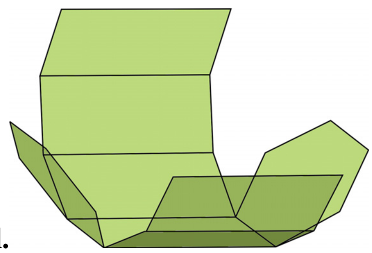
See interactive solids and their nets at the link below:
https://www.mathsisfun.com/geometry/polyhedron-models.html

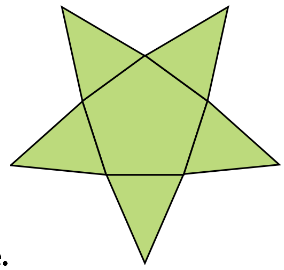
4. Name the solid that can be built from each net.

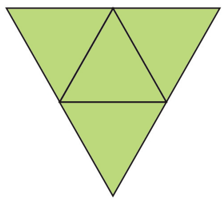
a. 

b. 

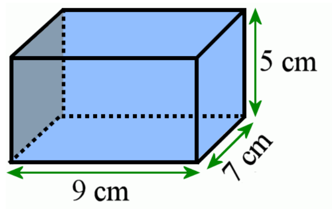
c. 

d. 

e. 

f. 

5. Which expression, (1), (2), or (3), can be used to calculate the surface area of this prism correctly? (You do *not* have to actually calculate the surface area.)



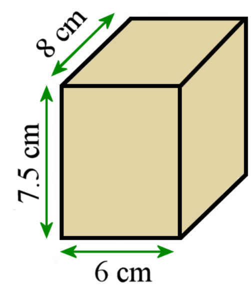
1. $2 \cdot 35\ \text{cm}^2\ +\ 2 \cdot 63\ \text{cm}^2\ +\ 2 \cdot 45\ \text{cm}^2$

2. 5 cm · 9 cm · 7 cm

3. 5 cm · 7 cm + 9 cm · 7 cm + 9 cm · 5 cm

6. Ryan organised the calculation of the surface area of this prism into three parts. Write down the intermediate calculations, and solve. This way, your teacher (or others) can follow your work. Remember also to include the units (cm or cm^2)!

Top and bottom:



Back and front:

The two sides:

Total:

7. The surface area of a cube is 96 square centimetres.

a. What is the area of one face of the cube?

b. How long is each edge of the cube?

c. Find the volume of the cube.

Puzzle Corner

Consider the rectangular prisms in problem #2. If the edges of the little cubes were double as long, how would that affect the surface area? Volume?

You can use the table below to investigate the situation.

Prism a.	Prism b.	Prism c.	Prism d.
A = ________ cm^2	A = ________ cm^2	A = ________ cm^2	A = ________ cm^2
V = ________ cm^3	V = ________ cm^3	V = ________ cm^3	V = ________ cm^3

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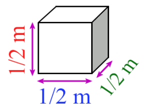
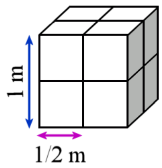
Volume of a Rectangular Prism with Sides of Fractional Length

Example 1. Let's imagine that the edges of this little cube each measure ½ m.



If we stack eight of them so that we get a bigger cube… we get this:



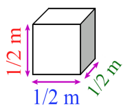
The bigger cube has <u>1 m edges, so its volume is 1 cubic metre</u>.

If eight identical little cubes make up this bigger cube, and its volume is 1 cubic metre, then <u>the volume of *one* little cube is 1/8 cubic metre</u>.

Notice: this is the same result that we get if we multiply the height, width and depth of the little cube:

$$\frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} = \frac{1}{8}\text{ m}^3$$

1. The edges of each little cube measure ½ m. What is the total volume, in cubic metres, of these figures?



a.

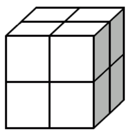
width = __1/2__ m

height = ________ m

depth = ________ m

__1__ little cube, 1/8 m^3

V = __________ m^3



b.

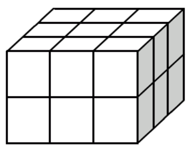
width = ________ m

height = ________ m

depth = ________ m

__8__ little cubes, each 1/8 m^3

V = __________ m^3



c.

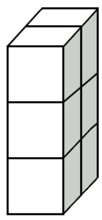
width = ________ m

height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3



d.

width = ________ m

height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3

2. Write a multiplication (width · depth · height) to calculate the volume of the figures (c) and (d) above, and verify that you get the same result as above.

a. V = ________ m · ________ m · ________ m

=

b. V = ________ m · ________ m · ________ m

=

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Understanding Distributions

A **statistical question** is a question where we expect a range of *variability* in the answers to the question.

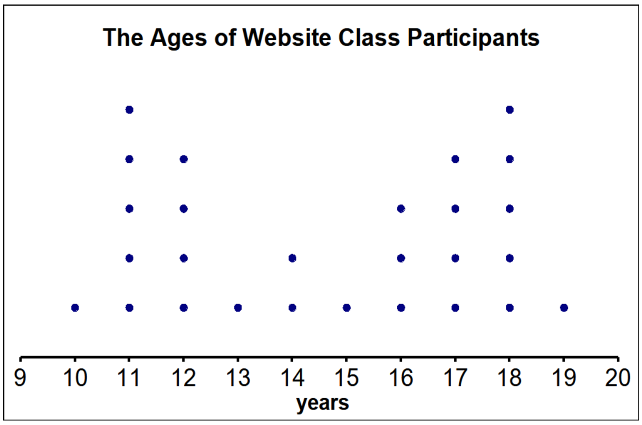
For example, "How old am I?" is *not* a statistical question (there is only one answer), but "How old are the students in my school?" *is* a statistical question because we expect the students' ages not to be all the same.

"How much does this TV cost?" is *not* a statistical question because we expect there to be just one answer.

"How much does this TV cost in various stores around town?" *is* a statistical question, because we expect a number of different answers: the prices in different stores will vary.

To answer a statistical question we collect a set of **data** (many answers). The data can be displayed in some kind of a graph, such as a bar graph, a histogram, or a dot plot.

This is a **dot plot** showing the ages of the participants in a website-building class. Each dot in the plot signifies one observation. For example, we can see there was one 13-year old and two 14-year olds in the class.



The dot plot shows us the **distribution** of the data: it shows how many times (the frequency) each particular value (age in this case) occurs in the data.

This distribution is actually **bimodal,** or "double-peaked". This means it has two "centres": one around 11-12 years, and another around 17-18 years.

1. Are these statistical questions? If not, change the question so that it becomes a statistical question.

a. What colour are my teacher's eyes?

b. How much money do the students in this university spend for lunch?

c. How much money do working adults in Romania earn?

d. How many children in the United States use a cell phone regularly?

e. What is the minimum wage in Ohio?

f. How many sunny days were there in August, 2020, in London?

g. How many pets does my friend have?

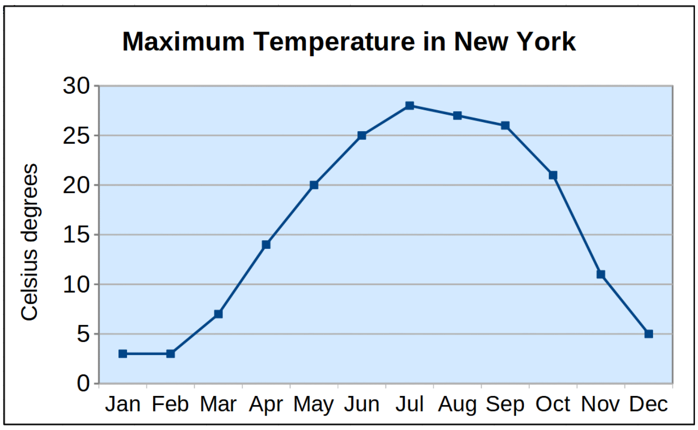
2. Is this graph based on a statistical question?

Why or why not?

3. The line graph shows the maximum temperatures in New York for each month of a certain year.

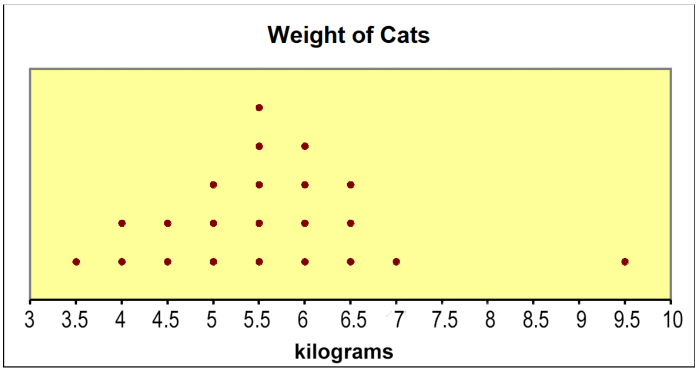
Is this graph based on a statistical question?

Why or why not?



4. The title of this dot plot is not the best. But, could the plot be based on a statistical question?

If yes, give it a better, more specific, title. Imagine what situation and what question might have produced the data.



5. Change each question from a non-statistical question to a statistical question, and vice versa.

a. What shampoo do you use?

b. How cold was it yesterday where you live?

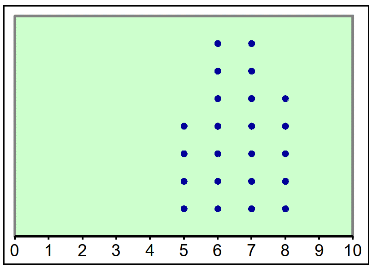
c. How old are people in Germany when they marry (the first time)?

d. How long does it take for our company's packages to reach the customers?

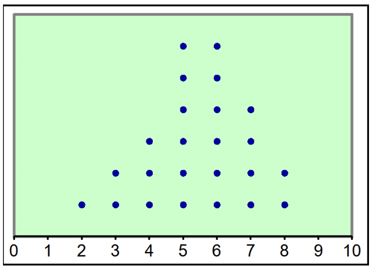
We are often interested in the **centre**, **spread** and **overall shape** of the distribution. Those three things can summarise for us what is important about the distribution.

The **centre** of a distribution has to do with where its peak is. We can use mean, median and mode to characterise the central tendency of a distribution. We will study those in detail in the next lesson.

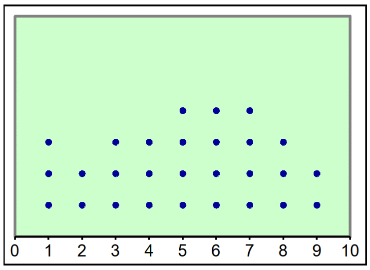
These three dot plots show how the **spread** of a distribution can vary. This means how the data items themselves are spread — whether they are "spread" all over, or tightly concentrated near some value, or somewhat concentrated around some value. We will study more about spread in another lesson.



little spread

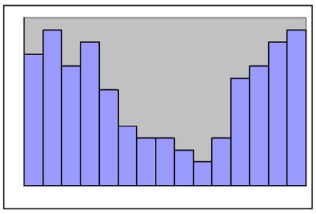


medium spread

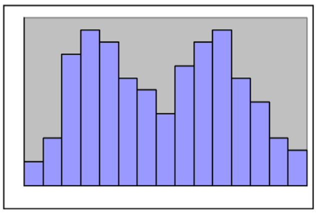


large spread

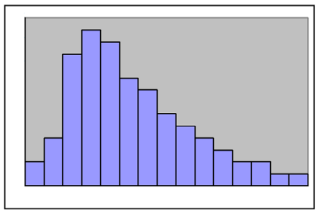
The distribution can have many varying overall **shapes.** For example:



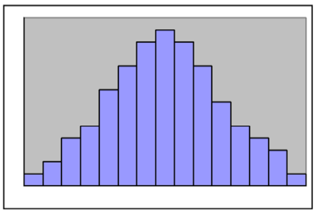
U-shaped



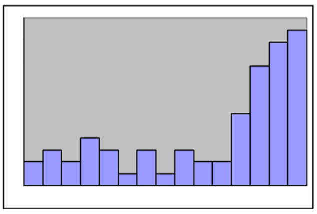
double-peaked (bimodal)



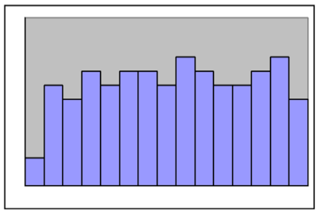
asymmetrical, right-tailed
(a.k.a. right-skewed)



bell-shaped



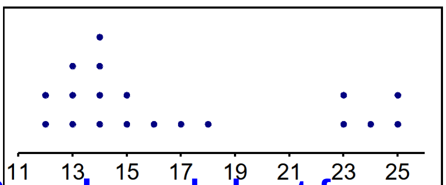
J-shaped
(can also be mirrored where most of the values are at the left)



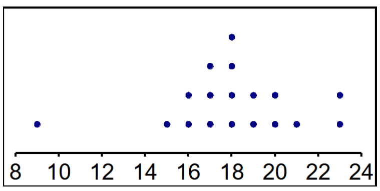
rectangular

In addition to its overall shape, a distribution may have a gap, an outlier, or a cluster:

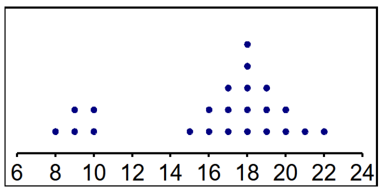
This distribution has a **gap** from 19 to 22:



In this distribution, 9 is an **outlier —** a data item whose value is considerably less or more than all the others.



This distribution has a bell shape overall (with a peak at 18), but also a **cluster** or a smaller peak at 8-10.



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Boxplots

Boxplots or **box-and-whisker plots** are simple graphs on a number line that use a box with "whiskers" to visually show the quartiles of the data. Boxplots show us a **five-number summary** of the data: the minimum, the 1st quartile, the median, the 3rd quartile and the maximum.

Example 1. These are prices of hair dryers in three stores (in dollars).

14 15 19 20 20 20 21 24 25 34 34 35 35 37 42 45 55

Five-number summary:

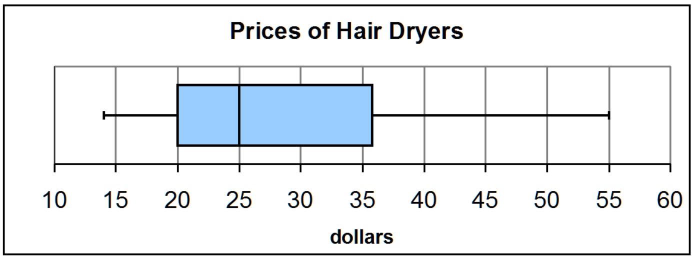
Minimum: $14

First quartile: $20

Median: $25

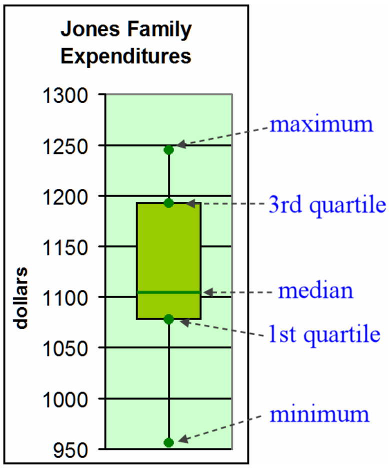
Third quartile: $36

Maximum: $55



The box itself starts at the 1st quartile and ends at the 3rd quartile. Therefore, its width is the interquartile range. We draw a line in the box marking the median ($25).

The boxplot also has two "whiskers". The first whisker starts at the minimum ($14) and ends at the first quartile. The other whisker is drawn from the third quartile to the maximum ($55).



Example 2. This boxplot shows the Jones family's monthly expenditures over a 12-month period. This time the boxplot is drawn vertically.

The box extends over half the data. This means that half the time, they spend from about $1 080 to $1 190 monthly. But sometimes they spend only about $950, and sometimes up to $1 245 in a month.

<u>Five-number summary:</u>

Minimum: $956

First quartile: $1 078

Median: $1 105

Third quartile: $1 193

Maximum: $1 245

1. **a.** Read the five-number summary from the boxplot.

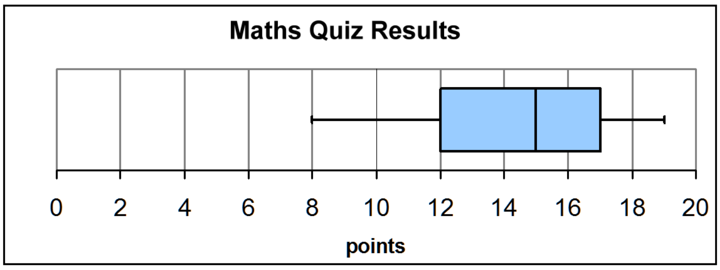
Minimum:

First quartile:

Median:

Third quartile:

Maximum:



b. Look at the box and fill in: Half the students got between _____ and _____ points in the quiz.

c. Do you think the quiz went well?

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Making Histograms

Histograms are like bar graphs, but the bars are drawn so they touch each other. Histograms are used only with numerical data.

Example. These are the prices of hair dryers in three stores again (in dollars). Make a histogram.

14 15 19 20 20 20 21 24 25 34 34 35 35 37 42 45 55

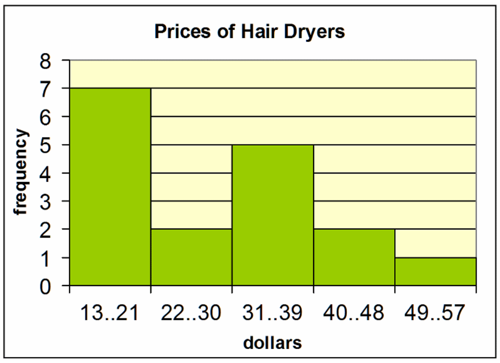
We need to decide how many bins to make and how "wide" they are. For that, we first calculate the **range**, or the difference between the greatest and smallest data item. It is 55 − 14 = 41. Then we divide the range into equal parts (bins) to get the *approximate* bin width.

If we make five bins, we get 41 ÷ 5 = 8.2 for the bin width. The bins would be 8.2 units apart. However, in this case it is nice to have the bins go by whole numbers, so we round 8.2 up to 9 and use 9 for the bin width.

The important part is that *each data item needs to be in one of the bins*. You may have to try out slightly different bin widths and starting points to see how it works. This time, starting the first bin at 13 makes the last bin to end at 57, which works, because the data will "fit" into the bins. (Starting at 14 would work, too.)

The **frequency** describes *how many data items fall into that bin*. Lastly, all we need is to draw the histogram, remembering that the bars touch each other.

Price ($)	Frequency
13..21	7
22..30	2
31..39	5
40..48	2
49..57	1



This is a **double-peaked** distribution and **skewed to the right** (the direction of skewness is where the "long tail" of the distribution is; in this case to the right). Since it definitely is *not* bell-shaped, the mean is *not* a good measure of centre. Therefore, the *median* is the better choice for measure of centre.

Consequently, we need to use *interquartile range*, and not mean absolute deviation, as a measure of variation.

The median is underlined below:

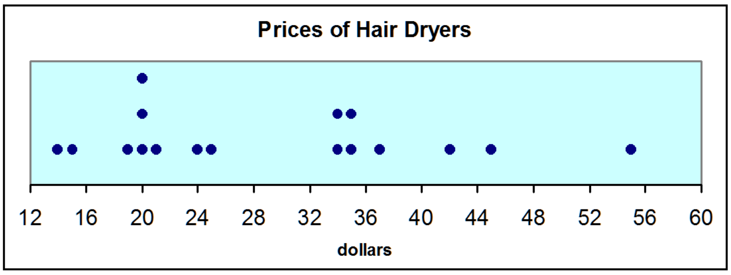
14 15 19 20 20 20 21 24 **<u>25</u>** 34 34 35 35 37 42 45 55

The 1st quartile is $20 and the 3rd quartile is $36 (verify those). So the interquartile range is $16.

This means that half of the data is found between 20 and 36 dollars. This price range is quite large for devices with a median price of only $25. A large range compared to the median describes data that is widely scattered. (We can also see that from the dot plot.)

Since the median is $25, which is nearer the low end of the interval from $20 to $36, the prices are somewhat more concentrated in the lower end of that interval.

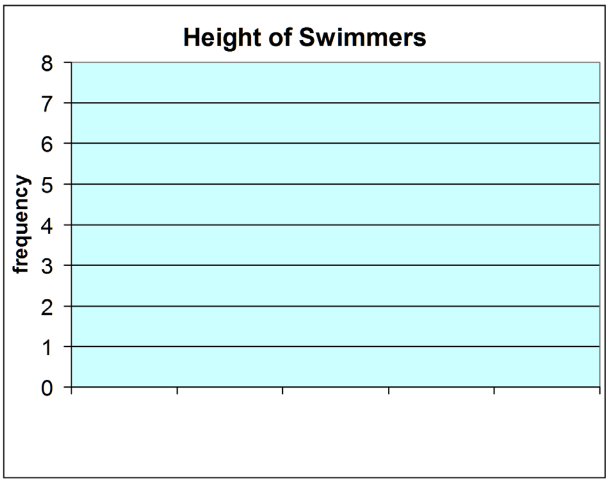
For a comparison, look at the dot plot as well. It has a similar shape to the histogram.



1. This data lists the heights of 24 swimmers in centimetres. Make a histogram with five bins.

155 155 156 157 158 159 159 160 162 162 163 163
164 165 166 167 167 168 168 170 172 174 175 177

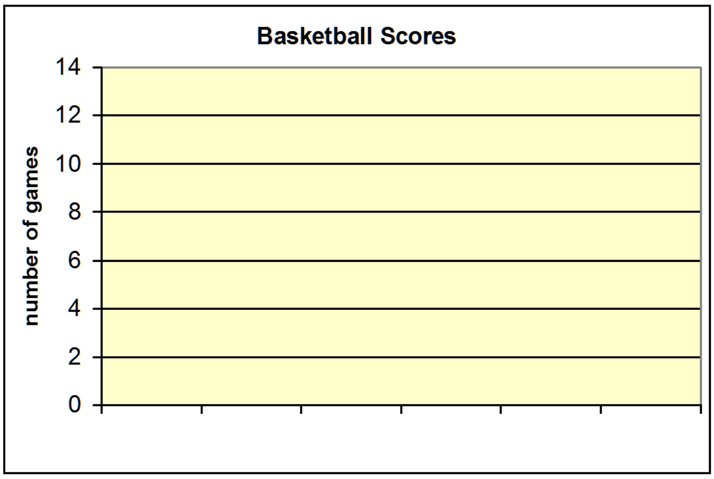
Height (cm) **Frequency**



2. Make a histogram from this data, which lists all the scores a basketball team had in the games in one season. Make six bins.

60 62 68 71 72 72 73 74 74 74 75 75 76 77 77 77
78 78 78 79 79 81 81 82 83 83 85 86 88 90 92 95

Score	Frequency



3. **a.** As this distribution in problem 2 has its peak near the centre, you could use either mean or median as a measure of centre. This time, find the median and the interquartile range.

median ____________ interquartile range ____________

b. Describe the variation in the data. Is it very scattered (varied), somewhat so, or not very much so?

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Stem-and-Leaf Plots

This lesson is optional.

A stem-and-leaf plot is made using the numbers in the data, and it looks a little bit like a histogram or a dot plot turned sideways.

In this plot, the tens digits of the individual numbers become the **stems**, and the ones digits become the **leaves**. For example, the second row 2 | 1 2 5 8 actually means 21, 22, 25, and 28. Notice how the leaves are listed in order from the smallest to the greatest.

Ages of the participants in the County Fair Karaoke Contest:

14 18 21 22 25 28 30 30
31 33 33 36 37 40 45 58

Stem	Leaf
1	4 8
2	1 2 5 8
3	0 0 1 3 3 6 7
4	0 5
5	8

4 | 5 means 45

Since stem-and-leaf plots show not only the *shape* of the distribution but also the individual *values*, they can be used to get a quick overview of the data. This distribution has a central peak and is somewhat skewed to the right.

You can also find the median fairly easily because you can follow the individual values from the smallest to the largest, and find the middle one.

Stem-and-leaf plots are most useful for numerical data sets that have 15 to 100 individual data items.

1. **a.** Complete the stem-and-leaf plot for this data:

   19 20 34 25 21 34 14 20 37 35 20 24 35 15 45 42 55

   (prices of hair dryers in three stores)

   b. What is the median?

Stem	Leaf
1	
2	
3	
4	
5	

5 | 4 means 54

2. **a.** Complete the stem-and-leaf plot for this data. This time, the stems are the first two digits of the numbers, and the leaves are the last digits.

   709 700 725 719 750 740 757 745 786 770 728 755

   (monthly rent, in dollars, for one-bedroom apartments in Houston, Texas)

   b. Find the median monthly rent.

   c. Find the interquartile range.

   d. Describe the spread of the distribution (is the data spread out a lot, a medium amount, a little, etc.)

Stem	Leaf
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	

71 | 9 means 719

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Statistics Revision

You may use a calculator in all questions in this lesson.

1. Are these statistical questions or not? If not, change the question so that it becomes a statistical question.

   a. Which kind of books do the visitors of this library like the best?

   b. How many pages are in the book *How to Solve It* by G. Polya?

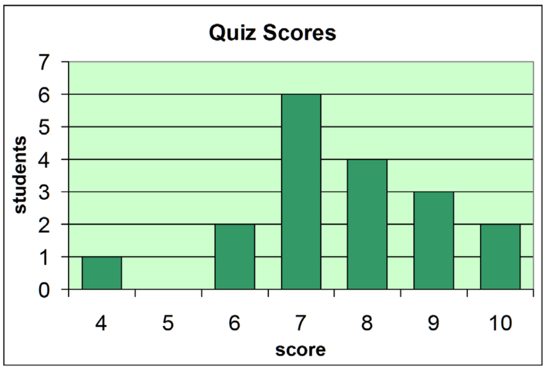
2. **a.** Find the mean, median and mode.
   Hint: recreate the list of the original data.

   Mean:

   Median:

   Mode:

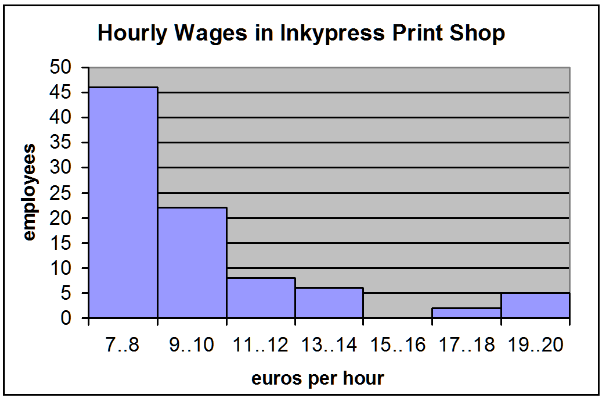
   b. Describe the shape and any striking features of the distribution.



3. This graph shows the hourly wages in euros per hour of the 89 employees in the Inkypress Print Shop.

   a. *About* what fraction of the people earn 7-8 euros/hour?

   b. Describe the shape and any striking features of the distribution.



   c. The mean is 9.66 euros/hour and the median is 8 euros/hour.
   Which is better in describing the majority's wages in this print shop?

   d. Which measure of variation should be used to describe this data, based on your answer to (c)?