math MAMMOTH

Grade 5-A Worktext
International Version

The four operations

Large numbers and the calculator

Problem solving

Decimals, part 1

Graphing and statistics

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Light Blue Series

Edition 1/2021

Contents

Chapter 4: Decimals, Part 1

Chapter 5: Statistics and Graphing

Foreword

Math Mammoth Grade 5, International Version comprises a complete maths curriculum for the fifth grade mathematics studies. This curriculum is essentially the same as the *Math Mammoth Grade 5* sold in the United States, only customised for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the fifth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not customary (imperial) units.
- The currency used in word problems is the Australian dollar.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

Fifth grade is when we focus on fractions and decimals and their operations in great detail. Students also deepen their understanding of whole numbers, are introduced to the calculator, learn more problem solving and geometry, and study statistical graphs. The main areas of study in Math Mammoth Grade 5 are:

1. Students develop fluency with addition and subtraction of fractions, and developing understanding of the multiplication and division of fractions.
2. Students finalise fluency with multi-digit addition, subtraction, multiplication, and division (including division with two-digit divisors), and they solve word problems involving all four operations.
3. Students develop a solid understanding of decimal place value and learn to use all four basic operations with decimals.
4. In geometry, students study the concept of volume, and how to measure and calculate it, and classify two-dimensional figures based on their properties.

Additional topics we study are some basic algebraic concepts, large numbers, graphs, prime factorisation, and converting measurements.

This book, 5-A, covers the four operations (chapter 1), large numbers (chapter 2), problem solving (chapter 3), decimals (chapter 4), and statistic (chapter 5). The rest of the topics are covered in the 5-B worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. In fifth grade, chapter 4 (decimals, part 1) should be studied before chapter 6 (decimals, part 2). and chapter 7 (fraction addition & subtraction) before chapter 8 (fraction multiplication & division). However, you can be flexible with all the other chapters and schedule them earlier or later.

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualise the instruction for your student.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For revision, the curriculum includes a worksheet maker (Internet access required), mixed revision lessons, additional cumulative revision lessons, and the word problems continually require usage of past concepts. Please see more information about revision (and other topics) in the FAQ at **https://www.mathmammoth.com/faq-lightblue.php**

I heartily recommend that you view the full user guide for your grade level, available at **https://www.mathmammoth.com/userguides/** and also included in the digital (download) version of the curriculum.

And lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

Chapter 1: The Four Operations
Introduction

We start fifth grade by studying the four basic operations. This includes studying the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought throughout this chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students write simple expressions for problems which they solve. They study the correct order of operations in an expression. An *equation* in mathematics consists of an expression that equals another expression (expression = expression). We also study simple equations, both with and without the help of visual bar models.

Next, we revise multi-digit multiplication, starting with partial products (multiplying in parts) and how that can be visualised geometrically. Then it is time for long division, especially practising long division with two-digit divisors. We also study why long division works, in the lesson *Long Division and Repeated Subtraction*. Throughout the lessons there are also word problems to solve.

Lastly, we study the topics of divisibility, primes, and factoring. Students learn the common divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10. In prime factorisation, we use factor trees.

Although the chapter is named “The Four Operations,” please notice that the idea is not to practise each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are trying to develop the students’ *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, “undo” operations, and so on. Many of the ideas in this chapter are preparing them in advance for algebra.

The Lessons in Chapter 1

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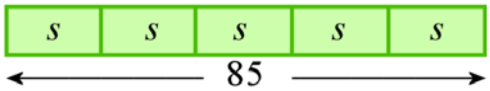
Revision: Multiplication and Division

Multiplication and division have to do with things or groups of the same size.

When you multiply the number of groups by the amount in each group (or the other way around), you get the total.

When you divide the total by the number of groups, you get the amount in each group.

When you divide the total by the amount in each group, you get the number of groups.



Five equal-sized groups make a total of 85. We can write a fact family:

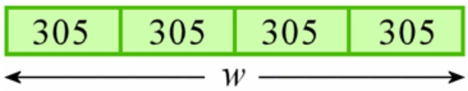
$5 \times s = 85$ $\quad$ $85 \div 5 = s$

$s \times 5 = 85$ $\quad$ $85 \div s = 5$

Which equation above can be used to find (or solve) the unknown s?

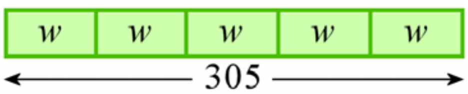
1. Write four equations for each bar model (a fact family). Then solve for w.

a.



w = ___________

b.



w = ___________

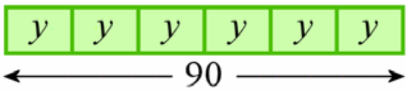
2. Which equation matches which bar model? Also solve for y.

Equations:

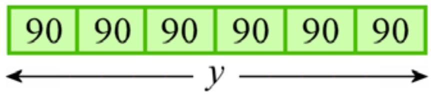
$6 \times y = 90$

$y \div 6 = 90$

a.



b.



3. Draw a bar model to represent the equations. Then solve them.

a. R ÷ 5 = 120

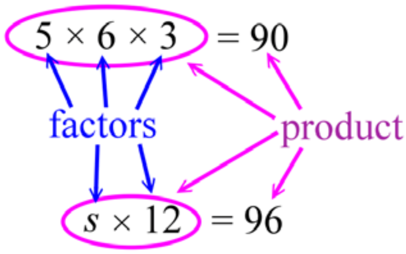
b. 5 × R = 120

c. $y \times 12 = 600$

d. $y \div 12 = 60$

Sample pages from
www.mathmammoth.com

Product and factors



The numbers that are being multiplied are called **factors**.

The result is called a **product**—even if you have not yet calculated it. So "5 × 6" is called a product.

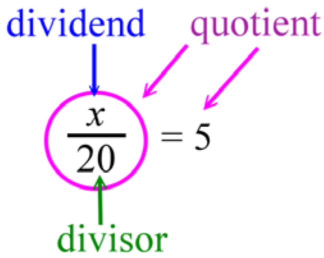
Examples:

5 × 6 is a product. 5 and 6 are the factors.

s × 12 is a product: it is the product of s and 12.

You can call 5 × 6 × 3 the *product written*, and the answer 90 you can call the product that has been *solved* or *calculated*.

Dividend, divisor, and quotient



The number you divide is called the **dividend**. The number you divide by is the **divisor**.

The result is the **quotient**, even if it has not yet been solved.
So "x ÷ 20" is a quotient (of x and 20).

Examples:

The quotient of 100 and 5 is written as 100 ÷ 5, or using the fraction line as $\frac{100}{5}$. We can solve or calculate that to get 20.

The quotient of x and 20 is written $x \div 20$ or $\frac{x}{20}$.

4. Write an expression or an equation to match each written sentence.

a. The product of 52 and 8	**b.** The quotient of 15 000 and 300
c. The product of 4, S, and 18	**d.** The quotient of 80 and x
e. The quotient of 240 and 8 is 30	**f.** The product of 3, 5, and T is 60

5. Write a division equation where the dividend is 280, the quotient is 4, and the divisor is unknown. Use a letter for the unknown. Then find the value of the unknown.

6. Write a division equation where the quotient is 3, the divisor is 91, and the dividend is unknown. Use a letter for the unknown. Then find the value of the unknown.

Look carefully at this expression: $3 \times 47 + 8 \times 47$. Think of it as three copies of 47, and another eight copies of 47. In total, we have 11 copies of 47, or 11×47.

Similarly, $9 \times 165 - 4 \times 165$ is like saying that we have 9 copies of the number 165, and we take away four copies of that number. What is left? Five copies of that number, or 5×165.

7. For each two expressions, decide if the answers are the same or not. Do *not* calculate the answers.

a. $3 \times 417 - 417$ 2×417	**b.** $6 \times 799 - 2 \times 799$ 3×799	**c.** $389 + 389 + 389 + 72 + 72 + 72$ $3 \times 389 + 3 \times 72$
d. 16×68 $9 \times 68 + 7 \times 68$	**e.** $500 - 25 + 19$ $500 - (25 + 19)$	**f.** $832 - 225 - 195$ $832 - (225 + 195)$

8. Which number sentence matches the problem? *You don't have to calculate the answer.*

The sides of a rectangular park measure 26 m and 43 m. Ashley ran around it three times. What is the distance she ran?

a. $(26 + 43) \times 3$ **b.** $3 \times 2 \times (26 + 43)$
c. $26 + 43 + 26 + 43$ **d.** $3 \times 26 + 43 + 26 + 43$

9. Look at the division equations. In each, the *dividend* is the unknown. Explain how you can find the unknown.
(You don't have to actually solve the equations; just explain *how* to solve them.)

$x \div 5 = 4$ $N \div 12 = 60$
$y \div 8 = 100$ $M \div 83 = 149$

10. Look at the division equations. In each, the *divisor* is the unknown. Explain how you can find the unknown.
(You don't have to actually solve the equations; just explain *how* to solve them.)

$16 \div x = 8$ $350 \div N = 50$
$72 \div y = 9$ $120 \div M = 6$

11. Solve for the unknown N or M.

a. $5 \times M = 20$	**b.** $M \div 3 = 5$	**c.** $45 \div M = 5$
d. $4 \times N = 8\ 800$	**e.** $N \div 20 = 600$	**f.** $64\ 000 \div N = 800$

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More Multiplication

Now we will study the multiplication algorithm with a 3-digit number on the bottom. This means we have three partial products to do, so the multiplication process takes three lines.

```
   2 6
   4 2 9
 × 2 2 7
 -------
 3 0 0 3
```

First multiply 7 × 429, ignoring the 2 and 2 in 227.

```
     1
   4 2 9
 × 2 2 7
 -------
 3 0 0 3
 8 5 8 0
```

Next multiply 20 × 429. Place a zero in the ones place, and then multiply as if it was just 2 × 429.

```
     1
     4 2 9
   × 2 2 7
   -------
   3 0 0 3
   8 5 8 0
 8 5 8 0 0
```

Then, 200 × 429. Since you are multiplying by 200, place a zero in the ones *and* in the tens places, and then multiply 2 × 429.

```
       4 2 9
     × 2 2 7
   ---------
     3 0 0 3
     8 5 8 0
 + 8 5 8 0 0
 -----------
   9 7 3 8 3
```

Lastly add.

1. Multiply. Remember: you will need to place *two zeros* in the third line.

a. 191 × 245

b. 409 × 228

c. 246 × 137

d. 815 × 723

e. 207 × 803

f. 125 × 662

2. Practise 4-digit by 2-digit and 5-digit by 2-digit multiplications.

a. $\begin{array}{r} 1\,4\,9\,1 \\ \times \quad 2\,7 \\ \hline \end{array}$	**b.** $\begin{array}{r} 2\,0\,8\,5 \\ \times \quad 3\,5 \\ \hline \end{array}$	**c.** $\begin{array}{r} 8\,1\,1\,6 \\ \times \quad 1\,8 \\ \hline \end{array}$
d. $\begin{array}{r} 2\,0\,5\,1\,6 \\ \times \quad 3\,7 \\ \hline \end{array}$	**e.** $\begin{array}{r} 3\,1\,4\,4\,7 \\ \times \quad 2\,9 \\ \hline \end{array}$	**f.** $\begin{array}{r} 1\,2\,7\,0\,4 \\ \times \quad 4\,5 \\ \hline \end{array}$

3. Solve.

A large shipping container can hold 7 500 kilograms. A company packs 155 boxes of windows in it, each weighing 16 kg. How much weight can they put in the container after that?

4. Revision! Multiply mentally. (Remember the shortcut? Multiply without the zeros, then tag as many zeros at the end of the answer as there are in the factors.)

a. 500 × 200 =	**b.** 30 × 210 =
c. 250 × 40 =	**d.** 2 000 × 400 =
e. 2 × 800 × 20 =	**f.** 30 × 40 × 50 =

When the factors end in zeros, we can take a shortcut! Study the examples carefully.

Example 1:

```
      1 1
      9 5 6
  ×   2 0 0
  ---------
  1 9 1 2 0 0
```

Here, you can first place two zeros in the ones and tens places in the answer, and then just multiply 2 × 956.

Example 2:

```
     4 1 1
         9 5 0
     ×   8 2 0
     ---------
         0 0 0
     1 9 0 0 0
 + 7 6 0 0 0 0
 -------------
   7 7 9 0 0 0
```

Be careful... the first "line" consists totally of zeros. On the second line, first place a zero, then multiply. On the third line, first place TWO zeros, then multiply.

```
   4 1 1
     9 5
   × 8 2
   -----
   1 9 0
 + 7 6 0 0
 ---------
   7 7 9 0
```

It is easier to multiply 82 × 95 and tag two zeros to the final answer to get 779 000.

5. Multiply.

a. 500 × 29 = ____________

Simply multiply 5 × 29, then tag _____ zeros to the final answer.

b. 340 × 210 = ____________

Multiply ______ × ______ , then tag ____ zeros to the final answer.

c. 280 × 700 = ____________

Multiply ______ × ______ , then tag ____ zeros to the final answer.

d. 99 × 9 900 = __________

e. 500 × 1 800 = _________

f. 24 500 × 30 = _________

You can estimate even when there are many operations. Round the numbers in such a way that you can calculate mentally.	$1\ 124 - 2 \times 243$ $\approx 1\ 100 - 2 \times 250 = 600$

6. Solve.

a. $1\ 754 - 5 \times 139$

Estimate: ______________________________

Exact: ______________________________

b. $58 \times (139 + 382)$

Estimate: ______________________________

Exact: ______________________________

7. The McKinleys earn $760 each week, of which they put $120 into savings. How much does the McKinley family put into savings in a year?

8. Multiply mentally in any order. Try to find the *easiest* order!

Note: In the problem $2 \times 18 \times 5$, it is easiest to first multiply 2 and 5!

And in the problem $4 \times 9 \times 25$, it is easiest to first multiply 4 and 25!

Can you see why?

a. $15 \times 2 \times 5 =$ ________ $4 \times 9 \times 25 =$ ________	**b.** $50 \times 7 \times 2 =$ ________ $4 \times 9 \times 0 \times 7 =$ ________
c. $3 \times 6 \times 2 =$ ________ $25 \times 37 \times 4 =$ ________	**d.** $2 \times 7 \times 2 \times 3 =$ ________ $4 \times 9 \times 5 \times 3 =$ ________
e. $50 \times 63 \times 2 =$ ________ $4 \times 19 \times 5 =$ ________	**f.** $11 \times 5 \times 6 =$ ________ $8 \times 9 \times 50 =$ ________

What is a leap year?

Every fourth year is a *leap year*. This means that instead of the year being 365 days long, it is 366 days long. In a leap year, February gets an extra day so it becomes 29 days long.

Leap years occur every four years. Here is a list of some recent and upcoming leap years:

2008, 2012, 2016, 2020, 2024, 2028, 2032, ...

Exception: when the year number is divisible by 100, the year is not a leap year—*unless* the year number is also divisible by 400. Thus, the years 1700, 1800, and 1900 were *not* leap years, whereas 2000 *was* a leap year since 2000 is divisible by 400.

Why do we need a leap year?

The time our Earth takes to go around the sun is not exactly 365 days, but about 365 1/4 days. That is why every four years we "get off" by one day, and we need to add that into the calendar.

9. **a.** How many days were there in the years from 1997 through 2000?

b. How many days were there in the years from 2001 through 2005?

10. (*optional challenge*) Figure out your age in days.
Remember that some years have been leap years.

Puzzle Corner

Try your skills with these two multiplications—they are real challenges!

Let an adult check them or check your work with a calculator.

a.

					4	0	2	7
				x	2	3	9	1
+								

b.

				1	5	8	4	1
		x			3	1	9	5

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Chapter 1 Revision

1. Solve (without a calculator).

a. 7 587 ÷ 27

b. 2 829 ÷ 41

c. 249 × 382

2. Solve 83 493 − y = 21 390.

3. Solve in the right order. You can enclose the operation to be done first in a “bubble” or a “cloud.”

a. 5 × (3 + 8) = __________	**b.** 20 + 240 ÷ 8 + 90 = __________
c. 100 − 2 × 5 × 7 = __________	**d.** 70 − 2 × (2 + 5) = __________

4. Divide mentally, and solve in the right order.

a. $\frac{3\,6\,3\,6}{6}$ =	**b.** $\frac{3\,6\,0\,8}{4}$ =	**c.** $\frac{4\,0\,5\,0}{5}$ =
d. $42 + \frac{255}{5}$ =	**e.** $\frac{4804}{(2+2)}$ =	

5. Find a number to fit in the box so the equation is true.

a. $25 = 7 + \square \times 2$	**b.** $72 \div 8 = (6 - 3) \times \square$	**c.** $(4 + \square) \div 3 = 2 + 2$

6. Write an expression *or* an equation to match each written sentence. You do not have to solve.

a. The difference of x and 9	**b.** The sum of y and 3 and 8 equals 28.
c. The quotient of 60 and b is equal to 12.	**d.** The product of 8, x and y

7. Which expression matches the problem? Also, solve the problem.

Three girls divided equally the cost of buying four sandwiches for $3.75 each. How much did each girl pay?

(1) 3 × $3.75 − 4	**(2)** 3 × $3.75 ÷ 4
(3) $3.75 ÷ 4 × 3	**(4)** 4 × $3.75 ÷ 3

8. Write a *single* expression (number sentence) for each problem, and solve.

a. Bonnie and Ben bought an umbrella for $12 and boots for $17, and divided the cost equally. How much did each one pay?

b. Henry bought five cartons of milk for $4.50 each. The grocer gave him $2 off the total cost. How much did Henry pay?

9. Draw a bar model to represent the equations. Then solve them.

a. R ÷ 4 = 544	
b. 4 × R = 300	

10. Mark an "x" if the number is divisible by 2, 3, 5, 6, or 9.

Divisible by	2	3	5	6	9
534					
123					

Divisible by	2	3	5	6	9
1 605					
2 999					

11. Factor the following numbers to their prime factors.

a. 21 / \	**b.** 12 / \	**c.** 38 / \
d. 75 / \	**e.** 124 / \	**f.** 89 / \

Chapter 2: Large Numbers and the Calculator
Introduction

In this chapter, we study large numbers and place value up to billions—that is, up to 12-digit numbers. Students will also add, subtract, and round large numbers, and learn about exponents and powers. Concerning exponents and powers, the focus is on powers of ten (such as 10^2, 10^5, 10^8, and so on), which is what the student should master in this grade level. If your student has difficulties with exponents in general, there is no need to worry. Exponents and powers are taught from scratch again in Math Mammoth grade 6.

In this chapter, students will be introduced to the calculator for the first time, and therefore they will need a simple calculator (preferably a physical one, not one on a computer or other device) about half-way through this chapter.

I have delayed the use of a calculator (as compared to many other maths curricula) for good reasons. I have received numerous comments on the harm that indiscriminate calculator usage can cause. If children are allowed to use calculators freely, their minds get "lazy," and they will start relying on calculators even for simple things such as 6 × 7 or 320 + 50. It is just human nature!

As a result, students may enter college without even knowing their multiplication tables by heart. Then they have trouble if they are required to use mental maths to solve simple problems.

Therefore, we educators need to *limit* calculator usage until the students are much older. Children *cannot* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is extremely useful, and students do need to learn how to use it. In this curriculum, I try to not only show the students how to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practise estimating the result before using a calculator to find the exact answer, and choosing whether mental maths or a calculator is the best "tool" for the calculation.

The Lessons in Chapter 2

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Adding and Subtracting Large Numbers

Just like 25 marbles + 54 marbles = 79 marbles, so will 25 million + 54 million = 79 million.

Just keep in mind:

A thousand thousands makes a million, and **a thousand millions makes a billion**.

800 000 + 200 000	**Half a million**
Think of it as 800 thousand + 200 thousand. The answer is 1 000 thousand or 1 000 000.	Think of it as half of a thousand thousands, or 500 thousands = 500 000.
34 999 000 + 1 000	**2 billion − 300 million**
This is 34 million 999 thousand + 1 thousand, making 34 million 1 000 thousand or 35 million.	Think of it as 2 000 million − 300 million, which makes 1 700 million or 1 700 000 000.

1. Add.

	a. 90 000	**b.** 99 000 000	**c.** 999 000
+ 1 000			
+ 10 000			
+ 100 000			
+ 1 000 000			

2. Match.

a.		**b.**	
1/2 million	750 000	1 million − 50 000	100 000 000
a hundred hundreds	100 000	1 million − 500 000	500 000
1/10 million	10^6	10^8	950 000 000
1/4 million	500 000	1 billion − 500 million	1/2 billion
3/4 million	10^4	1 billion − 50 million	950 000
a thousand thousands	200 000	1 million − 5 000	995 000
2/10 million	250 000	1 billion − 5 million	995 000 000

3. Add or subtract. Simply write the numbers under each other, lining up the digits and the spaces in the same places. Use the usual addition or subtraction algorithm, regrouping the same way as you have learned before.

a. 329 145 000 + 2 809 125 093	**b.** 5 049 + 45 390 000 + 5 483 700
c. 45 700 + 90 567 000 + 2 560 + 2 300 560	**d.** 290 800 + 254 000 230 + 56 391 + 2 381
e. 480 560 000 − 23 980 000	**f.** 1 000 000 − 156 990
g. 22 300 000 − 4 431 190	**h.** 7 014 289 000 − 3 103 559 391

4. Subtract and compare. Think of each million as a *thousand thousands*!
Use extra paper if necessary.

a. 1 million − 500 thousand =	**b.** 1 million − 50 thousand =
1 million − 100 thousand =	1 million − 10 thousand =
3 million − 400 thousand =	7 million − 20 thousand =

5. Continue counting for seven more numbers in each set:

a.	**b.**	**c.**
458 000 000	79 650 000	450 996 000
468 000 000	79 800 000	450 997 000
478 000 000	79 950 000	450 998 000
Each difference is ______________	Each difference is ______________	Each difference is ______________

6. Complete the addition path.

35 647 000 → add 10 000 → [ ] → add a million → [ ]

↓ add 100 thousand

[ ] ← add 10 million ← [ ] ← add a thousand ← [ ]

Puzzle Corner

Solve for the unknown x or N.

a. $x + 400\ 000 = 4\ 000\ 000$

$x =$ ____________________

b. $x - 350\ 000 = 2\ 000\ 000$

$x =$ ____________________

c. $200\ 000 + N + 600\ 000 = 7\ 000\ 000$

N = ____________________

d. $2 \times N = 3\ 000\ 000$

N = ____________________

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The Calculator

A **calculator** has buttons for each of the numbers from 0 to 9. The button with the plus (" + ") sign is used for addition. Similarly, the minus (" − ") button for subtraction, the times (" × ") or (" * ") button for multiplication, and divide (" ÷ ") or (" / ") button for division. To get an answer, press " = ".

For example, to calculate 34 × 2 492, press

[3] [4] [x] [2] [4] [9] [2] [=]

and the calculator should show you 84728.

In this lesson, use your calculator for *every exercise*. In other lessons, use a calculator only if you see the little calculator image next to the exercise.

1. Calculate with a calculator. Be careful when pushing the buttons! Go slowly, until you get used to the calculator and where the different buttons are.

a. 14 × 65	**b.** 15 624 ÷ 42
c. 539 + 8 409	**d.** 7 600 − 4 293

2. Find the decimal point on your calculator. Use it when you input prices.
Now, find the total cost of buying 15 chairs at $35.90 apiece.

3. Find the total cost. First, estimate it by using rounded numbers and mental maths. Then calculate the exact cost with a calculator.

a. six pairs of rubber boots for $14.90 each My estimation: ____________________ ______________________________ Exact answer: ____________________	**b.** 28 boxes of pencils for $2.25 each My estimation: ____________________ ______________________________ Exact answer: ____________________

4. **a.** Press [5] [−] [1] [=] on your calculator. It should show 4. Now, press [=] again. It should apply the *same operation* of subtracting one, so that you get 3. Press [=] again and again, until you reach zero. What happens if you continue the same way?

b. Do the same thing as in part (a), but this time start with 7, and subtract 2 each time.

c. Do the same thing as above, but this time start with [5] [x] [2] [=]. Then, press [=] repeatedly. What happens?

5. Calculate with a calculator. *Hint:* you can use the shortcut that you found in the previous exercise.

a.	**b.**	**c.**
$5^1 = 5$	$5^5 =$ ____________	$5^9 =$ ____________
$5^2 = 5 \times 5 = 25$	$5^6 =$ ____________	$5^{10} =$ ____________
$5^3 = 5 \times 5 \times 5 = 125$	$5^7 =$ ____________	$5^{11} =$ ____________
$5^4 = 5 \times 5 \times 5 \times 5 =$ ______	$5^8 =$ ____________	$5^{12} =$ ____________

6. Multiply 8 by itself repeatedly. *Note:* If the answers to the last problems do not fit into your calculator screen, just leave them empty.

a.	**b.**	**c.**
$8^1 = 8$	$8^4 =$ ____________	$8^7 =$ ____________
$8^2 = 8 \times 8 = 64$	$8^5 =$ ____________	$8^8 =$ ____________
$8^3 = 8 \times 8 \times 8 =$ ______	$8^6 =$ ____________	$8^9 =$ ____________

7. Look at the powers of 5 and of 8 that you calculated in the previous exercises.

a. Which power of 5 was the first one that was more than one million?

b. Which power of 8 was the first one that was more than one million?

8. For each given number, find the *smallest* power of the number that is more than or equal to one million. Note: For some of these, there is no answer. Write *impossible* as your answer.

a. $12^{\square} > 1\,000\,000$	**b.** $8^{\square} > 1\,000\,000$	**c.** $42^{\square} > 1\,000\,000$
d. $11^{\square} > 1\,000\,000$	**e.** $2^{\square} > 1\,000\,000$	**f.** $0^{\square} > 1\,000\,000$
g. $10^{\square} = 1\,000\,000$	**h.** $1^{\square} > 1\,000\,000$	**i.** $100^{\square} = 1\,000\,000$

9. **a.** Just to remind you, we have a *shortcut* for repeated addition, such as 20 + 20 + 20 + 20 + 20 + 20. What is it?

b. Now use your calculator. How many times do you need to add 20 repeatedly in order to reach 1 million?

c. How many times do you need to add 40 repeatedly in order to reach 1 million?

d. How many times do you need to add 5 000 repeatedly in order to reach 1 million?

> Most calculators don't know the order of operations. They always calculate first whatever operation you input first.
>
> This means that to calculate 3 × (52 − 9), you need to *first* press [5] [2] [−] [9] [=]
>
> Leave the answer "43" on the screen, and then press [×] [3] .

10. Does your calculator know the order of operations? It might, or it might not (calculators vary on this). Check and see using the following problems.

a. Input, in this order: 6 + 2 × 4 Will your calculator know to do 2 × 4 first, or will it calculate 6 + 2 first, and multiply that by 4? If it doesn't do the multiplication before addition, you can input the problem like this: **2 × 4 + 6**	**b.** Input, in this order: 40 − 3 × 10 Will your calculator know to do 3 × 10 first, or will it calculate 40 − 3 first, and multiply that by 10? If it doesn't do the multiplication before subtraction, you need to help it calculate this in the right order. First, input 3 × 10 and make a mental note of the answer (or learn to use the calculator's memory buttons). Then, input 40 − (your previous answer).

11. Calculate. (You may use a calculator... but mental maths might be quicker!)

a. 4 × (45 + 55)	**b.** 100 − 6 × 7	**c.** 100 ÷ 4 + 5 × 5
d. 60 + 3 × 5 + 8	**e.** 45 ÷ (7 + 8)	**f.** 4 × (11 + 10) + 20

12. First estimate using rounded numbers and mental maths. Then find the exact answer with a calculator.

a. What is your change, if you pay for seven drinks that cost $3.90 apiece with $40? My estimation: ____________________ __________________________________ Exact answer: ____________________	**b.** Kristen bought six boxes of crayons for $1.45 per box and one set of pencils for $9.80. What was the total cost? My estimation: ____________________ __________________________________ Exact answer: ____________________

13. Find a logical way to continue the number sequences, and then continue them for 5 more numbers.

a. 1 2 4 8 16

b. 12 72 432 2 592

c. 1 000 000 000 100 000 000 10 000 000 1 000 000

d. 1 048 576 524 288 262 144 131 072

Chapter 3: Problem Solving
Introduction

We start out this chapter by studying simple equations, presented as pan balance puzzles. The pan balance works very well for modelling the process of solving equations. In the second lesson, students use the bar model to help them solve equations. The equations on this level are very simple. More complex equations are presented in grades 6 and especially in grade 7 (pre-algebra).

The bulk of this chapter is then spent on the topic of problem solving, focusing on problems that involve a fractional part of a whole in some manner.

Encourage the student to draw a bar model for the problems, as it is such a helpful tool. Some of the problems here could even be found in regular Algebra 1 textbooks where naturally, they would be solved with algebra. However, the bar model enables us to solve them without algebra; yet, it helps the students' algebraic thinking. Essentially, one block in the bar model corresponds to the unknown x in an equation.

The Lessons in Chapter 3

Helpful Resources on the Internet

You can also access this list of links at **https://l.mathmammoth.com/gr5ch3**

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DISCLAIMER: *We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*

Pan Balance - Shapes
This interactive balance builds algebraic thinking. Find the weight of each shape by placing shapes on the two pans. Try to find situations where the weights are equal. One square always weighs 1 unit.
https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pan-Balance----Shapes/

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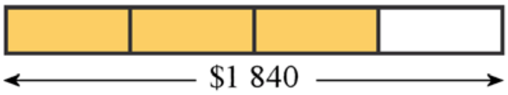
Problem Solving with Bar Models 1

A fractional part of a whole

Problem. Jackie earns $1 840 monthly and Jessie earns 3/4 as much. How much does Jessie earn?



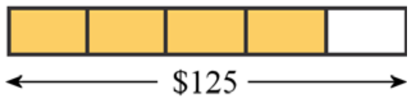
Solution. In the model, Jackie's salary is divided into four equal parts (blocks). To find 3/4 of it, first find 1/4 of it, which is **one block** in the model.

$1 840 ÷ 4 = $460

Then multiply that result by three:
3 × $460 = $1 380.
So, Jessie earns $1 380.

Solve. Draw a bar model. Write an expression (number sentence) for each calculation you do.

1. A camera that cost $125 was discounted by 1/5 of its price. What is the new price?



_______ ÷ _______ = _______

____

2. A pizza that weighs 680 g is divided into five equal pieces. How much do two pieces weigh?

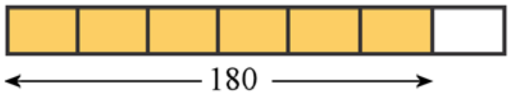
_______ ÷ _______ = _______

_______ × _______ = _______

3. A bottle of water costs 2/3 as much as a bottle of juice that costs $1.50. How much do *two* bottles of water and *two* bottles of juice cost?

A Fractional Part More

Problem. The school year in country A is 180 days long. In country B it is 1/6 part longer than that. How long is the school year in country B?



Solution. First, we divide the 180-day school year into 6 parts, to find how much one "block" is in the model:

180 ÷ 6 = 30. So, one block is 30 days.

Then we *add* one-sixth more to the whole bar model, and that is how long the school year is in country B.

180 + 30 = 210

So, the school year in country B is 210 days long.

Solve. Draw a bar model. Write an expression (number sentence) for each calculation you do.

4. The price of a train ride is $12. It went up by 1/6.
   What is the new price?

   _______ ÷ _______ = _______

   _______ + _______ = _______

5. A cafeteria lunch used to cost $4.50 but the price was increased by 1/5. What is the price now?

6. A one-way bus ride from Helen's home to town costs $1.
   The bus company will raise the price by 1/10 in June.

   a. How much will a one-way ride cost in June?

   b. How much more will a two-way ride (home to town to home) cost Helen in June than in May?

7. A T-shirt cost $10.50, but now it is discounted by 2/5 of its price.
   Annie bought *ten* shirts with the discounted price. What is the total cost?

8. Duckville has 3 687 inhabitants, which is 3/5
   of the number of inhabitants in Eagleby.
   How many people *in total* live in Eagleby and Duckville?

Puzzle Corner

A package of 10 small envelopes costs $2.50, and a package of 10 large ones costs 2/5 more. Find the total cost of buying 50 envelopes of each kind.

Mixed Revision Chapter 3

1. Draw a bar model where the total is 547, and the three parts are 119, 38, and x. Lastly solve for x.
(Revision: Addition and Subtraction/Ch.1)

2. The washer uses about 53 litres of water for a load of laundry. If you run the washer three times a week, how much water do you use in a year?

3. Write an equation to match each written sentence. (Revision: Addition and Subtraction/Ch.1)

a. The difference of 16 and 7 is 9.
b. The sum of 3, 9, and y is 20.

4. Find the missing number so that the equation is true. (The Order of Operations and Equations/Ch.1)

a. $42 = (7 + \square) \times 2$	**b.** $480 \div 8 = 10 \times 5 + \square$	**c.** $4 + \square = (200 - 50) \div 2$

5. Which of the following calculations can be used to check the division $458 \div 7 = 65$ R3?
(Long Division/Ch.1)

a. $3 \times 65 \times 7$ **b.** $65 + 7 \times 3$ **c.** $7 \times 65 + 3$ **d.** $(7 + 65) \times 3$

6. Determine if the two expressions have the same value without calculating anything.
(Revision: Multiplication and Division/Ch.1)

a. 3 289 − 144 − 276	**b.** 9 × 283 − 5 × 283	**c.** 5 × 636
3 289 − (144 + 276)	4 × 283	2 × 636 + 2 × 636

7. Factor the following composite numbers to their prime factors. (Prime Factorization/Ch.1)

a. 64 / \	**b.** 60 / \	**c.** 85 / \

8. Divide. Use the space to write a multiplication table for the divisor. Lastly check.
(A Two-Digit Divisor 1/Ch.1)

	$79\overline{)8\ 9\ 2\ 7}$	$\times\ 7\ 9$ _____

9. Fill in the missing parts. (Exponents and Powers/Ch. 2)

a. $2 \times 10^4 =$ ________	**b.** $712 \times 10^3 =$ ________	**c.** $55 \times 10^6 =$ ________
d. $6 \times 10^{\square} = 6\ 000$	**e.** $18 \times 10^{\square} = 180\ 000\ 000$	**f.** $69 \times \square^{\square} = 69\ 000\ 000$

10. First estimate the answer using rounded numbers and mental maths. Then find the exact answer with a calculator. (The Calculator/Ch. 2)

a. How many minutes are there in a year? Estimation: ________________ ________________ Exact answer: ________________	**b.** Emma bought five stuffed animals for $5.90 each and five sets of pencils for $2.35 each, to give as gifts. What was the total cost? Estimation: ________________ ________________ Exact answer: ________________

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Chapter 4: Decimals, Part 1
Introduction

In this first chapter about decimal arithmetic, students study place value with decimals, add and subtract decimals, and learn to multiply and divide decimals by whole numbers. We study more about decimal multiplication and division in chapter 6, along with conversions between measurement units. Some of the decimal lessons can appear boring, plus there are quite a few of them, so I hope that by breaking up the decimal topics into two chapters, students will not get "bogged down" by the number of topics to study. It can also help them retain the concepts, because we revise some topics from this chapter in chapter 6.

The first two lessons deal with place value, first with tenths and hundredths (up to two decimal digits), and then with thousandths (three decimal digits). Then we briefly look at decimals on a number line. These first lessons are very important, since understanding decimal place value is the foundation for understanding operations with decimals.

We start building on this foundation in the lesson *Add and Subtract Decimals—Mental Maths*. Students solve sums such as 0.8 + 0.06 based on their knowledge of place value. The value of that sum is 0.86, not 0.14, like students with a misconception could answer.

Adding and subtracting decimals in columns comes next. This is the common algorithm where the decimal points (or all places) need to be lined up before adding or subtracting. Students also learn to compare and round decimals.

Then lastly for this chapter, we study multiplying and dividing decimals by whole numbers, both using mental maths, and using column-multiplication and long division. The mental maths strategies are based on understanding decimal place value. One reason I include so many mental calculations is because they help students understand the concepts of decimal arithmetic and of place value.

You might wonder why *Math Mammoth Grade 5* presents decimals before fractions. The traditional way is to teach fractions first because then we can show that decimals are simply fractions of a specific type—namely, they are fractions with denominators that are powers of ten (for example, 0.45 is simply the fraction 45/100).

There are several reasons I present decimals before fractions. First, students have studied some about both decimals and fractions in earlier grades, so they should have the necessary background to comprehend that the decimals we are studying here *are* fractions. Therefore, I see no need to study all fraction arithmetic in 5th grade before decimal arithmetic.

Secondly, I feel that decimal arithmetic is somewhat easier than fraction arithmetic, and students already know more about it than they know about all the fraction arithmetic that is studied in 5th grade (in 5-B). Thus, studying decimal arithmetic first may be easier for some students.

The Lessons in Chapter 4

Helpful Resources on the Internet

You can also access this list of links at **https://l.mathmammoth.com/gr5ch4**

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DISCLAIMER: *We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*

Decimal Arithmetic - Videos by Maria
These are my videos where I explain all about decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Suitable for grades 5-6.
https://www.mathmammoth.com/videos/grade_5/5th-grade-videos.php#decimals

CONCEPT OF DECIMAL

Decimal Demonstrator
An interactive visual model that uses cups to demonstrate decimal numbers up to two decimal digits.
https://www.ictgames.com/mobilePage/decimalDemonstrator/

Zoomable Decimal Number Line
Click on this interactive number line to zoom in more and more and explore decimal numbers.
https://www.mathsisfun.com/numbers/number-line-zoom.html

Decimals Line
An illustrative tool that a teacher can use to demonstrate decimals on a number line.
https://www.transum.org/Software/SW/Number_Line/Decimals.asp

Scales
Move the pointer on the scale to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
https://www.interactivestuff.org/sums4fun/scales.html

Puppy Pull Game: Fractions to Decimals
Help your team win the tug contest by converting fraction words to decimal numbers!
https://www.mathplayground.com/ASB_Puppy_Pull_Decimals.html

Fractions & Decimals Matching Mystery Picture Game
Find matching pairs of fractions and decimals while uncovering a hidden picture.
https://www.mathmammoth.com/practice/fractions-decimals

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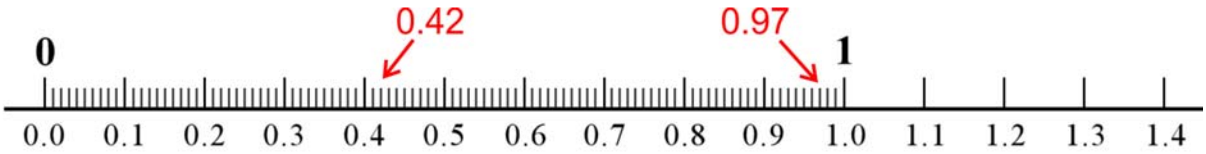
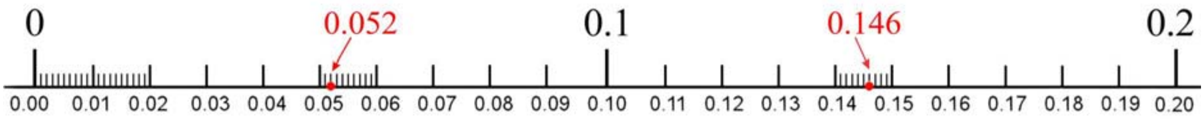
Decimals on a Number Line



This number line shows the interval from 0 to 1 divided into 100 parts—hundredths.
The tenths are marked, and in between each two tenths are little tick marks marking the hundredths.

Now we will zoom in to just a part of the above number line.

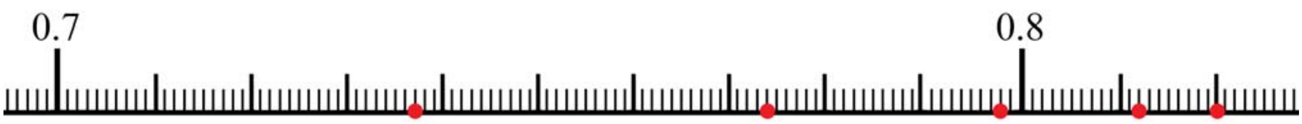


The labels 0.1 and 0.2 have been moved above the tick marks for clarity.
The tick marks for hundredths have been labelled, all the way from 0.00 to 0.20.

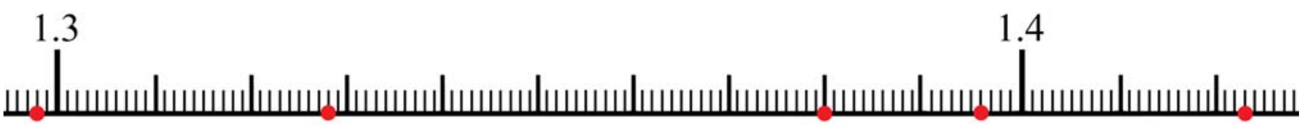
In between 0.00 and 0.01 (and also in between and 0.01 and 0.02) are new tick marks, marking *thousandths*. They divide the interval from 0.00 to 0.01 (which is one hundredth) into ten parts. The marks are so tiny, you may not even see them clearly, but they are there!

1. Mark these decimals on the number line: 3.6 3.43 3.89 4.11 2.98 4.05

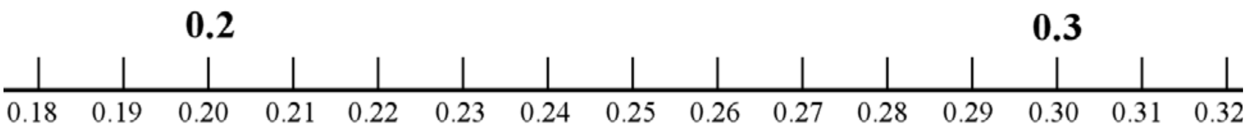
2. Now we have zoomed in to see the *thousandths*. Write what decimals are indicated by the dots on the number line.



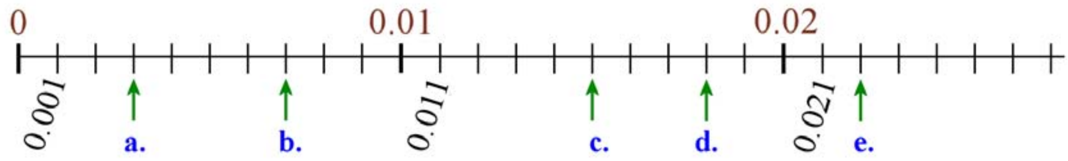
3. Write what decimals are indicated by the dots on the number line.



4. Mark these decimals on the number line: 0.187, 0.205, 0.252, 0.301, and 0.314 in approximate locations. Imagine little tick marks between each two neighbouring hundredths.



5. Write the decimals indicated by the arrows.



a. ____________ **b.** ____________ **c.** ____________ **d.** ____________ **e.** ____________

6. What kind of parts do you get?

When you divide one whole into ten equal parts, you get ________________________________.

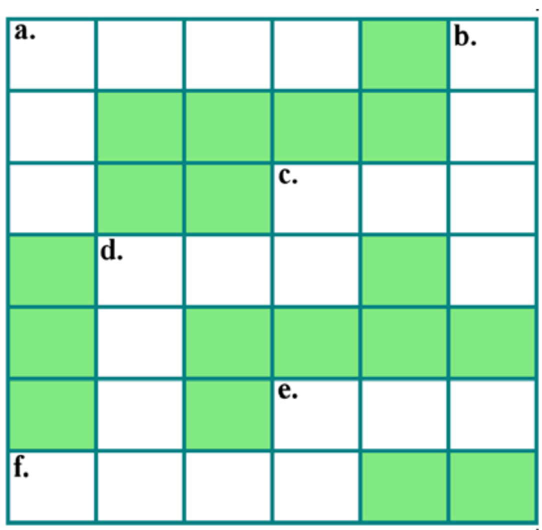
When you divide one whole into a hundred equal parts, you get ___________________________.

When you divide one tenth into ten equal parts, you get ________________________________.

When you divide one hundredth into ten equal parts, you get _____________________________.

7. Place the numbers in the cross-number puzzle. Place the decimal point in the same square as the value for the ones place. (It does not take up a square of its own.)

Across:	*Down:*
a. 2 and 49 thousandths	**a.** 2 and nine hundredths
c. 2 and 7 hundredths	**b.** 3 and 76 thousandths
d. 5 hundredths	**c.** 2 and five tenths
e. 71 hundredths	**d.** 3 thousandths
f. 392 thousandths	**e.** 2 tenths



Add and Subtract Decimals—Mental Maths

What number is one tenth (0.1) more than the number in the chart?

H	T	O		t	h	th
1	0	9	.	7	5	

Look at the **tenths digit** (which is 7), and add ONE to it.
We get eight... signifying 8 tenths. So, the new number is 109.85.

What number is three thousandths (0.003) more than the number in the chart?

H	T	O		t	h	th
		5	.	0	8	2

Look at the **thousandths digit** (which is 2), and add THREE to it.
We get five... signifying 5 thousandths. So, the new number is 5.085.

1. Use the place value chart to help you, and write the decimal that is...

a.

O		t	h	th
0	.	2	8	5

one tenth more than 0.285 __________

one hundredth more than 0.285 __________

one thousandth more than 0.285 __________

b.

T	O		t	h	th
		.			

one tenth more than 2.06 __________

one hundredth more than 2.06 __________

one thousandth more than 2.06 __________

c.

O		t	h	th
	.			

one tenth more than 0.605 __________

one hundredth more than 0.605 __________

one thousandth more than 0.605 __________

d.

H	T	O		t	h	th
			.			

one tenth more than 832.7 __________

one hundredth more than 832.7 __________

one thousandth more than 832.7 __________

2. Write the decimal that is the given amount more than the given decimal.

a.

O		t	h	th
0	.	3	9	

4 thousandths more: __________

b.

O		t	h	th
0	.	0	1	6

2 tenths more: __________

c.

O		t	h	th
1	.	0	4	

5 hundredths more: __________

d.

H	T	O		t	h	th
1	2	0	.	3		

1 hundredth more: __________

e.

T	O		t	h	th
5	5	.	6	7	8

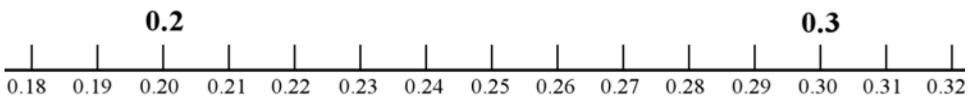
2 tenths more: __________

f.

O	O		t	h	th
4	1	.			

7 thousandths more: __________

0.2 + 0.04 = _____



If you are at 0.2 and you go four hundredths (0.04) further, where will you end up?
Some children think that the answer is 0.6 or 0.06. What do you think?

$$\frac{2}{10} + \frac{4}{100}$$
$$\downarrow \quad \downarrow$$
$$\frac{20}{100} + \frac{4}{100} = \frac{24}{100}$$

$$0.2 \quad + \; 0.04$$
$$\downarrow \qquad \downarrow$$
$$0.20 \; + \; 0.04 \; = 0.24$$

Let's write 0.2 and 0.04 as fractions: 2/10 and 4/100. They have different denominators (10 and 100). So, before we add them, we need to convert 2/10 into 20/100. Then we can add easily. (See the addition on the left.)

It turns out we can add 0.2 + 0.04 using a similar idea! We can "tag" a zero onto the end of 0.2 so it becomes 0.20 (this does *not* change its value). Now *both* decimals have hundredths. This is the exact same process as writing 2/10 as 20/100.

Then, it is easy to add: 20 hundredths and 4 hundredths makes 24 hundredths, or 0.20 + 0.04 = 0.24.

A "trick" or shortcut: "Tag" (decimal) zeros to the end of the decimal numbers so that all of them have the same amount of decimal digits. Then it is easy to add or subtract them.

Example 1. 0.8 + 0.003 = $0.8\underline{00}$ + 0.003 = 0.803

Notice how both numbers end up being so many *thousandths* (800 thousandths and 3 thousandths).

Example 2. 0.205 + 0.04 = 0.205 + $0.04\underline{0}$ = 0.245

Note 1: Essentially, this corresponds to lining up the decimal points and the different places, when adding decimals in columns.

Note 2: This trick does not work with whole numbers. For example, if we tag a zero to the end of 7, it becomes 70... its value changes! Tagging *decimal* zeros to a number does not change its value.

3. Add. First, make the decimals have the same amount of decimal digits by tagging zero(s) to them.

a. 0.009 + 0.006 = __________	**b.** 0.8 + 0.6 = __________
0.009 + $0.06\underline{0}$ = __________	0.8 + 0.06 = __________
0.009 + 0.6 = __________	0.8 + 0.006 = __________

4. Add or subtract.

a. 0.7 + 0.005 = __________	**b.** 0.7 − 0.5 = __________
0.07 + 0.5 = __________	0.7 − 0.07 = __________
0.007 + 0.05 = __________	0.05 − 0.007 = __________

5. Add or subtract.

a. 0.2 + 0.9 = ________ 0.02 + 0.9 = ________ 0.02 + 0.009 = ________	**b.** 0.77 − 0.3 = ________ 0.77 − 0.03 = ________ 0.77 − 0.003 = ________

6. These students have some misconceptions. Correct their answers—and learn from them!

a. Laura thinks: Both of these decimals have two zeros after the decimal point so I will put the same two zeros in the answer. 0.008 + 0.003 = 0.0011	**b.** Jessie reasons: Just add the decimal parts separately from the whole number parts: 0.7 + 0.7 = 0.14

1 whole = 10 tenths = 100 hundredths = 1000 thousandths
1 = 1.0 = 1.00 = 1.000

Example 3. Solve 1 − 0.64 = ?

Think of 1 whole as 100 hundredths. Then, the question becomes:
100 *hundredths* − 64 *hundredths* = ? It is like 100 apples − 64 apples!
The answer is 36 apples... oops... I meant, 36 hundredths :^) (0.36).

Example 4. Solve 0.219 + ______ = 1

This time, think of 1 whole as 1 000 thousandths, so the question becomes:
219 *thousandths* plus what makes 1 000 *thousandths*? It is 781 thousandths, or 0.781.

7. Complete the additions so the sum is 1.

a. 0.6 + ______ = 1	**d.** 0.99 + ______ = 1	**g.** 0.999 + ______ = 1
b. 0.60 + ______ = 1	**e.** 0.87 + ______ = 1	**h.** 0.002 + ______ = 1
c. 0.61 + ______ = 1	**f.** 0.22 + ______ = 1	**i.** 0.304 + ______ = 1

8. Subtract from 1.

a. 1 − 0.01 = ______	**d.** 1 − 0.001 = ______	**g.** 1 − 0.506 = ______
b. 1 − 0.04 = ______	**e.** 1 − 0.008 = ______	**h.** 1 − 0.56 = ______
c. 1 − 0.51 = ______	**f.** 1 − 0.021 = ______	**i.** 1 − 0.411 = ______

If you are 1.33 on the number line, and you go seven thousandths (0.007) further, where do you end up? You will be at 1.337.

This illustrates the addition

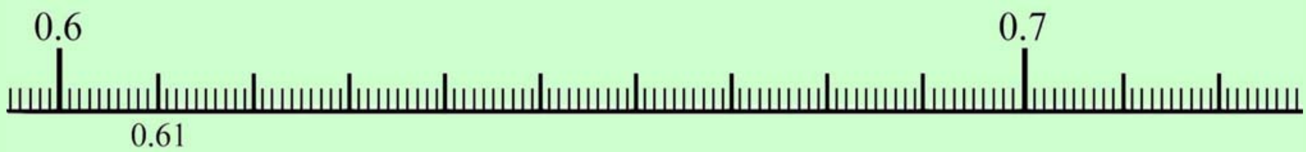
1.33 + 0.007 = <u>1.337</u>

If you are 1.4, and you go 11 thousandths further, where will you end up?
At 1.411.

The addition is:

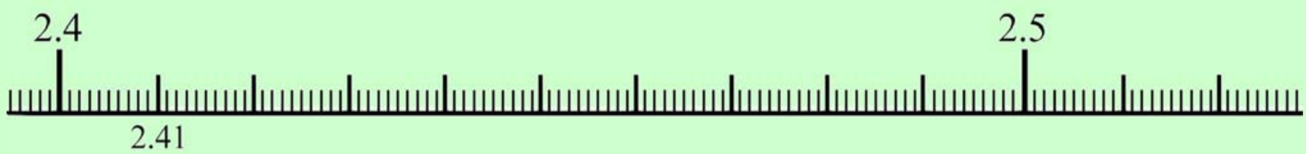
1.4 + 0.011 = <u>1.411</u>

9. Use the number line to add.



a. 0.6 + 0.004 = ________	**b.** 0.63 + 0.002 = ________	**c.** 0.68 + 0.001 = ________
d. 0.7 + 0.008 = ________	**e.** 0.71 + 0.013 = ________	**f.** 0.69 + 0.015 = ________

10. Use the number line to add.



a. 2.42 + 0.003 = ________	**b.** 2.47 + 0.008 = ________	**c.** 2.5 + 0.009 = ________
d. 2.4 + 0.001 = ________	**e.** 2.4 + 0.01 = ________	**f.** 2.45 + 0.03 = ________

11. Write in expanded form.

a. 24.09 =

b. 4 500.1 =

c. 0.294 =

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Divide Decimals by Whole Numbers 1

To divide a decimal by a whole number with long division is very easy.

Simply divide normally, as if there were no decimal point. Then, **put the decimal point in the quotient in the same place** as it is in the dividend.

See the example on the right. It is your task to finish checking the division by multiplication. Verify that the multiplication gives you the original dividend, 41.51.

```
    0 5.9 3
  ---------
7 ) 4 1.5 1
  - 3 5
  -----
      6 5
    - 6 3
    -----
        2 1
      - 2 1
      -----
          0
```

Check:

```
   5.9 3
×      7
--------
```

1. Divide. Check each division result with multiplication.

a. $5 \overline{)\,5.30}$ Check:	**b.** $6 \overline{)\,2.388}$ Check:
c. $19 \overline{)\,23.94}$ Check:	**d.** $23 \overline{)\,57.638}$ Check:

You know that when dividing whole numbers, there can be a remainder. For example, 24 ÷ 5 = 4 R4. But, we can continue such divisions into decimal digits. To do that, add decimal zeros to the dividend.	**Example 1.** This is the division 24 ÷ 5 but with 24 written as 24.0. It is actually an even division, with a quotient of 4.8.	$$\begin{array}{r} 0\ 4.8 \\ 5\overline{)2\ 4.0} \\ \underline{2\ 0} \\ 4\ 0 \\ \underline{-\ 4\ 0} \\ 0 \end{array}$$ **Check:** $$\begin{array}{r} {}^{4} \\ 4.8 \\ \times\ \ 5 \\ \hline 2\ 4.0 \end{array}$$

How do you know how many decimal zeros to add to the dividend, so the division will be even?

You cannot tell that before you divide. Just start with maybe 2-3 zeros, and see how the division goes. You can always add more zeros to the dividend if you need to. Besides, not every decimal division is even! We will see an example of that on the next page.

2. Divide in two ways: first by indicating a remainder, then by long division. Add a decimal point and decimal zeros to the dividend. Lastly, check your answer by multiplying.

a. 31 ÷ 4 = ______ R _____ $4\overline{)3\ 1}$ Check:	**b.** 56 ÷ 5 = ______ R _____ $5\overline{)5\ 6}$ Check:
c. 15 ÷ 8 = ______ R _____ $\overline{)}$ Check:	**d.** 45 ÷ 20 = ______ R _____ $\overline{)}$ Check:

Sometimes a decimal division is not even, but just keeps on going forever, like the one below! In that case, **stop the division** at some point, and **give the answer as a rounded number.**

Example 2. Seven people shared evenly the cost of a meal for $99.90. How much did each person pay?

This has to do with money, so the answer needs to have two decimal digits. That means we need to calculate the answer to three decimals (so we can then round it to two decimals).

So, we write 99.90 as 99.900 (with three decimal digits) before dividing.

The answer is then rounded: $14.271 ≈ $14.27. But, if each person pays $14.27, they would pay a total of 7 × 14.27 = $99.89. That is one cent short. So in reality, one person would pay $14.28 and the rest $14.27 each.

```
   1 4.2 7 1
7 )9 9.9 0 0
  -7
   2 9
  -2 8
     1 9
   - 1 4
       5 0
     - 4 9
         1 0
```

3. Divide. Add decimal zeros to the dividend, as necessary.

a. Continue the division to 3 decimals, then round your answer to 2 decimals.

7) 2 5

Check:

b. Continue the division to 2 decimals, then round your answer to 1 decimal.

6) 7 8 2

Check:

c. Round your answer to 2 decimals.

3) 4. 5 7

Check:

d. Round your answer to 3 decimals.

11) 2.3

Check:

Use the grid and extra paper for calculations.

4. Six friends shared as equally as they could, in paying for a meal that cost \$87.50. How much did each one pay?

5. The PE teacher divided a 2-kilometre track into seven equal parts. How long are the parts? Give your answer to <u>two</u> decimal digits, in kilometres. *Remember to write 2 with <u>three</u> decimal zeros before you divide.*

6. A recipe calls for 1.5 kg of beef and it makes six servings. How much beef is in one serving?

7. Mary checked the prices of four different hot sauces: \$2.55, \$2.69, \$2.95, and \$2.75. Calculate the average price. *Hint: to find the average, add all the items and divide by the number of the items*

Puzzle Corner

Figure out where the decimal point has to go when we multiply a decimal by a decimal! Estimation can help.

a. Now, 3.82 is about 4, and 7.1 is about 7. If 382 × 71 = 27 122, then what is 3.82 × 7.1?

b. If 45 × 309 = 13 905 what is 4.5 × 30.9? (Estimate!)

c. If 569 × 271 = 154 199 what is 56.9 × 2.71?

Chapter 5: Statistics and Graphing
Introduction

This chapter starts out with a study of the coordinate grid, but only in the first quadrant. Besides learning how to plot points, students also plot ordered pairs (points) from number patterns or rules. This is actually the beginning of the study of *functions*.

Practising the use of the coordinate grid is a natural "prelude" to the study of line graphs, which follows next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs and double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories ("bins"). The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorised.

Toward the end of the chapter, we study average (also called the *mean*) and mode, and how these two concepts relate to line and bar graphs. Other maths curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is plenty of time to learn that concept in subsequent grades. Introducing all three concepts at the same time tend to jumble the concepts together and confuse them — and all some students are able to grasp from that is only the calculation procedures. I feel it is better to introduce and contrast initially just the two concepts, the mean and the mode, in order to give the student a solid foundation. We will introduce the median later, and then compare and contrast it with the other two.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 5

Helpful Resources on the Internet

You can also access this list of links at **https://l.mathmammoth.com/gr5ch5**

DISCLAIMER: *We check these links a few times a year. However, we cannot guarantee that the links have not changed. Parental supervision is always recommended.*

SCAN ME

COORDINATE GRID

Cali and the Coordinate Grid
Move Cali on the given coordinates (only positive numbers). How long will it take you to feed Cali 10 times?
https://www.math10.com/en/math-games/games/geometry/games-cali-coordinate-system.html

Soccer Coordinates Game
Plot the coordinates on the coordinate grid correctly to block the soccer ball from entering the goal.
https://www.xpmath.com/forums/arcade.php?do=play&gameid=90

Coordinate Grid Quiz from ThatQuiz.org
Practise plotting a point and giving the coordinates of a given point (in the first quadrant).
https://www.thatquiz.org/tq-7/?-j48-l5-p0

Number Pattern Tables
Apply the rule to find the missing number in the table.
https://www.studyladder.com/games/activity/number-pattern-tables--20584

Interpret Relationships Between Number Patterns (from Khan Academy)
Generate patterns using given rules, identify relationships between terms, and graph ordered pairs consisting of corresponding terms from the patterns.
https://cutt.ly/relationships-number-patterns

Graph a Two-Variable Relationship
Practise identifying relationships between variables with this interactive exercise.
https://www.ixl.com/math/grade-5/graph-a-two-variable-relationship

GRAPHING AND GRAPHS

Easy Practice Problems for Reading Bar Graphs
First, customise your bar chart. Then, click on the buttons on the left side to get questions to answer.
https://www.topmarks.co.uk/Flash.aspx?f=barchartv2

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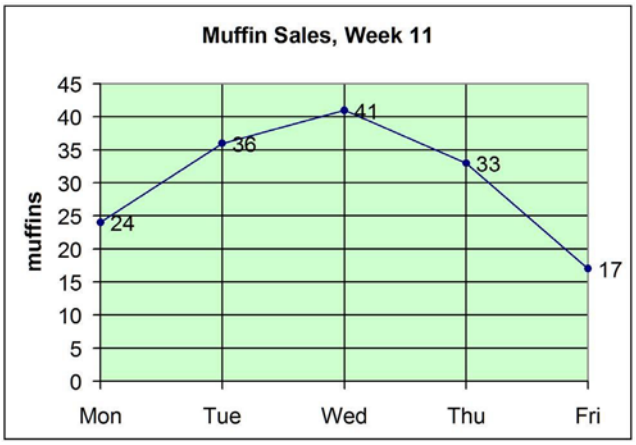
Line Graphs

Mary sold muffins every day at 2 pm in the school cafeteria. She recorded her sales in the table:

Muffin Sales, Week 11	
Day	**Muffins sold**
Mon	24
Tue	36
Wed	41
Thu	33
Fri	17



We can draw a line graph out of this, because the data (the numbers she recorded) is organised by *time* (days of the week). To do that, we first plot the individual data points in the grid. Then we draw lines to connect neighbouring points.

Besides that, the line graph also needs:

- **labels** for the **tick marks** on the two axes
- a **label** for the **vertical axis** (the *y*-axis)
- a **label** for the **horizontal axis** (the *x*-axis) unless it is very clear what it is about. In the graph above, the labels "Mon," "Tue," and so on show very clearly that they are days of the week, so we don't necessarily need a title "Days of the week" for the horizontal axis.
- a **title** at the top. Sometimes the graph might be quite clear without a title—because of the surrounding context or otherwise. It often helps the readers if the graph has a title.

Use a line graph for data that is organised by
some unit of time (hours, days, weeks, years, *etc*.)

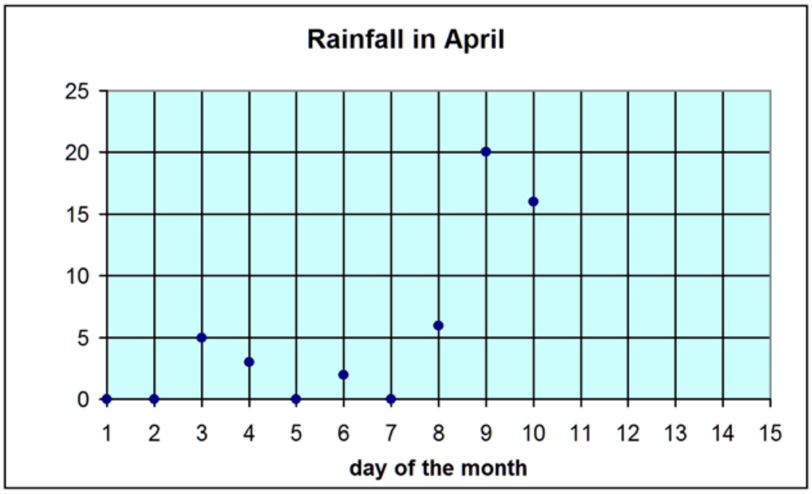
1. **a.** Add a label for the vertical axis that says "Rainfall (mm)".
(The "mm" stands for millimetres.)

b. Add five more data points to the graph from this data:

Day	11	12	13	14	15
Rainfall (mm)	9	0	0	13	2

c. Draw a line between each two consecutive points.

d. How many dry days were there in the first half of April?



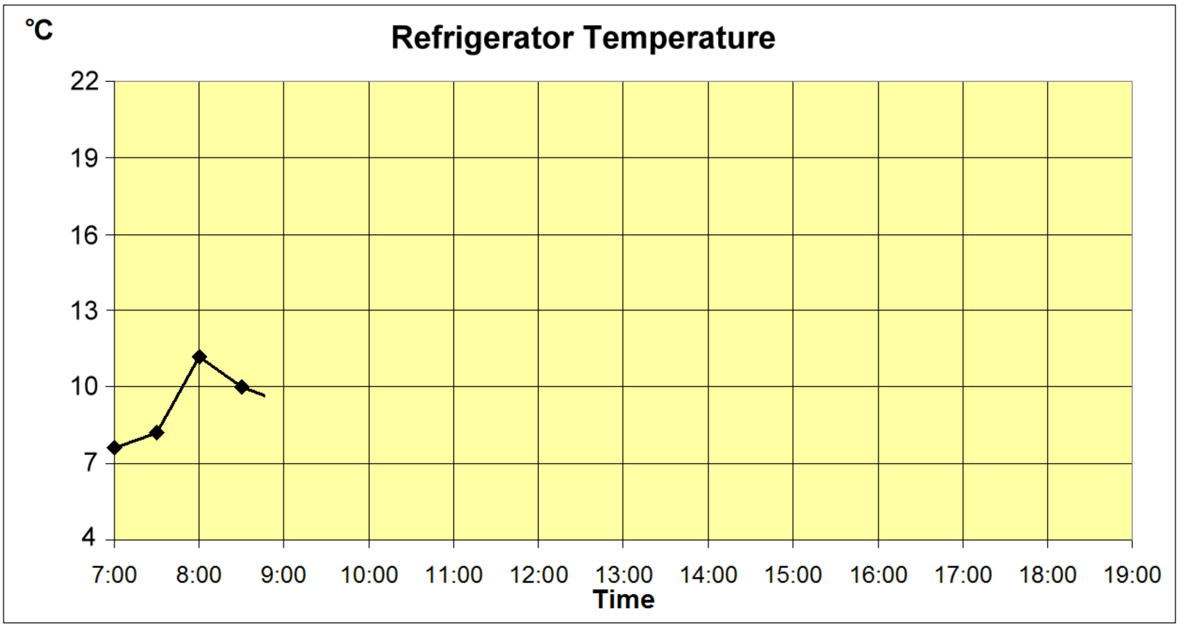
2. Jessie recorded the temperature of his fridge every 30 minutes during the day.
   a. Finish drawing the line graph.

   b. How did the temperature change around noon?
   Give a possible reason for that change.

   c. How did the temperature change around 5PM?
   Give a possible reason for that change.

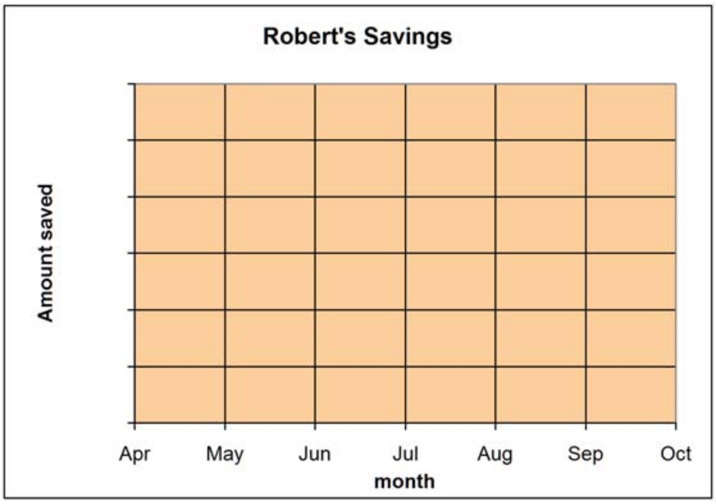
Time	7:00	7:30	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30
Temperature (°C)	8	8	11	10	9	8	10	10	9	13	17	13

Time	13:00	13:30	14:00	14:30	15:00	15:30	16:00	16:30	17:00	17:30	18:00	18:30	19:00
Temperature (°C)	10	9	8	11	10	9	12	13	18	16	13	10	8



3. Robert recorded his total savings at the end of each month. Draw a line graph of that data.
   Note: *You* need to choose the scaling for the vertical axis so that the largest number, $107, will fit on the grid. Think: should the gridlines go by five? By ten? By fifteen? By some other number?

Month	**Total savings**
Apr	$8
May	$22
Jun	$46
Jul	$61
Aug	$78
Sep	$95
Oct	$107

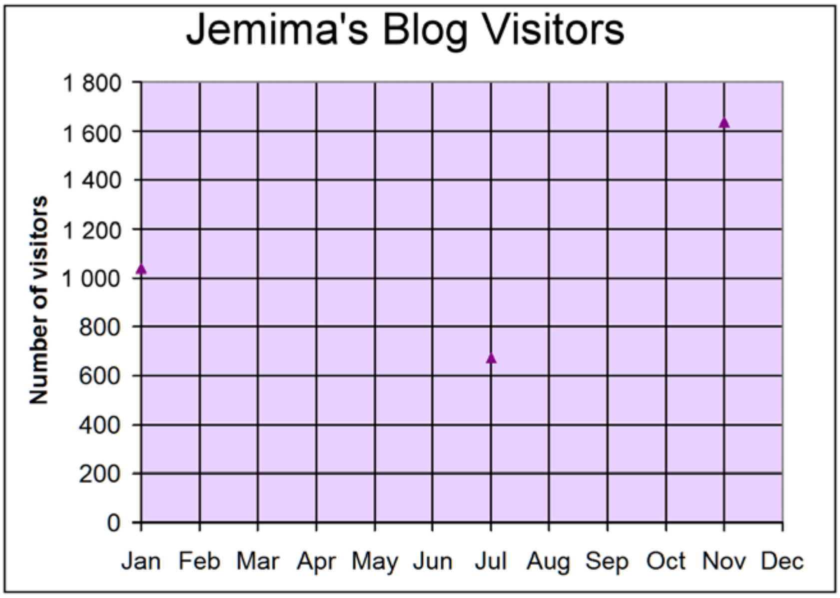


4. The table below shows the monthly visitor count to Jemima's blog. Three of the data points are already plotted. Your task is to plot the rest and finish the line graph.

Note 1: Since the vertical gridlines go by 200s, you cannot make an exact dot at, say, 1442. You need to round the numbers first. Round them to the nearest 50. Then plot the points.

Note 2: To find the three months with most visitors, look at the actual numbers given in the table.

Month	Visitors	rounded to the nearest 50
Jan	1 039	1 050
Feb	1 230	1 250
Mar	1 442	
Apr	1 427	
May	1 183	1 200
Jun	823	
Jul	674	650
Aug	924	
Sep	1 459	
Oct	1 540	
Nov	1 638	
Dec	1 149	



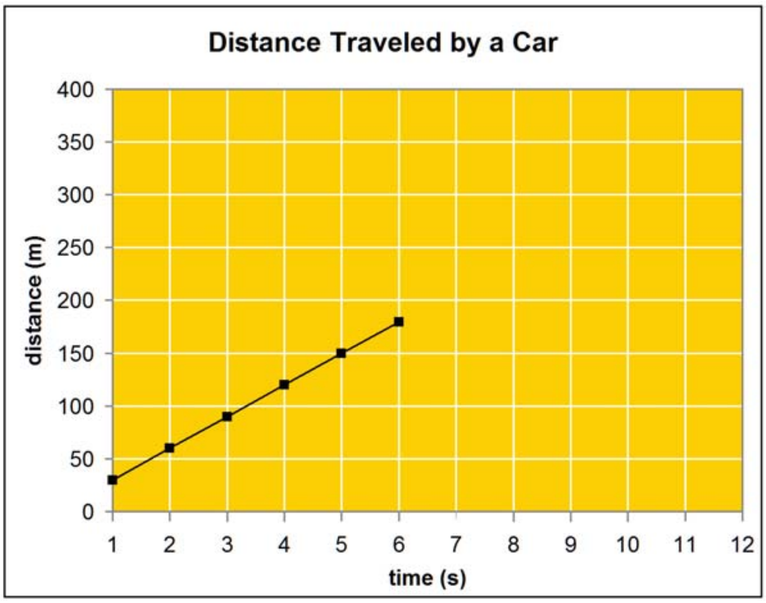
In the ____________________ Jemima's blog had far fewer visitors than in the spring or fall.

The three months with the fewest visitors were ____________, _____________, and ___________.

The three months with the most visitors were _____________, _____________, and ___________.

5. A car travels at a constant speed of 30 metres per second.

Time	Distance
0 s	0 m
1 s	30 m
2 s	60 m
3 s	90 m
4 s	120 m
5 s	150 m



a. Fill in the table, and continue the graph till 12 seconds.

b. When will the car have travelled 3 km?

6. **a.** Draw a line graph of the data on the right.

- First draw the two axes, one at the bottom and the other at the left side. Use a ruler so the graph looks neat.
- Label the axes. Label the horizontal axis as "year" (not as "*x*"). Label the vertical axis as "members" (not as "*y*").
- Label the whole graph by writing at the top: "After-School Sports Club Members from 1998 to 2005."
- Since the horizontal axis is for the years, draw tick marks on that axis for the years, but use *three* squares between each tick mark because the numbers for the years are so long (four digits).
- Then choose a scaling for the vertical axis. Because the member counts vary from 27 to 63, it makes sense to mark the vertical axis in fives, starting from 0. In other words, let each grid square be 5 members.
- Now you are ready to plot the points and draw the line graph.

After-School Sports Club	
Year	**Members**
1998	56
1999	63
2000	60
2001	35
2002	27
2003	32
2004	57
2005	63

b. What do you think might have caused the drop in membership in 2001 - 2003?

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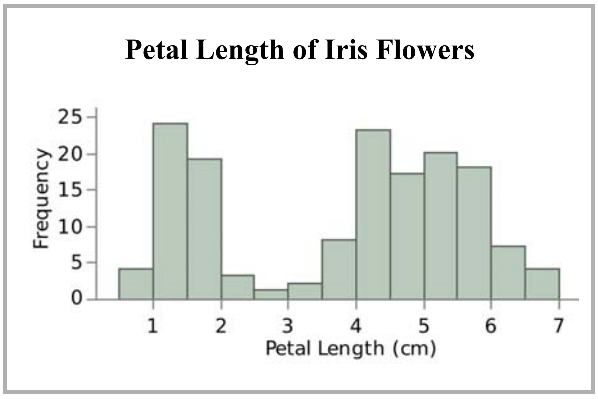
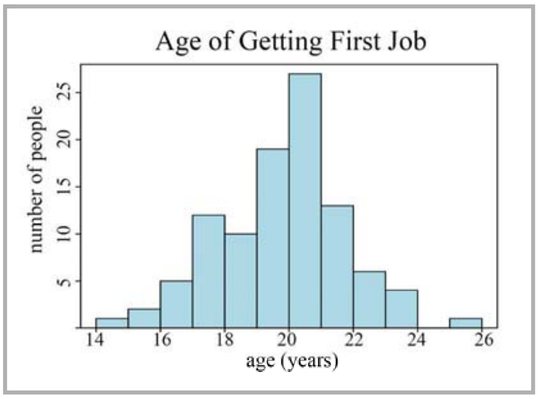
Making Histograms

Histograms are like bar graphs, but the bars are drawn so they touch each other. Histograms are used with numerical data.

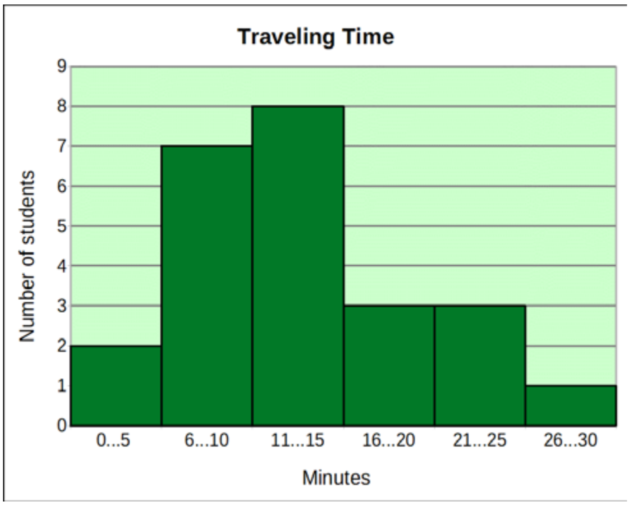




The vertical axis of a histogram shows us the **frequency**, or **the number of items**. For example, in the histogram above on the left, there are four flowers with a petal length between 0.5 and 1 cm, and 24 flowers with a petal length between 1 and 1.5 cm.

The label for this vertical axis can be "Frequency", or it can be the phrase "Number of...", such as "Number of people", or something similar.

Example 1. Jenny asked her classmates how long it took them to travel to school that day, and made a histogram from the results.



Notice how in this histogram, the categories for the horizontal axis are not single numbers, but a *range* (such as from 11 to 15 minutes).

From the graph, we can read that

- Two students took between 0 and 5 minutes to travel to school. From the graph, we cannot tell the exact time it took those students; just that there were two of them who took from 0 to 5 minutes to get to school.
- Seven students took between 6 and 10 minutes to get to school.
- Eight students took between 11 and 15 minutes to get to school.
- And so on.

We can also tell that *most* of the students took from 6 to 15 minutes to get to school.

How? Because the two highest bars are for 6...10 minutes and 11...15 minutes, and they have 7 + 8 = 15 students, which is *most* of the students, since the total number of students is 2 + 7 + 8 + 3 + 3 + 1 = 24.

Histograms are interesting, because from the same data, you can draw several histograms! It depends on how you choose the number ranges for your "bins" (categories).

Example 2. Let's look at the maths test scores of a fifth grade class:

13 40 32 38 32 28 21 30 45 17 22 26 33 25 27 36 42 19 21

First we need to make **bins** (categories) for this data. Only after that can we draw the histogram.

Let's make *five* bins for the test score data. These bins are shown on the right. The first bin is from 12 to 18 points, the second bin is from 19 to 25 points, and so on.

point count	frequency
12-18	
19-25	
26-32	
33-39	
40-46	

How did we come up with those limits for the bins?

First, we find the smallest value and the greatest value in the data. Those are 13 and 45. The first bin has to include 13 and the last bin has to include 45.

If we want five bins, we find the difference between those numbers and divide that by 5. The result will give us the **width** of each bin.

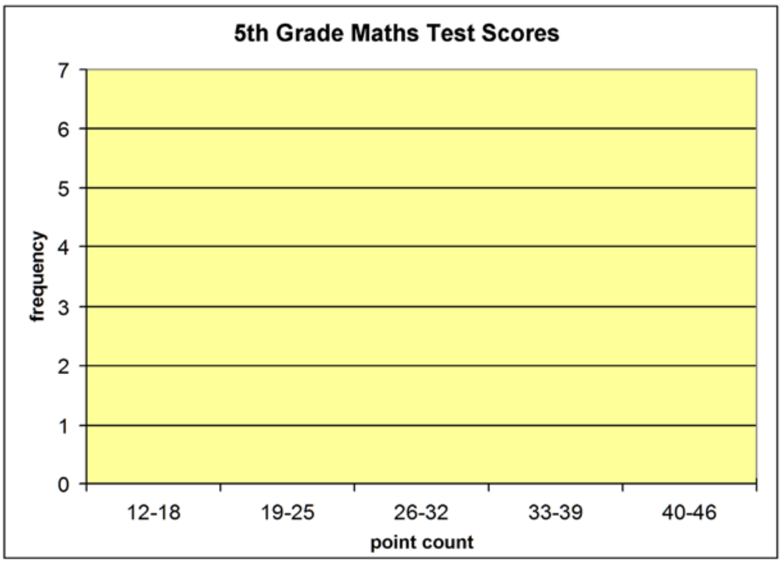
We get $45 - 13 = 32$ and $32 \div 5 = 6.4$. The width could be 6.4 points. However, usually it is more reasonable to use a whole number for the bin width, so we need to round 6.4 **up** to 7 points "wide." (If we round the 6.4 down to 6, then the last bin will not reach to the highest number.)

We can start the first category at 13 (the lowest score) or even a little bit before that, at 12. The important thing is that the last bin has to be able to include 45, our highest number.

Notice how each bin starts at 7 points higher than the previous one: the second at 19, the third at 26, and so on.

1. Finish filling in the table above, and draw the histogram.

   You will need to count *how many* individual test scores "fall into," or belong in, each bin—that is the **frequency**. Then you will be ready to draw the histogram! Draw it so that the bars touch each other without leaving gaps in between.



2. Henry determined the height of a bunch of saplings in his orchard (in centimetres). Here is his data, already organized from the smallest to the largest number:

42 43 45 51 52 55 55 55 56 56 58 61 62 73

77 85 85 86 86 88 88 90 92 92 92 94 95 98

Our first task in making a histogram is to decide how many categories there will be. For 28 data items it makes sense to use <u>five</u> categories.

a. Next, determine the bin width: Find the largest and the smallest numbers in the data, calculate the difference between them, divide that difference by five, and round the result up to the next whole number.

b. Determine the starting point of the first bin, and from that, calculate the starting points of the other bins (by simply adding the bin width). The first bin *can* start at the smallest number, 42, but it can also start lower. You need to check and test whether the highest number, 98, fits into the last bin.

Below are three different empty frequency charts for you to test which starting point will work. You will only use ONE of them for making the final histogram.

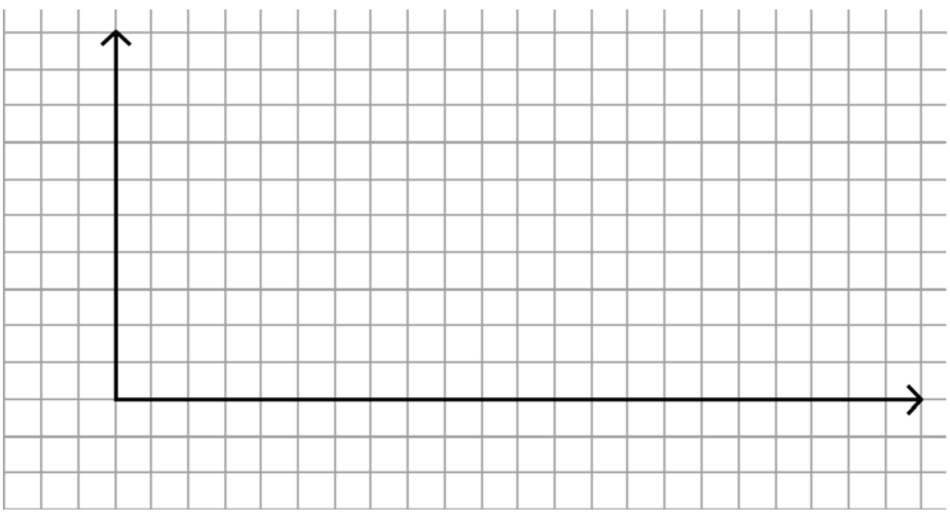
plant height	frequency

plant height	frequency

plant height	frequency

c. Write your categories in the frequency chart and fill it in.

d. Lastly, draw the histogram.

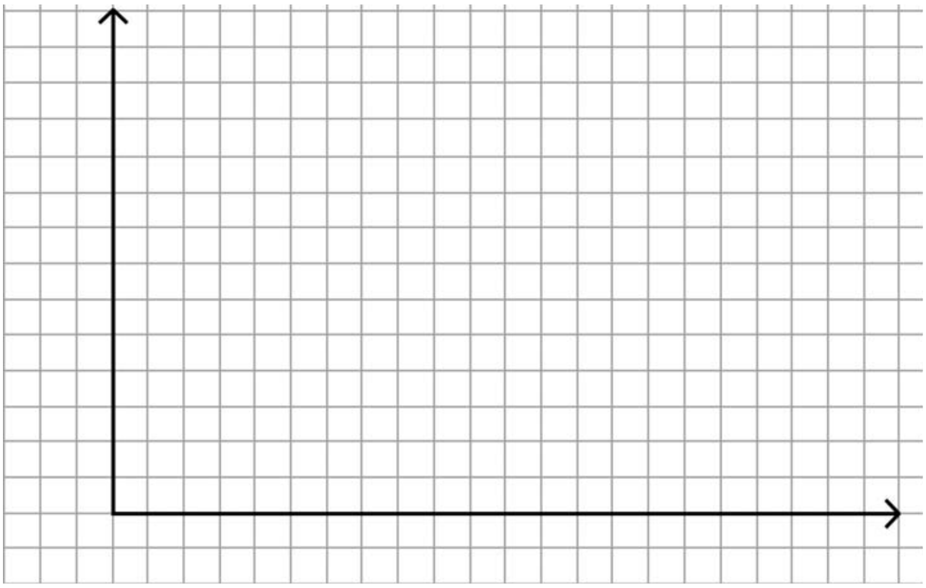


3. Draw a histogram with *four* categories using the following data.

The weights of 15 healthy female German Shepherd dogs (in pounds):

65 72 62 60 66 67 65 73 70 64 66 63 68 58 63

weight	frequency



4. Draw a histogram with five categories from the following data.

Researchers determined by their molars, the age of 26 African elephants, living in three herds. Here is the data (each number is the age of one elephant):

3 0 6 23 12 0 1 15 9 8 43 2 4 10 22 38 5 17 3 8 18 27 19 7 4

age	frequency