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## **Foreword**

Math Mammoth Grade 4, International Version comprises a complete maths curriculum for the fourth grade mathematics studies. This curriculum is essentially the same as the Math Mammoth Grade 4 sold in the United States, only customised for international use in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not properly align to the fourth grade standards in your country. However, you can probably find material for any missing topics in the neighbouring grades of Math Mammoth.

The International version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not customary (imperial) units.
- The currency used in word problems is the Australian dollar.
- The spelling conforms to British international standards.
- The pages are formatted for A4 paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (The decimals are formatted with a decimal point, as in the US version.)

The main areas of study in Math Mammoth Grade 4 are:

- 1. Students develop understanding and fluency with multi-digit multiplication, and use efficient multiplication procedures to solve problems.
- 2. They develop understanding of division to find quotients involving multi-digit dividends (long division), and they solve word problems involving division, including division with a remainder.
- 3. Students develop an understanding of fraction equivalence and some operations with fractions. They learn to add and subtract fractions with same denominators, and to multiply a fraction by a whole number.
- 4. Students learn the concept of angle. They draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Additional topics we study are place value, time, measuring, graphs, and decimals.

This book, 4-B, covers division (chapter 5), geometry (chapter 6), fractions (chapter 7), and decimals (chapter 8). The rest of the topics are covered in the 4-A worktext.

I wish you success in teaching maths!

Maria Miller, the author

### **User Guide**

Note: You can also find the information that follows online, at https://www.mathmammoth.com/userguides/.

#### **Basic principles in using Math Mammoth Complete Curriculum**

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

• The two student worktexts (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at <a href="https://www.mathmammoth.com/videos/">https://www.mathmammoth.com/videos/</a>. There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach the student!
- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to a corresponding list.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file which requires Internet access for use.
- You can use the free online exercises at <a href="https://www.mathmammoth.com/practice/">https://www.mathmammoth.com/practice/</a>
  This is an expanding section of the site, so check often to see what new topics we keep adding!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- Answer keys are provided for everything.

#### How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. You can also use the videos at <a href="https://www.mathmammoth.com/videos/">https://www.mathmammoth.com/videos/</a>

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

#### Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 2 or 3 pages long, intended for 1-2 days. Some lessons are 4-5 pages and can be covered in two days. There are also some optional lessons (not included in the tables on the right).

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Workte	Worktext 4-A						
Chapter 1	24 days						
Chapter 2	16 days						
Chapter 3	30 days						
Chapter 4	19 days						
TOTAL	89 days						

Worktext 4-B								
Chapter 5	33 days							
Chapter 6	20 days							
Chapter 7	20 days							
Chapter 8	11 days							
TOTAL	84 days							

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

#### Example:

Grade level	Lesson pages		Days for tests and revisions		Pages to study per day	Pages to study per week
4-A	174	88	8	80	2.18	10.9
4-B	183	92	8	84	2.18	10.9
Grade 4 total	357	180	16	164	2.18	10.9

The table below is for you to fill in. Allow several days for tests and additional revision before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Days for tests and revisions	Pages to study per day	Pages to study per week
4-A	174			
4-B	183			
Grade 4 total	357			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of particular problems) and particular problems. If your student gets it with less amount of https://www.mathmammoth.com

exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on maths. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards maths.

### Working space, the usage of additional paper and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g.  $200 + \underline{\hspace{1cm}} = 1000$ ). Typically, I have intended that such exercises to be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at <a href="https://www.mathmammoth.com/lessons/practical\_tips\_mental\_math">https://www.mathmammoth.com/lessons/practical\_tips\_mental\_math</a> also gives you a summary of some of those principles.

#### **Using tests**

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

#### Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revision concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the printed version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

- 1. Check if the worksheet maker lets you make worksheets for that topic.
- 2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
- 3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
- 4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
- 5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
- 6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

#### Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

#### **Math Stars Problem Solving Newsletter (grades 1-8)**

https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link: https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

#### Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

# **Chapter 5: Division Introduction**

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, average, divisibility, and problem solving. It is a long chapter, because division and long division are "in focus" in fourth grade. Therefore, feel free to mix the lessons from this chapter with lessons from some other chapter, essentially using the curriculum in a somewhat spiral manner. This is especially advisable if your student has difficulties retaining the material or starts feeling bored with these topics.

For further help in teaching these topics, check out the free videos matched to the curriculum at <a href="https://www.mathmammoth.com/videos/">https://www.mathmammoth.com/videos/</a>. Remember not to automatically assign all the exercises. Instead, adjust the amount of exercises according to the student's needs. The rest can be used later for revision.

We start out by revising basic division facts by single-digit numbers (such as  $24 \div 4$  or  $56 \div 7$ ). After that, we study terminology of division and dividing numbers by whole tens and hundreds (such as  $400 \div 20$ ). Next students practise the order of operations again—this time with division as one of the operations.

Then we study the concept of remainder, preparing students for the upcoming lessons on long division. At first, the concept of remainder is presented visually. Soon, students solve simple division problems with a remainder, written with the long division symbol (or long division "corner", as I like to call it).

Next comes a set of lessons intended to teach long division in several small steps. We start with divisions where each of the digits in the dividend (thousands, hundreds, tens, and ones) can be divided evenly by the divisor (for example,  $3096 \div 3$ ). As the next step, there is a remainder in the ones. Then, the divisions have a remainder in the tens. Finally, there is a remainder in the hundreds and in the thousands, and this completes the step-by-step learning process for long division. The lessons also include lots of word problems to solve.

After long division, we study the concept of average, which is a nice application of division, and problems that involve finding a fractional part of a quantity using division. For example, we can find 3/4 of a number by first finding 1/4 (dividing by 4) and then multiplying the result by 3. Students get help from visual bar models to solve the problems.

The last section deals with elementary number theory. We study basic divisibility rules (though not all of them), prime numbers, and finding all factors of a given two-digit number.

### **Pacing Suggestion for Chapter 5**

This table does not include the chapter test, as it is found in a different place. Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 5	page	span	suggested pacing	your pacing
Revision of Division (optional)	13	(3 pages)	2 days	
Division Terms and Division with Zero	16	2 pages	1 day	
Dividing with Whole Tens and Hundreds	18	3 pages	2 days	
Order of Operations and Division	21	2 pages	1 day	
The Remainder, Part 1	23	3 pages	2 days	
The Remainder, Part 2	26	2 pages	1 day	
The Remainder, Part 3	28	2 pages	1 day	
Long Division 1	30	4 pages	2 days	
Long Division 2	34	3 pages	1 day	
Sampile Worksheet from	37	4 pages	2 days	
https://www.mathmammoth.com				

Long Division with 4-Digit Numbers	41	4 pages	2 days	
More Long Division	45	3 pages	1 day	
Remainder Problems	48	4 pages	2 days	
Long Division with Money	52	2 pages	1 day	
Long Division Crossword Puzzle (optional)	54	(1 page)	1 day	
Average	55	3 pages	2 days	
Finding Fractional Parts with Division	58	3 pages	2 days	
Problems with Fractional Parts	61	2 pages	1 day	
Problems to Solve	63	3 pages	2 days	
Divisibility	66	4 pages	2 days	
Prime Numbers	70	3 pages	2 days	
Finding Factors	73	2 pages	1 day	
Mixed Revision Chapter 5	75	2 pages	1 day	
Revision, Chapter 5	77	2 pages	1 day	
Chapter 5 Test (optional)				
TOTALS		62 pages	33 days	
with optional content		(66 pages)	(36 days)	

### **Helpful Resources on the Internet**

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- online practice for concepts;
- online games, or occasionally, printable games;
- animations and interactive illustrations of maths concepts;
- articles that teach a maths concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://links.mathmammoth.com/gr4ch5



# **Revision of Division**

**Multiplication** has to do with equal-size groups:  $2 \times 4$  means 2 groups of 4.

**Division** is the opposite operation of multiplication, and it *also* has to do with equal-size groups:  $8 \div 4$  can mean, "How many groups of 4 are in 8?"

It can also mean, "How many in each group, when 8 things are put into 4 groups?"

Division has two "meanings":

- Dividing to find how many are in each group.
- Dividing into groups of a certain size.



 $2 \times 6 = 12$ 

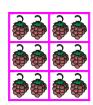
"12 divided into 2 groups; how many in each group?"

$$12 \div 2 = 6$$

OR

"How many sixes are in 12?"

$$12 \div 6 = 2$$



 $6 \times 2 = 12$ 

"12 divided into 6 groups; how many in each group?"

$$12 \div 6 = 2$$

<u>OR</u>

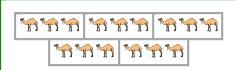
"How many twos are in 12?"

$$12 \div 2 = 6$$

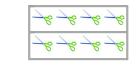
1. Write a multiplication sentence and two division sentences.



a.



h



c.

2. Fact families: write two division and two multiplication sentences.

**a.** 21 7 and 3

**b. 24** 4 and

13

**c. 36** 4 and

[This page is intentionally left blank.]

# **Long Division 1**

### Divide hundreds, tens, and ones separately.

Write the dividend inside the long division "corner", and the quotient on top.

$$64 \div 2 = ?$$

Divide tens and ones separately:

$$6 \text{ tens} \div 2 = 3 \text{ tens (t)}$$

 $4 \text{ ones } \div 2 = 2 \text{ ones (o)}$ 



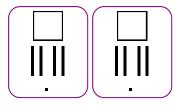
$$\frac{3}{3}\frac{2}{2}$$

### $282 \div 2 = ?$

2 hundreds  $\div$  2 = 1 hundred (h)

$$8 \text{ tens} \div 2 = 4 \text{ tens (t)}$$

$$2 \div 2 = 1$$
 (o)



1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.

<b>a.</b> Make 2 groups	<b>b.</b> Make 3 groups	<b>c.</b> Make 3 groups	<b>d.</b> Make 4 groups
1			
2)62	3)	3)	4)

2. Divide thousands, hundreds, tens, and ones separately.

a. 
$$4)84$$

b. 
$$3)393$$

a. 
$$4)84$$
 b.  $3)393$  c.  $3)660$  d.  $4)8040$ 

e. 
$$3\overline{)66}$$
 f.  $2\overline{)6042}$  g.  $3\overline{)330}$  h.  $4\overline{)4804}$ 

h. 
$$4)4804$$



Four does not go into 2. You can put zero in the quotient in the hundreds place or omit it. Four does go into 24, six times. Put 6 in the quotient.

Five does not go into 3. You can put zero in the quotient. Five does go into 35, seven times.

### **Explanation:**

The 2 of 248 is 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient will not have any whole hundreds.

Then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result, 6 tens goes as part of the quotient.

Check the final answer:  $4 \times 62 = 248$ .

#### **Explanation:**

 $3000 \div 5$  will not give any whole thousands to the quotient because the answer is less than 1000.

But 3 thousands and 5 hundreds make 35 hundreds together. You can divide  $3500 \div 5 = 700$ , and place 7 as part of the quotient in the hundreds place.

Check the final answer:  $5 \times 701 = 3505$ .

If the divisor does not "go into" the first digit of the dividend, look at the <u>first two digits</u> of the dividend.

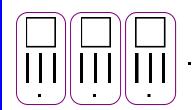
3. Divide. Check your answer by multiplying the quotient and the divisor.

a. 
$$3 ) 1 2 3$$

h. 
$$4)2404$$

31

#### The ones division is not even. There is a remainder.



$$\frac{1 \ 3 \ 1 \ R2}{3 \ 3 \ 9 \ 5}$$

 $395 \div 3 = 131 \text{ R2}$ 

3 goes into 3 one time. 3 goes into 9 three times.

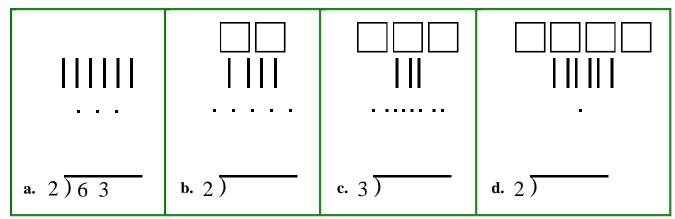
3 goes into 5 one time, but not evenly. Write the remainder 2 after the quotient.

Four does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

Four goes into 16 four times. Four goes into 5 once, with a remainder of 1.

Eight does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3200). Eight goes into 32 four times  $(3200 \div 8 = 400)$ Eight goes into 0 zero times (tens). Eight goes into 7 zero times, with a remainder of 7.

4. Divide into groups. Find the remainder.



5. Divide. Indicate the remainder if any.

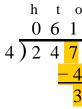
a. 
$$4)847$$
 b.  $2)69$  c.  $3)367$  d.  $4)89$ 

**b.** 
$$2)69$$

e. 
$$2)$$
 1 2 1 f.  $6)$  1 8 0 5 g.  $7$  2 1 5 h.  $8$  2 4 8 2

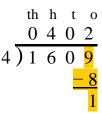
h. 
$$8)2482$$

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:



When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$ 



When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.

Check:  $4 \times 402 + 1 = 1609$ 

6. Practise some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

**b.** 
$$3)95$$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

**a.** 
$$440 \div 4 =$$

$$820 \div 2 =$$

**b.** 
$$3600 \div 400 =$$

$$369 \div 3 =$$

**c.** 
$$824 \div 2 =$$

$$560 \div 90 =$$

[This page is intentionally left blank.]

# **Divisibility**

A number n is **divisible** by another number m, if the division  $n \div m$  is exact (no remainder).

For example,  $18 \div 3 = 6$ , so 18 is divisible by 3.

Also, 18 is divisible by 6, because we can write the other division  $18 \div 6 = 3$ .

So, 18 is divisible by *both* 6 and 3. We say 6 and 3 are **divisors** of 18.

You can use long division to check if a number is divisible by another.

4)67

For example,  $67 \div 4 = 16$ , R3. There is a remainder, so 67 is *not* divisible by 4.

 $\frac{-4}{2}$  7

Also, from this we learn that neither 4 nor 16 are divisors of 67.

<u>- 2 4</u>

1. Divide and determine if the number is divisible by the other number.

**a.** 
$$21 \div 3 =$$
\_\_\_\_\_

Is 21 divisible by 3?

**b.** 
$$40 \div 6 =$$

Is 40 divisible by 6?

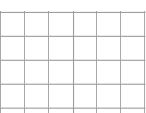
**c.** 
$$17 \div 5 =$$
\_\_\_\_\_

Is 5 a divisor of 17?

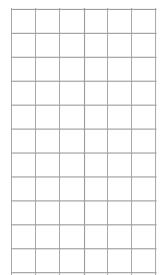
Is 7 a factor of 84?

2. Answer the questions. You may need long division.

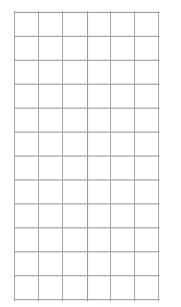
**a.** Is 98 divisible by 4?



**b.** Is 603 divisible by 7?



**c.** Is 3 a factor of 1256?



Sample worksheet from

https://www.mathmammoth.com

In any multiplication, the numbers that are multiplied are called **factors** and the result is called a **product**.

factor factor product  $7 \times 6 = 42$ 

For example, since  $6 \times 7 = 42$ , 6 and 7 are **factors** of 42.

From this multiplication fact we can write two divisions:  $42 \div 6 = 7$  and  $42 \div 7 = 6$ . So, this means that 6 and 7 are also divisors of 42.

From this we can notice the following:

#### If a number is a factor of another number, it is also its divisor.

There is yet one more new word to learn that ties in with all of this: multiple.

We say 42 is a multiple of 6, because 42 is some number times 6 (namely  $7 \times 6$ ).

And of course 42 is also a multiple of 7, because 42 is some number times 7 (namely,  $\underline{6} \times 7$ )!

3. Fill in.

We know that  $8 \times 9 = 72$ . So, 8 is a \_\_\_\_\_\_ of 72, and so is 9.

Also, 72 is a \_\_\_\_\_\_ of 8, and 72 is a \_\_\_\_\_ of 9.

And, 72 is \_\_\_\_\_\_ by 8 and by 9.

4. Fill in.

a. Is 5 a factor of 55?	<b>b.</b> Is 8 a divisor of 45?			
Yes, because ÷ =	No, because ÷ =			
<b>c.</b> Is 36 a multiple of 6?	<b>d.</b> Is 34 a multiple of 7?			
, because ÷ =	, because ÷ =			
e. Is 7 a factor of 46?	<b>f.</b> Is 63 a multiple of 9?			
, because	, because			

**Multiples of 6** are all those numbers we get when we multiply 6 by other numbers. For example, we can multiply  $0 \times 6$ ,  $7 \times 6$ ,  $11 \times 6$ ,  $109 \times 6$ , and so on. The resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, and so on.

- 5. **a.** Make a list of multiples of 11, starting at 0 and continue at least to 154.
  - **b.** Make a list of multiples of 111, starting at 0. Continue as long as you can in this space!

#### **Divisibility by 2**

Numbers that are divisible by 2 are called **even** numbers. Numbers that are NOT divisible by 2 are called **odd** numbers.

Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

#### **Divisibility by 5**

Numbers that end in 0 and 5 are divisible by 5.

For example, 10, 35, 720, and 3675 are such numbers.

6. Mark an "x" if the number is divisible by 2 or by 5.

number	divisible		number	divisible		number	divi	sible	number	divis	sible
number	by 2	by 5	number	by 2	by 5	number	by 2	by 5	number	by 2	by 5
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

### **Divisibility by 10**

Numbers that end in 0 are divisible by 10.

For example, 10, 60, 340, and 2570 are such numbers.

7. Mark an "x" if the number is divisible by 2, by 5, or by 10.

number		divisib	le	divisible		number	divisible				
number	by 2	by 5	by 10	number	by 2	by 5	by 10	number	by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in a zero, so it is ALSO divisible by \_\_\_\_ and \_\_\_\_.

8. <b>a.</b>	Write a	list of	numbers	that are	divisible	by 2,	from	0 to	60.
--------------	---------	---------	---------	----------	-----------	-------	------	------	-----

\_\_\_\_\_

This is also a list of \_\_\_\_\_\_ of 2.

- **b.** In the list above, *underline* those numbers that are divisible by 4. What do you notice?
- **c.** In the list above, *colour* those numbers that are divisible by 6. What do you notice?
- **d.** Which numbers are divisible by both 4 and by 6?
- 9. **a.** Write a list of numbers that are divisible by 3, from 0 to 60.

This is also a list of \_\_\_\_\_ of 3.

- **b.** In the list above, *underline* those numbers that are divisible by 6. What do you notice?
- **c.** In the list above, *colour* those numbers that are divisible by 9. What do you notice?
- 10. Use the lists you made in (8) and (9). Find numbers that are divisible by both 2 and 9.
- 11. What number is a factor of every number?
- 12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers. Which ones?

Who am I? (Hint: I am less than 50.)



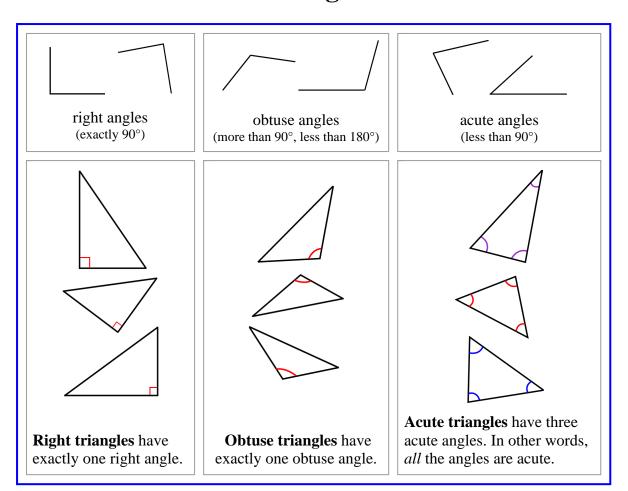
Who am I? (Hint: I am less than 100.)



Divided by 9, I leave a remainder of 6. Divided by 4, I leave a remainder of 1. Divided by 10, I leave a remainder of 3. I am a multiple of 3, 4, 5, and 6. I am a factor of 120. Divided by 7, I leave a remainder of 4.

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# **Triangles**

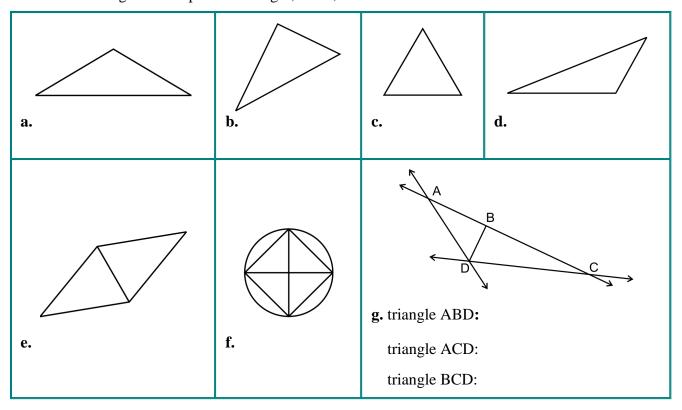


- 1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.
  - **b.** Draw another, different right triangle.
  - **c.** A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?

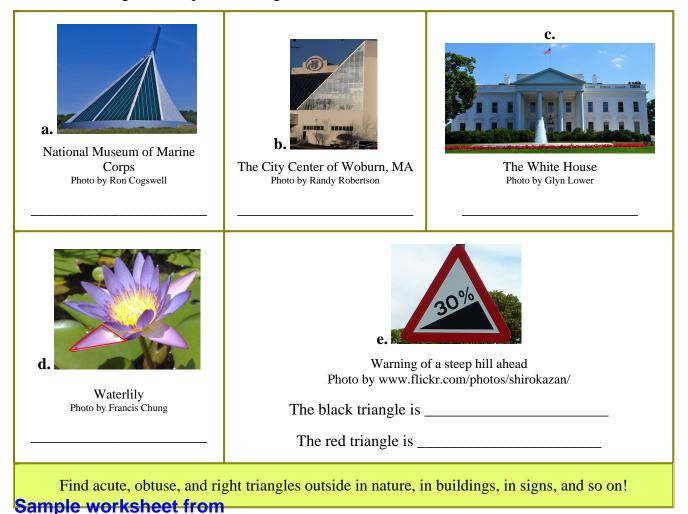
A right triangle has one right angle. The other two angles are \_\_\_\_\_

Ther obtu	w an obtuse angle. In make it into an Ise triangle by Ise in the third side.				
	w another, different se triangle.				
obtu two	obtuse triangle has one se angle. Are the other angles in an obtuse gle acute, right, or se?				
An ob	tuse triangle has one obtu	se angle. The other two angles are			
3. <b>a.</b> Drav	w any acute triangle.				
<b>b.</b> Mea	sure its angles.				
They	y measure°,				
	°, and°.				
4. Observ	. Observe all that you have done in this lesson thus far, and fill in the blanks below.				
	Right triangles have exa	actly one,			
	and the other two angles	are			
	_	xactly one,			
	and the other two angles	are			
	Acute triangles have	angles.			

5. Label the triangles in the pictures as right, acute, or obtuse.



6. Label the triangles in the pictures as right, acute, or obtuse.



7. **a.** Draw a triangle with  $35^{\circ}$  and  $40^{\circ}$  angles. The 35° angle is already drawn for you. **b.** Measure the third angle. It is \_\_\_\_\_ degrees. **c.** What kind of triangle is it? (acute, right, obtuse) 8. a. Draw a triangle with  $125^{\circ}$  and  $40^{\circ}$  angles. **b.** Measure the third angle. It is \_\_\_\_\_ degrees. **c.** What kind of triangle is it? (acute, right, obtuse) 9. a. Draw a triangle with  $55^{\circ}$  and  $35^{\circ}$  angles. **b.** Measure the third angle. It is \_\_\_\_\_ degrees. **c.** What kind of triangle is it? (acute, right, obtuse)

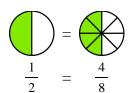
#### New Terms

- · an acute triangle
- an obtuse triangle
- · a right triangle

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# **Equivalent Fractions**

If you eat half of a pizza, or if you eat 4/8 of a pizza, you have eaten the same amount.



1/2 and 4/8 are equivalent fractions.



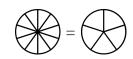
The two fraction strips show an equal amount. So, we can write an equal sign between the

two mixed numbers:  $1\frac{1}{5} = 1\frac{2}{10}$ .

1. Colour the first fraction. Shade the same amount of pie in the second picture. Write the second fraction.

**a.** 
$$\frac{1}{2}$$
 =

**b.** 
$$\frac{3}{4}$$
 =



**c.** 
$$\frac{6}{10}$$
 =

**d.** 
$$\frac{8}{12}$$
 =

**e.** 
$$\frac{1}{3}$$
 =

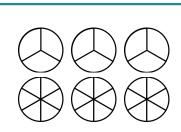
**f.** 
$$1\frac{2}{3}$$



**g.** 
$$1\frac{10}{12}$$



2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.



**a.** 
$$\frac{3}{3} =$$

**b.** 
$$\frac{4}{3} =$$

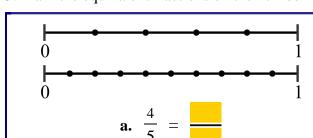
c. 
$$\frac{7}{3}$$
 =

**d.** 
$$2\frac{1}{3} =$$

**e.** 
$$1\frac{2}{3} =$$

**f.** 
$$2\frac{2}{3}$$
 =

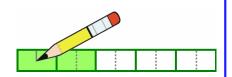
3. Mark the equivalent fractions on the number lines.



$$\frac{3}{9} = \frac{3}{9}$$

**Example 1.** The fraction strip illustrates  $\frac{2}{5}$ . If you split each

piece (both the coloured and white pieces) into *two* new pieces, what fraction do you get?

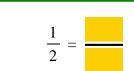


You get  $\frac{4}{10}$ : four coloured pieces, and ten pieces total.

You have two times as many coloured pieces, and two times as many total pieces as before.

4. Split both the coloured and white pieces as instructed. Write the fraction after you change it.

**a.** Split all the pieces into two new ones.



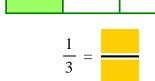
**b.** Split all the pieces into four new ones.



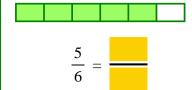
**c.** Split all the pieces into three new ones.



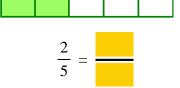
**d.** Split all the pieces into three new ones.



**e.** Split all the pieces into two new ones.

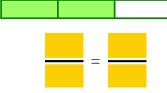


**f.** Split all the pieces into three new ones.

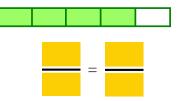


### Do you notice a *shortcut* for finding the second fraction?

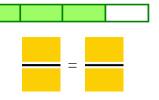
**g.** Split all the pieces into four new ones.



**h.** Split all the pieces into two new ones.



i. Split all the pieces into three new ones.



If you found the shortcut, explain how it works in these problems:

Split all the pieces into three new ones.

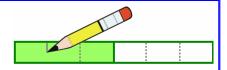


Split all the pieces into two new ones.



**Example 2.** The fraction strip illustrates  $\frac{1}{2}$ . If we split each

piece into *three* new pieces, we get  $\frac{3}{6}$ .



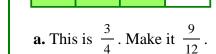
Now we have *three* times as many coloured pieces, and *three* times as many pieces in total as we had before. Look at the right side of this box, to see how we can illustrate it this way  $\rightarrow$ 

 $\frac{\times 3}{2} = \frac{3}{6}$ 

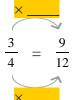
We multiply both the top and bottom number in a fraction by 3.

We get an equivalent fraction—it is the same amount, just cut into more pieces. *This does not mean we multiply the whole fraction by 3*.

5. Split the pieces. Fill in the missing parts.



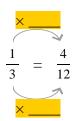
Each piece is split into \_\_\_\_ new ones.





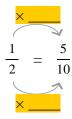
**b.** This is  $\frac{1}{3}$ . Make it  $\frac{4}{12}$ .

Each piece is split into \_\_\_\_\_ new ones.

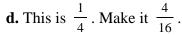


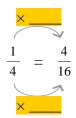
**c.** This is  $\frac{1}{2}$ . Make it  $\frac{5}{10}$ .

Each piece is split into \_\_\_\_ new ones.

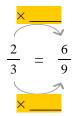




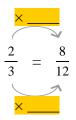




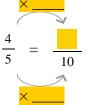
**e.** This is 
$$\frac{2}{3}$$
. Make it  $\frac{6}{9}$ .



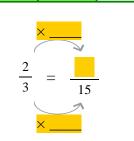
**f.** This is 
$$\frac{2}{3}$$
. Make it  $\frac{8}{12}$ .



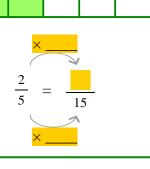




h.



i.



6. Write the equivalent fraction. Use multiplication.

**a.** Split all the pieces into three new ones.

$$\frac{5}{6} = \frac{}{}$$

**b.** Split all the pieces into five new ones.

$$\frac{3}{4} = \frac{}{}$$

**c.** Split all the pieces into four new ones.

$$\frac{2}{5} = \frac{2}{2}$$

**d.** Split all the pieces into ten new ones.

$$\frac{9}{10} = \frac{}{}$$

7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.

**a.** Pieces were split into \_\_\_\_\_ new ones.

**b.** Pieces were split into \_\_\_\_\_ new ones.

$$\frac{3}{10} = \frac{30}{}$$

**c.** Pieces were split into \_\_\_\_\_ new ones.

$$\frac{2}{5} = \frac{2}{30}$$

**d.** Pieces were split into \_\_\_\_ new ones.

$$\frac{7}{8} = \frac{35}{1}$$

- **e.**  $\frac{2}{3} = \frac{1}{6}$
- **f.**  $\frac{3}{5} = \frac{9}{1}$
- $\mathbf{g} \cdot \frac{5}{6} = \frac{12}{12}$
- **h.**  $\frac{1}{3} = \frac{9}{9}$

8. Write the fractions that have tenths with hundredths instead.

- **a.**  $\frac{1}{10} = \frac{1}{100}$
- **b.**  $\frac{3}{10} =$
- **c.**  $\frac{6}{10} =$
- **d.**  $\frac{4}{10} =$
- **e.**  $\frac{13}{10} =$

9. Connect the equivalent fractions with a line.

- a.  $\begin{array}{c|cccc}
  \frac{2}{3} & \frac{1}{3} \\
  \frac{1}{4} & \frac{1}{2} \\
  \frac{5}{10} & \frac{2}{8} \\
  \frac{2}{6} & \frac{6}{9}
  \end{array}$
- b.  $\begin{array}{c|cccc}
   & \frac{1}{2} & \frac{2}{10} \\
   & \frac{3}{4} & \frac{1}{3} \\
   & \frac{1}{5} & \frac{6}{12} \\
   & \frac{4}{12} & \frac{9}{12}
  \end{array}$

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_	_	
	$\frac{3}{6}$	$\frac{3}{12}$
0	$\frac{1}{4}$	$\frac{1}{2}$
c.	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$
	•	

10. Write chains of equivalent fractions!

**a.**  $\frac{1}{2} = \frac{1}{4} = \frac{1}{6} = \frac{1}{8} = \frac{1}{6} = \frac{1}{8}$ 

### We can use equivalent fractions to add fractions that have different denominators.

**Example 3.** Add  $\frac{2}{10} + \frac{17}{100}$ . First, write 2/10 as 20/100 (an equivalent fraction).

Then you can add, because the fractions now have the same denominator:  $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$ .

#### 11. Add.

_			
	<b>a.</b> $\frac{1}{10} + \frac{8}{100}$	<b>b.</b> $\frac{7}{10} + \frac{3}{100}$	<b>c.</b> $\frac{45}{100} + \frac{3}{10}$
	$\downarrow$ $\downarrow$	$\downarrow$ $\downarrow$	
	$\frac{8}{100} + \frac{8}{100} =$	$\frac{1}{100} + \frac{1}{100} =$	
	<b>d.</b> $\frac{9}{10} + \frac{9}{100}$	<b>e.</b> $\frac{7}{10} + \frac{23}{100}$	<b>f.</b> $\frac{24}{100} + \frac{9}{10}$
	<b>g.</b> $\frac{7}{100} + 1\frac{4}{10}$	<b>h.</b> $2\frac{28}{100} + 1\frac{5}{10}$	<b>i.</b> $\frac{6}{10} + \frac{35}{100} + \frac{7}{100}$

12. Draw a picture showing that 1/3 and 4/12 are equivalent fractions.

# Puzzle Corner

Add. This is challenging. *Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.* 

**a.** 
$$\frac{3}{4} + \frac{1}{2}$$

**b.** 
$$\frac{1}{5} + \frac{3}{10}$$

c. 
$$\frac{2}{3} + \frac{2}{9}$$

# **Subtracting Fractions and Mixed Numbers**

Here are five sixths. If you take away two of them, how many sixths are left? Three sixths, of course!

**Example 1.** 
$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$



In this example, we can simply subtract the whole numbers and the fractions separately: 6 - 1 = 5, and

**Example 2.** 
$$6\frac{7}{8} - 1\frac{2}{8} = 5\frac{5}{8}$$

 $\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$ . It works, because from 7 eighths we *can* take away 2 eighths.

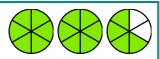
If it was the other way around, we would need a different approach.

1. Subtract. You can cross out parts from the images to help you.

**a.** 
$$\frac{9}{10} - \frac{1}{10} =$$

**b.** 
$$\frac{11}{12} - \frac{7}{12} =$$

**c.** 
$$2\frac{4}{6} - \frac{2}{6} =$$



**d.** 
$$2\frac{5}{9} - 1\frac{3}{9} =$$





$$e. \frac{9}{4} - \frac{3}{4} =$$

**f.** 
$$2\frac{7}{8} - \frac{3}{8} =$$

**g.** 
$$5\frac{9}{12} - 2\frac{5}{12} =$$

**h.** 
$$\frac{7}{10} - \frac{5}{10} =$$

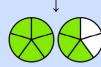
i. 
$$10\frac{7}{12} - 7\frac{3}{12} =$$

**j.** 
$$1 - \frac{7}{8} =$$

Example 3.



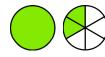
$$1\frac{3}{5} - \frac{4}{5}$$



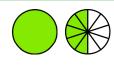
$$\frac{8}{5} - \frac{4}{5} = \frac{4}{5}$$

Here it helps to first change the mixed number 1 3/5 into the fraction 8/5. It is like cutting the whole pie into fifths. Now it is easy to subtract 4 fifths.

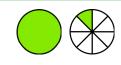
2. Subtract. To help you, you can "cut" the whole pie into pieces first.

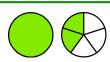






**b.**  $1\frac{5}{10} - \frac{9}{10}$ 



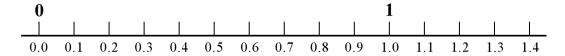


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# Adding and Subtracting with Tenths

You already know how to add or subtract decimals that have tenths, such as $0.8 + 0.5$ . They are just fractions with a denominator of 10. Compare the two additions in each box. One of them is written with decimals and the other with fractions.	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
There is one tricky thing: $0.6 + 0.7$ is <b>NOT</b> 0.13!	0.6 + 0.7 = 1.3	1.5 + 0.9 = 2.4
To see why, add the corresponding fractions. Notice that six-tenths and seven-tenths makes thirteen-tenths, which is more than one!	$\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each "number-line jump.



**a.** You are at 0.7, and you jump *five tenths* to the right.

**b.** You are at 0.6, and you jump *eight tenths* to the right.

**c.** You are at 1.1, and you jump *eight tenths* to the left.

**d.** You are at 1.3, and you jump *four tenths* to the left.

**e.** You are at 0.2, and you jump *eleven tenths* to the right.

2. Solve the fraction additions, and then write them using decimals.

**a.** 
$$\frac{2}{10} + \frac{7}{10} =$$
 **b.**  $\frac{5}{10} + \frac{6}{10} =$  **c.**  $\frac{9}{10} + \frac{8}{10} =$  0.2 +

3. Add or subtract.

a.
 b.
 c.
 d.

 
$$0.9 + 0.2 =$$
 $0.5 + 0.7 =$ 
 $0.8 + 0.7 =$ 
 $1.8 - 0.9 =$ 
 $1.9 + 0.2 =$ 
 $3.5 + 0.7 =$ 
 $0.8 + 2.7 =$ 
 $5.8 - 0.9 =$ 

Sample worksheet from

#### 4. Calculate.

 a.
 b.
 c.
 d.

 2.3 + 0.9 = \_\_\_\_\_\_
 1.5 + 0.7 = \_\_\_\_\_\_
 6.6 - 0.5 = \_\_\_\_\_\_
 4.7 - 1.7 = \_\_\_\_\_\_

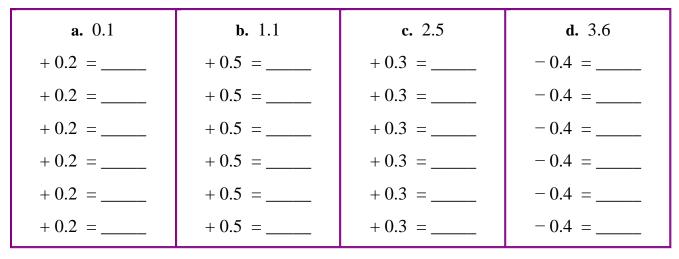
- 5. Write the numbers.
  - **a.** 3 tenths, 5 ones
  - **b.** 7 tens, 8 ones, 4 tenths
  - c. 4 tenths, 3 ones, 6 tens

T O te 4 7 . 5

In this place value chart, "T" means tens, "O" means ones, and "te" means tenths.

We can see that the number 47.5 has 4 tens, 7 ones, and 5 tenths.

6. Continue the patterns by adding or subtracting the same number repeatedly.



- 7. Remember: 1 millimetre is one-tenth of a centimetre. Or, 1 mm = 0.1 cm.
- a. Draw a line that is 4.7 cm long.

  b. Measure the line in centimetres.
  Use a decimal.

  1 2 3 4 5
- 8. In (a) and (b), convert. In (c), add and give your answer in centimetres.
  - **a.**  $0.5 \text{ cm} = \underline{\qquad} \text{ mm}$  **b.**  $7 \text{ mm} = \underline{\qquad} \text{ cm}$  **c.**  $5 \text{ mm} + 0.9 \text{ cm} = \underline{\qquad} \text{ cm}$   $1.2 \text{ cm} = \underline{\qquad} \text{ mm}$   $35 \text{ mm} = \underline{\qquad} \text{ cm}$   $4 \text{ cm} + 3.4 \text{ cm} = \underline{\qquad} \text{ cm}$
- 9. The two sides of a rectangle measure 6.5 cm and 3.6 cm. Draw the rectangle on blank paper. What is its perimeter?

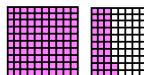
# **Two Decimal Digits—Hundredths**



This is 3 hundredths (3/100).

As a decimal, we write **0.03**.

Read 0.03 as "three hundredths."



This is 1 and 32 hundredths (1 32/100). As a decimal, we write **1.32**.

Read 1.32 as "one and 32 hundredths."

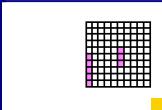


This is 20 hundredths (20/100). As a decimal, we write it as **0.20**.

It is *also* two tenths (2/10 or 0.2), because it is two columns, and each column is one-tenth of the whole. So, 0.20 = 0.2, or 20 hundredths equals 2 tenths.

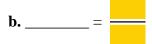
The two decimal digits after the decimal point indicate hundredths.

1. Write the number that each picture illustrates as a decimal *and* as a fraction or mixed number. Then read the decimals aloud.



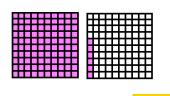
a. \_\_\_\_\_ = \_\_\_\_







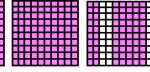














2. Colour to illustrate the decimals. Then write them as fractions. Read the decimals aloud.



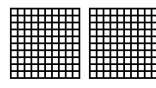
**a.** 0.52 =



**b.** 0.7 =



 $\mathbf{c.} \ 0.09 =$ 



**d.** 1.08 =