math MAMMOTH

Grade 5-B Worktext
International Version

Decimals, part 2

Fractions: add and subtract

Fractions: multiply and divide

Geometry

Light Blue Series

By Maria Miller

Edition 1/2021

Contents

Chapter 8: Fractions: Multiply and Divide

Chapter 9: Geometry

Foreword

Math Mammoth Grade 5, International Version comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 5* sold in the United States, only customised for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the fifth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not customary (imperial) units.
- The currency used in word problems is the Australian dollar.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
(The decimals are formatted with a decimal point, as in the US version.)

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study statistical graphs. The main areas of study in Math Mammoth Grade 5 are:

1. Students develop fluency with addition and subtraction of fractions, and developing understanding of the multiplication and division of fractions.

2. Students finalize fluency with multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors), and they solve word problems involving all four operations.

3. Students develop a solid understanding of decimal place value and learn to use all four basic operations with decimals.

4. In geometry, students study the concept of volume, and how to measure and calculate it, and classify two-dimensional figures based on their properties.

Additional topics we study are some basic algebraic concepts, large numbers, graphs, prime factorisation, and converting measurements.

This book, 5-B, covers more on decimals (chapter 6), fraction addition & subtraction (chapter 7), fraction multiplication and division (chapter 8), and geometry (chapter 9). The rest of the topics are covered in the 5-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. In fifth grade, chapter 4 (decimals, part 1) should be studied before chapter 6 (decimals, part 2). and chapter 7 (fraction addition & subtraction) before chapter 8 (fraction multiplication & division). However, you can be flexible with all the other chapters and schedule them earlier or later.

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualise the instruction for your student.

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For revision, the curriculum includes a worksheet maker (Internet access required), mixed revision lessons, additional cumulative revision lessons, and the word problems continually require usage of past concepts. Please see more information about revision (and other topics) in the FAQ at **https://www.mathmammoth.com/faq-lightblue.php**

I heartily recommend that you view the full user guide for your grade level, available at **https://www.mathmammoth.com/userguides/** and also included in the digital (download) version of the curriculum.

And lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Decimals, Part 2
Introduction

In this chapter, we focus on decimal multiplication and division, and conversions between measurement units.

We start out with the topic of multiplying decimals by decimals. This is typically an easy topic, as long as students remember the rule concerning the decimal digits in the answer. This rule could be confused with the other rules of decimal arithmetic that we also study in this chapter. In 6th grade, I provide a proof for the rule using fraction multiplication. I didn't include it here, because the chapter already contains so many new topics for students. So, including the justification for the rule may just cause an overload, plus, students haven't studied fraction multiplication yet.

Then we learn about multiplication as *scaling*. We cannot view decimal multiplications, such as 0.4×1.2, as repeated addition. Instead, they are viewed as scaling—shrinking or enlarging—the number or quantity by a scaling factor. So, 0.4×1.2 is thought of as scaling 1.2 by 0.4, or as four-tenths *of* 1.2. You may recognize this as the same as 40% of 1.2.

Next, we learn about decimal division that can be done with mental maths. Students divide decimals by whole numbers (such as $0.8 \div 4$ or $0.45 \div 4$) by relating them to equal sharing. They divide decimals by decimals in situations where the divisor goes evenly into the dividend, thus yielding a whole-number quotient (e.g. $0.9 \div 0.3$ or $0.072 \div 0.008$).

In the next lesson, *More Division with Decimals*, we simply revise long division with decimals, where the divisor is a whole number.

The following topic is multiplying and dividing decimals by powers of ten. This is presented with the help of place value charts. The actual concept is that the number being multiplied or divided *moves* in the place value chart, as many places as there are zeros in the power of ten. As a shortcut, we can move the decimal point. However, the movement of the decimal point is an "illusion"—that is what seems to happen—but in reality, the number itself got bigger or smaller; thus, its digits actually changed positions in the place value system.

Next, we study the metric system and how to convert various metric units (within the metric system), such as converting kilograms to grams, or dekalitres to hectolitres. The first of the two lessons mainly deals with very commonly used metric units, and we use the meaning of the prefix to do the conversion. For example, centimetre is a hundredth part of a metre, since the prefix "centi" means 1/100. Knowing that, gives us a means of converting between centimetres and metres.

The second lesson deals with more metric units, even those not commonly used, such as dekalitres and hectograms, and teaches a method for conversions using a chart. These two methods for converting measuring units within the metric system are sensible and intuitive, and help students not to rely on mechanical formulas.

Next, we turn our attention to dividing decimals by decimals, which then completes our study of all decimal arithmetic. The principle here is fairly simple, but it is easy to forget (multiply both the dividend and the divisor by a power of ten, until you have a whole-number divisor). After learning that, students practice rounding measurements, and some generic problem solving with decimals. Recall that not all students need all the exercises; use your judgment.

Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

The Lessons in Chapter 6

Helpful Resources on the Internet

You can also access this list of links at **https://l.mathmammoth.com/gr5ch6**

SCAN ME

DISCLAIMER: *We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*

Decimal Arithmetic - Videos by Maria
These are my videos where I explain all about decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Suitable for grades 5-6.
https://www.mathmammoth.com/videos/grade_5/5th-grade-videos.php#decimals

MULTIPLICATION

Exploring Multiplication of Decimals
Enter two numbers with one decimal digit, and you will see the product as a rectangular area.
http://www.hbschool.com/activity/elab2004/gr6/1.html

Decimals Workshop
Practise adding, subtracting, multiplying, or dividing decimals with this customisable interactive exercise.
https://mrnussbaum.com/search?q=Decimals+Workshop

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Multiply Decimals by Decimals

Multiplying decimals is easy! You simply multiply as if there were no decimal points. Then place the decimal point in the answer following this rule:

The answer will have **as many decimal places/digits** as there are, IN TOTAL, in all of the factors.

Example 1. 0.05 × 0.7 Multiply in your head: 5 × 7 = 35. The factor 0.05 has **two** and 0.7 has **one** decimal digit. The answer has to have **three**, so the answer is 0.035.	**Example 2.** 12 × 2 × 0.3 × 0.2 Multiply mentally: 12 × 2 × 3 × 2 = 144. The factors have 0, 0, 1, and 1 decimal digits—a total of 2. The answer has to have 2 decimal digits/places, so the answer is 1.44.

1. Multiply first as if there were no decimal points. Then add the decimal point to the answer.

a. 0.5 × 0.3 = ________	**c.** 0.4 × 0.08 = ________	**e.** 8 × 0.3 = ________
b. 0.9 × 0.6 = ________	**d.** 0.7 × 0.02 = ________	**f.** 0.1 × 2.7 = ________
g. 0.2 × 0.1 = ________	**i.** 0.9 × 0.01 = ________	**k.** 0.7 × 0.3 = ________
h. 0.8 × 0.1 = ________	**j.** 9 × 0.06 = ________	**l.** 7 × 0.03 = ________

The answer to a decimal multiplication may end in one or more *decimal* zeros. That is no problem. You may **simplify the final answer** by dropping the ending decimal zeros.

Example 3. To solve 50 × 0.006, first multiply in your head 50 × 6 = 300. The factors (50 and 0.006) have 0 and 3 decimal places, so the answer will have **3**. Therefore, the answer is 0.300, but it *simplifies* to 0.3.	**Example 4.** To solve 400 × 0.05, we first multiply 400 × 5 = 2 000. The factors have 0 and 2 decimal digits, so the answer will have **2**. The answer is 20.00. You can simplify that to 20.

2. Multiply. Simplify your final answer.

a. 0.4 × 0.5 = *0.20* = *0.2*	**e.** 40 × 0.05 = ________ = ________
b. 20 × 0.06 = ________ = ________	**f.** 0.6 × 0.2 × 0.5 = ________ = ________
c. 3 × 0.2 × 0.5 = ________ = ________	**g.** 600 × 0.004 = ________ = ________
d. 300 × 0.009 = ________ = ________	**h.** 0.4 × 0.5 × 60 = ________ = ________

The shortcut to decimal multiplication

1) Multiply as if there were no decimal points.

2) Place the decimal point in the answer. The **number of decimal digits** in the answer is the **SUM** of the number of decimal digits in the factors.

Example 5. To solve 0.81 × 2.5, multiply **as if it was 81 × 25**. In other words, ignore the decimal points. (Also, "081" is the same as 81 so we can ignore the zero, too.)

Since 0.81 has *two* decimal digits, and 2.5 has *one*, the answer will have *three*. The final answer is therefore 2.025.

$$\begin{array}{r} 81 \\ \times\ 25 \\ \hline 405 \\ +\ 1620 \\ \hline 2025 \end{array}$$

Does that make sense? We can estimate:

0.81 × 2.5 ≈ 0.8 × 3 = 2.4
or 0.81 × 2.5 ≈ 0.8 × 2.5 = 2.

Yes, a final answer of 2.025 makes sense, since it is close to our estimates.

Example 6. This time, we include the decimal points when writing calculating 1.49 × 0.7, but even so, we multiply **as if it was 149 × 7**. Imagine the decimal points are not there! And there is NO need to align the decimal points like in addition and subtraction.

The final answer has *three* decimal places, since the factors have two and one, respectively.

$$\begin{array}{r} {}^{3\ 6} \\ 1.49 \\ \times\ 0.7 \\ \hline 1.043 \end{array}$$

← **two** decimal places
← **one** decimal place
← **three** decimal places

Estimate: 1.49 × 0.7 ≈ 1.5 × 0.7 = 1.05.

If the estimate was *not* close to our final answer, there would probably be an error somewhere.

3. Solve with long multiplication. Also, estimate. Write the longer number on top.

a. 0.3 × 1.19

Estimate: __________

X

b. 0.9 × 51.7

Estimate: __________

X

c. 204.5 × 0.4

Estimate: __________

X

d. 2.2 × 0.72

Estimate: __________

X

e. 5.6 × 2.8

Estimate: __________

X

f. 3.34 × 4.2

Estimate: __________

X

Example 7. Strawberries cost $5.15/kg, and you buy 0.7 kg. What is the total cost?

If you were to buy 3 kilograms of strawberries, you would multiply 3 × $5.15 to find the total price. When you buy 0.7 kg, you do the SAME: **multiply the price by 0.7.** The total cost is **0.7 × $5.15 = 3.605** (see the multiplication on the right).

$$\begin{array}{r} {}^{1}\ {}^{3} \\ 5\ 1\ 5 \\ \times\quad\ 7 \\ \hline 3\ 6\ 0\ 5 \end{array}$$

Since this is a money amount, it needs rounded to two decimals: $3.61.

Note that the answer also makes sense: 0.7 kg of strawberries should cost less than 1 kg of strawberries, which was $5.15.

4. Find the total cost. Write a multiplication.

a. Ribbon costs $1.10 per metre, and you buy 0.4 metres.

Cost: ________ × ________ = _____________

b. Nuts cost $8 per kilogram. You buy 0.3 kg.

Cost: ________ × ________ = _____________

c. A phone call costs $7 per hour. You talk for 1.2 hours.

Cost: ________ × ________ = _____________

d. Lace costs $2.20 per metre, and you buy 1.5 metres.

Cost: ________ × ________ = _____________

5. Lauren went to a farmer's market, and bought 0.4 kg of blueberries at $9.20/kg, 3.5 kg of potatoes at $1.20/kg, and 1.2 kg of carrots at 1.85/kg. What is the total cost?

6. You are trying to find out how much it costs to feed your pet parrot for a year.

You have figured that your parrot eats about 90 grams, which is 0.09 kg, of parrot feed in a day. Also, parrot feed costs $7.08/kg. So, what is the cost of feeding your parrot for a year (not just a day)? Round your answer to the nearest dollar.

7. Multiply.

a. $0.7 \times 1.1 =$ ________	**c.** $0.002 \times 9 =$ ________	**e.** $0.4 \times 4 \times 0.2 =$ ________
b. $0.02 \times 0.5 =$ ________	**d.** $1.1 \times 0.3 =$ ________	**f.** $6 \times 0.06 \times 0.2 =$ ________

8. Find the missing factors.

a. $0.5 \times$ ________ $= 0.35$	**d.** $4 \times$ ________ $= 2.4$	**g.** $0.9 \times$ ________ $= 7.2$
b. $0.5 \times$ ________ $= 0.035$	**e.** $4 \times$ ________ $= 0.024$	**h.** $0.9 \times$ ________ $= 0.081$
c. $0.05 \times$ ________ $= 0.035$	**f.** $4 \times$ ________ $= 0.24$	**i.** $0.9 \times$ ________ $= 0.54$

9. Solve by long multiplication or in your head. Place the letter of each problem under the right answer, and solve the riddle.
Hint: Sometimes you can figure out the correct answer by estimating.

0.8×2.7	U	10×0.042	S	2.09×1.7	R	200×0.008	O
1.6×2.05	D	1.5×1.5	E	70×0.05	S	$0.5 \times 0.4 \times 0.7$	Y
10×0.04	L	3.45×0.7	I	0.05×2.4	E	$0.01 \times 600 \times 0.2$	F
		1.06×0.4	E	15.2×0.18	B		

Riddle: *What did number 22 say to 21.21?* "*YOU'RE...*

2.736	0.424	0.42	2.415	3.28	2.25		0.14	1.6	2.16	3.553	3.5	0.12	0.4	1.2

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Divide Decimals by Decimals 1

You have learned...

...how to divide decimals *by whole numbers*, using either mental maths or long division:

2.04 ÷ 2 = ________

0.24 ÷ 6 = ________

5.2 ÷ 10 = ________

```
   0 2.4 6
7 ) 1 7.2 2
  -1 4
     3 2
   - 2 8
       4 2
     - 4 2
         0
```

...and how to divide decimals *by decimals* mentally, thinking of how many times it fits:

2.5 ÷ 0.5 = ________

0.021 ÷ 0.003 = ________

But how can we divide if the divisor is a decimal, yet the division won't be even?

Examples of that type of divisions are 4.6 ÷ 0.029 and 0.23 ÷ 0.07.
Such decimal divisions are based on the following principle:

> We **change the decimal division into a new division** problem that has the *same answer*, yet it has a *whole-number* divisor. This new problem can be solved with regular long division.

Let's now explore how to do that change.

1. Solve, thinking how many times the divisor "fits into" the dividend. **Notice something special!**

a. 60 ÷ 20 = ________	**e.** 350 ÷ 50 = ________	**i.** 2 000 ÷ 10 = ________
b. 6 ÷ 2 = ________	**f.** 35 ÷ 5 = ________	**j.** 200 ÷ 1 = ________
c. 0.6 ÷ 0.2 = ________	**g.** 3.5 ÷ 0.5 = ________	**k.** 20 ÷ 0.1 = ________
d. 0.06 ÷ 0.02 = ________	**h.** 0.35 ÷ 0.05 = ________	**l.** 2 ÷ 0.01 = ________

What did you notice?

The answers are the same, because 0.02 fits into 0.06 as many times as 0.2 fits into 0.6, as many times as 2 fits into 6, and as many times as 20 fits into 60.

2. Solve first the easier of the two problems in each box. The answers to both are the same.

a. 5 ÷ 0.2 = ________	**b.** 7 ÷ 0.35 = ________	**c.** 36.9 ÷ 3 = ________
50 ÷ 2 = ________	700 ÷ 35 = ________	0.369 ÷ 0.03 = ________

The way to transform a difficult decimal division problem, such as 3.439 ÷ 5.6, into a problem with the *same answer*, but with a *whole-number* divisor, is this:

- **Multiply both the dividend and the divisor by 10 repeatedly, until the divisor becomes a whole number.** Each problem you make this way will have the same answer!

Example 1. Solve 2.104 ÷ 0.04.

We multiply both numbers in the problem by 10 until the divisor is a whole number →

2.104 ÷ 0.04	(This is the original problem.)
21.04 ÷ 0.4	(The divisor is not a whole number yet.)
210.4 ÷ 4	← Now the divisor is a whole number!

(Remember also, the shortcut for multiplying by 10 is to move the decimal point — in this case, in *both* numbers).

Then, we take the last problem, 210.4 ÷ 4, and solve it with long division →
Notice that the dividend does not have to be a whole number.

```
   0 5 2.6
4 )2 1 0.4
  -2 0
     1 0
   -   8
       2 4
```

The answer is 52.6. So, the answer to the original problem, 2.104 ÷ 0.04, is *also* 52.6. Check by multiplying (using the *original* problem):

```
    1 2
    5 2.6     ← one decimal digit
×   0.0 4     ← two decimal digits
  2.1 0 4     ← three decimal digits
```

3. First, multiply both the dividend and the divisor by 10, repeatedly, until you get a *whole-number divisor*. (You can use the shortcut of moving the decimal point to do that.) Then, divide. Lastly, check your answer by multiplying (but use the numbers from the *original* problem to do the check).

a. 0.445 ÷ 0.05 = ________

4.45 ÷ 0.5 (This does not work.)

44.5 ÷ 5 (This works! →)

5) 4 4.5

Check:

b. 2.394 ÷ 0.7 = ________

)

Check:

4. Solve. Check your final answer by multiplying (but use the numbers from the *original* problem to do the check).

a. 9.735 ÷ 0.003 = ________

Check:

b. 0.477 ÷ 0.09 = ________

Check:

c. 546.6 ÷ 1.2 = ________

Check:

d. 1.764 ÷ 0.006 = ________

Check:

Chapter 7: Fractions: Add and Subtract

Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade.

This chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to revise and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a "stumbling block." Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions. We first revise how to convert fractions into other equivalent fractions. These lessons use a visual model of splitting pie pieces further, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models, and then gradually advance toward the abstract. Several lessons are devoted to understanding and practising the basic concept, and also to applying this new skill to mixed numbers.

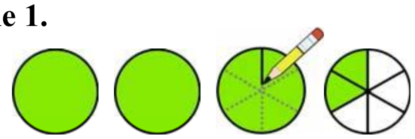
The lesson *Comparing Fractions* revises some mental maths methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions.

The Lessons in Chapter 7

Subtracting Mixed Numbers 1

Strategy 1: Renaming / regrouping. In this method we first cut one of the whole pies into slices, and join these slices with the existing slices. After that, we can subtract.

Example 1.



$3\frac{2}{6} - 1\frac{5}{6}$

↓

$2\frac{6}{6} + \frac{2}{6} - 1\frac{5}{6}$

↓

$2\frac{8}{6} - 1\frac{5}{6} = 1\frac{3}{6}$

At first we have three uncut pies and 2/6 more. We cut one of the whole pies into sixths. We end up with only two whole (uncut) pies and 8 sixths.

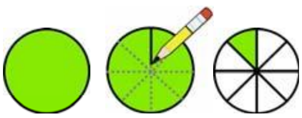
We say that 3 2/6 has been **renamed** as 2 8/6. Now we can subtract 1 5/6 easily.

We can also solve this problem by writing the mixed numbers one under the other.

$$\begin{array}{r} 2\ \frac{8}{6} \\ \not{3}\ \not{\frac{2}{6}} \\ -\ 1\ \frac{5}{6} \\ \hline 1\ \frac{3}{6} \end{array}$$

We **regroup** (borrow) 1 whole pie as 6 sixths. There are already 2 sixths in the fractional parts column, so we add the 6/6 and 2/6 and write 8/6 in place of 2/6. Now we can subtract the 5/6.

Example 2.



$2\frac{1}{8} - \frac{5}{8}$

↓

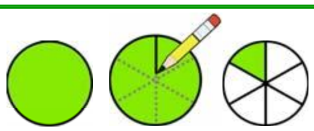
$1\frac{9}{8} - \frac{5}{8} = 1\frac{4}{8}$

Or:

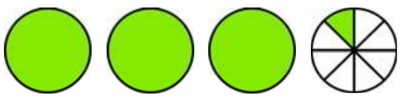
We need to regroup 1 whole as 8/8. The 8/8 and the existing 1/8 make a total of 9/8.

$$\begin{array}{r} 1\ \frac{9}{8} \\ \not{2}\ \not{\frac{1}{8}} \\ -\ 1\ \frac{5}{8} \\ \hline \frac{4}{8} \end{array}$$

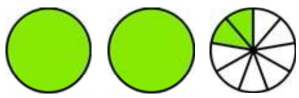
1. Don't subtract anything. Divide ONE whole pie into fractional parts and rename the mixed number.



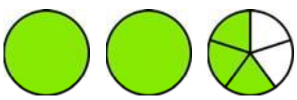
a. $2\frac{1}{6}$ is renamed as



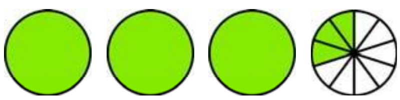
b. $3\frac{1}{8}$ is renamed as



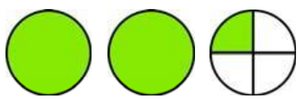
c. $2\frac{2}{9}$ is renamed as



d. $2\frac{3}{5}$ is renamed as

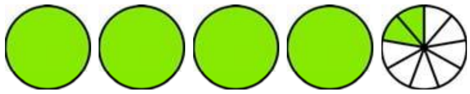


e. $3\frac{3}{10}$ is renamed as



f. $2\frac{1}{4}$ is renamed as

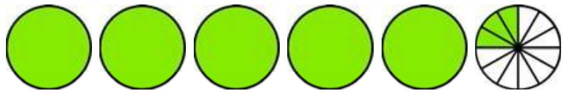
2. Rename, then subtract. Be careful. Use the pie pictures to check your calculation.



a. $4\frac{2}{9} - 1\frac{8}{9}$

↓

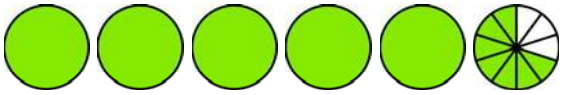
$= 3\frac{11}{9} - 1\frac{8}{9} =$



b. $5\frac{3}{12} - 2\frac{7}{12}$

↓

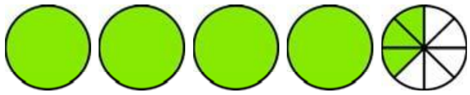
$= 4\frac{}{12} - 2\frac{7}{12} =$



c. $5\frac{7}{10} - 3\frac{9}{10}$

↓

$= \underline{\quad}\,\frac{}{} - 3\frac{9}{10} =$



d. $4\frac{3}{8} - 1\frac{7}{8}$

↓

$= \underline{\quad}\,\frac{}{} - 1\frac{7}{8} =$

3. Regroup (if necessary) and subtract.

a. $\overset{2}{\cancel{3}}\ \overset{\frac{}{9}}{\cancel{\frac{4}{9}}}$ $-\ \frac{8}{9}$	**b.** $7\frac{4}{9}$ $-\ 2\frac{7}{9}$	**c.** $12\frac{9}{12}$ $-\ 6\frac{11}{12}$	**d.** $8\frac{3}{14}$ $-\ 5\frac{9}{14}$
e. $14\frac{7}{9}$ $-\ 3\frac{5}{9}$	**f.** $11\frac{5}{21}$ $-\ 7\frac{15}{21}$	**g.** $26\frac{4}{19}$ $-\ 14\frac{15}{19}$	**h.** $10\frac{3}{20}$ $-\ 5\frac{7}{20}$

Strategy 2: Subtract in Parts. First, subtract what you can from the fractional part of the minuend. Then subtract the rest from one of the whole pies. Study the examples.

Example 3. $2\frac{1}{8} - \frac{5}{8}$

$$= 2\frac{1}{8} - \frac{1}{8} - \frac{4}{8}$$

$$= 2 - \frac{4}{8}$$

$$= 1\frac{8}{8} - \frac{4}{8} = 1\frac{4}{8}$$

First we take away only 1/8, which leaves 2 whole pies. Then we subtract the rest (4/8) from one of the whole pies.

Example 4. $3\frac{2}{9} - 2\frac{7}{9}$

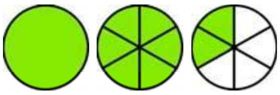
$$= 3\frac{2}{9} - 2\frac{2}{9} - \frac{5}{9}$$

$$= 1 - \frac{5}{9}$$

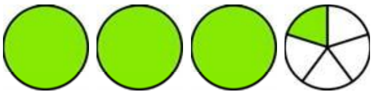
$$= \frac{9}{9} - \frac{5}{9} = \frac{4}{9}$$

We cannot subtract 7/9 from 2/9. So, first we subtract 2 and 2/9, which leaves 1 whole pie. The rest, 5/9, is subtracted from the last whole pie.

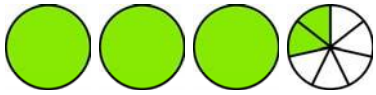
4. Subtract in parts. Remember: you can *add* to check a subtraction problem.



a. $2\frac{2}{6} - \frac{5}{6}$

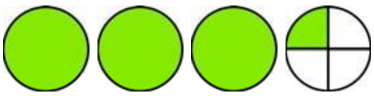


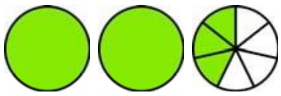
b. $3\frac{1}{5} - 2\frac{3}{5}$

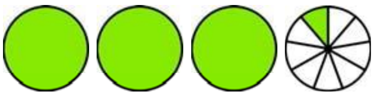


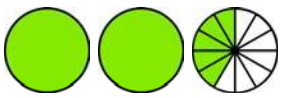
c. $3\frac{2}{7} - 2\frac{6}{7} =$

5. Subtract in two parts. Write a subtraction sentence.

a.  Cross out $\frac{3}{4}$.

b.  Cross out $1\frac{5}{7}$.

c.  Cross out $1\frac{5}{9}$.

d.  Cross out $1\frac{11}{12}$.

Example 5. Look at Mia's maths work: $7\frac{1}{6} - 2\frac{5}{6} = 9\frac{6}{6} = 10$. Can you see why it is wrong?

If you have 7-and-a-fraction, and you subtract 2-and-a-fraction, you cannot get 10 as an answer! Neither can you get 2-and-a-fraction or 3-and-a-fraction. The answer to this problem should be either 5-and-a-fraction or 4-and-a-fraction.

In reality, Mia was *adding* instead of subtracting. (If you have ever done that, you are not alone—it is a common error.)

Always check if your answer is reasonable (not too small, or too big).

6. Subtract. Check that your answer is reasonable.

a. $8\frac{1}{5} - 3\frac{3}{5} =$	**b.** $4\frac{2}{8} - 1\frac{7}{8} =$	**c.** $12\frac{4}{13} - 9\frac{8}{13} =$
d. $11\frac{2}{15} - 6\frac{6}{15} =$	**e.** $7\frac{1}{20} - 3\frac{7}{20} =$	**f.** $6\frac{14}{100} - 2\frac{29}{100} =$

7. Two sides of a triangle measure 3 5/8 m, and the perimeter of the triangle is 10 1/8 m. How long is the third side of the triangle?

8. Ellie had 4 m of material. She needed 7/8 m for making a skirt, and she made two. How much material is left?

9. A recipe for chocolate cookies calls for 1 ¾ cups of flour and Harry is making a double batch. However, he *only* has ¾ cup of flour! How much more flour would Harry need to have enough to make the cookies?

Puzzle Corner $5\frac{2}{9} - \frac{5}{9} - 1\frac{8}{9} =$

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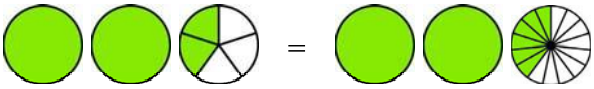
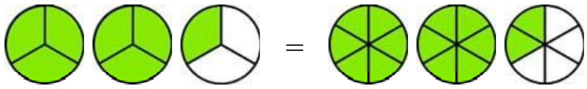
Equivalent Fractions 2



$2\,\frac{2}{5} = 2\,\frac{6}{15}$

Here you see the mixed number 2 2/5 changed into an equivalent mixed number 2 6/15. Actually, we only changed the fractional part, 2/5, into the equivalent fraction 6/15 and obviously the whole-number part did not change.

These pictures show the fraction 7/3 converted into an equivalent fraction 14/6. Now, 7/3 is a fraction, not a mixed number. You can see that from the picture because the whole pies have been split into fractional pieces. We consider it as seven thirds (slices). Also, 7/3 is an **improper fraction** because its value is 1 or more. (Of course, 14/6 is also.)



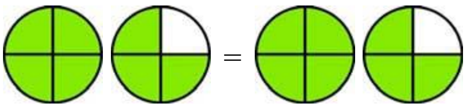
$\frac{7}{3} = \frac{14}{6}$

A **proper fraction** is a fraction whose value is less than 1.

We use equivalent fractions also with mixed numbers and with improper fractions.

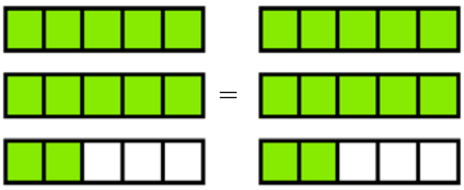
1. These are improper fractions. Split the slices in the right-hand picture.
Write the equivalent fractions.

a. Split each slice into three.



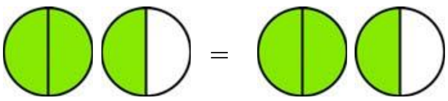
$\frac{7}{4} \overset{\times ☐}{=} \frac{\;}{\;}$

b. Split each piece in two.



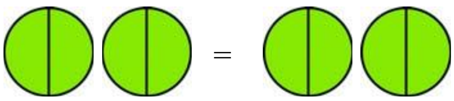
$\frac{12}{5} \overset{\times ☐}{=} \frac{\;}{\;}$

c. Split each slice into four.



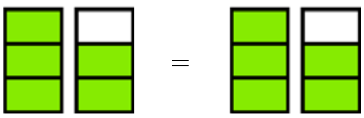
$\frac{3}{2} = \frac{\;}{\;}$

d. Split each slice into two.



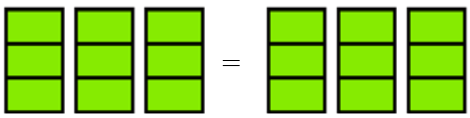
$\frac{4}{2} = \frac{\;}{\;}$

e. Split each piece in two.



$\frac{5}{3} = \frac{\;}{\;}$

f. Split each piece into two.



$\frac{9}{3} = \frac{\;}{\;}$

2. Fill in the missing numbers in these equivalent fractions and mixed numbers.

a. $5\,\frac{7}{10} = ☐\,\frac{☐}{80}$	**b.** $5\,\frac{7}{10} = ☐\,\frac{28}{☐}$	**c.** $6\,\frac{2}{9} = ☐\,\frac{12}{☐}$	**d.** $\frac{7}{1} = \frac{☐}{6}$	**e.** $\frac{8}{3} = \frac{☐}{15}$

3. Here, we have written the number 3 as a *fraction*, as 3/1. Write the number 3 as a fraction using other kinds of parts, also.

whole pies	halves	thirds	fourths	fifths	tenths	hundredths
$\frac{3}{1}$	$\frac{\square}{2}$					

4. Write the number 2 1/2 as a fraction using...

halves	fourths	sixths	eighths	tenths	twentieths	hundredths
$\frac{\square}{2}$						

5. If you can find an equivalent fraction, write it. If you cannot, cross out the whole problem.

a. $\frac{5}{7} = \frac{\quad}{28}$ The pieces were split into ____ .	**b.** $\frac{2}{5} = \frac{\quad}{18}$ The pieces were split into ____ .	**c.** $\frac{1}{4} = \frac{\quad}{14}$ The pieces were split into ____ .	**d.** $\frac{2}{3} = \frac{\quad}{12}$ The pieces were split into ____ .	**e.** $\frac{5}{6} = \frac{8}{\quad}$ The pieces were split into ____ .
f. $\frac{1}{6} = \frac{\quad}{28}$ The pieces were split into ____ .	**g.** $\frac{2}{9} = \frac{\quad}{63}$ The pieces were split into ____ .	**h.** $\frac{5}{4} = \frac{\quad}{32}$ The pieces were split into ____ .	**i.** $\frac{1}{3} = \frac{5}{\quad}$ The pieces were split into ____ .	**j.** $\frac{3}{8} = \frac{8}{\quad}$ The pieces were split into ____ .

6. Explain in your own words when a problem of equivalent fractions is *not possible* to do. Use an example problem or problems in your explanation.

7. Make chains of equivalent fractions. Pay attention to the *patterns* in the numerators and in the denominators.

a. $\frac{3}{4} = \frac{\square}{8} = \frac{\square}{12} = \quad = \quad = \quad = \quad = \quad =$

b. $\frac{5}{3} = \frac{10}{6} = \frac{\square}{9} = \quad = \quad = \quad = \quad = \quad =$

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Adding and Subtracting Mixed Numbers

In this lesson, we will be adding and subtracting **mixed numbers with unlike fractional parts.** Here's how:

1. First convert the unlike fractional parts into like fractions.
2. Then add or subtract the mixed numbers.

Example 1.

$$2\frac{1}{2} + 1\frac{7}{8} \Rightarrow 2\frac{4}{8} + 1\frac{7}{8} = 3\frac{11}{8} \Rightarrow 4\frac{3}{8}$$

Notice that the answer, 3 11/8, has a fractional part that is more than one (an improper fraction). Therefore, we need to write it as 4 3/8.

1. First convert the fractional parts into like fractions, then add.

a. $6\frac{2}{3} + 3\frac{1}{5} \Rightarrow 6\frac{\square}{15} + 3\frac{\square}{\square} = $ ______

b. $10\frac{1}{8} + 3\frac{2}{5} \Rightarrow$ ______

c. $17\frac{1}{16} + 3\frac{3}{8} \Rightarrow$ ______

2. First convert the fractional parts into like fractions, then add. Lastly, change your final answer so that the fractional part is not more than 1 whole.

a. $4\frac{1}{2} + 3\frac{4}{5} \Rightarrow 4\frac{\square}{10} \;\; 3\frac{\square}{10}$ ______ $\Rightarrow$

b. $5\frac{5}{6} + 7\frac{2}{3} \Rightarrow$ ______ $\Rightarrow$

c. $3\frac{5}{6} + 2\frac{7}{8} \Rightarrow$ + ______ $\Rightarrow$

d. $9\frac{5}{7} + 7\frac{2}{3} \Rightarrow$ ______ $\Rightarrow$

Example 2. Study how we can write the same problem and its solution either horizontally or vertically.

Horizontally:

$$2\tfrac{1}{2} - 1\tfrac{2}{3} = 2\tfrac{3}{6} - 1\tfrac{4}{6}$$

$$= 1\tfrac{9}{6} - 1\tfrac{4}{6} = \tfrac{5}{6}$$

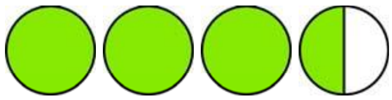
Notice how 2 3/6 is **renamed** as 1 9/6. This is the same process as regrouping in the "vertical" solution.

Vertically:

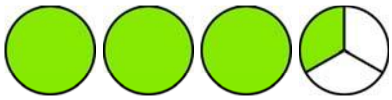
$$\begin{array}{r} 2\tfrac{1}{2} \\ -\ 1\tfrac{2}{3} \\ \hline \end{array} \Rightarrow \begin{array}{r} 1\ \tfrac{9}{6} \\ \not{2}\ \tfrac{\not{3}}{\not{6}} \\ -\ 1\tfrac{4}{6} \\ \hline \tfrac{5}{6} \end{array}$$

3. Solve. You can use the pies to help.

a. $2\tfrac{3}{4} - 1\tfrac{3}{8}$



b. $3\tfrac{1}{2} - 1\tfrac{1}{3}$



c. $3\tfrac{1}{3} - 1\tfrac{4}{9}$

4. First convert the fractional parts into like fractions, then subtract. You may need to regroup.

a. $\begin{array}{r} 5\tfrac{1}{2} \\ -\ 2\tfrac{4}{5} \\ \hline \end{array} \Rightarrow$ ______

b. $\begin{array}{r} 15\tfrac{4}{8} \\ -\ 8\tfrac{5}{6} \\ \hline \end{array} \Rightarrow$ ______

c. $\begin{array}{r} 16\tfrac{5}{9} \\ -\ 10\tfrac{1}{2} \\ \hline \end{array} \Rightarrow$ ______

d. $\begin{array}{r} 4\tfrac{1}{6} \\ -\ 2\tfrac{3}{5} \\ \hline \end{array} \Rightarrow$ ______

e. $\begin{array}{r} 11\tfrac{1}{12} \\ -\ 3\tfrac{1}{4} \\ \hline \end{array} \Rightarrow$ ______

f. $\begin{array}{r} 8\tfrac{2}{9} \\ -\ 2\tfrac{3}{4} \\ \hline \end{array} \Rightarrow$ ______

5. First convert the fractional parts into like fractions. Then add or subtract.

a. $7\frac{2}{7} - 2\frac{1}{2}$	**b.** $8\frac{9}{15} + 5\frac{4}{5}$
c. $3\frac{2}{9} - 1\frac{1}{3}$	**d.** $6\frac{2}{3} - 1\frac{1}{7}$

6. Sally needs 1 1/4 metres of material to make a blouse and 8/10 of a metre to make a skirt.

a. Find how many metres of material she needs for both of them.

b. Now use *decimals* to solve the same problem. Which way do you feel is easier?

7. Seth's two heaviest books weigh 1 1/4 kg and 1 2/5 kg.

a. What is their total weight in *kilograms*? Use fraction addition.

b. Change the two weights into grams.

c. Now find the total weight in grams.

(**d.** Do the same with your heaviest books.)

Chapter 8: Fractions: Multiply and Divide
Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 9 on geometry in between chapters 7 and 8.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 7, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication (2/3) × 18 km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as 5 ÷ 3. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, 20 ÷ 6 = 3 2/6).

Then students encounter the shortcut or rule for fraction division: each division is actually changed into a *multiplication* by the reciprocal of the divisor. While this rule is presented here in 5th grade, students are not required to master fraction division in all cases (such as with mixed numbers). We focus on divisions where the divisor and dividend are whole numbers unit fractions. However, I have also included some problems with non-unit fractions.

After introducing the shortcut, students study two main contexts and real-life applications for fraction division: sharing divisions and so-called “measurement divisions”, where we think how many times the divisor “fits into” the dividend.

The Lessons in Chapter 8

Helpful Resources on the Internet

You can also access this list of links at **https://l.mathmammoth.com/gr5ch8**

SCAN ME

DISCLAIMER: *We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*

Videos for Fraction Multiplication and Division
Author's own videos that cover the topics of this chapter.
https://www.mathmammoth.com/videos/grade_5/5th-grade-videos.php#fractions2

REDUCING/SIMPLIFYING FRACTIONS

Cancelling Demonstration
Watch a movie that uses circles to demonstrate how to rename to lowest terms with cancelling.
https://www.visualfractions.com/cancel/

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
https://www.homeschoolmath.net/worksheets/fraction.php

Speedway Fractions
Add and subtract fractions, simplifying your answer. Power up your race car and win first place!
https://www.mathplayground.com/ASB_Speedway.html

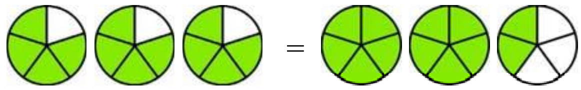
Multiply Fractions by Whole Numbers

To multiply a whole number and a fraction, find the total number of pieces.
This means you multiply the whole number and the top number (numerator) of the fraction.
The denominator stays the same.

Example 1. $3 \times \frac{4}{5}$ is three copies of $\frac{4}{5}$.

How many fifths are there in total?
Multiply 3 × 4 to find out the total number of fifths. We get 12 fifths.

Lastly, we give the final answer as a mixed number and in lowest terms.

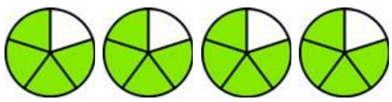


$3 \times \frac{4}{5} = \frac{12}{5} = 2\frac{2}{5}$

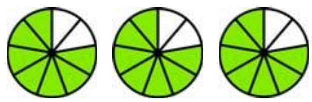
Example 2. $8 \times \frac{3}{4}$ means 8 × 3 pieces, or 24 pieces. Each piece is a fourth. So, we get $\frac{24}{4}$.

We need to write the answer as a mixed number. This time, $\frac{24}{4}$ happens to be the whole number 6.

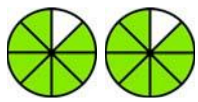
1. Write a multiplication sentence to match the picture, and solve.



a. $\underline{\ 4\ } \times \frac{\square}{\square} =$



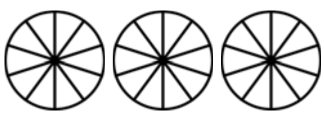
b. ____ $\times \frac{\square}{\square} =$



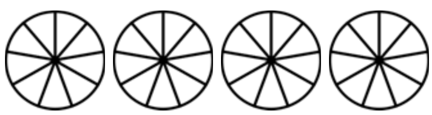
c. ____ $\times \frac{\square}{\square} =$

2. Multiply. Remember to give your answer as a mixed number.
   You may use the pie pictures to help.

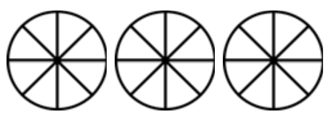
a. $3 \times \frac{7}{10} =$



b. $4 \times \frac{7}{9} =$



c. $3 \times \frac{5}{8} =$



3. Solve. Give your answer in lowest terms (simplified) and as a mixed number. Study the example.

a. $6 \times \frac{4}{9} = \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}$	**b.** $4 \times \frac{7}{10} =$
c. $2 \times \frac{11}{20} =$	**d.** $9 \times \frac{2}{15} =$

Example 3. Multiplication can be done in either order. (In other words, multiplication is *commutative*.)

So, $\frac{3}{10} \times 5$ is the same as $5 \times \frac{3}{10}$. They both equal $\frac{5 \times 3}{10} = \frac{15}{10}$. This simplifies to $\frac{3}{2}$, which is $1\frac{1}{2}$.

4. Solve. Give your answer in lowest terms (simplified) and as a mixed number.

a. $\frac{15}{6} \times 2 =$	**b.** $6 \times \frac{7}{100} =$
c. $\frac{1}{12} \times 16 =$	**d.** $2 \times \frac{35}{100} =$
e. $\frac{9}{20} \times 10 =$	**f.** $\frac{7}{15} \times 7 =$

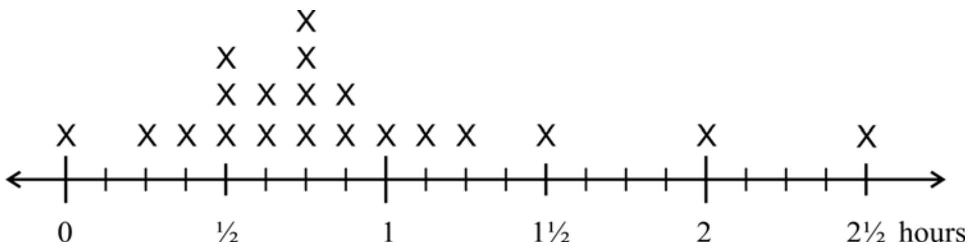
5. Erica has beverage glasses that hold 3/8 litres each.
How much water does she need to fill four of them?

6. Marlene wants to triple this recipe (make three times as much). How much of each ingredient will she need?

Brownies

3/4 cup butter
1 1/2 cups brown sugar
4 eggs
1 1/4 cups cocoa powder
1/2 cup flour
2 tsp vanilla

7. William asked 20 fifth graders how much time they spent on housework and chores the day before. He then rounded the answers to the nearest 1/8 hour. The line plot shows his results. Each x-mark corresponds to one fifth grader.



a. Exclude the three students who did the least housework and three who did the most, and fill in:

Most students used between ___________ and ___________ hours for housework and chores.

b. The average for this data is 7/8 hours. Use this to calculate how many hours these 20 fifth graders used for housework in total.

A REMINDER

A fraction *of* a number means that **fraction TIMES the number.** In other words, the word "of" translates into multiplication.

Example 4. $\frac{3}{10}$ of \$120

$\downarrow$

$\frac{3}{10} \times \$120$

You have previously learned how to find 3/10 of \$120 using division:

- First, divide \$120 by 10 to find 1/10 of it. It is \$12.
- Then, multiply that by 3 to get 3/10 of \$120. You get \$36.

We get the same answer with **fraction multiplication**: $\frac{3}{10} \times \$120 = \frac{3 \times \$120}{10} = \frac{\$360}{10} = \$36.$

Both methods are essentially the same: you divide by 10 and multiply by 3, it is just done in two different orders.

8. Find the following quantities.

   a. 2/5 of 35 kg

   b. 4/9 of 180 km

9. Dad is building a shelf that is 4 metres long. He wants to use 2/5 of it for gardening supplies and the rest for tools. How long is the section of the shelf that is for gardening supplies? (*Hint: Use centimetres.*)

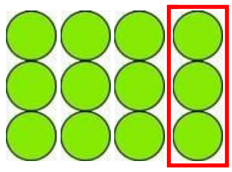
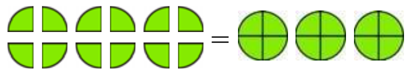
10. **a.** Janet and Sandy earned \$81 for doing yard work. They divided the money unequally so that Janet got 2/3 of it and Sandy got the rest. How much money did each girl get?

    b. What happens if the amount they earned is \$80 instead?

11. Allen drew a 10 cm by 16 cm rectangle on paper. Then he drew a second rectangle that was 3/4 as long and wide as the first one.

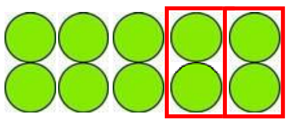
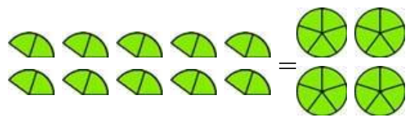
    a. How long and how wide was Andrew's second rectangle?

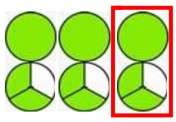
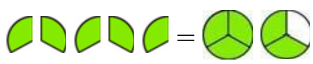
Epilogue (optional): There is something interesting about multiplying "a fraction times a whole number" and multiplying "a whole number times a fraction." Let's compare.

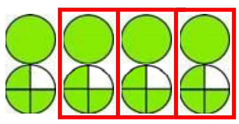
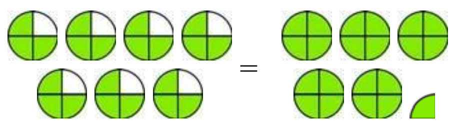
 $\frac{1}{4} \times 12$ means **a fourth part of 12,** which is 3.	 $12 \times \frac{1}{4}$ means **12 copies of 1/4**, which makes 3 whole pies.

Notice: Both $\frac{1}{4} \times 12$ and $12 \times \frac{1}{4}$ equal 3. That makes sense, because multiplication can be done in any order. But they mean different things (a fourth part of 12, and 12 copies of 1/4).

Fill in the missing parts. (Optional.)

a. A two-fifth part of 10	**10 copies of 2/5**
 $\frac{\square}{\square} \times 10$ means a two-fifth part of 10, which is equal to ____.	 $10 \times \frac{\square}{\square}$ means 10 copies of ____, which is equal to ____.

b. A third part of 5	**5 copies of 1/3**
 $\frac{\square}{\square} \times 5$ means a third part of 5, which is equal to ____.	 $5 \times \frac{\square}{\square}$ means 5 copies of ____, which is equal to ____.

c. A three-fourth part of 7	**7 copies of 3/4**
 $\frac{\square}{\square} \times 7$ means three fourths of 7, which is equal to ____.	 $7 \times \frac{\square}{\square}$ means 7 copies of ____, which is equal to ____.

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Dividing Fractions: The Shortcut

It may sound kind of strange, but the quickest way to calculate the answer to most fraction division problems is to **change the division problem into a *multiplication*.** We will explore that thought in this lesson.

How can a division be changed into a multiplication?

Think about this: We have studied how finding half of a number can be calculated by multiplying the number by 1/2. But, finding half of a number is the SAME as *dividing* the number by 2.

Similarly, to multiply a number by 1/3 will give us one third of that number. But another way to find 1/3 of any number is to divide that number by 3.

And lastly, keep in mind that the two factors in multiplication can be written in either order.

Multiplication	Division
$\frac{1}{2} \times 68 = 34$ *or* $68 \times \frac{1}{2} = 34$	$68 \div 2 = 34$
$\frac{1}{3} \times 240 = 80$ *or* $240 \times \frac{1}{3} = 80$	$240 \div 3 = 80$
$\frac{1}{6} \times 42 = 7$ *or* $42 \times \frac{1}{6} = 7$	$42 \div 6 = 7$

To be precise, we change a division by 3 into a multiplication by $\frac{1}{3}$. Similarly, we can change a division by 5 into a multiplication by $\frac{1}{5}$, or a division by 10 into a multiplication by $\frac{1}{10}$, ... and so on, for all unit fractions (fractions with a numerator of 1).

1. Change each division into a multiplication, and solve.

a. $30 \div 5$
↓ ↓ ↓
$30 \times \frac{1}{5} = 6$

b. $\frac{1}{9} \div 3$
↓ ↓ ↓
$\frac{1}{9} \times \frac{1}{3} = \frac{\quad}{\quad}$

c. $\frac{1}{4} \div 2$
↓ ↓ ↓
$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$

d. $\frac{1}{7} \div 3$
↓ ↓ ↓
$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$

e. $32 \div 8$
↓ ↓ ↓

f. $\frac{1}{5} \div 4$
↓ ↓ ↓

The numbers 3 and $\frac{1}{3}$ are **reciprocal numbers.** So are 5 and $\frac{1}{5}$.

Two numbers are reciprocal numbers if, when you multiply them, you get 1.

For example, the numbers $\frac{1}{9}$ and 9 are reciprocal numbers, because $\frac{1}{9} \times 9 = \frac{9}{9} = 1$.

Similarly, $\frac{2}{5}$ and $\frac{5}{2}$ are reciprocal numbers, because $\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$.

Example 1. You can find a reciprocal of any fraction by "*flipping*" it (switching the numerator and the denominator).

The reciprocal of $\frac{6}{13}$ is $\frac{13}{6}$.

Example 2. To find the reciprocal of a whole number, first write it as a fraction with a denominator of one: $27 = \frac{27}{1}$. Then, "flip" it, and you get $\frac{1}{27}$ as its reciprocal.

2. Find the reciprocals of these numbers.

a. $\frac{1}{6}$	**b.** $\frac{1}{100}$	**c.** 5	**d.** 21	**e.** $\frac{7}{8}$
f. $\frac{11}{3}$	**g.** $\frac{1}{42}$	**h.** 1	**i.** 13	**j.** $\frac{5}{6}$

The shortcut for fraction division is:

Change the division into a multiplication, and the divisor into its reciprocal.

Example 3.
$$\frac{6}{7} \div 2$$
↓ ↓ ↓
$$\frac{6}{7} \times \frac{1}{2} = \frac{6}{14} = \frac{3}{7}$$

3. Change each division into a multiplication, and solve.

a. $\frac{1}{2} \div 6$ ↓ ↓ ↓	**b.** $\frac{1}{8} \div 5$ ↓ ↓ ↓	**c.** $\frac{1}{30} \div 3$ ↓ ↓ ↓
d. $\frac{4}{5} \div 2$ ↓ ↓ ↓	**e.** $\frac{3}{8} \div 3$ ↓ ↓ ↓	**f.** $\frac{8}{15} \div 4$ ↓ ↓ ↓

The shortcut also works when the divisor is a fraction (in fact, it always works... no matter what kind of numbers you have for the dividend and the divisor). Study the examples.

Example 4. $7 \div \frac{1}{2}$ ↓ ↓ ↓ $7 \times 2 = 14$	**Example 5.** $\frac{3}{4} \div \frac{2}{5}$ ↓ ↓ ↓ $\frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\frac{7}{8}$

4. Change each division into a multiplication, and solve.

a. $8 \div \frac{1}{2}$ ↓ ↓ ↓	**b.** $7 \div \frac{1}{3}$ ↓ ↓ ↓	**c.** $11 \div \frac{1}{5}$ ↓ ↓ ↓
d. $\frac{4}{5} \div \frac{1}{2}$ ↓ ↓ ↓	**e.** $\frac{1}{9} \div 3$ ↓ ↓ ↓	**f.** $\frac{4}{5} \div \frac{8}{9}$ ↓ ↓ ↓

5. Let's revise fraction addition and subtraction for a bit!

a. $\frac{5}{6} + \frac{3}{8}$	**b.** $\frac{5}{6} - \frac{3}{8}$
c. $3\frac{1}{7} + 1\frac{1}{2}$	**d.** $10\frac{2}{7} - 3\frac{1}{5}$

6. Jackie walks 3/8 of a km to school every day, and the same distance back.
What distance does she walk in a five-day school week?

7. A recipe for ginger cookies calls for 2 ¼ cups of flour. If you make the recipe 1 ½ times, how much flour do you need?

8. A piece of land was split between two brothers so that one got 3/5 of the land and the other got 2/5. Then, the brother who got more, decided to use half of his land for growing crops.

If you multiply the two fractions, $\frac{3}{5}$ and $\frac{1}{2}$, what does the answer to that multiplication tell you?

Puzzle Corner

The shortcut for fraction division works also with mixed numbers. You just need to convert mixed numbers to fractions before dividing. See how well you do with these problems!

a. $\frac{29}{10} \div 1\frac{2}{5}$

b. $1\frac{7}{8} \div \frac{3}{4}$

c. $3\frac{5}{6} \div 1\frac{1}{2}$

Mixed Revision Chapter 8

1. Subtract. (Adding and Subtracting Mixed Numbers/Ch.7)

a. $4\frac{2}{6} - 1\frac{5}{6} =$	**b.** $3\frac{2}{9} - 1\frac{7}{9} =$
c. $4\frac{2}{3} - 2\frac{1}{4} =$	**d.** $7\frac{1}{6} - 1\frac{3}{5} =$

2. Write the numbers in expanded form (as a <u>sum</u> of their different "parts" according to place value).
(Place Value up to Billions/Ch. 2)

a. 453 763 = ___ × 100 000 + ___ × 10 000 + ___ × 1 000 + ___ × 100 + ___ × 10 + ___ × 1

b. 63 170 = ___ × 10 000 + ____________________

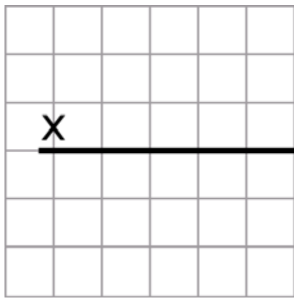
c. 3 910 603 = ___ × 1 000 000 + ____________________

3. The price of Shirt A is $6.29. It is one-third of the price of Shirt B. Find the price of Shirt B.

4. Solve by multiplying in columns. Estimate first.
(Multiply Decimals by Decimals 1/Ch. 6)

2.11 × 6.8

Estimate: __________



5. Use *decimal* multiplication to find these amounts. (Multiplication as Scaling/Ch. 6)

a. 3/10 of 13 kg	**b.** 7/10 of 1.5 litres	**c.** 22/100 of 4 km

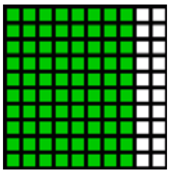
6. Multiply and divide. (Multiply and Divide by Powers of Ten 1 and 2/Ch. 6)

a. 0.34 ÷ 10 = ________ 2.1 ÷ 100 = ________	**b.** 100 × 0.098 = ________ 1 000 × 46.7 = ________	**c.** 19 ÷ 10^3 = ________ 10^4 × 0.03 = ________

7. Convert between the measuring units. (The Metric System 1/Ch. 6)

a. 5 070 g = __________ kg 2.5 kg = __________ g	**b.** 0.6 L = __________ ml 10 500 ml = __________ L	**c.** 0.06 km = __________ m 2 600 m = __________ km

8. Write the number illustrated by the picture in two ways as a decimal (as tenths and as hundredths) and in two ways as a fraction (as tenths and as hundredths). How are the different ways read?
(Revision: Tenths and Hundredths/Ch. 4)



9. Write these numbers. (Place Value up to Billions/Ch. 2)

a. 200 + 30 thousand + 32 million =
b. 500 billion + 5 + 500 thousand =
c. 300 + 87 million + 612 billion + 2 thousand =

10. You are placing chairs that are 55 cm wide in a row in a large dining room. The room is 9 m wide. If you leave 90 cm walking space at both ends of your row, how many chairs can you fit in the row?
(The Metric System 1/Ch. 6)

11. Mum was making a double recipe applesauce at a time. Each recipe would fill four jars. After 9 times, how many jars of applesauce had she made?

12. Calculate the products mentally. (Exponents and Powers/Ch. 2)

a. 2×10^5	**c.** 45×10^6
b. 8×10^4	**d.** 785×10^3

13. Twenty-six kilograms of strawberries are packaged evenly into five boxes.

a. How much does each box weigh?

b. If the strawberries cost $3 per kilogram, how much does one box cost?

14. Divide. (Divide Decimals by Decimals 1 and 2/Ch. 6)

a. $82.50 \div 0.06$	**b.** $48.302 \div 0.2$

Puzzle Corner Solve the equations. Mental maths should be sufficient!

a. $0.50 + x = 0.677$ **b.** $x + 1.52 = 2$ **c.** $1 - x = 0.378$ **d.** $x - 0.5 = 1.27$

Chapter 9: Geometry
Introduction

This chapter includes many problems that involve drawing geometric figures, because drawing is an excellent help towards achieving a conceptual understanding of geometry. Most of those are marked with “”, meaning the exercise is to be done in a notebook or on blank paper.

The chapter starts out with lessons that revise topics from previous grades, such as measuring angles, the vocabulary of basic shapes, how to draw a perpendicular line through a given point on a line, and how to draw a triangle with given angle measurements. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students are introduced to a compass, and they draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of a rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson also includes several drawing problems where students draw triangles that match the given information.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognise volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

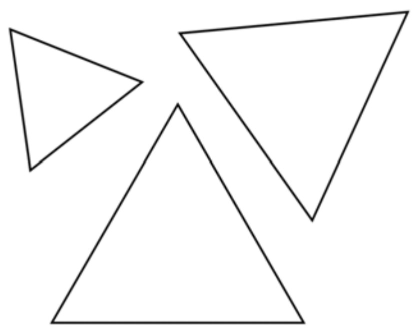
The Lessons in Chapter 9

Equilateral, Isosceles, and Scalene Triangles

Classification according to sides

If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

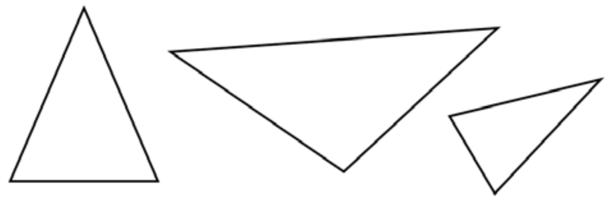
"*Equi-*" refers to things that are the same or equal, and "*lateral*" refers to sides. Think of it as a "same-sided" triangle.



If only *two* of a triangle's sides are congruent, then it is called an **isosceles triangle**.

Think of it as a "same-legged" triangle, the "legs" being the two sides that are the same length.

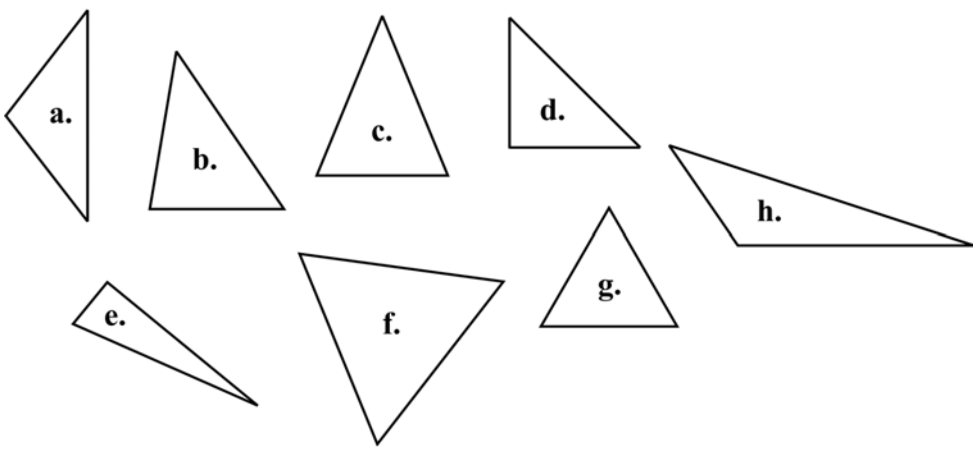
MARK the two congruent sides of each isosceles triangle:



Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.

1. Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

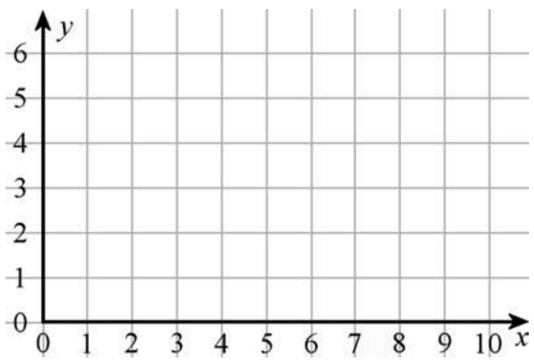
   You can mark each triangle with an "*e*," "*i*," or "*s*" correspondingly.



2. Plot the points (0, 0), (3, 5), (0, 5) , and connect them with line segments to form a triangle.

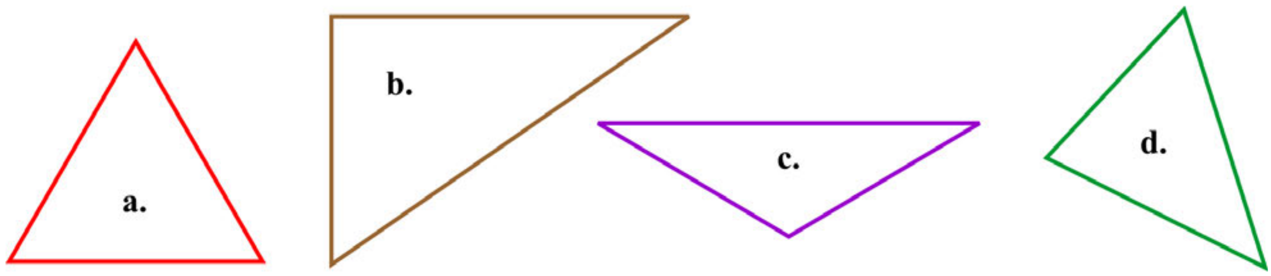
   Classify your triangle by its sides. Is it equilateral, isosceles, or scalene?

   ____



Classification according to angles
Remember, we can also classify a triangle according to its <u>angles</u>. • A **right triangle** has one right angle. • An **obtuse triangle** has one obtuse angle. • An **acute triangle** has three acute angles.

3. Classify the triangles as “acute,” “right,” or “obtuse” (by their angles), and also as “equilateral,” “isosceles,” or “scalene” (by their sides).



Triangle	Classification by the angles	Classification by the sides
a.		
b.		
c.		
d.		

4. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles <u>and</u> sides.

Triangle 1: (0, 0), (4, 0), (0, 4)

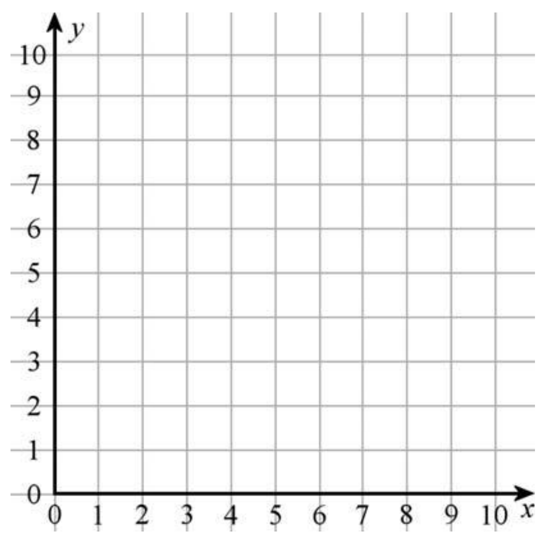
__________________________ and

____

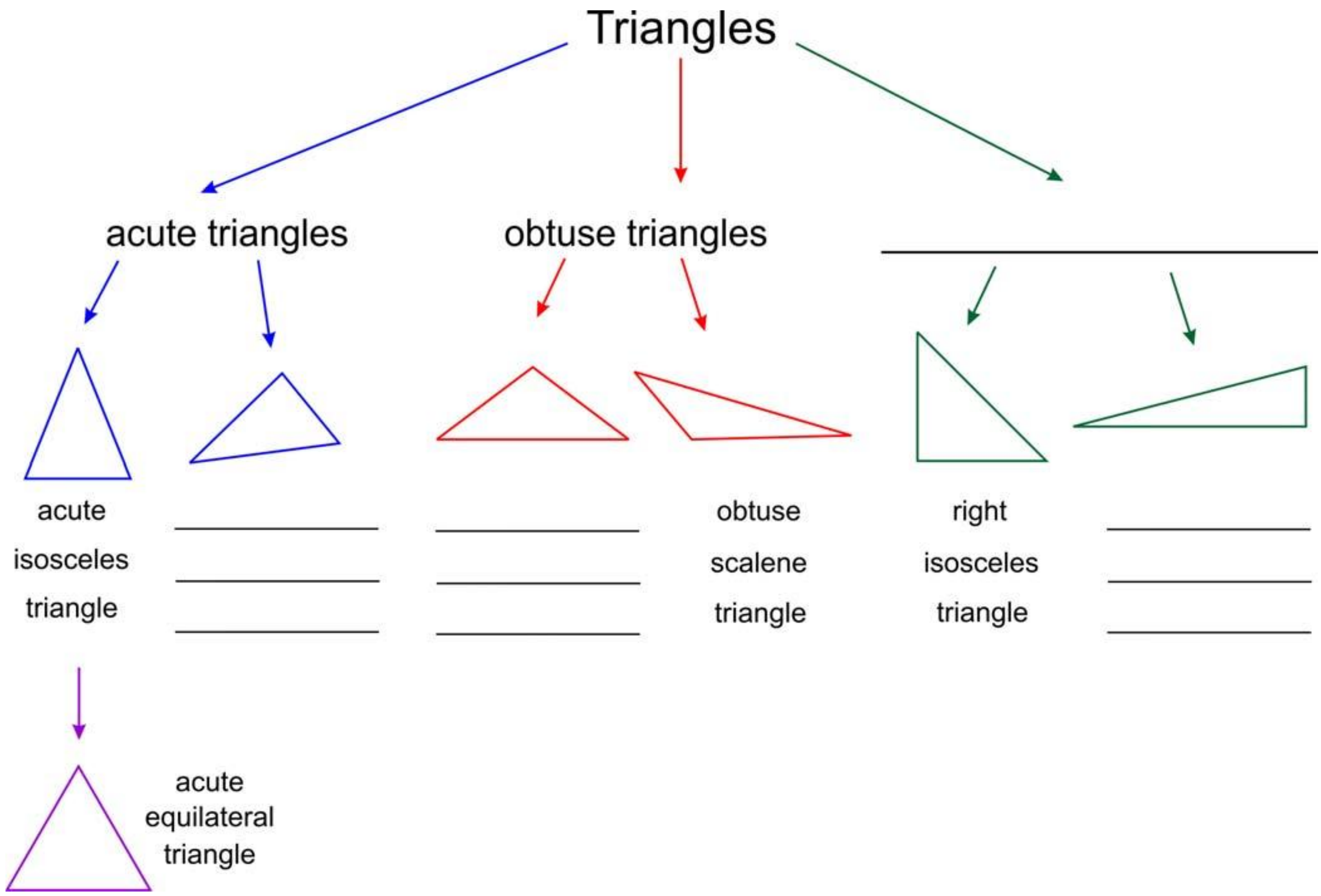
Triangle 2: (5, 5), (1, 8), (9, 4)

__________________________ and

____



5. Fill in the missing parts in this tree diagram classification for triangles.



6. Sketch an example of the following shapes. You don't need to use a ruler.

a. an obtuse isosceles triangle

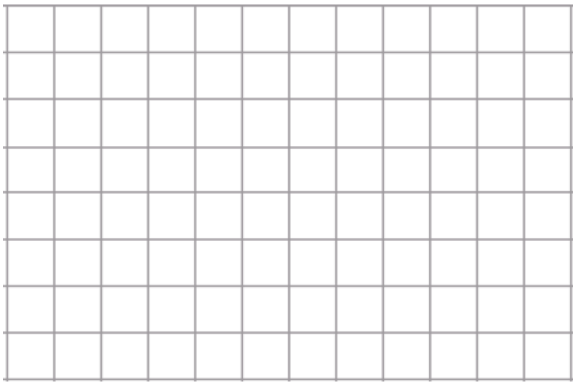
b. an obtuse scalene triangle

c. a right scalene triangle

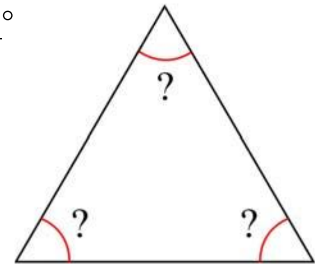
7. Plot in the grid

a. a right isosceles triangle

b. an acute isosceles triangle.



8. Make a guess about the angle measures in an equilateral triangle: _______°
Measure to check.



9. **a.** Could an equilateral triangle be a right triangle?
If yes, sketch an example. If not, explain why not.

b. Could a scalene triangle be obtuse?
If yes, sketch an example. If not, explain why not.

c. Could an acute triangle be scalene?
If yes, sketch an example. If not, explain why not.

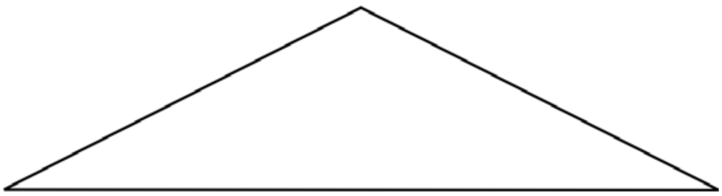
10. State whether or not it is possible to draw the following figures. (You don't have to draw any.)

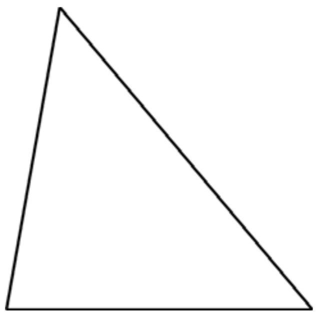
a. an obtuse equilateral triangle

b. a right equilateral triangle

c. an acute isosceles triangle

11. Measure all the angles in these isosceles triangles. Continue their sides, if necessary.
Mark the angle measures near each angle.

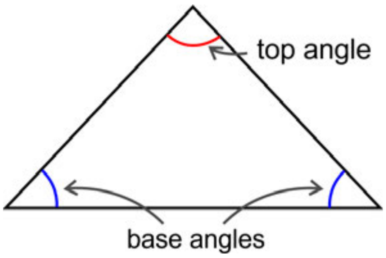
a. 

b. 

12. **a.** Draw any isosceles triangle.
Hint: Draw any angle. Then, measure off the two congruent sides, making sure they have the same length. Then draw the last side.

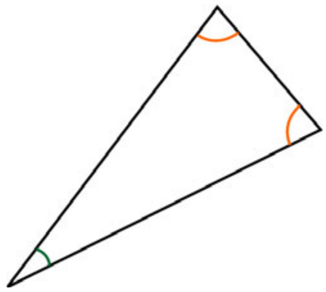
b. Measure the angles of your triangle. They measure ________ °, ________ °, and ________ ° .

13. Based on the last two exercises, can you notice something special about the angle measures?



There are two angles in an isosceles triangle that have the SAME angle measure. They are called the **base angles.**

The remaining angle is called the **top angle**.



Can you find the top angle and the base angles in this isosceles triangle?

14. Draw an isosceles triangle with 75° base angles.
(The length of the sides can be anything.)
Hint: Start by drawing the base side (of any length).
Then, draw the 75° angles.

15. **a.** Draw an isosceles right triangle whose two sides measure 5 cm.
Hint: Draw a right angle first. Then, measure off the 5-cm sides. Then draw in the last side.

b. How long is the third side?

c. What is the measure of the base angles?

16. Draw a scalene obtuse triangle where one side is 3 cm and another is 7 cm.
Hint: Draw the 7-cm side first, then the 3-cm side forming any obtuse angle with the first side.

Puzzle Corner

a. Draw two isosceles triangles with a 50° top angle.
Your two triangles should not be identical.

b. What is the angle measure of the base angles?

New Terms
• *equilateral triangle* • *isosceles triangle* • *scalene triangle*

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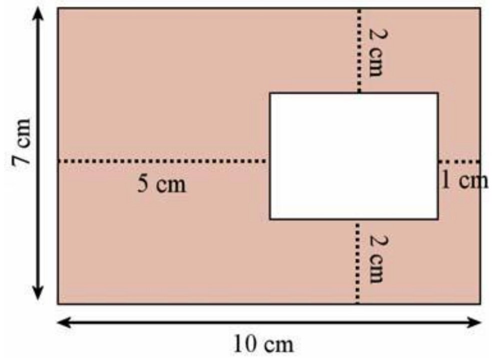
Area and Perimeter Problems

Example 1. Find the area of the shaded figure.



The easiest way to do this is:
(1) Find the area of the larger outer rectangle.
(2) Find the area of the white inner rectangle.
(3) Subtract the two.

1. The area of the large rectangle is
   7 cm × 10 cm = 70 cm^2.

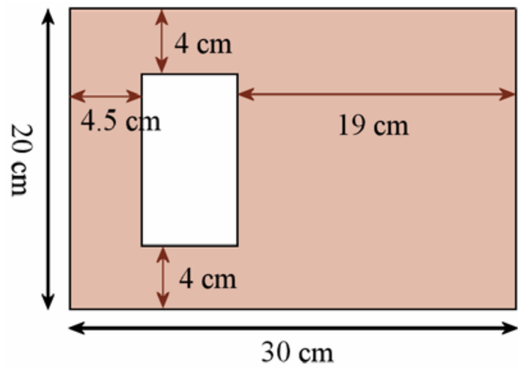
2. First we find the *sides* of the white rectangle by subtracting. The longer side of the white rectangle is 10 cm − 5 cm − 1 cm = 4 cm.
   The shorter side is 7 cm − 2 cm − 2 cm = 3 cm.

   So, the area of the white rectangle is 4 cm × 3 cm = 12 cm^2.

3. Lastly we subtract the two areas to find the shaded area: 70 cm^2 − 12 cm^2 = 58 cm^2.

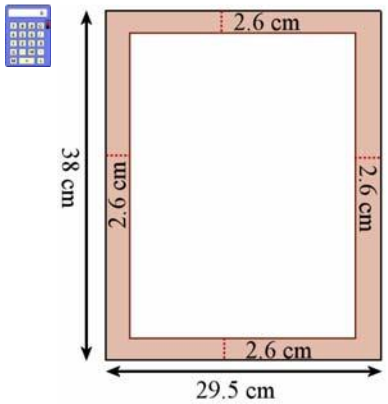
<u>Note:</u> In the following problems, record your work—all the calculations—carefully. That way you are much less likely to make mistakes! Use a notebook, if necessary, for additional space.

1. **a.** Find the area of the *white* rectangle.
   All lines meet at right angles.



b. Find the area of the shaded figure.

2. The image on the right shows a picture frame.
   Find the area of the actual frame (that is, of the shaded part).
   Give your answer to the nearest whole square centimetre.
   (All lines meet at right angles.)



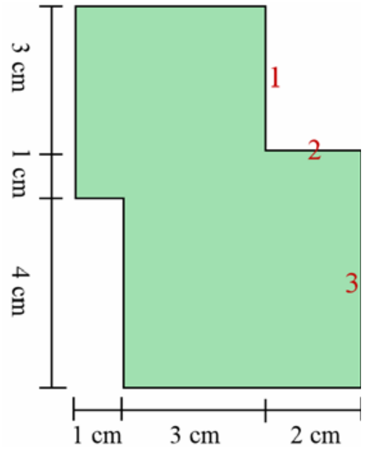
3. The perimeter of a rectangle is 42 cm.
   If the long side of the rectangle is 11 cm,
   how long is the shorter side?

Example 2. Find the perimeter of the figure.

We need to find the length of *each* side and then add the lengths. Start, for example, at the side marked with 1, then go to the side marked with 2, then to side 3, and so on, until you have "traveled" all the way around the figure.

Side 1 is 3 cm. Side 2 is 2 cm. Side 3 is 5 cm.
The total perimeter is:

3 cm + 2 cm + 5 cm + 5 cm + 4 cm + 1 cm + 4 cm + 4 cm = 28 cm.



Example 3. Find the area of the figure.

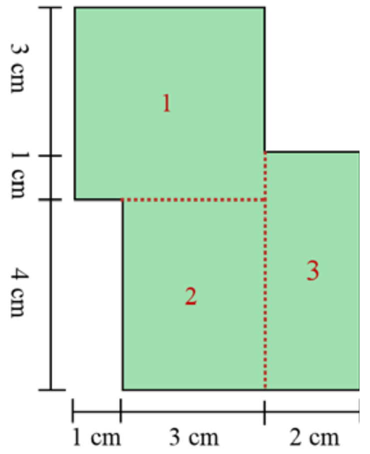
Divide the figure into rectangles by drawing in it some additional lines.

Rectangle 1 has an area of $4\text{ cm} \times 4\text{ cm} = 16\text{ cm}^2$.

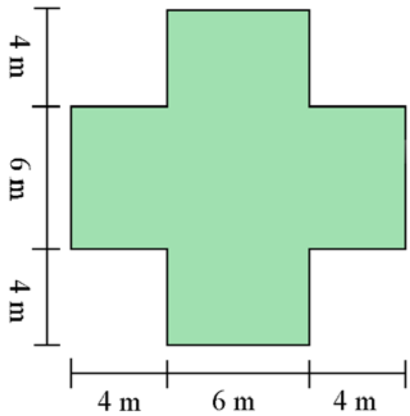
Rectangle 2 has an area of $3\text{ cm} \times 4\text{ cm} = 12\text{ cm}^2$.

Rectangle 3 has an area of $2\text{ cm} \times 5\text{ cm} = 10\text{ cm}^2$.

The total area is: $16\text{ cm}^2 + 12\text{ cm}^2 + 10\text{ cm}^2 = 38\text{ cm}^2$.

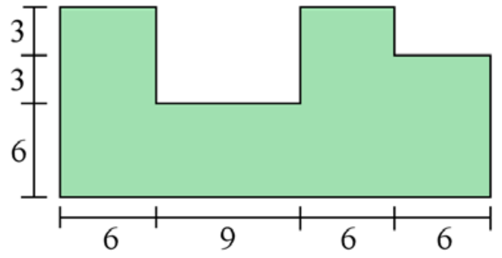


4. Find the area and the perimeter of this figure.
All lines meet at right angles.



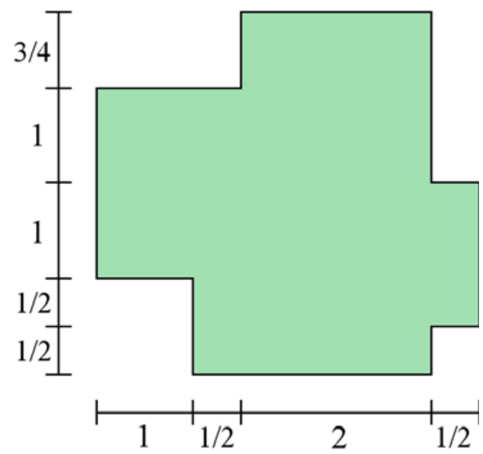
Sample pages from
www.mathmammoth.com

5. Find the area and the perimeter of this figure.
All lines meet at right angles.
The dimensions are given in centimetres.



6. A farmer fenced a rectangular field with 910 m of fencing.
One side of that field measures 330 metres.
How long is the other side of the field?

7. Find the area and the perimeter of this figure.
All lines meet at right angles.
The dimensions are given in metres.



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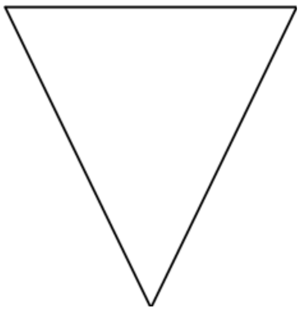
Chapter 9 Revision

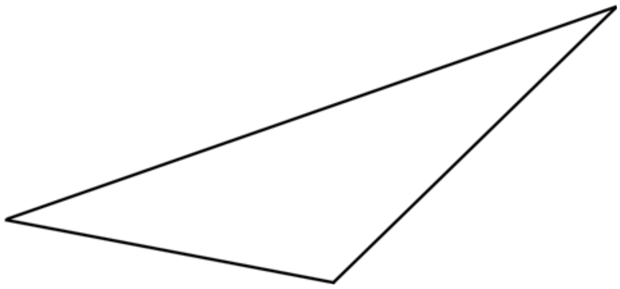
1. Measure all the angles of the triangles. Then classify the triangles.



a. Angles: ________ °, ________ °, ________ °

Acute, obtuse, or right?

Equilateral, isosceles, or scalene?



b. Angles: ________ °, ________ °, ________ °

Acute, obtuse, or right?

Equilateral, isosceles, or scalene?

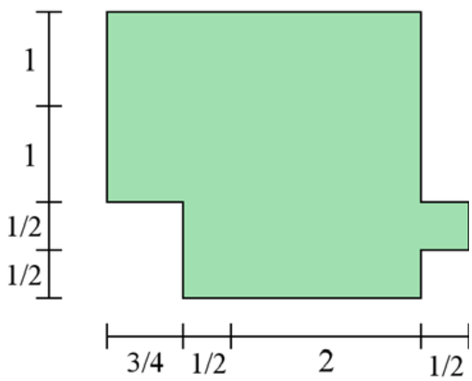
2. **a.** Draw an isosceles triangle with 50° base angles and a 7 cm base side (the side between the base angles).

b. Measure the top angle.

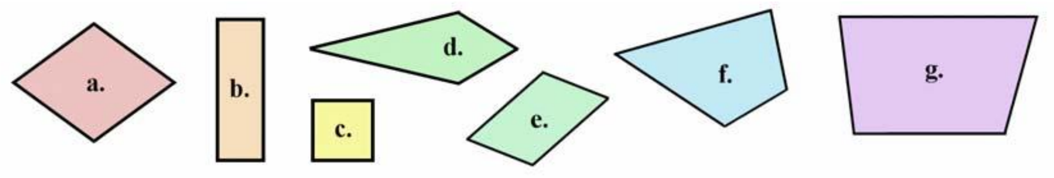
It is ________ ° .

c. Find the perimeter of your triangle in millimetres.

3. Find the perimeter and area of this figure. All measurements are in m.



4. Name the different types of quadrilaterals.



a.

b.

c.

d.

e.

f.

g.

5. Name the quadrilateral that...

a. is a parallelogram and has four right angles.

b. is a parallelogram and has four sides of the same length.

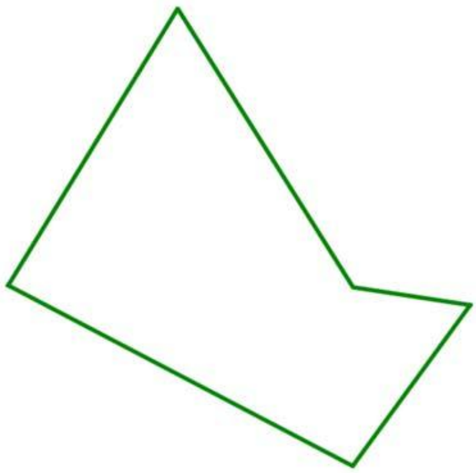
c. has two parallel sides and two sides that are not parallel.

6. **a.** What is this shape called?

b. Divide the shape into three triangles by drawing two diagonals inside it.

c. Number each of the triangles.

d. Classify each triangle according to its sides (equilateral, isosceles, scalene) and according to its angles (acute, obtuse, right).

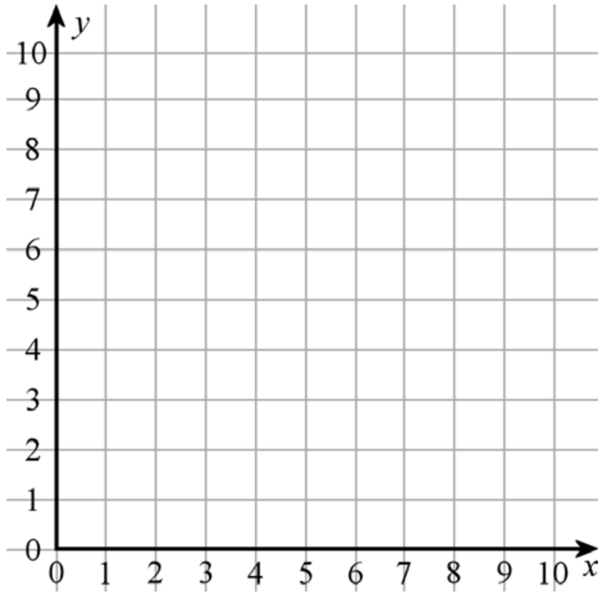


7. **a.** Draw an isosceles obtuse triangle.

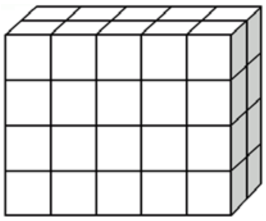
b. Draw a scalene acute triangle.

8. **a.** Draw a circle with its centre at (2, 3) and a radius of 2 units. Use a compass.

   b. Draw another circle with its centre at (6, 5) and a *diameter* of 8 units.



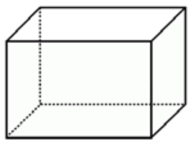
9. Find the volume of this rectangular prism, if…



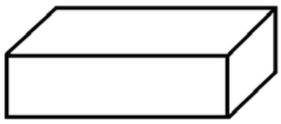
   a. …the edge of each little cube is 1 cm.

   b. …the edge of each little cube is 2 cm.

10. What is the height of this box, if its bottom dimensions are 2 cm × 4 cm and its volume is 32 cubic centimetres?



11. A gift box is 12 cm wide, 6 cm deep, and 4 cm tall. How many of these boxes do you need to have a total volume of 864 cubic cm?



Puzzle Corner

The area of the bottom face of a cube is 16 cm^2. What is its volume?