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Foreword

Math Mammoth Grade 7, International Version, comprises a complete maths curriculum for the seventh grade mathematics studies. This is a pre-algebra course, and students can continue to an Algebra 1 curriculum after completing it.

This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 7, only customised for international audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the seventh grade standards in your country. However, you can probably find material for any missing topics in neighbouring grades.

The international version of Math Mammoth has been customised for international audiences in these aspects:

- The curriculum uses metric measurement units. Imperial units, such as miles and pounds, are not used, with the following exceptions. One exception is a word problem about a phone's screen size in Chapter 5, in the lesson "Word Problems and Equations, Part 3". Since phone screen sizes are measured in inches worldwide, the problem uses inches. Another exception is the usage of cups in several word problems referring to serving sizes and recipes.
- The spelling conforms to British international standards.
- Paper size is A4.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
(The decimals are formatted with a decimal point, the same as the US version.)

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-A, covers the language of algebra (chapter 1), integers (chapter 2), one-step equations (chapter 3), rational numbers (chapter 4), and equations and inequalities (chapter 5). The rest of the topics are covered in the 7-B worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching maths!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth complete curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your student's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

In seventh grade, the first five chapters should be studied in order. Also, chapter 7 (Percent) should be studied before studying any of the chapters 8, 9, or 10 (Geometry, Probability, Statistics). You can be flexible with the rest of the scheduling.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" onto the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your student!

- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the maths lessons, for learning maths facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- The answer keys are included in the digital download version. They are sold as a separate book for the printed version.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for your student to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely will vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include the optional lessons nor tests.)

Most lessons are 3 or 5 pages long, intended for one day. Some 5-page lessons can take two days. There are also a few optional lessons.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Worktext 7-A	
Chapter 1	8 days
Chapter 2	13 days
Chapter 3	9 days
Chapter 4	16 days
Chapter 5	16 days
TOTAL	**62 days**

Worktext 7-B	
Chapter 6	16 days
Chapter 7	11 days
Chapter 8	22 days
Chapter 9	10 days
Chapter 10	12 days
TOTAL	**71 days**

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A	83	10	199	73	2.73	13.6
7-B	97	10	240	87	2.76	13.8
Grade 7 total	180	20	439	160	2.74	13.7

The table below is for you to use.

Grade level	School days	Days for tests and revisions	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A			199			
7-B			240			
Grade 7 total			439			

Let's say you determine that your student needs to study about 2.7 pages a day, or 14 pages a week on average in order to finish the curriculum in a year. As the student studies each lesson, keep in mind that sometimes most of the page might be reserved for workspace to solve equations or other problems. You might be able to cover more than the average number of pages on such a day. Some other day you might assign the student only one page of word problems.

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, seventh graders might spend 45-90 minutes a day on maths. If your student finds maths enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can affect the student's attitude towards maths.

Working space, the usage of additional paper and mental maths

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1000). Typically, I have intended that such exercises be done using MENTAL MATHS.

However, there are some students who struggle with mental maths (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning maths.

Students struggling with mental maths will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental maths strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the printed version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any maths topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. By then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the MathMammoth.com website.

Chapter 1: The Language of Algebra

Introduction

In the first chapter of *Math Mammoth Grade 7* we both revise basic algebra topics from sixth grade, and also go deeper into them, plus study the basic properties of the four operations. Since a good part of this chapter is revision, it serves as a gentle introduction to 7th grade maths, laying a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers. When we study equations in chapters 3 and 5, and also in subsequent grade levels, students will use the skills from this chapter (such as simplifying expressions, using the distributive property) in solving equations.

The main topics are the order of operations, writing and simplifying expressions, and the properties of the four operations, including the distributive property. Students have studied most of these in 6th grade. The main principles are explained and practised both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

Please note that it is not recommended to assign all the exercises by default. Use your judgeement, and try to vary the number of assigned exercises according to the student's needs. See the user guide at the beginning of this book or at **https://www.mathmammoth.com/userguides/** for some further thoughts on using and pacing the curriculum.

You can find matching videos for topics in this chapter at **https://www.mathmammoth.com/videos/** (choose grade 7).

Good Mathematical Practices

- The student is embarking on a wonderful journey into algebra — learning to do arithmetic with letters. The familiar properties of the four operations still hold, just like they do with numbers. Algebra is such a wonderful tool because it allows us to abstract a given situation and represent it symbolically, and then manipulate the representing symbols as if they have a life of their own. It is the foundational tool that allows us to model real-world situations with mathematics.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 1	**page**	**span**	**suggested pacing**	**your pacing**
Exponents and the Order of Operations	13	*4 pages*	1 day	
Expressions and Equations	17	*3 pages*	1 day	
Properties of the Four Operations	20	*4 pages*	1 day	
Simplifying Expressions	24	*4 pages*	1 day	
Growing Patterns 1	28	*3 pages*	1 day	
The Distributive Property	31	*5 pages*	2 days	
Chapter 1 Revision	36	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*25 pages*	8 days	

Games at Math Mammoth Online Practice

Hexingo Game — Order of Operations
Practise the order of operations with the four basic operations, brackets, and exponents.
https://www.mathmammoth.com/practice/order-operations#num=3&operations=add,sub,mult,div,exponents,parens

Expression Exchange
This online activity includes THREE separate work areas where you can explore how simple algebraic expressions work, and then one game. In the work areas, you can learn how to add and subtract simple algebraic terms in order to form an expression. In the game, you will go through practice exercises, forming the asked expressions from parts.
https://www.mathmammoth.com/practice/expression-exchange

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of maths concepts;
- **articles** that teach a maths concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr7ch1

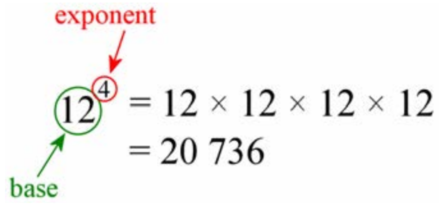
SCAN ME

Exponents and the Order of Operations

Let's revise! Exponents are a shorthand for writing repeated multiplications by the same number.

For example, $0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9 \cdot 0.9$ is written 0.9^5.

The tiny raised number is called the **exponent**.
It tells us how many times the **base** number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 0.7^8 is read as "seven tenths to the eighth power" or "zero point seven to the eighth."

The "powers of 6" are simply expressions where 6 is raised to some power: for example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the area of a square with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the volume of a cube with an edge 1.5 units long.

1. Evaluate.

a. 4^3 **b.** 10^5 **c.** 0.1^2

d. 0.2^3 **e.** 1^{100} **f.** 100 cubed

2. Write these expressions using exponents. Find their values.

a. $0 \cdot 0 \cdot 0 \cdot 0 \cdot 0$ **b.** $0.9 \cdot 0.9$

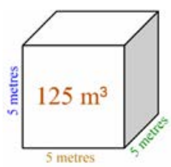
c. $5 \cdot 5 \cdot 5 \ + \ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

d. $6 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \ - \ 9 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

The expression $(5\text{ m})^3$ means that we multiply 5 metres by itself three times:

$(5\text{ m})^3 = 5\text{ m} \cdot 5\text{ m} \cdot 5\text{ m} = 125\text{ m}^3$

Notice that $(5\text{ m})^3$ is different from 5 m^3. The latter has no brackets, so the exponent (the little 3) applies only to the unit "m" and not to the whole quantity 5 m.



3. Find the value of the expressions. Include the proper unit.

a. $(2\text{ cm})^3$ **b.** $(11\text{ m})^2$ **c.** $(1.2\text{ km})^2$ **d.** $(6\text{ cm})^2$

4. Match each of (a) and (b) with one expression on the right.

a. The volume of a cube with edges 2 cm long.

b. The volume of a cube with edges 8 cm long.

(i) 8 cm^3 **(ii)** $(8\text{ cm})^3$ **(iii)** 512 cm

The Order of Operations — BE[MD][AS] 1) Solve what is within brackets **(B)**. 2) Solve exponents **(E)**. 3) Solve the multiplicative operations — this includes both multiplications **(M)** and divisions **(D)** — from left to right. 4) Solve the additive operations — this includes both additions **(A)** and subtractions **(S)** — from left to right.	**Example 1**. In $15 - 2 + 3 \cdot 3$, we do $3 \cdot 3$ first, then the subtraction, and lastly the addition. You can remember BEMDAS with the silly mnemonic *Big Elephants Meet Daily And Sing.* Or make up your own!

5. Find the value of each expression.

a. $120 - (9 - 4)^2$ **b.** $120 - 9 - 4^2$	**c.** $4 \cdot 5^2$ **d.** $(4 \cdot 5)^2$	**e.** $10 \cdot 2^3 \cdot 5^2$ **f.** $10 + 2^3 \cdot 5^2$
g. $(0.2 + 0.3)^2 \cdot (5 - 5)^4$	**h.** $0.7 \cdot (1 - 0.3)^2$	**i.** $20 + (2 \cdot 6 + 3)^2$

Example 2. Simplify $(10 - (5 - 2))^2$. Here we have double brackets. First calculate what is within the *inner* brackets: $5 - 2 = 3$. Then the expression becomes $(10 - 3)^2$. The rest is easy: $(10 - 3)^2 = 7^2 = 49$.	**Example 3.** Simplify $2 + \dfrac{1+5}{40-6^2}$. The fraction line works just like brackets, as a grouping symbol, grouping both what is above the line and also what is below it. Therefore, first solve what is in the numerator and in the denominator (in either order). $2 + \dfrac{1+5}{40-6^2} = 2 + \dfrac{6}{4} = 2 + \dfrac{2}{3} = \dfrac{8}{3}$

6. Find the value of each expression.

a. $(12 - (9 - 4)) \cdot 5$	**b.** $12 - (9 - (4 + 2))$	**c.** $(10 - (8 - 5))^2$	**d.** $3 \cdot (2 - (1 - 0.4))$

7. Find the value of each expression.

a. $\dfrac{4 \cdot 5}{2} \cdot \dfrac{9}{3}$	**b.** $\dfrac{4 \cdot 5}{2} + \dfrac{9}{3}$	**c.** $\dfrac{4+5}{2} + \dfrac{9}{3-1}$

In algebra and beyond, the fraction line is preferred over the ÷ symbol, and it acts as a grouping symbol (just like brackets).

Compare how each of these expressions looks when written either with the division symbol or with the fraction line. The latter usually makes the expressions easier to read.

$46 \div 2 + 50 \div 5$ vs. $\frac{46}{2} + \frac{50}{5}$ $\qquad$ $48 \div (1 + 5) \cdot 3$ vs. $\frac{48}{1+5} \cdot 3$

Notice how only what comes directly after the ÷ symbol, whether a single number or an expression in brackets, goes to the denominator.

Example 4. Rewrite the expression (10 + 8) ÷ 4 + 3 using the fraction line.

The denominator is just 4, not 4 + 3. The 10 + 8 will not need brackets anymore because the fraction line in itself is a grouping symbol. So, this is written as $\frac{10+8}{4} + 3$.

The additions and subtractions that are done last (*not* additions and subtractions in brackets or in the numerator/denominator) separate the expression into subsections that we call *terms*.

Example 5. This expression has four terms, separated by a +, then a −, and lastly a + sign.

$$3^2 + \frac{2}{4} - \frac{30}{6+2} + 4 \cdot 8$$

Example 6. Rewrite the expression 2 ÷ 4 + 3 ÷ (7 + 2) using the fraction line.

Now there are *two* divisions: the first by 4 and the second by (7 + 2), separated by an addition. This means we will use two fractions, or two terms, in the expression. It is written as $\frac{2}{4} + \frac{3}{7+2}$.

8. Match the expressions that are the same.

2 ÷ 3 · 4 $\qquad$ 2 ÷ (3 · 4) $\qquad$ 1 + 3 ÷ (4 + 2) $\qquad$ 1 + 3 ÷ 4 + 2 $\qquad$ (1 + 3) ÷ 4 + 2 $\qquad$ (1 + 3) ÷ (4 + 2)

$1 + \frac{3}{4} + 2$ $\qquad$ $\frac{1+3}{4+2}$ $\qquad$ $\frac{2}{3} \cdot 4$ $\qquad$ $\frac{2}{3 \cdot 4}$ $\qquad$ $\frac{1+3}{4} + 2$ $\qquad$ $1 + \frac{3}{4+2}$

9. Rewrite each expression using the fraction line and then find its value.

a. 56 ÷ 7 + 6	**b.** 7 ÷ (2 + 6)	**c.** 16 ÷ (2 + 6) − 2	**d.** 4 ÷ 5 − 1 ÷ 3

To **evaluate an expression** means to find (calculate) its value.

Example 7. Evaluate the expression $x^2 - \frac{2+y}{y}$ when x is 10 and y is 3.

This means we substitute 10 for x and 3 for y in the expression and then calculate its value according to the order of operations:

$$x^2 - \frac{2+y}{y} = 10^2 - \frac{2+3}{3} = 100 - \frac{5}{3} = 98\ 1/3$$

However, in algebra and beyond, it is customary to *not* give answers as mixed numbers but as fractions, to avoid confusion. After all, 98 1/3 could easily be mistaken for 981/3. So let's go back to the expression 100 − (5/3) and simplify it so it becomes a fraction:

$$100 - \frac{5}{3} = \frac{300}{3} - \frac{5}{3} = \frac{295}{3}$$ (This is the final value as a fraction.)

10. Find the value of these expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $\frac{9^2}{9} \cdot 6$	**b.** $\frac{2^3}{3^2}$	**c.** $\frac{(5-3)\cdot 2}{8-1+2} + 3$

11. Evaluate the expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $2x^2 - x$, when $x = 4$	**b.** $\frac{3s}{5} - \frac{2t}{5}$, when $s = 10$ and $t = 4$
c. $\frac{x^2}{x+1}$, when $x = 3$	**d.** $\frac{a+b}{b} + 2$, when $a = 1$ and $b = 3$

12. Why is it wrong to write the expression $2 + 5 \cdot 2 \div 4$ as $\frac{2 + 5 \cdot 2}{4}$?

Expressions and Equations

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as *x*, *y*, *a*, T, and so on.

Note: Expressions do *not* have an "equals" sign!

Examples of expressions: 5 $\frac{xy^4}{2}$ $T - 5 + \frac{x}{7}$

An **equation** has two expressions separated by an equals sign:

(expression 1) = (expression 2)

Examples: $0 = 0$ $2(a - 6) = b$

$9 = -8$ (a false equation) $\frac{x + 3}{2} = 1.5$

What do we do with expressions?

We can find the *value* of an expression (*evaluate* it). If the expression contains variables, we cannot find its value unless we know the value of the variables.

For example, to find the value of the expression $2x$ when x is 6/7, we simply substitute 6/7 in place of x.
We get $2x = 2 \cdot (6/7) = 12/7$.

Note: When we write $2x = 2 \cdot (6/7) = 12/7$, the equals sign is *not* signalling an equation to solve. (In fact, we already know the value of x!) It is simply used to show that the value of the expression $2x$ here is the same as the value of $2 \cdot (6/7)$, which is in turn the same as 12/7.

What do we do with equations?

If the equation has a variable (or several) in it, we can try to *solve* the equation. This means we find the values of the variable(s) that make the equation true.

For example, we can solve the equation $0.5 + x = 1.1$ for the unknown x.

The value 0.6 makes the equation true: $0.5 + 0.6 = 1.1$. We say $x = 0.6$ is the **solution** or the **root** of the equation.

1. This is a revision. Write an expression.

a. $2x$ minus the sum of 40 and x.

b. The quantity 3 times x, cubed.

c. s decreased by 6

d. five times b to the fifth power

e. seven times the quantity x minus y

f. the difference of t squared and s squared

g. x less than 2 cubed

h. the quotient of 5 and y squared

i. 2 less than x to the fifth power

j. x cubed times y squared

k. the quantity $2x$ plus 1 to the fourth power

l. the quantity x minus y divided by the quantity x squared plus one

To read the expression $2(x + y)$, use the word ***quantity***:
"two times the quantity x plus y."

There are other ways, as well, just not as common:

"two times the sum of x and y," or
"the product of 2 and the sum x plus y."

Some equations are *true*, and others are *false*. For example, 0 = 9 is a false equation.

Some equations are neither. The equation $x + 1 = 7$ is neither false nor true in itself.
However, if x has a specific value, then we can tell if the equation becomes true or false.

Indeed, solving an equation means finding the values of the variables that make the equation *true*.
The solutions of an equation are also called its **roots**.

Example. Find the root of the equation $20 - 2y^2 = 2$ in the set {1, 2, 3, 4}.

We try each number from the set, checking to see if it makes the equation true:

$20 - 2 \cdot 1^2 \stackrel{?}{=} 2$	$20 - 2 \cdot 2^2 \stackrel{?}{=} 2$	$20 - 2 \cdot 3^2 \stackrel{?}{=} 2$	$20 - 2 \cdot 4^2 \stackrel{?}{=} 2$
$18 \neq 2$	$12 \neq 2$	$2 = 2$	$-12 \neq 2$

In the given set, the only root of the equation is 3.

2. Write an equation. Then solve it by guess and check and logical reasoning.

a. 78 decreased by some number is 8.

b. The difference between a number and 2/3 is 1/4.

c. A number divided by 7 equals 3/21.

Equation	Solution

3. **a.** Find the root(s) of the equation $n^2 - 9n + 14 = 0$ in the set on the right.

1 10 3 6 2 7

b. Find the root(s) of the equation $9x - 5 = 2x$ in the set on the right.

1/5 5/7 7/9

4. Which of the numbers 0, 1, 3/2, 2 or 5/2 make the equation $\dfrac{y}{y-1} = 3$ true?

5. **a.** Ann is 5 years older than Tess, and Tess is n years old.
Write an <u>expression</u> for Ann's age.

b. Let A be Alice's age and B be Betty's age.
Which equation matches the sentence
"Alice is 8 years younger than Betty"?

A = 8 − B **A = B − 8** **B = A − 8**

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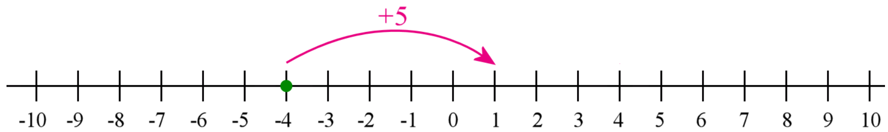
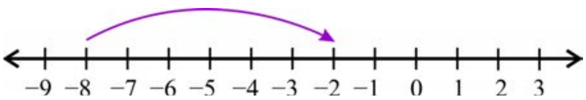
Addition and Subtraction on the Number Line 1

Addition can be modelled on the number line as a movement to the *right*.

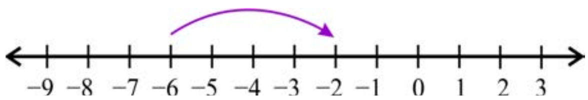
Suppose you are at −4, and you jump 5 steps to the right. You end up at 1. We write the addition −4 + 5 = 1.



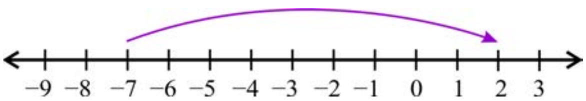
1. Write an addition equation to match each number line jump.



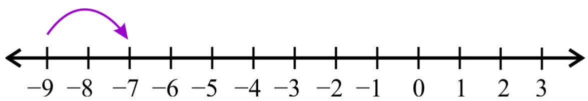
a. _______ + _______ = _______



b. _______ + _______ = _______

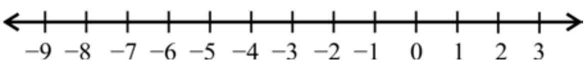


c. _______ + _______ = _______

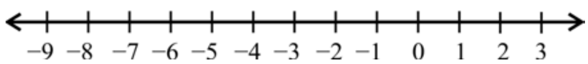


d. _______ + _______ = _______

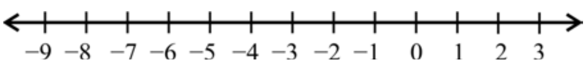
2. Draw a number line jump for each addition equation and solve.



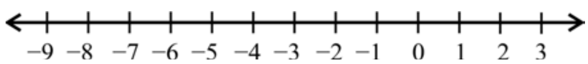
a. −8 + 3 = _______



b. −2 + 5 = _______



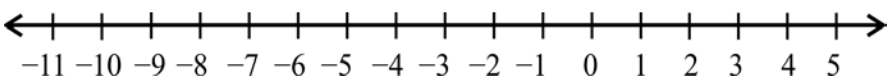
c. −4 + 4 = _______



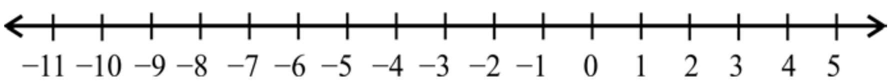
d. −10 + 12 = _______

3. What about adding more than one number? How could these additions be illustrated by number line jumps?

a. −4 + 2 + 3

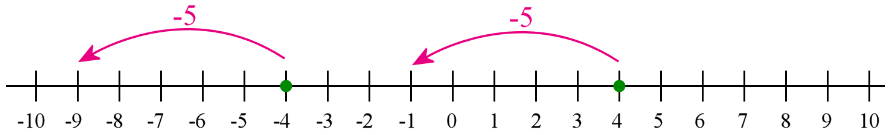


b. −11 + 6 + 4



Subtraction can be shown on the number line as a movement to the *left*.

You are at −4, and you jump 5 steps <u>to the left</u>. You end up at −9. We write the subtraction $-4 - 5 = -9$.

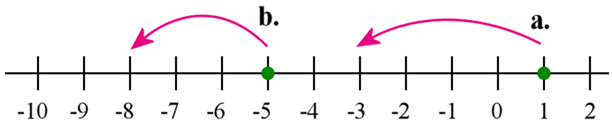


What subtraction does the other jump show? (Check the bottom of the page for the answer.)

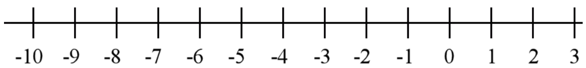
4. Write a subtraction to match each number line jump.

a. ________ − ________ = ________

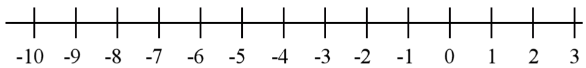
b. ________ − ________ = ________



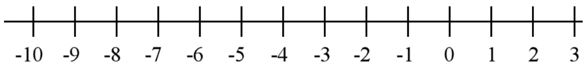
5. Draw a number line jump for each subtraction and complete the subtraction sentence.



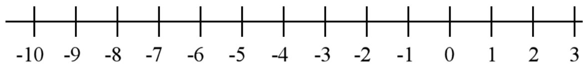
a. $2 - 6 =$ _______



b. $-2 - 5 =$ _______

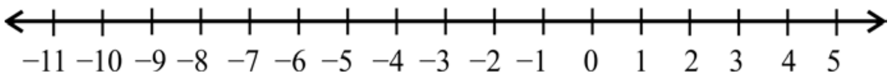


c. $0 - 7 =$ _______



d. $-5 - 4 =$ _______

6. Write an addition or a subtraction equation. You can use the number line to help.

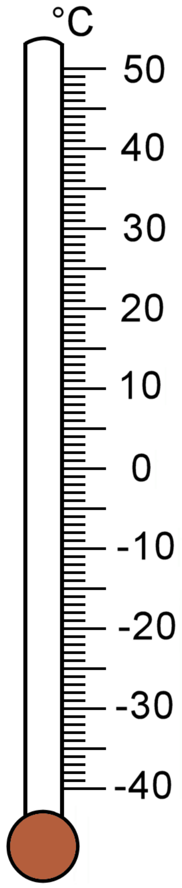


	Equation:
a. You are at $^{-}3$. You jump 6 to the right. You end up at _______.	
b. You are at $^{-}3$. You jump 6 to the left. You end up at _______.	
c. You are at 2. You jump 7 to the left. You end up at _______.	
d. You are at $^{-}10$. You jump 9 to the right. You end up at _______.	
e. You are at $^{-}7$. You jump 4 to the right. You end up at _______.	

Answer to the question in the second teaching box: $4 - 5 = -1$.

7. The temperature changes from what it was before. Find the new temperature.

before	2° C	0° C	1° C	−2° C	−12° C	−7° C
change	drops 3° C	drops 7° C	drops 5° C	rises 5° C	rises 6° C	rises 3° C
now						



8. Explain how each addition or subtraction can model a change in temperature.

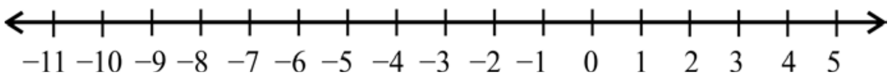
a. $-2 + 4 = 2$

b. $-2 - 4 = -6$

c. $-3 + 3 = 0$

9. Add or subtract. Think of jumps on the number line.

a. $2 - 7 =$	**b.** $^{-}2 - 2 =$	**c.** $^{-}3 + 3 =$	**d.** $^{-}8 + 6 =$
$1 - 5 =$	$^{-}6 - 3 =$	$^{-}6 + 3 =$	$^{-}12 + 4 =$
$5 - 9 =$	$^{-}5 - 2 =$	$^{-}15 + 5 =$	$^{-}11 + 13 =$



10. Find the number that is missing from the equations. Think of jumps on the number line.

a. 1 − _______ = $^{-}2$	**c.** $^{-}7$ + _______ = $^{-}4$	**e.** 1 − ______ = $^{-}6$	**g.** $^{-}5$ + _______ = 0
b. 3 − _______ = $^{-}5$	**d.** $^{-}9$ + _______ = $^{-}4$	**f.** 0 − ______ = $^{-}9$	**h.** $^{-}7$ + _______ = 7

11. The expression $1 - 3 - 5 - 7$ can be thought of as a person making jumps on the number line. Where does the person end up?

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Chapter 2 Revision

1. Match the equations with the situations and complete the missing parts.

a. A ball was dropped from 9 m above sea level; it fell 4 m.
Now the ball is at __________ m.

b. John had a $4 debt. He earned $9. Now he has ______________.

c. John had $4. He had to pay his dad $9. Now he has ______________.

d. A diver was at the depth of 9 m. Then he rose 4 m.

Now he is at __________ metres.

e. The temperature was −4° C and fell 9°. Now it is _____ ° C.

4 − 9 = ______

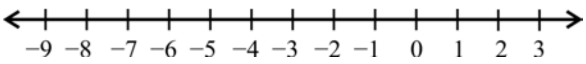
−4 + 9 = ______

−9 + 4 = ______

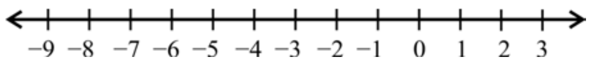
−4 − 9 = ______

9 − 4 = ______

2. Illustrate each addition on the number line.



a. −9 + 5 = ______



b. −1 + (−3) = ______

3. Add or subtract.

a. (−12) + (−1) = ______	**b.** −12 − (−1) = ______	**c.** 7 − 12 = ______
d. 21 + (−48) = ______	**e.** 41 + (−38) = ______	**f .**−610 + 900 = ______
g. (−2) + 7 + (−7) + (−1) = ______	**h.** 4 + (−10) + (−12) + 1 = ______	

4. Complete the equations, using <u>one positive</u> and <u>one negative</u> integer. There are many possible solutions.

a. _____ + _____ = −2 _____ + _____ = −2	**b.** _____ + _____ = 0 _____ + _____ = 0	**c.** _____ + _____ = 3 _____ + _____ = 3

5. Change each subtraction into an addition and solve.

a. 1 − (−7) ↓ ____ + ____ = ______	**b.** 2 − 11 ↓ ____ + ____ = ______	**c.** −20 − (−6) ↓ ____ + ____ = ______	**d.** −3 − 8 ↓ ____ + ____ = ______

6. Which king ruled the Persian Empire longer, Xerxes I, who ruled from 486 to 465 BC,
or Darius II, who ruled from 424 to 404 BC?

7. Iodide is an ion with 53 protons and 54 electrons.
Write a sum to represent the total electric charge of this ion.

8. The chart shows you the high and low temperatures during two winter days. Which day saw a greater difference in the high and low temperatures?

Day	**Monday**	**Friday**
High temperature	−13° C	−5° C
Low temperature	−18° C	−8° C

9. Give a real-life context for each expression, and find its value.

a. $-6 + (-8)$

b. $5 - 12$

c. $4 \cdot (-20)$

d. $-50 \div 2$

10. Draw lines to connect the expressions that have the same *value*.

$-50 - 20$ $\quad$ $50 - 20$ $\quad$ $-50 - (-20)$

$50 - (-20)$ $\quad$ $-50 + 20$ $\quad$ $50 + (-20)$

$-50 + (-20)$ $\quad$ $50 + 20$

11. The distance between b and -28 is fifteen units.
Find two possible values for b.

12. True or false?

a. Any integer more than 6 has an absolute value more than 6.

b. Any integer less than 6 has an absolute value less than 6.

c. A number and its opposite have the same absolute value.

d. The absolute value of the opposite of a number is the same as the opposite of the absolute value of the same number.

13. Multiply.

a. $-2 \cdot (-4) =$ _______ $-2 \cdot 4 =$ _______	**b.** $(-3) \cdot (-8) =$ _______ $7 \cdot (-12) =$ _______	**c.** $(-3) \cdot 3 \cdot (-1) =$ _______ $-7 \cdot (-2) \cdot (-2) =$ _______

14. Divide.

a. $-10 \div (-5) =$ _______ $24 \div (-3) =$ _______	**b.** $(-12) \div (-4) =$ _______ $21 \div (-3) =$ _______	**c.** $-56 \div 7 =$ _______ $-120 \div (-10) =$ _______

15. Find the missing numbers.

a. $-5 \cdot$ _______ $= -30$	**b.** $2 \cdot$ _______ $= -18$	**c.** $-8 \cdot$ _______ $= 48$
d. $-42 \div$ _______ $= 6$	**e.** $-64 \div$ _______ $= -8$	**f.** $81 \div$ _______ $= -9$

16. Solve the equations by thinking logically.

a. $5y = -100$ $y =$ _______	**b.** $-4b = -48$ $b =$ _______	**c.** $\frac{35}{y} = -5$ $y =$ _______

17. Divide and simplify if possible.

a. $1 \div (-6)$	**b.** $-3 \div 15$	**c.** $-6 \div (-7)$

18. Find the value of each expression.

a. $8 - (-2) \cdot 1 + 6$	**b.** $30 - 3 \cdot (-5)$	**c.** $-2 + 5 \cdot (2 - 3)$
d. $-17 + \frac{(-32)}{4}$	**e.** $\frac{8}{(-2)} - 15$	**f.** $2 + \frac{30}{-3 - 3}$

19. Find the value of the expressions when $x = -3$ and $y = 4$.

a. x^2	**b.** $-5xy$	**c.** $2 - (y + x)$

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Constant Speed

If an object travels with a constant speed, we have three quantities to consider: *speed or velocity* (v), *time* (t), and *distance* (d). The formula $d = vt$ tells us how they are interconnected.

Does that formula make sense?

Let's say John rides his bicycle at a constant speed of 12 km per hour for four hours. How far can he go? The formula says you multiply the speed (12 km/h) by the time (4 h) to get the distance (48 km). So the formula does make sense — that is how our "common sense" tells us to calculate it also.

Example 1. A boat travels at a constant speed of 15 km/h. How long will it take the boat to go a distance of 21 km?

The problem gives us the speed and the distance. The time (t) is unknown.

We can solve the unknown time by using the formula $d = vt$. We simply substitute the given values of v and d in it, and we will get an equation that we can solve:

$$\begin{array}{ccc} d & = & v\,t \\ \downarrow & & \downarrow \\ 21 & = & 15\,t \end{array}$$

To make it easier, we will leave off the units while solving the equation. We can do that, since both the velocity and the distance involve kilometres.

Next, we solve this equation:

$21 = 15t$	Flip the sides.
$15t = 21$	Divide both sides by 15.
$\frac{15t}{15} = \frac{21}{15}$	The 15 in numerator and denominator cancel.
$t = 7/5$	

The final answer $t = 7/5$ is in *hours*, because the unit for speed was kilometres per hour.

Let's work on that answer a bit more and change 7/5 hours to minutes.

How much is 1/5 of an hour? That's right, it is 12 minutes. And 7/5 hours is 84 minutes. So the final answer is 1 hour, 24 minutes.

You may use a calculator for all the problems in this lesson.

1. Use the formula $d = vt$ to solve the problems.

a. A caterpillar crawls along at a constant speed of 20 cm/min. How long will take it to travel 34 cm?

$$\begin{array}{ccccc} d & = & v & & t \\ \downarrow & & \downarrow & & \downarrow \end{array}$$

b. Father leaves at 7:40 a.m. to drive 20 km to work. If his average speed is 48 km/h, when will he arrive at work?

$$\begin{array}{ccccc} d & = & v & & t \\ \downarrow & & \downarrow & & \downarrow \end{array}$$

How to change hours and minutes into fractional and decimal hours and vice versa	
Example 2. Change 14 minutes into hours. Since there are 60 minutes in an hour, 14 minutes is simply 14/60 of an hour. This simplifies to 7/30 of an hour. You can also change it into a decimal by dividing 7 by 30, to get 0.233 hours (rounded to three decimals).	**Example 3.** Change 4.593 hours into hours and minutes. How many minutes are in the decimal part? Since one hour is 60 minutes, 0.593 hours is 0.593 · 60 minutes = 35.58 minutes ≈ 36 minutes. So 4.593 hours ≈ 4 hours 36 minutes.

2. Convert the given times into hours in decimal format. Round your answers to three decimal digits.

a. 35 minutes	**b.** 44 minutes
c. 2 h 16 min	**d.** 4 h 9 min

3. Give these times in hours and minutes.

a. 2.4 hours	**b.** 0.472 hours
c. 3 3/5 hours	**d.** 16/50 hours

4. The average speed of a bus is 64 km/hour. What distance can it travel in 4 hours and 15 minutes?

5. Sam is an athlete who can run 16 km in an hour. How long will it take him to run home from the shopping centre, a distance of 3.8 km?

6. A train travelled a distance of 600 kilometres between two towns so that in the first half of the distance, its average speed was 150 km/h, and in the second half, only 120 km/h. How long did it take to travel from the one town to the other?

7. Elijah wants to use the extra time between classes for exercising. He plans to jog for 25 minutes in one direction, turn, and jog back to school. What is the distance Elijah can jog in 25 minutes if his average jogging speed is 9 km/h?

Example 4. Andy drove from his home to his workplace, which was 39 km away, in 26 minutes. What was his average speed?

The average speed of cars is usually given in miles per hour or kilometres per hour. This unit of speed actually gives us a **formula for calculating speed**:

speed	is	km	per	hour
↓	↓	↓	↓	↓
velocity	=	distance	/	time

(The word "per" indicates division.)

In symbols, $v = \frac{d}{t}$.

His average speed is therefore

$$v = \frac{d}{t} = \frac{39 \text{ km}}{26 \text{ minutes}} = \frac{3}{2} \text{ km per minute}$$

$= 1.5$ kilometres per minute.

The problem is that the average speed is usually given in km per *hour*, not km per minute. How can we fix that?

One way is to multiply our answer by 60.

Doing that, we get

$$1.5 \frac{\text{km}}{\cancel{\text{min}}} \cdot 60 \frac{\cancel{\text{min}}}{\text{hour}} = 90 \frac{\text{km}}{\text{hr}} .$$

Another way is to change the original time of 26 minutes into hours before using the formula.

Now, 26 minutes is simply 26/60 hours, which simplifies to 13/30 hours. We can write

$$v = \frac{39 \text{ km}}{13/30 \text{ hours}}$$

This is a **complex fraction**: a fraction that has another fraction in the numerator, denominator, or both.

One way to calculate its value with a calculator is to use brackets and input it as 39 ÷ (13 ÷ 30). Check out what happens if you input it as 39 ÷ 13 ÷ 30.

Tip: Instead of brackets, you can use the reciprocal button (1/*x*) on your calculator. First calculate the value of the fraction inverted (upside-down): $\frac{13/30}{39}$. This is the **reciprocal** of the fraction. You can input it as 13 ÷ 30 ÷ 39.

Once you've calculated the reciprocal, push the 1/*x* button to convert it into the answer that you want.

8. Find the average speed in the given units.

a. A duck flies 5 kilometres in 6 minutes.
Give your answer in kilometres per hour.

b. A lion runs 900 metres in 1 minute.
Give your answer in kilometres per hour.

c. Henry sleds 75 metres down the hill in 1.5 minutes.
Give your answer in metres per second.

d. Rachel swims 400 metres in 32 minutes.
Give your answer in kilometres per hour.

9. Jake's grandparents live 150 km away from his home. One day it took him 2 h 14 min to get there and 1 h 55 min to come back home. In the questions below, round your answers to one decimal digit.

a. What was his average speed going there?
Hint: Change the time in hours and minutes into decimal hours.

b. What was his average speed coming back?

c. What was his overall average speed for the whole trip?

10. Another day Jake visited his grandparents again. Let's say that, because of the traffic, Jake achieved an average speed of 75 km/h going there but an average speed of only 65 km/h coming back. How much longer did Jake spend driving home from his grandparents' place than going there?

How to remember the formula *d* = *vt*

I will show you how to *derive* that formula!
Then you do not really have to memorise it.

But you *do* need to remember the formula $v = d / t$.

You can remember *that* formula with the trick I explained earlier—by thinking of the common unit for measuring speed (km per hour):

speed	is	km	per	hour
↓	↓	↓	↓	↓
velocity	=	distance	/	time

In symbols: $v = d / t$.

Just solve for *d* from the formula *v* = *d*/*t*

$v = \frac{d}{t}$ We want *d* alone, so we multiply both sides by *t*.

$vt = \frac{d}{t} \cdot t$ The *t*'s on the right side cancel.

$vt = d$ We have it!

Turning it around we get ***d* = *vt***, which is the most common formula to show how velocity (v), time (t), and distance (d) are related.

11. Compare the average speeds to find which bird is faster: a seagull that flies 16 km in 24 minutes or an eagle that flies 22 km in half an hour?

12. A train normally travels at a speed of 120 km per hour. One day, the conditions were so icy and cold that it had to slow down to travel safely. So the train travelled the first half of its 160-km journey at half its normal speed. Then the weather improved, and the train was able to go faster again. It sped the remaining distance at twice its normal speed to make up time.

    a. How long did the train take to travel the whole distance (160 km)?

    b. How long would the train have taken if it had travelled the whole trip at its normal speed?

    c. What was the train's average speed for the trip on this cold and icy day?

13. How long will it take Charlotte to ride her bike from the music store to her home — a distance of 4.5 km — if she rides 1/3 of it at 12 km/h and the rest at 15 km/h?

The next problems are more challenging.

14. Your normal walking speed is 6 km per hour. One day you walk slowly, at 3 km per hour, half the distance from home to the swimming pool. Can you now make up for your slow walking by walking the remaining distance at double your normal speed?

(Hint: Make up a distance between your home and the swimming pool for an example calculation. Choose an easy number.)

15. An aeroplane normally flies at a speed of 1000 km/h. Due to some turbulence, it has to travel at a lower speed of 800 km/h for the first 40 minutes of a 1600-km trip. How fast should it fly for the rest of the trip so as to make up for the lost time?

(Hint: You will also need to calculate the normal travelling time for this trip.)

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Rational Numbers

If you can write a number as a *ratio of two integers*, it is a **rational number.**

For example, 4.3 is a rational number because we can write it as the ratio $\frac{43}{10}$ or 43:10.

Note: To represent rational numbers, we usually indicate the ratio with a fraction line rather than a colon.

Examples of rational numbers

Since −10 can be written as $\frac{-10}{1}$, it is a rational number. It can also be written as $\frac{10}{-1}$.

Since 0.1 can be written as $\frac{1}{10}$, it is a rational number.

Since 3.24 can be written as $\frac{324}{100}$, it, too, is a rational number.

Negative fractions

The ratio of the integers 7 and −10 gives us the fraction $\frac{7}{-10}$. As we studied earlier, we usually write this as $-\frac{7}{10}$ and read it as "negative seven tenths."

Obviously, all fractions, whether negative or positive, are rational numbers.

Negative fractions give us negative decimals.

For example, $-\frac{8}{10}$ is written as a decimal as −0.8, and $-5\,\frac{21}{100} = -5.21$.

You can write a rational number as a ratio of two integers in many ways.

For example, the decimal −1.4 can be written as a ratio of two integers in all these ways (and more!):

$$-1.4 = \frac{-14}{10} = \frac{-28}{20} = \frac{28}{-20} = \frac{42}{-30} = \frac{-42}{30} = \frac{-7}{5}$$

So −1.4 is *definitely* a rational number! ☺ But the same holds true for all rational numbers—you can always write them as a ratio of two integers in multitudes of ways.

1. Write these numbers as a ratio (fraction) of two integers.

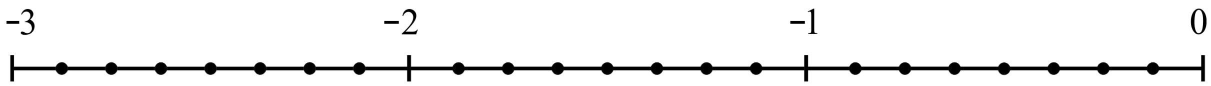
a. 6	**b.** −100	**c.** 0	**d.** 0.21
e. −1.9	**f.** −5.4	**g.** −0.56	**h.** 0.022

2. Are all percents, such as 34% or 5%, rational numbers? Justify your answer.

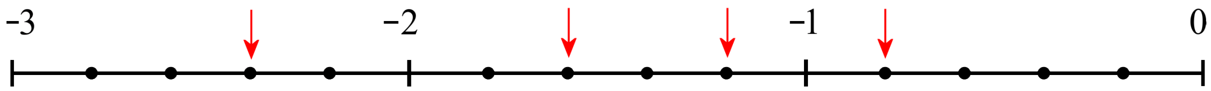
3. Form a fraction from the two given integers. Then convert it into a decimal.

a. 8 and 5	**b.** −4 and 10	**c.** 89 and −100
d. −5 and 2	**e.** 91 and −1000	**f.** −1 and −4

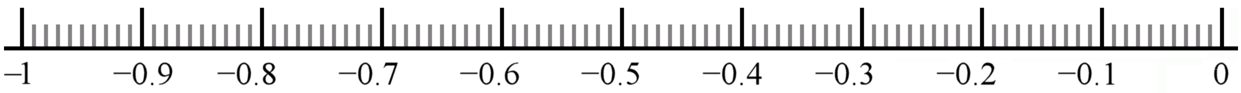
4. Mark the fractions and mixed numbers on the number line below: $-\frac{1}{2}$, $-\frac{7}{8}$, $-1\frac{5}{8}$, $-\frac{9}{4}$, $-2\frac{3}{4}$



5. Write the fractions marked by the arrows.



6. Mark the decimals on the number line: −0.11, −0.58, −0.72, −0.04



7. Sketch a number line from −3 to 0, with tick marks at every tenth. Then mark the following numbers on your number line: −0.2, −1.5, −2.8, −3/5, and −5/2.

8. Write these rational numbers as ratios of two integers (fractions) in a lot of different ways.

a. $-2 = -\frac{2}{1} =$
b. $0.6 = \frac{6}{10} =$

9. Compare, writing < or > in between the numbers.

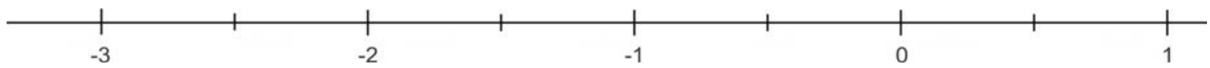
a. $-\frac{7}{8}$ -1	**b.** $-\frac{3}{4}$ $\frac{1}{2}$	**c.** $-\frac{15}{2}$ -7	**d.** -0.98 -1.4

10. Order these rational numbers in order, from the smallest to the greatest.

2.1 $-\frac{1}{8}$ -1 $-\frac{7}{3}$ -2.01 1 $\frac{1}{3}$ -0.5

11. Mark the decimals *and* the fractions on the number line, approximately.

0.3 $-\frac{2}{5}$ -0.8 $-\frac{10}{4}$ -2.1 $-1\frac{1}{2}$ $-\frac{17}{10}$ 0.95



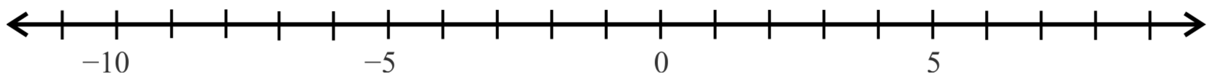
Recall that the absolute value of a number is its distance from zero.

Below, the thickened line shows the set of numbers that are greater than −3 and at the same time, less than 3. We can write it like this: the set of numbers x so that $-3 < x < 3$.

These are the numbers whose absolute value is less than 3, in other words the set of numbers for which $|x| < 3$. Their distance to zero is less than 3. For example, −2.8 and 0.492 and −6/5 belong to this set.

Note that 3 and −3 are not part of this set; that is why we use an open circle at 3 and −3.

12. **a.** Show on the number line the set of numbers x for which $|x| < 1.5$



b. List three rational numbers in that set that are not integers.

13. List three rational numbers r so that $|r| < 2$ and $r > -1$.

Repeating Decimals

As you already know, sometimes it is easy to write a fraction as a decimal. For example, 3/10 = 0.3 and 1/4 = 0.25. However, if you don't know of any other way to to find the decimal equivalent of a fraction, the technique that works all the time is to **treat the fraction as a division** and divide.

Example 1. Write $\frac{31}{40}$ as a decimal.

We will use long division. Note how we add many decimal zeros to the dividend (31) so that we can continue the division into the decimal digits.

This division **terminates** (comes out even) after just three decimal digits.

We get $\frac{31}{40}$ = 0.775. This is a **terminating decimal**.

```
       0 0.7 7 5
    40)3 1.0 0 0 0
      -2 8 0
         3 0 0
      -  2 8 0
           2 0 0
         - 2 0 0
               0
```

1. Write as decimals, using long division. Continue the division until it terminates.

a. $\frac{3}{16}$	**b.** $\frac{51}{32}$	**c.** $\frac{17}{80}$

2. Use long division to write these fractions as decimals. Continue the division to at least 6 decimal digits. Notice what happens!

a. $\frac{2}{3}$	**b.** $\frac{7}{11}$	**c.** $\frac{8}{9}$

Example 2. Write $\frac{18}{11}$ as a decimal.

```
     0 1.6 3 6 3
11 ) 1 8.0 0 0 0
    -1 1
       7 0
     - 6 6
         4 0
       - 3 3
           7 0
         - 6 6
             4 0
           - 3 3
               7
```

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient and the remainders 40 and 70 start repeating.

So $\frac{18}{11}$ = 1.636363... . We can use an ellipsis (three dots, or "…") to indicate that the decimal is non-terminating. A better notation is to draw a **bar** (a line) over the digits that repeat: 1.636363… = $1.\overline{63}$.

This number is called a **repeating decimal** because the digits "63" repeat forever!

The decimal form of ANY rational number is either a terminating decimal or a repeating decimal.

This is an important fact. It says that when you write any fraction as a decimal, there are only two possibilities: either the decimal terminates or it repeats.

The converse is also true: if a decimal terminates or is a repeating decimal, it *can* be written as a fraction, thus is a rational number.

Example 3. The repeating decimal 1.9051050505… is written as $1.9051\overline{05}$. Notice that the bar marks only the digits that repeat ("05"). The digits "9051" that don't repeat are not included under the bar.
(If you're curious, as a fraction, this number is 1 886 054/990 000.)

Example 4. A calculator gives the decimal expansion of 5/13 as 0.38461538461538461538461538461538...

The repeating part is the digits "384615". So, 5/13 = $0.\overline{384615}$.

Example 5. The decimal 0.095 is a terminating decimal, but we *can* write it with an unending decimal expansion if we write zeros for all the decimal places after thousandths:

0.095 = 0.095000000000...

In other words, we can think of it as repeating the digit zero. In that sense, 0.095 = $0.095\overline{0}$.
However, as you know, we normally write terminating decimals without the extra zeros.

3. Write each decimal using a line over the repeating part.

a. 0.09090909… **b.** 5.6843434343… **c.** 0.19866666666…

4. Do it the other way around: write the repeating digits several times followed by an ellipsis (three dots).

a. $0.0\overline{887}$ **b.** $0.245\overline{6}$ **c.** $2.1\overline{7234}$

5. Which decimal is greater?

a. Which is more, $0.\overline{3}$ or 0.3? How much more?	**b.** Which is more, $0.\overline{55}$ or $0.\overline{5}$? How much more?
c. Which is more, $0.45\overline{0}$ or 0.45? How much more?	**d.** Which is more, $0.\overline{12}$ or 0.12? How much more?

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Adding and Subtracting Rational Numbers

Adding a mix of positive and negative numbers

1. First add all the positive numbers and all the negative numbers separately.
2. Lastly add these two totals as you learned to do in the previous lesson.

Example 1. Add −9.5 + 2.4 + 0.5 + (−4.3) + (−0.8).

Adding all the negative decimals, we get −9.5 + (−4.3) + (−0.8) = **−14.6**.

The positives total to 2.4 + 0.5 = **2.9**.

Lastly, we add those two totals: **2.9** + (**−14.6**) = −11.7.

1. Add.

a. −0.5 + 0.6 + (−1.2) + (−1.4) + 1.6	**b.** −\$1.08 + (−\$4.30) + \$0.56 + \$0.99 + (−\$0.25)
c. $\frac{1}{2} + \left(-\frac{1}{3}\right) + \left(-\frac{4}{3}\right) + \frac{1}{6}$	**d.** $\frac{5}{8} + \left(-\frac{1}{2}\right) + \frac{3}{4} + \left(-\frac{3}{2}\right) + \left(-\frac{7}{8}\right)$

Subtracting rational numbers

Sometimes you might be able to figure out the subtraction by using a number line. At other times, you may need to use the definition of subtraction, and change the subtraction into an addition:

The difference of numbers *a* and *b* is the <u>sum</u> of *a* and the opposite of *b*: $\boldsymbol{a - b = a + (-b)}$

In other words, instead of subtracting a number, you add its opposite.

Example 2. Solve $-\frac{11}{12} - \left(-\frac{1}{3}\right)$.

We simply change the subtraction into addition, and get $-\frac{11}{12} + \frac{1}{3}$.

Then, $-\frac{11}{12} + \frac{1}{3} = -\frac{11}{12} + \frac{4}{12} = \frac{-11 + 4}{12} = \frac{-7}{12} = -\frac{7}{12}$.

2. Solve.

a. 0.97 − 1.67	**b.** −5.61 − 0.9	**c.** 2.5 − (−4.2) + (−0.3)

3. Mark has $55.20. He wants to buy a fan that costs $77.90. His mum said he can owe her the part that he cannot pay now. Write an expression to represent Mark's money situation (balance) after the purchase, and find its value.

4. Alexander has $105 left on his credit card. He goes to the store and they have batteries on sale for $11.45 a package. Alex buys 12 packages because it's such a good deal. What is the balance on his card after this purchase?

5. Solve.

a. $-\frac{5}{8} + \left(-\frac{1}{5}\right)$	**b.** $-\frac{11}{10} - \left(-\frac{4}{3}\right)$
c. $-\frac{2}{9} - \left(-\frac{2}{3}\right) - \frac{5}{9}$	**d.** $\frac{5}{8} + \left(-\frac{1}{8}\right) + \frac{1}{4} + \left(-\frac{1}{2}\right) + \left(-\frac{9}{8}\right)$

6. Give a real-life situation for the sum −\$50 + (−\$12.90) + \$85 and find its value.

7. Recall that the distance between two numbers *a* and *b* is | *a* − *b* |. Evaluate this expression for the given values of *a* and *b*. Check that you get a reasonable answer.

a. *a* = 0.7 and *b* = −0.7 | ____ − ____ | =	**b.** *a* = −7.8 and *b* = −5.4
c. *a* = 1/4 and *b* = −3 1/4	**d.** *a* = −4/10 and *b* = −9/10

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Two-Step Equations, Part 1

In two-step equations, we need to apply two different operations to both sides of the equation.

Example 1. On the side of the unknown (left), there is a multiplication by 3 and an addition of 2. To isolate the unknown, we need to undo those two operations, in two steps.

$$3x + 2 = 25 \quad | -2$$
$$3x = 23 \quad | \div 3$$
$$x = 23/3$$

Check:

$$3 \cdot (23/3) + 2 \stackrel{?}{=} 25$$
$$23 + 2 \stackrel{?}{=} 25$$
$$25 = 25 \checkmark$$

What if you divide first? That is possible:

$$3x + 2 = 25 \quad | \div 3$$
$$\frac{3x + 2}{3} = \frac{25}{3}$$
$$x + \frac{2}{3} = \frac{25}{3} \quad | -2/3$$
$$x = 23/3$$

Note that this leads to fractions in the middle of the solution process which is more error-prone. Then, the 2 on the left side <u>also has to be divided by 3</u> (to become 2/3). This is something that is easy to forget and is therefore another reason why subtracting first is the "safer" way, in this case.

If this was a real-life application, we would probably give the answer as a decimal, rounded to a reasonable accuracy. Since it is a mathematical problem, we leave the answer as a fraction. (Why not as a mixed number? It is not wrong, but fractions are less likely to be misread. The mixed number 7 2/3 can easily be misread as 72/3.)

1. Solve. Check your solutions (as always!).

a. $5x + 2 = 67$	**b.** $3y - 2 = 70$	**c.** $3x + 11 = 74$
d. $8z - 2 = 98$	**e.** $75 = 12x + 3$	**f.** $55 = 4z - 11$

Example 2. This equation has decimals. The solution process works the same way. However, we give the final answer as a rounded decimal. Note that we first simplify $0.5n + 0.7n$ as $1.2n$.

$$\begin{aligned} 0.5n + 0.7n - 9.7 &= 0.45 \\ 1.2n - 9.7 &= 0.45 \quad && | + 9.7 \\ 1.2n &= 10.15 \quad && | \div 1.2 \\ n &\approx 8.5 \end{aligned}$$

When checking an equation with a rounded answer, we don't require precise equality. Near equality is taken as the equation checking.

$$1.2 \cdot 8.5 - 9.7 \stackrel{?}{=} 0.45$$

$$0.5 \approx 0.45 \checkmark$$

In mathematics, the usage of a fraction typically implies that the value is precise, whereas a decimal implies that the number might be a rounded, approximate number and not precise. As you know, real-life applications often use decimals. So, in the case of an equation like this, we give the final answer rounded. In 8th grade, you will learn precise rounding rules governing these situations. For now, unless otherwise stated, round the final answer to the same accuracy as the least accurate decimal in the original equation.

2. Solve. Give the solutions to two decimal digits. Check your solutions (as always!).

a. $6.3y - 0.4 = 3$	**b.** $5.5 = 0.4y - 2.8$	**c.** $0.77s - 0.12 = 0.43$
d. $62.4 + 6x + 4x = 72.78$	**e.** $0.825 = 0.2y + 0.05y + 0.3$	**f.** $t + 1.27t - 3.12 = 3.098$

3. Check each solution on the right. If it is incorrect, find the error, and correct it.

a.
$$\begin{aligned} 10x - 14 &= 31 \\ 10x &= 17 \\ x &= 1.7 \end{aligned}$$

b.
$$\begin{aligned} 31 &= x + 15 + x \\ 31 &= 2x + 15 \\ 46 &= 2x \\ x &= 23 \end{aligned}$$

These equations have negative numbers. The solution process works the same way. Just be careful to follow the rules of integer arithmetic.

Example 3a.

$$\begin{aligned} -2x + 7 &= -4 && \mid -7 \\ -2x &= -11 && \mid \div(-2) \\ x &= 11/2 \end{aligned}$$

Check:

$$-2(11/2) + 7 \overset{?}{=} -4$$
$$-11 + 7 \overset{?}{=} -4$$
$$-4 = -4 \ ✓$$

Example 3b.

$$\begin{aligned} 2x - 7x - 11 &= 40 \\ -5x - 11 &= 40 && \mid +11 \\ -5x &= 51 && \mid \div(-5) \\ x &= -51/5 \end{aligned}$$

Check:

$$-5(-51/5) - 11 \overset{?}{=} 40$$
$$51 - 11 \overset{?}{=} 40$$
$$40 = 40 \ ✓$$

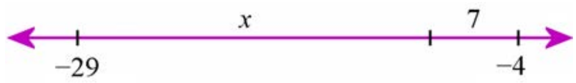
4. Solve. Check your solutions (as always!).

a. $-6x + 2 = -30$	**b.** $-y - 2 = 26$	**c.** $9x - 11 = -8$
d. $12z - 44 = -98$	**e.** $-100 = 12x - 15 + 18x$	**f.** $-6 = -2t - 11 - 2t$

5. Solve. Compare the two equations.

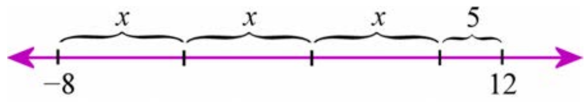
a. $-5y + 2 = -11$	**b.** $5y - 2 = -11$

6. The number line diagram on the right illustrates the equation $-29 + x + 7 = -4$. Figure out the solution using the diagram.

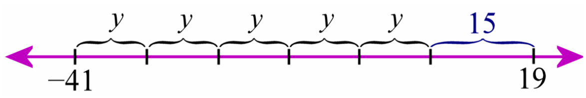


Next, write an equation to match each number line diagram below and find the value of the unknown.

a.



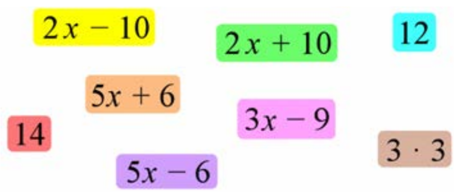
b.



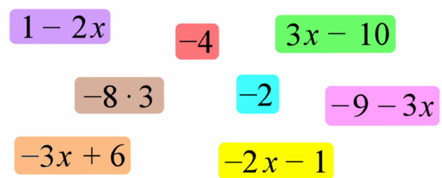
7. Use these for more practice as needed. Use blank paper as necessary. Round the solutions for (b) and (c) to two decimal digits.

a. $3y - 20 = 65$	**b.** $6z + 5 = 2.2$	**c.** $5.2x + 6.25 = 108$
d. $-4s + (-2s) - 2 = 40$	**e.** $2a + 1 + 3a = -20$	**f.** $-3t + 2 = -9$

a. Choose from the expressions below to build an equation that has the root $x = 2$.



b. Choose from the expressions below to build an equation that has the root $x = 5$.

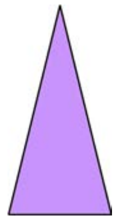


Puzzle Corner

Problems to Solve

Example. The perimeter of an isosceles triangle is 26 cm and its base measures 5 cm. How long are the two sides that are equal (congruent)?

(To help you, label the image with the data from the problem.)



Read the two solutions below. Notice how neatly the steps tie in with each other!

Solution 1: an equation

Let x *be* the unknown side length. We get:

$$\text{perimeter} = x + x + 5$$

$$26 = 2x + 5$$

Next we solve the equation:

$$\begin{aligned} 2x + 5 &= 26 && \mid -5 \\ 2x &= 21 && \mid \div 2 \\ x &= 10.5 \end{aligned}$$

The two other sides measure 10.5 cm each.

Solution 2: Logical thinking

The perimeter is 26 cm. This means that the two unknown sides and the 5-cm side add up to 26 cm.

Therefore, if we subtract the base side of 5 from the perimeter, the two unknown sides must add up to 21 cm.

So one side is half of that, or 10.5 cm.

You may use a calculator in all the problems of this lesson.

1. Solve the problem below in two ways: write an equation, and use logical reasoning.

A quadrilateral has three congruent sides. The fourth side measures 1.4 m.
If the perimeter of the quadrilateral is 7.1 metres, what is the length of each congruent side?

Equation:	Logical thinking (and mental maths):

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Equations with Fractions, Part 1

When an equation of the form $p(x + q) = r$ involves a fraction, we can use the same technique as before: first distribute the multiplication and go on from there. You will need to use fraction arithmetic.

However, it is also possible to start by **multiplying both sides by the denominator of the fraction**, as this will **eliminate the fraction**, and then the solution process will only involve whole numbers (until possibly the last step). This is therefore often the simpler way. Study the examples carefully.

Example 1a: First distribute.

$3(x - 1) = \frac{4}{5}$	Distribute the multiplication.
$3x - 3 = \frac{4}{5}$	**+ 3**
$3x = 3\frac{4}{5}$	Change 3 4/5 into a fraction.
$3x = \frac{19}{5}$	**÷ 3**
$x = \frac{19}{15}$	

Check: $3(19/15 - 1) \stackrel{?}{=} \frac{4}{5}$

$3(4/15) \stackrel{?}{=} \frac{4}{5}$

$12/15 = \frac{4}{5}$ ✓

Example 1b:
First multiply by the denominator of the fraction.

$3(x - 1) = \frac{4}{5}$	**· 5**	(Multiply both sides by the denominator of the fraction.)
$5 \cdot 3(x - 1) = 5 \cdot \frac{4}{5}$		Next, simplify.
$15(x - 1) = 4$		Distribute the multiplication.
$15x - 15 = 4$	**+ 15**	
$15x = 19$	**÷ 15**	
$x = \frac{19}{15}$		

1. Solve. You can choose which way to start. Also, check your work, using blank paper if necessary.

a. $4(x - 3) = \frac{1}{8}$	**b.** $2(b - 8) = \frac{2}{3}$	**c.** $2(a + 3) = \frac{11}{4}$

2. Use these for more practice (use blank paper).

a. $3(x + 1) = \frac{2}{5}$ **b.** $2(w - 4) = -\frac{3}{4}$ **c.** $4(y + 1) = \frac{36}{5}$ **d.** $\frac{11}{3} = 7(x - 3)$

Example 2. Here we have a fraction times a quantity in brackets. Again, compare the two ways. Which seems simpler?

Example 2a: First distribute.

$\frac{1}{4}(a+5) = 2$

$\frac{1}{4}a + \frac{1}{4} \cdot 5 = 2$

$\frac{1}{4}a + \frac{5}{4} = 2 \quad \Big| - 5/4$

$\frac{1}{4}a = 2 - \frac{5}{4}$

$\frac{1}{4}a = \frac{3}{4} \quad \Big| \cdot 4$

$a = 3$

Example 2b: First multiply by the denominator of the fraction.

$\frac{1}{4}(a+5) = 2 \quad \Big| \cdot 4$

$\cancel{4} \cdot \frac{1}{\cancel{4}}(a+5) = 4 \cdot 2$

$1(a+5) = 8$

$a + 5 = 8$

$a = 3$

Check:

$\frac{1}{4} \cdot (3+5) \stackrel{?}{=} 2$

$\frac{1}{4} \cdot 8 \stackrel{?}{=} 2$

$2 = 2$ ✓

3. What was done in each step? Fill in the missing parts.

a.

$\frac{3}{5}(x+7) = -2 \quad \Big|$ ______

$5 \cdot \frac{3}{5}(x+7) = 5 \cdot (-2)$

______ $(x+7) =$ ______

$3x +$ ______ $= -10 \quad \Big|$ ______

$3x = -31 \quad \Big|$ ______

$x = -31/3$

b.

$2(x-3) = \frac{3}{8} \quad \Big|$ ______

$8 \cdot 2(x-3) = 8 \cdot \frac{3}{8}$

$16(x-3) =$ ______

$16x - 48 =$ ______ $\quad \Big|$ ______

$16x = 51 \quad \Big|$ ______

$x = 51/16$

4. Solve. You can choose which way to start. Also, check your work, using blank paper if necessary.

a. $\frac{1}{3}(x+7) = 24$	**b.** $\frac{2}{5}(y-5) = 1$	**c.** $\frac{2}{3}(z-3) = 5$

5. Find the errors in these solutions, and correct them.

a.

$\frac{5}{6}(y - 2)$	=	1	$\cdot$ **6**
$5y - 2$	=	6	**+ 2**
$5y$	=	8	**÷ 5**
y	=	8/5	

b.

$3(x + 7)$	=	$\frac{11}{4}$	$\cdot$ **4**
$3x + 21$	=	11	**− 21**
$3x$	=	−10	**÷ 3**
x	=	−10/3	

6. Use these for more practice as needed. Use blank paper.

a. $\frac{1}{8}(x - 7) = 2$	**b.** $8(r - 3) = \frac{7}{4}$	**c.** $10(a - 4) = \frac{5}{4}$
d. $3 = \frac{5}{6}(x - 2)$	**e.** $\frac{4}{5}(x + 1) = 3$	**f.** $3(y + 4) = \frac{24}{5}$

7. Here's a riddle to discover by solving the equations.

F. $3(x - 1) = \frac{6}{5}$	**O.** $2(x + 1) = \frac{10}{3}$	**W.** $\frac{1}{8}(x - 3) = 1$
T. $-45 = 3(x - 5)$	**R.** $0.1(x - 5) = 3$	**G.** $-2 = 2(x + 4)$
D. $2(x + 4) = -11$	**O.** $6(x - 1) = -1$	**O.** $0.6(x + 12) = 4.2$

Why do hummingbirds hum?

	7/5	−5	35	−5	5/6	−10		11	2/3	35	−9.5	
Because they							the					S