MATH MAMMOTH

Grade 6-A

Complete Worktext

- The basic operations and place value
- Ratios, proportions, and problem solving
- Decimals
- Number theory
- Fractions

By Maria Miller

www.MathMammoth.com

Contents

Foreword 6

Chapter 1: Basic Operations and Place Value

Introduction 7
Mental Math Review.......... 9
Review of the Four Operations 11
Terminologies for the Four Operations 13
Powers and Exponents 15
The Order of Operations 18
Multiplying and Dividing in Parts 20
Word Problems 23
Place Value 24
Rounding 28
Chapter 1 Review 30

Chapter 2: Ratios and Proportions

Introduction 32
Expressions 35
Equations 37
Using Two Variables - Functions 40
Ratios 44
Solving Problems Using Equivalent Ratios 47
Ratio Problems and Bar/Block Models 1 49
Ratio Problems and Bar/Block Models 2 51
Ratios in Rectangles 55
Solving Proportions 1: Equivalent Rates 57
Solving Proportions 2: Cross Multiplying 59
Why Cross-Multiplying Works 62
Solving Proportions 3: Practice 63
Scaling Figures 1 67
Scaling Figures 2 70
Floor Plans 72
Chapter 2 Review.......... 74

Chapter 3: Decimals

Introduction 79
Place Value with Decimals 82
Comparing Decimals 84
Add and Subtract Decimals 86
Rounding Decimals 88
Review: Multiply and Divide Decimals Mentally 91
Review: Multiply Decimals by Decimals 93
Review: Long Division with Decimals 95
Problem Solving with Decimals 97
Fractions to Decimals 99
Multiply and Divide by Powers of Ten 102
Scientific Notation 104
Divide Decimals by Decimals 1 107
Divide Decimals by Decimals 2 110
Problems with Customary Measuring Units 112
Metric System Prefixes 116
Convert Units in the Metric System 118
Convert Between Customary and Metric 122
Chapter 3 Review 124

Chapter 4: Number Theory

Introduction 129
Divisibility 131
Factoring and Primes 1 135
Factoring and Primes 2 137
Factoring and Primes 3 141
Simplifying Fractions Using Factoring 144
Greatest Common Factor (GFC) 147
The Least Common Multiple (LCM) 150
Chapter 4 Review 154

Chapter 5: Fractions

Introduction **156**
Fraction Terminology **161**
Review: Fractions and Mixed Numbers **162**
Subtracting Mixed Numbers **164**
Adding Unlike Fractions **166**
Review: Simplifying Fractions **170**
Add and Subtract Fractions: More Practice **171**
Multiplying Fractions **173**
Simplify Before Multiplying **176**
Divide Fractions **178**
Many Operations and Fraction Equations **181**
Comparing Fraction and Decimal Division **184**
Multiplication, Division, and Fractions **187**
Problems with Fractional Parts **189**
Ratio Problems Involving Fractions **193**
Scaling in Maps **195**
Chapter 5 Review **198**

Foreword

Math Mammoth Grade 6-A and *Grade 6-B* worktexts comprise a complete math curriculum for the sixth grade mathematics studies. It contains eight main chapters. Each chapter has an introduction, which contains notes to the teacher; the actual lessons then follow with problems. Each chapter ends in a review lesson. The chapter tests are found in a separate folder and are printed separately. This product also includes an HTML page that you can use to make extra practice worksheets for computation.

Sixth grade is a time to deepen the student's understanding of fractions and decimals, and to study ratios, proportions, and percent as new topics. Also studied are integers, geometry, statistical graphs, and probability.

The *Math Mammoth Grade 6 Complete Curriculum* starts out with a review of the four operations with whole numbers and place value. Students are also introduced to exponents. While most of the first chapter is review, we also emphasize the ability to deal with expressions such as $600 \div 4 + 10^3$.

Next is ratios and proportions, which are a major part of study in sixth grade. Students are already somewhat familiar with ratios and finding fractional parts from earlier grades, and now it is time to advance that knowledge into the study of proportions, which arise naturally from the study of fractions and equivalent ratios. We show several ways of solving proportions. As applications, students study scaling geometric figures, floor plans, and scaling in maps.

The next several chapters deal with fractions and decimals. They have already been studied extensively in fifth grade. Now in sixth grade we review those topics, using more decimal digits (decimals) and larger denominators (fractions). Students also study factoring, the least common multiple, and the greatest common factor, and can use those concepts when dealing with fraction simplification or addition.

Percent is an important topic to understand thoroughly so we devote a whole chapter to it. It ties in with decimals, fractions, ratios, *and* proportions, and of course has lots of practical applications.

The study of integers is preparing students for pre-algebra and algebra. Here we study all four operations with integers, and also graphing simple functions.

In geometry, students encounter angle problems and calculations. We also study congruent transformations and similar figures. Area and volume calculations are reviewed from fifth grade. Pi and the area of a circle are studied as new topics.

Then, we turn our attention to statistics and probability. Students will analyze and create a variety of statistical graphs, and study mean, median, and mode. Probability is a topic that until recently was not a part of grade-school curriculum. Thus, we start slowly, introducing the probability topics in a simple manner. Students will encounter these topics again in pre-algebra and algebra.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 1: Basic Operations and Place Value
Introduction

The goal of this first chapter is to review the four basic operations with whole numbers, review the order of operations and place value, and to learn about exponents, including the order of operations with exponents.

A lot of this chapter is just review, with the exception of the topic of exponents. This should hopefully provide a gentle start for 6th grade math where students are allowed to review some important topics, and yet learn something important as well.

Some major goals for 6th grade math that will be studied later are a mastery of all fraction and decimal operations, a solid understanding of ratio, proportion, and percent, and being able to solve common problems that involve these concepts.

The Lessons in Chapter 1

	page	span
Mental Math Review	9	*2 pages*
Review of the Four Operations	11	*2 pages*
Terminologies for the Four Operations	13	*2 pages*
Powers and Exponents	15	*3 pages*
The Order of Operations	18	*2 pages*
Multiplying and Dividing in Parts	20	*3 pages*
Word Problems	23	*1 page*
Place Value	24	*4 pages*
Rounding	28	*2 pages*
Chapter 1 Review	30	*2 pages*

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations.
http://education.jlab.org/smdeluxe/index.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

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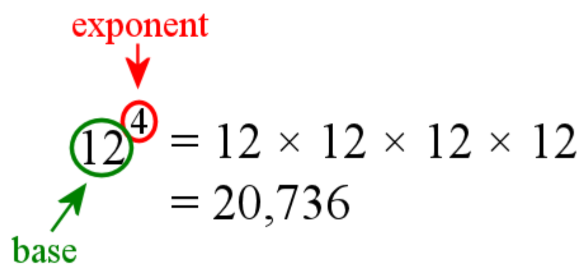
Powers and Exponents

Exponents are a kind of "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read "two raised to the fifth power," "two to the fifth power," or even just "two to the fifth." Similarly, 7^9 is read "seven raised to the ninth power," "seven to the ninth power," or "seven to the ninth." The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

However, expressions with powers of 2 and 3 are almost always read differently:
The expression 11^2 is usually read as "eleven squared" because it describes the area of a square with sides 11 units long. Similarly, 31^3 is generally read as "thirty-one cubed" because it gives the volume of a cube with sides 31 units long.

1. Write out these expressions as multiplications, then solve them.

a. $3^2 = \underline{\ 3 \times 3 = 9\ }$

b. 1^6

c. 4^3

d. 10^6

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^3

j. 10^5

2. Rewrite these expressions as multiplication. Then use a calculator to solve them.

a. 6^4

b. 11^3

c. 13^3

d. 27^5

3. Rewrite each expression using an exponent, then solve it. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

The expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with sides 7 units long.

For example, if the sides of a square are 5 cm long, then its area is 5 cm × 5 cm = 25 cm^2.

Notice that the symbol for "square centimeters" is $\mathbf{cm^2}$. This means "centimeter × centimeter." We are, in effect, squaring the measuring unit! In fact, we do the same thing when we use the units "square meters" and "square kilometers."

We could also write that expression as $(5\text{ cm})^2$ or "the quantity, five centimeters, squared." This means that both the 5 and the unit "cm" are squared, which makes 25 cm^2. Without the parenthesis it would be 5 cm^2 and mean "five square centimeters," which is something very different.

We can do the same thing with the traditional units of inches, feet, and miles. People often write "sq. in." for square inches, or "sq. ft." for square feet, instead of $\mathbf{in^2}$ and $\mathbf{ft^2}$, but both ways are correct.

Similarly, 7^3 is read "seven *cubed*" because it gives the volume of a *cube* with sides 7 units long.

For example, if the sides of a cube are 10 cm long, then its volume is $(10\text{ cm})^3$ = 1,000 cm^3, or "one thousand cubic centimeters.".

4. Express the area using exponents and solve.

a. A square with a side of 12 kilometers: The area is $(12\text{ km})^2$ = __________	**b.** A square with sides 6 m long: Its area is
c. A square with sides each 6 inches long: Its area is	**d.** A square with a side with a length of 12 ft: The area is

5. Express the volume using exponents and solve.

a. A cube with a side of 2 cm: The volume is	**b.** A cube with sides each 10 inches long:
c. A cube with sides 1 ft in length:	**d.** A cube with edges that are all 5 m long:

6. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its side?

c. The area of a square is 121 m^2. What is its perimeter?

d. The *area* of one face of a cube is 64 in^2. What is its volume?

Notice something special about powers of 0 and powers of 1.

$0^5 = 0 \times 0 \times 0 \times 0 \times 0$ is simply 0! You can easily see that 0^3 , 0^7 , 0^{21}, and all of the other powers of 0 (0 raised to any whole-number power) are equal to 0.

$1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1$ is simply 1! It's easy to see that 1^3 , 1^9 , 1^{65}, and all of the other powers of 1 (1 raised to any whole-number power) are equal to 1.

The powers of 10 are also very special—and very easy!

Notice that the exponent tells us *how many zeroes* there are in the answer.

$10^1 = 10$	$10^4 = 10{,}000$
$10^2 = 10 \times 10 = 100$	$10^5 = 100{,}000$
$10^3 = 10 \times 10 \times 10 = 1{,}000$	$10^6 = 1{,}000{,}000$

7. Fill in the patterns. In part (d), choose your own number to be the base.
   Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

8. Look at the patterns above. Think carefully about how each step comes from the previous one. Then answer the questions.

   a. If you are given that $3^7 = 2{,}187$,
   how can you use that result to find 3^8 ?

   b. Find 3^8 without a calculator.

   c. If you are given that $2^{45} = 35{,}184{,}372{,}088{,}832$,
   how can you use that result to find 2^{46} ?

   d. Find 2^{46} without a calculator.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method? Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?
Try this same pattern for at least one other base number, *n*. What value do you calculate for n^0? Do you think it will come out this way for every base number? Why or why not?

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Chapter 2: Ratios and Proportions
Introduction

In this chapter we concentrate on two important concepts: ratios and proportions and word problems involving those. We also study a little bit about expressions, equations, and graphing simple functions—elementary algebra concepts—but only on an introductory level. All of these topics are important in further studies of algebra.

This chapter has quite a bit of reading to do, probably more than in some other math curricula. This is because the text will often explain *why* the mathematical process works. Another reason is because we do a lot of problem solving in this chapter.

We start out by studying expressions and very simple equations. This is just a brief introduction to algebra, and these two topics are definitely studied a lot more in prealgebra and algebra courses. Now, ratios and proportions are *also* studied in prealgebra and algebra courses, so they will be reviewed in subsequent grades. However, my aim is to provide students with a thorough understanding of ratios and proportions here in 6th grade, not only because that is the norm, but also because they are used so much in everyday-life applications, and because they are a natural extension to go to after the student understands the basics of fractions.

After the little bit of expressions and equations, we study ratios, which should be a familiar topic from *Math Mammoth 5th grade*. The focus is on equivalent ratios because those will lead us into proportions just a few lessons later. After the introduction to ratios, we study various kinds of word problems involving ratios, and use a block or bar model to solve these problems. The lesson *Ratios in Rectangles* has applications about the aspect ratio.

Solving proportions is divided into three separate lessons. In the first one, we solve proportions by thinking through equivalent ratios. In the second one, the usual method of cross-multiplying is introduced. Then follows a lesson that explains just why cross-multiplying is allowed. Lastly, there is more practice with solving proportions and word problems.

As a last topic in the chapter, we study scaling geometric figures and floor plans, which are simple applications of proportions.

The Lessons in Chapter 3

	page	span
Expressions	35	*2 pages*
Equations	37	*3 pages*
Using Two Variables - Functions	40	*4 pages*
Ratios	44	*3 pages*
Solving Problems Using Equivalent Ratios	47	*2 pages*
Ratio Problems and Bar/Block Models 1	49	*2 pages*
Ratio Problems and Bar/Block Models 2	51	*4 pages*
Ratios in Rectangles	55	*2 pages*

Solving Proportions 1: Equivalent Rates	57	*2 pages*
Solving Proportions 2: Cross Multiplying	59	*3 pages*
Why Cross-Multiplying Works	62	*1 pages*
Solving Proportions 3: Practice	63	*4 pages*
Scaling Figures 1	67	*3 pages*
Scaling Figures 2	70	*2 pages*
Floor Plans	72	*2 pages*
Chapter 2 Review	74	*5 pages*

Helpful Resources on the Internet

Equation Match
Playing on level 1, you need to match simple equations based on them having the same solution.
http://www.bbc.co.uk/education/mathsfile/shockwave/games/equationmatch.html

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/Algebra_Puzzle.html

Battleship
Choose the right solution for a 1-step equation every time you hit the enemy's ship. Some of the equations involve negative solutions; however since the game is interesting, some students might be willing to play it anyway (you can always guess at the right solution since it is a multiple choice game).
http://www.quia.com/ba/36544.html

Algebra Meltdown
Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.
http://www.mangahigh.com/en_gb/games/algebrameltdown

Words into Equations Battleship Game
Practice expressions such as quotient, difference, product, and sum.
http://www.quia.com/ba/210997.html

Balance when Adding and Subtracting Game
The interactive balance illustrates simple equations. Your task is to add or subtract x's, and add or subtract 1's until you have x alone on one side.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

Algebra Balance Scales
Similar to the one above, but you need to first put the x's and 1's in the balance to match the given equation.
http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html — only positive numbers
http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html — includes negative numbers

Practice with Ratios
An online quiz from Regents Exam Prep Center
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm

Practice with Proportions
An online quiz from Regents Exam Prep Center
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracProp.htm

Ratio Stadium
A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.
http://www.arcademicskillbuilders.com/games/ratio-stadium/

Dirt Bike Proportions
A racing game where you need to find the unknown in a simple proportion. This game would actually work equally well for practicing equivalent fractions, because the proportions are quite simple.
http://www.arcademicskillbuilders.com/games/dirt-bike-proportions/dirt-bike-proportions.html

Challenge Board
Choose questions from the challenge board about rates, ratios, and proportions.
http://www.quia.com/cb/158527.html

Ratio and Proportion Game From BBC Skillswise
Write the simplified ratio of red to black marbles. Answer simple questions about ratios and marbles.
http://www.bbc.co.uk/skillswise/numbers/wholenumbers/ratioandproportion/ratio/game.shtml

Ratio Pairs Matching Game
Match cards representing equivalent ratios.
Easy: http://nrich.maths.org/4824 Challenge: http://nrich.maths.org/4821

Equivalent Ratios Workout
10 online practice problems.
http://www.math.com/school/subject1/practice/S1U2L1/S1U2L1Pract.html

All About Ratios - Quizzes
Online quizzes about same and different ratios.
http://math.rice.edu/~lanius/proportions/index.html

Free Ride
An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=178

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Solving Problems Using Equivalent Ratios

Example. If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form an equivalent rate. However, it's not easy to go from 45 minutes to 60 minutes (1 hour). Therefore, let's first figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

$$\frac{21 \text{ miles}}{45 \text{ minutes}} \overset{\div 3}{=} \frac{\square \text{ miles}}{15 \text{ minutes}} \overset{\times 4}{=} \frac{\square \text{ miles}}{60 \text{ minutes}}$$

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4.

This is now easy to solve: $\frac{21 \text{ miles}}{45 \text{ minutes}} = \frac{7 \text{ miles}}{15 \text{ minutes}} = \frac{28 \text{ miles}}{60 \text{ minutes}}$. He can ride 28 miles in 1 hour.

1. If Jake can ride 8 miles in 14 minutes, how long will he take to ride 36 miles?
   Use the equivalent rates below.

$$\frac{8 \text{ miles}}{14 \text{ minutes}} = \frac{4 \text{ miles}}{\square \text{ minutes}} = \frac{36 \text{ miles}}{\square \text{ minutes}}$$

2. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\_\_\_}{1 \text{ hr}} = \frac{\_\_\_}{15 \text{ min}} = \frac{\_\_\_}{45 \text{ min}}$

b. $\frac{\$6}{45 \text{ min}} = \frac{\_\_\_}{15 \text{ min}} = \frac{\_\_\_}{1 \text{ hr}} = \frac{\_\_\_}{1 \text{ hr } 45 \text{ min}}$

c. $\frac{3 \text{ in}}{8 \text{ ft}} = \frac{\_\_\_}{2 \text{ ft}} = \frac{\_\_\_}{12 \text{ ft}} = \frac{\_\_\_}{20 \text{ ft}}$

d. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\_\_\_}{1 \text{ min}} = \frac{\_\_\_}{3 \text{ min}}$

3. A car can go 50 miles on 2 gallons of gasoline.

   a. How many gallons of gasoline would the car need for a trip of 60 miles?
   Use the equivalent rates below.

$$\frac{50 \text{ miles}}{2 \text{ gallons}} = \frac{5 \text{ miles}}{\square \text{ gallons}} = \frac{60 \text{ miles}}{\square \text{ gallons}}$$

   b. How far can the car travel on 15 gallons of gasoline?
   (Hint: First consider the case with just 1 gallon of gasoline.)

Example. You get 20 erasers for $1.90.
How much would 22 erasers cost?

We cannot *easily* find a number by which to multiply to get from 20 to 22. It is possible, but we will solve this problem in a different way.

$$\frac{\text{20 erasers}}{\$1.90} \underset{\times\,??}{\overset{\times\,??}{=}} \frac{\text{22 erasers}}{?}$$

We will *first* figure out the cost of just 2 erasers! To find that, divide the price by 10. Then, from 2 erasers to 22 erasers, the cost will increase 11-fold:

11 × $0.19 = $2.09. So 22 erasers would cost $2.09.

$$\frac{\text{20 erasers}}{\$1.90} \underset{\div\,10}{\overset{\div\,10}{=}} \frac{\text{2 erasers}}{\$\,0.19} \underset{\times\,11}{\overset{\times\,11}{=}} \frac{\text{22 erasers}}{\$\;\_\_\_\_}$$

Another way to come up with the same answer is just to add $0.19 (the cost of 2 erasers) to $1.90 (the cost of 20 erasers). Yet another way is to find the cost of 1 eraser first and multiply that cost by 22.

4. On the average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots?

$$\frac{\text{9 baskets}}{\text{12 shots}} =$$

5. You get 30 pencils for $4.50.
How much would 52 pencils cost?

$$\frac{\text{30 pencils}}{\$4.50} =$$

6. A train travels at a constant speed of 80 miles per hour.

a. How far will it go in 140 minutes? Use equivalent rates.

$$\frac{\text{80 miles}}{\text{1 hour}} =$$

b. How long will it take for the train to travel 50 miles? Use equivalent rates.

$$\frac{\text{80 miles}}{\text{1 hour}} =$$

7. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to the lowest terms.

b. Assuming the same ratio holds true in another group of 125 people, how many of those people can we expect to like blue?

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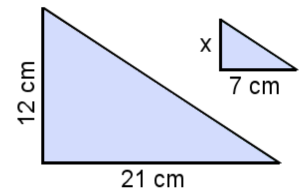
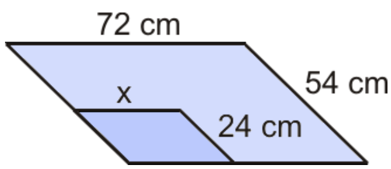
Scaling Figures 2

Example. The two triangles are similar. Find the side marked with *x*.

Here, we do *not* know the scale ratio, but we can solve it from the two corresponding sides that measure 7 cm and 21 cm. The scale is 21:7 or 3:1. Now we can figure out that the side length 12 cm needs to be divided by 3. We get *x* = 4 cm.



Example. The two parallelograms are similar. Find the length of the side marked by *x*.



Solution 1. We can get the scale ratio from the two corresponding sides whose lengths are known—the 54-cm and 24-cm sides—and form a proportion to solve *x*. The ratio *x* : 72 has to equal the ratio 24 : 54.

$$\frac{x}{72} = \frac{24}{54}$$ We simplify the fraction 24/54 before continuing.

$$\frac{x}{72} = \frac{4}{9}$$ Now cross-multiply.

$$9x = 4 \cdot 72$$ Multiply the right side.

$$9x = 288$$ Divide both sides by 9.

$$\frac{9x}{9} = \frac{288}{9}$$ Divide the right side.

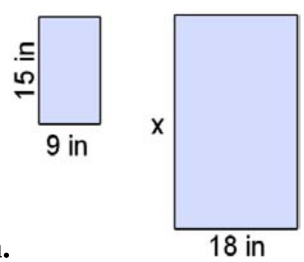
$$x = 32$$ So, x is 32 cm.

Solution 2. Like in solution 1, we first find and simplify the scale ratio using the two corresponding sides whose lengths are known. The scale ratio is 24:54 = 4:9 (simplified).

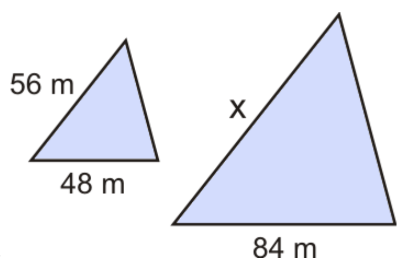
Now we use the scaling ratio 4:9 and compare it to the 72-cm side, which corresponds to the unknown side. Imagine that the 72-cm side is divided into 9 parts. Each part is 72 cm ÷ 9 = 8 cm. Similarly, the unknown side has four parts, so it is 4 × 8 cm = 32 cm.

1. The figures are similar. Find the side lengths marked with *x*. First figure out the scale ratio.

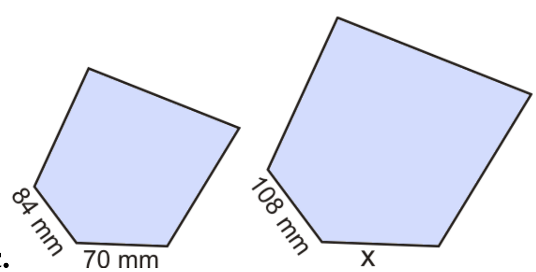
a.



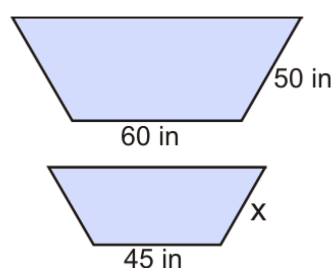
b.



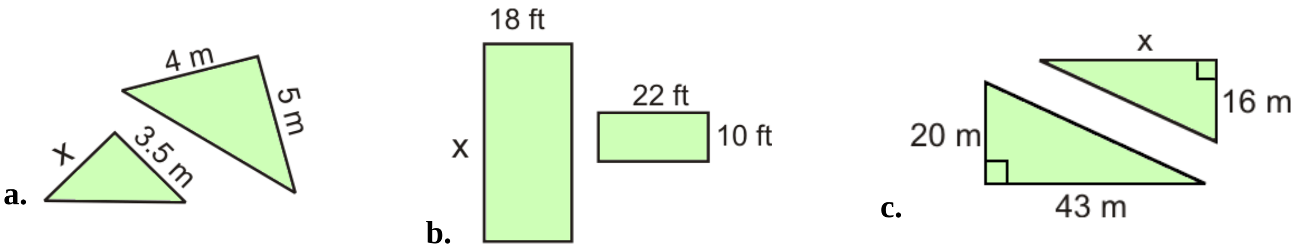
c.



d.



2. The figures are similar. Find the side lengths marked with *x*.



3. The sides of a rectangle measure 3″ and 4 1/2″.
The shorter side of another, similar rectangle is 3/4″.
How long is the longer side of that rectangle?

4. The rectangles 1 through 4 in the table on the right are similar.

a. Follow the example given and find these ratios in whole numbers in lowest terms:

- The scale ratio between rectangle #3 and rectangle #4

  It is $\frac{2}{2.5} = \frac{4}{5}$.

- The scale ratio between rectangle #1 and rectangle #3

- The scale ratio between rectangle #1 and rectangle #4

	Length	**Width**
Rectangle 1	1 cm	
Rectangle 2	1.5 cm	
Rectangle 3	2 cm	
Rectangle 4	2.5 cm	7.5 cm

b. Use the scale ratios to find the width of each rectangle, and fill in the rest of the table. Draw the rectangles in your notebook or on a separate sheet of paper.

c. What is the *aspect ratio* (*i.e.*, the ratio of length to width) of each rectangle?

5. The area of a square is 36 cm^2. The square is shrunk in a ratio of 4:3. What is the area of the resulting square?

6. A rectangle with the sides of 2 1/2 in. and 1 3/4 in. is enlarged in a ratio of 1:4. Find the area of the resulting rectangle.

7. The aspect ratio of a rectangle is 2:3 and its perimeter is 50 cm.
Find the area of the rectangle after it is shrunk in a scale ratio of 5:2.

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Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but the lessons and exercises were mostly using numbers with a maximum of three decimal digits. This time, there is no such restriction and the decimals used can have many more decimal digits than that. However, if the student has a good grasp of decimals already (for example because of having studied the *Math Mammoth Grade 5 Complete Curriculum* or through the *Math Mammoth Decimals 2* book), consider assigning only 1/3 - 1/2 of the problems because he should be able to go through this chapter quickly.

We start out by studying place value with decimals and comparing decimals, up to six decimal digits. The next several lessons contain mainly review, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, using mental math for multiplying and dividing decimals, long division with decimals, fractions to decimals, and multiplying and dividing decimals by the powers of ten.

Scientific notation is a new topic. Most 7th grade math curricula also cover it. After that, we turn our attention one more time to dividing decimals by decimals. I have tried to explain the principle behind the common shortcut or rule ("Move the decimal point in both the divisor and the dividend so many steps that the divisor becomes a whole number"). The principle here has to do with multiplying the divisor and the dividend by a power of ten, and it even ties with equivalent fractions. Many school books never explain this principle in connection with decimal division.

The last lessons in this chapter deal with measuring units and the metric system, and nicely round up our study of decimals.

The Lessons in Chapter 3

	page	span
Place Value with Decimals	82	*2 pages*
Comparing Decimals	84	*2 pages*
Add and Subtract Decimals	86	*2 pages*
Rounding Decimals	88	*3 pages*
Review: Multiply and Divide Decimals Mentally	91	*2 pages*
Review: Multiply Decimals by Decimals	93	*2 pages*
Review: Long Division with Decimals	95	*2 pages*
Problem Solving with Decimals	97	*2 pages*
Fractions to Decimals	99	*3 pages*
Multiply and Divide by Powers of Ten	102	*2 pages*
Scientific Notation	104	*3 pages*
Divide Decimals by Decimals 1	107	*3 pages*
Divide Decimals by Decimals 2	110	*2 pages*
Problems with Customary Measuring Units	112	*4 pages*
Metric System Prefixes	116	*2 pages*

Convert Units in the Metric System .... 118 *4 pages*
Convert Between Customary and Metric .... 122 *2 pages*
Chapter 3 Review .... 124 *5 pages*

Helpful Resources on the Internet

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

Smaller and Smaller Maze
Practice ordering decimal numbers to find your way through the maze.
www.mathsyear2000.org/magnet/kaleidoscope/smaller/index.html

Decimal and Whole Number Jeopardy
Review place value and comparing and rounding numbers. Also, practice number patterns.
www.quia.com/cb/8142.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
themathgames.com/arithmetic-games/place-value/decimal-place-value-math-game.php

Sock
Push the green blocks into the holes to make the target number.
www.interactivestuff.org/sums4fun/sock.html

Decimal Squares Blackjack
Play cards with decimals, trying to get as close to 2 as possible without going over.
www.decimalsquares.com/dsGames/games/blackjack.html

A Decimal Puzzle
Make every circle add up to 3.
nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.htmlsopen=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football
Simple games for addition, subtraction, multiplication, and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.
funbrain.com

Exploring Division of Decimals
Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.
www.hbschool.com/activity/elab2004/gr6/1.html

Decimal Speedway
Practice decimal multiplication in this fun car-racing game.
www.decimalsquares.com/dsGames/games/speedway.html

Fractions - Decimals calculator
Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any decimal places, which normal calculators don't do.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html

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Fractions to Decimals

One way to change a fraction into a decimal is to find an equivalent fraction with a denominator of 10, 100, 1000, etc. This works with *some* fractions, but not most.	× 5 $\frac{13}{20} = \frac{65}{100} = 0.65$ × 5	÷ 3 $\frac{27}{30} = \frac{9}{10} = 0.9$ ÷ 3	× 8 $\frac{31}{125} = \frac{248}{1{,}000} = 0.248$ × 8

1. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1000.

a. $\frac{1}{4}$ **b.** $\frac{1}{5}$ **c.** $1\frac{1}{20}$ **d.** $3\frac{9}{25}$

e. $\frac{12}{200}$ **f.** $8\frac{3}{4}$ **g.** $4\frac{3}{5}$ **h.** $\frac{13}{20}$

i. $\frac{13}{25}$ **j.** $\frac{11}{125}$ **k.** $\frac{24}{400}$ **l.** $\frac{95}{500}$

2. Change to decimals and calculate mentally.

a. $0.2 + \frac{1}{4}$ **b.** $0.34 + 1\frac{1}{5}$ **c.** $2\frac{3}{5} + 1.3$ **d.** $\frac{2}{10} - 0.09$

When the method above doesn't work, simply remember that the fraction line indicates division and DIVIDE the numerator on top by the denominator on the bottom.

So to change, for example, $\frac{1}{7}$ or $\frac{25}{32}$ or $\frac{6}{17}$ into decimals, use long division (or a calculator).

Example. Find $\frac{18}{11}$ as a decimal. To do that, we write 18 as 18.0000 in the long division "corner" and divide by 11. We notice the digits "63" start repeating.

```
     0 1.6 3 6 3
11 )1 8.0 0 0 0
    1 1
      7 0
     - 6 6
        4 0
       - 3 3
          7 0
         - 6 6
            4 0
           - 3 3
              7
```

Therefore, $\frac{18}{11} = 1.636363...$

This can also be written by drawing a line over the repeating digits: $1.636363... = 1.\overline{63}$.

Example. Find $\frac{8}{31}$ to five decimal places. We cannot see any repeating pattern in the long division in the first six decimals.
(There *is* a pattern, but it is 15 digits long!). Therefore, we stop the division after six decimals, and round the number to five decimals.

```
     0.2 5 8 0 6 4
31 )8.0 0 0 0 0 0
     6 2
     1 8 0
    -1 5 5
       2 5 0
      - 2 4 8
           2 0
         -   0
          2 0 0
         - 1 8 6
            1 4 0
            1 2 4
              1 6
```

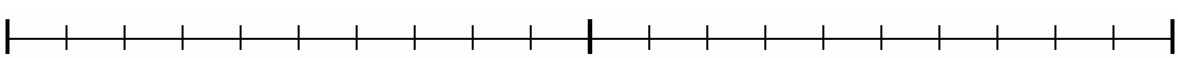
We get $\frac{8}{31} \approx 0.25806$.

3. Write as decimals. The ones you don't already know by heart, calculate by long division. Calculate to six places, then round to five, unless the division comes out even or you find a repeating pattern.

a. $\frac{2}{3}$	**b.** $1\frac{1}{3}$	**c.** $\frac{8}{9}$
d. $5\frac{17}{21}$	**e.** $\frac{19}{24}$	**f.** $\frac{1}{6}$
g. $\frac{3}{8}$	**h.** $2\frac{3}{11}$	**i.** $\frac{3}{7}$

4. Mark the following numbers on this number line that starts at 0 and ends at 2.

0.2, $\frac{1}{4}$, 0.65, $1\frac{1}{3}$, 0.04, $\frac{2}{5}$, 1.22, $1\frac{3}{4}$, 1.95, $1\frac{4}{5}$



5. Write as decimals. Use a notebook to do long division. Not all of these are repeating decimals. Of those that are, can you find a pattern in the repeating parts of the decimals?

a.	**b.**	**c.**	**d.**
$1 \div 3 = 0.\overline{3}$	$1 \div 9$	$1 \div 4$	$1 \div 6$
$2 \div 3$	$2 \div 9$	$2 \div 4$	$2 \div 6$
$3 \div 3$	$3 \div 9$	$3 \div 4$	$3 \div 6$
$4 \div 3$	$4 \div 9$	$4 \div 4$	$4 \div 6$
$5 \div 3$	$5 \div 9$	$5 \div 4$	$5 \div 6$
$6 \div 3$	$6 \div 9$	$6 \div 4$	$6 \div 6$
$7 \div 3$	$7 \div 9$	$7 \div 4$	$7 \div 6$
e.	**f.**	**g.**	**h.**
$1 \div 7 = 0.\overline{142857}$	$1 \div 8$	$1 \div 5$	$1 \div 11$
$2 \div 7$	$2 \div 8$	$2 \div 5$	$2 \div 11$
$3 \div 7$	$3 \div 8$	$3 \div 5$	$3 \div 11$
$4 \div 7$	$4 \div 8$	$4 \div 5$	$4 \div 11$
$5 \div 7$	$5 \div 8$	$5 \div 5$	$5 \div 11$
$6 \div 7$	$6 \div 8$	$6 \div 5$	$6 \div 11$
$7 \div 7$	$7 \div 8$	$7 \div 5$	$7 \div 11$

6. Divide. All of these either have a repeating pattern, or the decimal terminates.

a. $34.4 \div 12$

b. $66 \div 9$

c. $0.76 \div 11$

d. $0.23 \div 4$

7. When the fraction $\frac{1}{3}$ is written as a decimal, it is 0.33333… .
This could be rounded to three decimals (0.333), or to six decimals (0.333333), or to any other amount of decimals.
Find the *difference* between the rounded versions, if 0.33333… is first rounded to five decimals and then to only two decimals.

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Metric System Prefixes

The metric system has one basic unit for each thing we might measure: For length, the unit is the **meter**. For weight, it is the **gram**. And for volume, it is the **liter**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. These prefixes tell us what multiple of the basic unit the derived unit is.

Prefix	Abbreviated	Meaning
kilo-	k	1,000
hecto-	h	100
deca-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometer	km	1,000 meters
hectometer	hm	100 meters
decameter	dam	10 meters
meter	m	(the basic unit)
decimeter	dm	1/10 meter
centimeter	cm	1/100 meter
millimeter	mm	1/1000 meter

Unit	Abbr	Meaning
kilogram	kg	1,000 grams
hectogram	hg	100 grams
decagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kiloliter	kl	1,000 liters
hectoliter	hl	100 liters
decaliter	dal	10 liters
liter	l	(the basic unit)
deciliter	dl	1/10 liter
centiliter	cl	1/100 liter
milliliter	ml	1/1000 liter

1. Write these amounts in basic units (meters, grams, or liters) by "translating" their prefixes:
e.g., the "centi" in cm means "hundredths," so 3 cm is three hundredths of a meter (3/100 m).

a. 3 cm = 3/100 m = 0.03 m	**b.** 2 cg = _______ g = ________ g
5 mm = _______ m = _______ m	6 mL = _______ L = _______ L
7 dL = ________ L = ________ L	1 dg = _______ g = ________ g

2. Write the amounts in basic units (meters, grams, or liters) by "translating" the prefixes.

a. 3 kL = _______ L	**b.** 2 dam = _______ m	**c.** 70 km = ________ m
8 dag = _______ g	9 hL = _______ L	5 hg = _______ g
6 hm = _______ m	7 kg = _______ g	8 dal = _______ L

3. Write the amounts using derived units (prefixes).

a. 3,000 g = 3 kg	**b.** 0.01 m = _______ ____	**c.** 0.04 L = _______ ____
800 L = _______ ____	0.2 L = _______ ____	0.8 m = _______ ____
60 m = _______ ____	0.005 g = _______ ____	0.007 L = _______ ____

4. These measurements are all mixed up! Organize them in the table in descending order from the heaviest weight to the lightest.

Item	Mass

Grandpa 75 kg full suitcase

3 kg 30 kg

baby 200 g Cell phone

10 kg bucket of water

5. Write using prefixed units so that the number of units is the smallest possible whole number.

a. 0.04 meters = 4 cm

b. 0.005 grams

c. 0.037 meters

d. 400 liters

e. 0.6 meters

f. 2,000 meters

g. 0.206 liters

h. 20 meters

6. Change into the basic unit (either meter, liter, or gram). Think of the meaning of the prefix.

a. 45 cm = 0.45 m

b. 65 mg

c. 2 dm

d. 81 km

e. 6 mL

f. 758 mg

g. 2 kL

h. 8 dL

7. Find the total …

a. … weight of books that weigh individually:
1.2 kg, 1.04 kg, 520 g, and 128 g.

b. … volume of containers whose individual volumes are:
1.4 L, 2.25 L, 550 mL, 240 mL, and 4 dL.

8. Each measurement has a flub, either in the unit or in the decimal point. Correct them.

a. The length of a pencil: 13 m

b. The length of an eraser: 45 cm

c. Length of Dad's waist: 9.8 m

d. The height of a room: 0.24 m

e. Jack's height: 1.70 mm

f. Jenny's height: 1.34 cm

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Chapter 4: Number Theory
Introduction

Number theory has to do with the study of whole numbers and their special properties. In this chapter, we study divisibility rules, prime numbers, factoring, the greatest common factor (GCF), and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why this chapter includes a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list below for Internet resources.

Similarly, the main use for the least common multiple in arithmetic is with the addition of fractions, and we study that topic in this chapter also.

The concepts of factoring and the GCF are also important to understand because they will be carried over to algebra, where students will factor polynomials.

Primes are fascinating "creatures", and you can let students read more about them using the Internet resources listed below.

The Lessons in Chapter 4

	page	span
Divisibility	131	*4 pages*
Factoring and Primes 1	135	*2 pages*
Factoring and Primes 2	137	*4 pages*
Factoring and Primes 3	141	*3 pages*
Simplifying Fractions Using Factoring	144	*3 pages*
Greatest Common Factor (GCF)	147	*3 pages*
The Least Common Multiple (LCM)	150	*4 pages*
Chapter 4 Review	154	*4 pages*

Helpful Resources on the Internet

Arrays and factors
Drag rectangles to show the factorizations of a given number.
http://www.shodor.org/interactivate/activities/factors2/index.html

Factor Game
A fun, interactive game where you practice divisibility among numbers 1-100. You can play against the computer or against a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possibe divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

MathGoodies Interactive Factor Tree Game
Type in a missing number to the factor tree, and the program will find the other factor, and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.spacetime.us/arcade/play.php?game=2

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Primes, Factors and Divisibility - Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes/

Prime Number Calculator
This calculator tests if a number is a prime, and tells you its smallest divisor if it is not prime.
http://www.basic-mathematics.com/prime-number-calculator.html

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many are there, and more.
http://primes.utm.edu/

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips, and historical information how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.
http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20

Primality of 1 from Wikipedia
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and against the primality of 1
http://primefan.tripod.com/Prime1ProCon.html

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Factoring and Primes 2

Some numbers have *only two* divisors: 1 and the number itself. Such numbers are called **prime numbers.** 11 is one of them.

factor		factor		product
1	**×**	**11**	**=**	**11**

In the last lesson, we found that the prime numbers between 1 and 20 are **2**, **3**, **5**, **7**, **11**, **13**, **17** and **19**. 1 is usually not counted as a prime number. (See the last lesson for an explanation of why not.)

Prime factorization using a factor tree

A factor tree is a handy way to factor numbers to their prime factors. The factor tree starts at the root and grows upside down!

We want to factor 24, so we write 24 at the top. Then we factor 24 into 4 × 6. But 4 and 6 are not prime numbers, so we *continue* to factor. We factor four into 2 × 2 and six into 2 × 3.

```
(root)          24
               /  \
              4  ×  6
             / \   / \
(leaves)    2 × 2 × 2 × 3
```

We can't factor 2 or 3 any farther because they are prime numbers. Once you get to the primes in your "tree," they are the "leaves," and you stop factoring in that "branch." So 24 = 2 × 2 × 2 × 3. This is called the **prime factorization** of 24.

Examples:

```
  30
 /  \
5 × 6
   / \
  2 × 3
```

The number 5 is prime. It's a "leaf." Once you're done factoring, you "pick the leaves." (You can even circle them to see them better!) So 30 = 2 × 3 × 5.

```
  21
 /  \
3 × 7
```

Both 3 and 7 are prime numbers, so we cannot factor them any further.

So 21 = 3 × 7.

```
   66                 66
  /  \               /  \
11 × 6     OR       2 × 33
    / \                 / \
   2 × 3              11 × 3
```

You can start the factoring process any way you want. The end result is always the same: 66 = 2 × 3 × 11.

```
        72
       /  \
     12  ×  6
    /  \   / \
   3 × 4 × 2 × 3
      / \
     2 × 2
```

Since 72 has a lot of factors,

factoring takes several steps.

72 = 2 × 2 × 2 × 3 × 3

We also could have started by writing 72 = 2 × 36 or 72 = 4 × 18.

```
 57
/  \
```

How can you get started? Check:
- Is 57 in any of the times tables?
- Is it divisible by 2? By 3? By 5?

```
 65
/  \
```

How can you get started? Check:
- Is 65 in any of the times tables?
- Is it divisible by 2? By 3? By 5?

1. Factor the following numbers to their prime factors.

a. 18 / \	**b.** 6 / \	**c.** 14 / \
d. 8 / \	**e.** 12 / \	**f.** 20 / \
g. 16 / \	**h.** 24 / \	**i.** 27 / \
j. 25 / \	**k.** 33 / \	**l.** 15 / \

Prime numbers are like the building blocks of all numbers. They are the first and foremost, and other numbers are "built" from them. "Building numbers" is like factoring backwards. We start with the building blocks—the primes—and see what number we get:

```
2 × 5 × 2 × 2
 \ /     \ /
 10  ×    4
   \     /
     40
```

```
2 × 3 × 2 × 3 × 2
 \ /     \ /    |
  6   ×   6   × 2
  |        \   /
  6   ×     12
   \       /
      72
```

```
5 × 2 × 7
 \ /    |
 10  ×  7
   \   /
    70
```

```
2 × 7 × 2×3
 \ /    \ /
 14  ×   6
   \    /
     84
```

By using the process above (building numbers starting from primes) you can build ANY whole number there is! Can you believe that?

We can say this in another way: ALL numbers can be factored so the factors are prime numbers. That is sort of amazing! This fact is known as the *fundamental theorem of arithmetic*. And it is indeed fundamental.

So no matter what the number is—992 or 83,283 or 150,282—it can be written as a product of primes.

992 is factored on the right. $992 = 2^5 \times 31$. For 83,283 we get $3 \times 17 \times 23 \times 71$ and $151{,}282 = 2 \times 3^3 \times 11^2 \times 23$.
To find these factorizations, you need to test-divide the numbers by various primes, so it is a bit tedious. Of course, today's computer programs can do the division very quickly.

```
          992
         /   \
     4   ×   248
    / \      /  \
   2 × 2  ×  4 × 62
            / \   / \
           2 × 2 × 2 × 31
```

2. Build numbers from these sets of primes.

a. 2 × 5 × 11 \ / |	**b.** 3 × 2 × 2 × 2 \ / \ /	**c.** 2 × 3 × 7 \ / |
d. 11 × 3 × 2 | \ /	**e.** 3 × 3 × 2 × 5 \ / \ /	**f.** 2 × 3 × 17

3. Build more numbers from primes.

a. 2 × 5 × 13	**b.** 13 × 13 × 2	**c.** 19 × 3 × 3

4. Try it on your own! Pick 3 or 4 or more primes (you can even use the same prime several times), and see what number gets built from them.

Puzzle Corner

Ready for a challenge? Use your knowledge of divisibility tests and the calculator to factor these numbers into their prime factors:

a. 2,145 **b.** 3,680 **c.** 10,164

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Chapter 5: Fractions
Introduction

The aim of this chapter is to *review* all fraction arithmetic. The goal is that the student will become “fluent” with basic fraction operations, if he isn't already. The four operations of fractions have been studied in-depth in 5th grade, so the coverage here is quicker than in 5th grade.

For example, the lessons don't always delve into the reasons *why* a certain shortcut works—a lot of that is explained in the fifth grade material. While I consider it very important that the student understands fractions conceptually and understands why certain things are done the way they are done, the material here is building on the material for the earlier grades, where the students have been exposed to those thoughts and ideas.

If the student has a good grasp of fractions already (for example because of having studied the *Math Mammoth Grade 5 Complete Curriculum*), consider assigning only 1/3 - 1/2 of the calculation exercises because he should be able to go through this chapter quickly. However, many students may need the thorough review if they have forgotten these topics since 5th grade, so use your judgment.

The lesson “Comparing Fraction and Decimal Division” is optional. Often, you can solve the same division problem using either fraction or decimal division, if you can convert the numbers from decimals to fractions or vice versa. This lesson just examines the difference between fraction division and decimal division, and best suits advanced and interested students.

The page about fraction equations is also optional and can be omitted.

Besides re-studying fraction operations, students also have several problem-solving lessons to study. A lot of the problems in these lessons review and reinforce already studied concepts, such as ratio and using bar diagrams to solve problems with fractional parts. As a new—and hopefully interesting—application, we study scaling in maps.

The Lessons in Chapter 5

	page	span
Fraction Terminology	161	*1 page*
Review: Fractions and Mixed Numbers	162	*2 pages*
Subtracting Mixed Numbers	164	*2 pages*
Adding Unlike Fractions	166	*4 pages*
Review: Simplifying Fractions	170	*1 page*
Add and Subtract Fractions: More Practice	171	*2 pages*
Multiplying Fractions	173	*3 pages*
Simplify Before Multiplying	176	*2 pages*
Divide Fractions	178	*3 pages*
Many Operations and Fraction Equations	181	*3 pages*
Comparing Fraction and Decimal Division	184	*3 pages*
Multiplication, Division, and Fractions	187	*2 pages*

Problems with Fractional Parts	189	*4 pages*
Ratio Problems Involving Fractions	193	*2 pages*
Scaling in Maps	195	*3 pages*
Chapter 5 Review	198	*4 pages*

Helpful Resources on the Internet

Fractions and Mixed Numbers

Clara Fraction's Ice Cream Shop
A game in which you convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors on the cone.
www.mrnussbaum.com/icecream/index.html

Simplifying & Equivalent Fractions

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
http://www.homeschoolmath.net/worksheets/fraction.php

Equivalent Fractions video
A video by the author that ties in with the equivalent fraction lessons in this book.
http://www.youtube.com/watch?v=NF57T60CSPs

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Addition and Subtraction

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding fractions
Illustrates with pictures finding the common denominator.
http://matti.usu.edu/nlvm/nav/frames_asid_106_g_2_t_1.html

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
Computer deals you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fishy Fractions
Select the correct answer and the pelican catches the fish. Options for fraction addition or subtraction, like or unlike denominators, simplifying, comparing, and more.
www.iknowthat.com/com/L3?Area=FractionGame

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
www.fuelthebrain.com/Game/play.php?ID=47

Order Fractions
On each round, you drag five given fractions in the correct order.
http://www.bbc.co.uk/schools/ks2bitesize/maths/activities/fractions.shtml

Comparing Fractions - XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Fractional Hi Lo
Computer has selected a fraction. You make guesses and it tells you if your guess was too high or too low.
www.theproblemsite.com/games/hilo.asp

Multiplication and Division

Division of Fractions Conceptually, part 1
A video by the author that ties in with the division lessons in this book. The first part explains about "easy" divisions that can be solved mentally.
http://www.youtube.com/watch?v=41FYaniy5f8

Division of Fractions Conceptually, part 2
A video by the author that ties in with the division lessons in this book. This second part explains about reciprocal numbers and the general "shortcut" for fraction division.
http://www.youtube.com/watch?v=RaT1mDd0_6w

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiply and Reduce Fractions Battleship Game
When you hit the enemy's battleship, you need to solve a fraction multiplication problem.
http://www.quia.com/ba/57713.htm

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Number line bars
Fraction bars that illustrate visually how many times a fraction "fits into" another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

Fractions vs. Decimals (and Percents)

Fraction Pie
The user selects the numerator and denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=45

Comparing Fractions, Decimals, and Percentages
This site has factsheets, a nice matching pairs game, online quiz, and printable worksheets.
http://www.bbc.co.uk/skillswise/numbers/fractiondecimalpercentage/comparing/comparingall3/index.shtm

Fraction Decimal Conversion
Practice fractions vs. decimal numbers online with a matching game, concentration, or flash cards.
www.quia.com/jg/65724.html

Fraction/Decimal Worksheets
Change fractions to decimal numbers or decimal numbers to fractions.
http://www.homeschoolmath.net/worksheets/fraction-decimal.php

Fractions Vs. Decimals Calculator
www.counton.org/explorer/fractions/

Fraction Model
Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44

All Aspects

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also couple of games, for example: make cookies for Grampy.
www.visualfractions.com

Conceptua Math
Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.
www.conceptuamath.com

Who Wants Pizza?
Explains the concept of fraction, addition and multiplication with a pizza example, then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction math topics. Look for the lesson links in the left sidebar.
www.mathexpression.com/understanding-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Fractioncity
Make "fraction streets" and help kids with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
www.teachnet.com/lesson/math/fractioncity.html

Online Fraction Calculator
Add, subtract, multiply or divide fractions and mixed numbers.
www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
www.homeschoolmath.net/worksheets/fraction.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
www.homeschoolmath.net/worksheets/fraction-b.php

Games and Activities to learn Fractions
Commercial games for teaching fractions, including Equivalent Fractions Card Game, Pie and Pizza Fractions, Fractions bingo, teaching fractions with chocolate.
http://lesson-plan.org/lesson-plan/math-lesson-plan.html?offer=frewebhost&pid=0

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Simplifying Before Multiplying

You've already learned to use **factoring** when simplifying.

The example on the right shows simplifying 96/144.

You have also learned how to simplify **"criss-cross."** To simplify 45/150, we cancel the 5s from the numerator and the denominator. Then we simplify 9 and 30 into 3 and 10.

$$\frac{96}{144} = \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{12}}}{\underset{3}{\cancel{12}} \times \underset{1}{\cancel{12}}} = \frac{2}{3}$$

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \times \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \times \underset{1}{\cancel{5}}} = \frac{3}{10}$$

In a similar manner, you can simplify fractions *before* multiplying. This just saves you one step in writing. Why?

Compare the two examples on the right. If you write the numerator and the denominator multiplications separately, and then simplify, it just adds one extra step of writing. You might as well simplify before writing it out that way since it makes no difference in the result.

$$\frac{7}{\underset{2}{\cancel{6}}} \times \frac{\overset{1}{\cancel{3}}}{9} = \frac{7}{18}$$

$$\frac{7}{6} \times \frac{3}{9} = \frac{7 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 9} = \frac{7}{18}$$

1. Simplify before multiplying.

E. $\frac{3}{10} \times \frac{1}{3} =$

A. $\frac{5}{6} \times \frac{2}{4} =$

P. $\frac{4}{8} \times \frac{1}{3} =$

O. $\frac{2}{6} \times \frac{5}{7} =$

L. $\frac{2}{9} \times \frac{9}{11} =$

R. $\frac{2}{6} \times \frac{3}{9} =$

M. $\frac{4}{10} \times \frac{1}{3} =$

E. $\frac{3}{10} \times \frac{3}{9} =$

W. $\frac{4}{5} \times \frac{1}{6} =$

I. $7 \times \frac{5}{21} =$

N. $\frac{16}{24} \times 8 =$

S. $\frac{7}{40} \times 15 =$

	$\frac{5}{12}$	$\frac{1}{9}$	$\frac{1}{10}$		$\frac{21}{8}$	$\frac{5}{3}$	$\frac{2}{15}$	$\frac{1}{6}$	$\frac{2}{11}$	$\frac{1}{10}$		$\frac{16}{3}$	$\frac{5}{21}$	$\frac{2}{15}$	
These problems															!

You can simplify several times before multiplying.			
$\frac{\overset{1}{\cancel{3}}}{15} \times \frac{5}{\underset{2}{\cancel{6}}}$	$\frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} = \frac{1}{6}$	$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}}} \times \frac{7}{14}$	$\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{14}}} = \frac{1}{10}$
First simplify 3 and 6 into 1 and 2.	Then simplify 5 and 15 into 1 and 3.	First simplify 3 and 15 into 1 and 5.	Then simplify 7 and 14 into 1 and 2.

2. Simplify before you multiply.

a. $\frac{8}{12} \times \frac{6}{12}$ **b.** $\frac{3}{10} \times \frac{2}{18}$ **c.** $\frac{2}{30} \times \frac{10}{11}$

d. $\frac{7}{21} \times \frac{3}{4}$ **e.** $\frac{2}{16} \times \frac{8}{9}$ **f.** $\frac{18}{24} \times \frac{8}{9}$

g. $\frac{5}{36} \times \frac{24}{45}$ **h.** $\frac{16}{30} \times \frac{25}{24}$ **i.** $\frac{14}{25} \times \frac{35}{42}$

3. Try your simplifying skills with multiplying three fractions.

a. $\frac{5}{4} \times \frac{12}{9} \times \frac{3}{15}$ **b.** $\frac{8}{10} \times \frac{15}{27} \times \frac{9}{16}$ **c.** $\frac{1}{18} \times \frac{24}{33} \times \frac{9}{20}$

Puzzle Corner

a. Figure out how this was simplified.

$$\frac{\overset{\overset{1}{\cancel{5}}}{\cancel{15}}}{\underset{\underset{4}{\cancel{8}}}{\cancel{24}}} \times \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{60}}}{\underset{\underset{2}{\cancel{10}}}{\cancel{100}}} = \frac{3}{8}$$

b. Simplify:

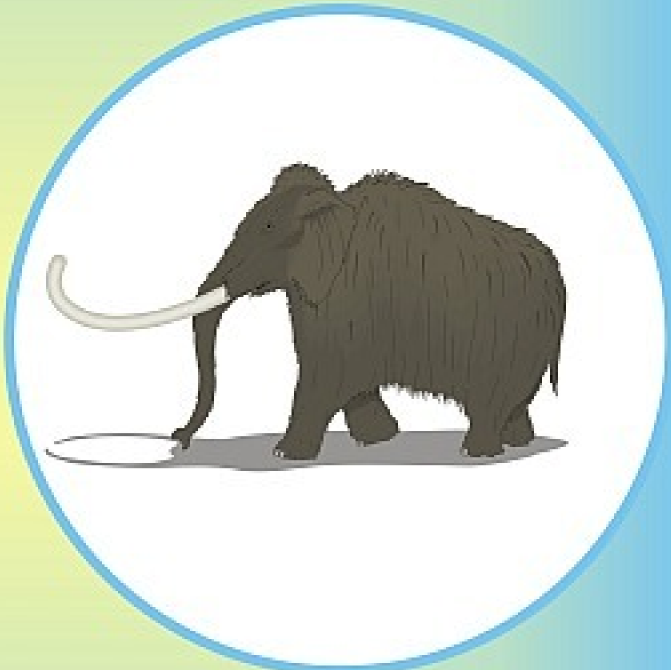
$$\frac{60}{48} \times \frac{36}{90} =$$

MATH MAMMOTH

Grade 6-B

Complete Worktext

- Percent
- Geometry: angles, similar & congruent figures, area, and volume
- Integers & coordinates
- Probability
- Statistics



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Contents

Chapter 6: Percent

Introduction 5
Percent 9
Percentage of a Number 13
Percentage of a Number: Using Decimals 16
Discounts 19
Sales Tax 21
Practice with Percent 23
"Backwards" Questions with Percent 26
Tenth of a Percent 27
Ratios, Fractions and Percent 29
Circle Graphs 32
Percent of Change 34
Percent of Change, Part 2 37
Percent of Change: Applications 39
Comparisons with Percent 41
Review Percent 45

Chapter 7: Geometry

Introduction 49
Angle Relationships 55
Classify Triangles 59
Angles in a Triangle 61
Quadrilaterals Review 63
Angles in Polygons 65
Drawing Problems 67
Congruent and Similar Figures 69
Similar Figures and Scale Ratio 71
Congruent Transformations 74
Transformations in the Coordinate Grid 76
Review: Area of Polygons 1 80
Review: Area of Polygons 2 83
Circumference of a Circle 85
Area of a Circle 88

Area and Perimeter Problems 91
Converting Between Metric Area Units 94
Converting Between Customary Area Units 98
Volume of Prisms and Cylinders 102
Volume of Pyramids and Cones 105
Surface Area 108
Converting Between Units of Volume 111
Geometry Review 113

Chapter 8: Integers

Introduction 118
Integers 122
Addition and Subtraction as Movements 124
Adding Integers 1: Counters 127
Adding Integers 2 130
Subtracting a Negative Integer 133
Add & Subtract Roundup 135
Multiplying Integers 138
Dividing Integers 141
Multiply & Divide Roundup 143
Coordinate Grid Practice 146
Graphing Linear Functions 150
Integers Review 154

Chapter 9: Probability and Statistics

Introduction 158
Simple Probability 163
Probability Problems from Statistics 166
Counting the Possibilities 168
Compound Probability 171
More Practice with Probability 175
Data Analysis 177
Mean, Median and Mode 182
Using Mean, Median and Mode 185
Stem-and-Leaf Plots 188
Making Histograms 190
Range 192
Probability and Statistics Review 194

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Chapter 6: Percent
Introduction

The concept of percent builds on a student's understanding of fractions and decimals. Specifically, students should already be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students who have used Math Mammoth have been practicing that concept since 4th grade, and one reason why I have emphasized finding a fractional part of a whole so much in the division and fraction materials in the earlier grades is specifically to lay a groundwork for the concept of percent.

Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, studying percent should not be difficult.

The first lesson, *Percent*, practices the concept of percent as 1/100, and how to write fractions and decimals as percentages. The second lesson, *Percentage of a Number*, teaches how to find a certain percentage of a quantity using mental math techniques. For example, students find 10% of $400 by dividing $400 by 10. In the next lesson, students find a percentage of a quantity using decimal multiplication, including using a calculator. For example, to find 17% of 45 km, students multiply 0.17 × 45 km.

Then follow lessons about discounts and sales tax, important applications in everyday life. Next we go on to the lesson *Practice with Percent*, which contrasts the two types of problems: questions that ask for a certain percentage of a number (the percentage is given), and questions that ask for the percentage. For example, the first type of question could be “*What is 70% of $380?*”, and the second type could be “*What percentage is $70 of $380?*”

I also present one optional lesson titled *Backwards Questions with Percent*, where students need to figure out “the whole” when a partial amount and a percentage are given. For example: “Three-hundred twenty students, which is 40% of all students, take additional PE. How many students are there in total?”

Then follows one lesson concentrating on a tenth of a percent. Thus far, all of the material has been with whole percents. From this lesson on, we will also use a tenth of a percent (such as 13.4%). We go on to compare ratios, fractions, and percent in one lesson. Next, students study how to make a circle graph.

The last major topic is percent of change. This deals of course with decreases and increases in quantities (such as prices). We also study how to find the percent of change when the original and new amount are known.

Tying in with percent of change, there is one lesson on *Comparisons with Percent*. It ties in, because the way to solve comparisons involving percent (such as how many percent less/more is one thing than another) is identical to finding percent of change.

Note: The first few lessons in this chapter are very similar to the percent lessons in Math Mammoth Grade 5-B curriculum. They are similar, but the problems use different numbers. This is on purpose: If your student did study from Math Mammoth Grade 5-B, then these first lessons will be review, hopefully fairly easy. However, if your student didn't study Math Mammoth in 5th grade, then these lessons will provide a foundation for the concept of percent.

The Lessons in Chapter 6

	page	span
Percent	9	*4 pages*
Percentage of a Number	13	*3 pages*
Percentage of a Number: Using Decimals	16	*3 pages*
Discounts	19	*2 pages*
Sales Tax	21	*2 pages*
Practice with Percent	23	*3 pages*
"Backwards" Questions with Percent	26	*1 page*
Tenth of a Percent	27	*2 pages*
Ratios, Fractions and Percent	29	*3 pages*
Circle Graphs	32	*2 pages*
Percent of Change	34	*3 pages*
Percent of Change, Part 2	37	*2 pages*
Percent of Change: Applications	39	*2 pages*
Comparisons with Percent	41	*4 pages*
Review Percent	45	*4 pages*

Helpful Resources on the Internet

Games & Tools

Virtual Manipulative: Percentages
Interactive tool where you fill in any two of the three 'boxes' (whole, part, and percent) and it will calculate the missing part and show the result visually in two ways.
http://matti.usu.edu/nlvm/nav/frames_asid_160_g_2_t_1.html

Mission: Magnetite
Hacker tries to drop magnetite on Motherboard. To stop him, match up percentages, fractions, and images showing fractional parts.
http://pbskids.org/cyberchase/games/percent/percent.html

Fractions and Percent Matching Game
A simple matching game: match fractions and percentages.
http://www.mathplayground.com/matching_fraction_percent.html

Fraction/Decimal/Percent Jeopardy
Answer the questions correctly, changing between fractions, decimals, and percentages.
http://www.quia.com/cb/34887.html

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Percentage of a Number

100% of something means all of it. 1% of something means 1/100 of it.
To calculate a percentage of some quantity is like calculating a fractional part of that quantity because *percent* simply means a hundredth part. Therefore, percentages are just fractions.
How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg. **To find 1% of something (1/100 of something), divide by 100.** Remember how to divide by 100 mentally: Just move the decimal point two places to the left. For example, 1% of 540 is 5.4. And 1% of 8.30 is 0.083.
To find 2% of some quantity, first find 1% of it, and double that. For example, let's find 2% of \$6. Since 1% of 6 is \$0.06, then 2% of 6 is \$0.12.
To find 10% of some quantity, divide by 10. Why does that work? 10% is 10/100. And, 10/100 is equal to 1/10. So we just find 1/10 of a quantity! For example, 10% of \$780 is \$78. Or, 10% of \$6.50 is \$0.65. (To divide by 10 mentally, just move the decimal point one place to the left.)
Can you think of a way to find 20% of a number? (Hint: Start with finding 10% of the number.) ____

1. Find 10% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

2. Find 1% of these numbers.

a. 700 ________ **b.** 321 ________ **c.** 60 ________ **d.** 7 ________

3. One percent of Mom's paycheck is \$22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage / number	**1,200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by ________. **10% is $\frac{1}{\square}$. To find 10% of a number, divide by ________.** **1% is $\frac{1}{\square}$. To find 1% of a number, divide by ________.**	**50% of 244 is __________.** **10% of 47 is __________.** **1% of 530 is __________ .**
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • First find _________% of the number and • then multiply by 2, 3, 4, 6, 7, 8, or 9.	10% of 120 is __________. 30 % of 120 is __________. 60 % of 120 is __________.

6. Find the percentages. Use mental math.

a. 10% of 60 kg ________ 20% of 60 kg ________	**b.** 10% of $14 ________ 30% of $14 ________	**c.** 10% of 5 mi ________ 40% of 5 mi ________
d. 1% of $60 ________ 4% of $60 ________	**e.** 10% of 110 cm ________ 70% of 110 cm ________	**f.** 1% of $1,330 ________ 3% of $1,330 ________

7. David pays a 20% income tax on his $2,100 salary.
How many dollars is the tax?

How much money does he have left after paying the tax?

8. Nancy pays 30% of her $3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of $55. Peter's solution: 10% of $55 is $5.50 So subtract 100% − $5.50 = $94.50	**b.** Find 6% of $1,400. Patricia's solution: 1% of $1,400 is $1.40. So, 6% is six times that, or $8.40.

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Discounts

Other than figuring sales tax, the area of life in which you will probably most often need to use percentages is in calculating discounts when you are shopping.

Example 1. *A laptop that costs $600 is 20% off. What is the sale price?*

Simply put, we calculate 20% of $600. That is the discounted amount in *dollars*. Then we subtract that from the original price, $600.

20% of $600 is $120. So $600 − $120 = $480.

Another way: Since 20% of the price has been removed, 80% of the price is left. So by calculating 80% of the original price, you will get the new discounted price:

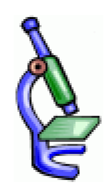
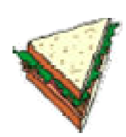
$0.8 \times \$600 = \$480$

1. All of these items are on sale. Calculate the discount in dollars and the resulting sale price.

a.  Price: $90 20% off Discount amount: $ 18 Sale price: $____________	**b.**  Price: $5 40% off Discount amount: $________ Sale price: $____________	**c.**  Price: $15 30% off Discount amount: $________ Sale price: $__________

2. A $25 swimsuit was on sale for 20% off.
   Monica calculated the discounted price this way:
   $25 − $20 = $5.
   What went wrong? Find the correct discounted price.

3. All the items are on sale. Find the discounted price.

a.  Price: $1.20 25% off Discounted price: $_______	**b.**  Price: $18 25% off Discounted price: $_______	**c.**  Price: $150 30% off Discounted price: $_______
d.  Price: $20 40% off Discounted price: $________	**e.**  Price: $2.20 10% off Discounted price: $_______	**f.**  Price: $1.30 50% off Discounted price: $________

You can often use **estimation** when calculating the discounted price, especially if you don't have a calculator with you while shopping.
Example. A \$198.95 bicycle is discounted by 25%. What is the discounted price? To estimate it, round the original price of the bicycle to \$200. 25% of \$200 is 1/4 of \$200, or \$50. So the discounted price is \$150.
Example. A \$425.90 laptop is discounted by 28%. What is the discounted price? Round the discount percentage to 30%, and the price of the laptop to \$430. 10% of \$430 is \$43. 30% of \$430 is three times that much, or \$129. Subtract using rounded numbers: \$430 − \$130 = \$300.

4. Estimate the discounted price.

a. 30% off of a \$39.90 book **b.** 17% off of a \$12.50 block of cheese **c.** 75% off of a \$75.50 pair of shoes

5. Which is a better deal? Estimate using rounded numbers and mental math.

a. 75% off of a \$199 brand-name mp3 player
OR an off-brand mp3 player for \$44.99

b. 40% off of a new, \$89 textbook
OR a used copy of the same textbook for \$39.90.

6. Which of these methods work for calculating the discounted price for 25% off of \$46?

$0.75 \times \$46$	$\$46 - 0.25 \times \$46$	$\$46 - 0.75 \times \$46$	$3 \times \$46 \div 4$
$0.25 \times \$46$	$\$46 - \frac{\$46}{4}$	$\frac{\$46}{4}$	$\frac{\$46}{4} \times 3$

7. A company sells a computer program for \$39.99. They estimate they could sell 50 copies of it in a week, with that price. But if they discount the price by 25%, they think they could sell 100 copies. *Estimate* which way would they earn the most money.

8. The original price of a phone was \$60, and the new, discounted price was \$48. How many percent was the discount?

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Percent of Change

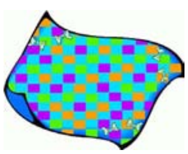
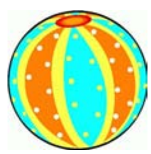
Percent of change has to do with situations where a price or some other quantity *increases* or *decreases* (changes) by a percentage. First we'll review discounts and price increases. Then we'll study how to calculate the percent of change — that is, how to find how many percent the price or other quantity changed.

You've already studied discounts, where the price of an item is discounted by 10%, 15%, or some other percentage. Similarly, the price of an item can also *increase* by a certain percentage.

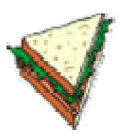
Example. An airline ticket costs $120 now. Next week it goes up by 10%. What will the new price be?

First, calculate 10% of $120. That's $12. Since the price is going *up*, we *add* that to the current price: $120 + $12 = $132. So the new price is $132.

1. Let's review. All these items are on sale. Calculate the new, discounted price.

 **a.** Price: $9 20% off New price: $___________	 **b.** Price: $6 25% off New price: $____________	 **c.** Price: $90 30% off New price: $__________

2. The price of these items goes up. Find the new price.

 **a.** Price: $5,000 10% increase New price: $__________	 **b.** Price: $110 20% increase New price: $__________	 **c.** Price: $90 30% increase New price: $_________
 **d.** Price: $3 15% increase New price: $__________	 **e.** Price: $2 30% increase New price: $_________	 **f.** Price: $1.50 50% increase New price: $__________

3. A jacket costs $50. First, its price increases by 20%.
Then, it is discounted by 20%.
Calculate the final price. Notice: it will NOT be $50!

Example. A phone cost \$50 at first. Now it is discounted, and costs \$45.
How many percent was the discount?

Note: The problem is asking for the *percentage*—not for the new price! It is solved in two steps:

1. We find how much was subtracted from the original price to get the discounted price. This means finding the difference \$50 − \$45 = \$5. This \$5 is the amount of the discount in dollars.
2. Then we find how many percent this \$5 difference is of the original \$50 price. How many percent is \$5 of \$50? It is 5/50 = 1/10 = 10%.

So the discount was 10%.

4. Fill in the blanks, and calculate how many percent the discount is in each case.

a. A toy construction set costs \$12. It is discounted and costs only \$8 now. How many percent is the discount? 1. First we find out how much was subtracted from \$12 to get \$8 (the DIFFERENCE). It is \$_________ 2. Then we find how many percent \$________ is of the original price, \$12:	**b.** A sewing kit costs \$20. It is discounted and costs only \$16 now. How many percent is the discount? 1. First we find out how much was subtracted from \$_______ to get \$_______ (the DIFFERENCE). It is \$_________ 2. Then we find how many percent \$________ is of the original price, \$20:

5. A portable reading device costs \$250. Now it is discounted and costs \$225. How many percent was the discount?

Compare these two problems carefully:

A shirt costs \$24 at first.
Now it is discounted by 25%.
What is the new price?

1. Calculate 25% of \$24. Since 25% = 1/4, this is 1/4 of \$24, or \$6.
2. Subtract \$24 − \$6 = \$18. That is the new price.

A shirt costs \$24 at first. Now it is discounted, and costs \$18.
How many percent was the discount?

1. Find how much was subtracted from \$24 to get \$18 (the difference). That's \$6. So it was discounted by \$6.
2. Find how many percent \$6 is of the original price, \$24. It is 6/24 = 1/4 = 25%. So the discount was 25%.

When we find the percentage of discount from a discount price (on the right), we work "backwards" compared to when we find the discounted price from a percentage discount (on the left).

Percent of increase

When the price increases, we can calculate that increase as a percent, too. Compare:

Gasoline cost $3/gallon last week. Now it has gone up by 5%. What is the new price?

1. Calculate 5% of $3. Since 10% of $3 is $0.30, then 5% is half of that, or $0.15.
2. Add $3 + $0.15 = $3.15/gallon. That is the new price.

Gasoline cost $3 last week. Now it costs $3.15. How many percent was the price increase?

1. Find how much was added to $3 to get $3.15 (the difference). That is $0.15.
2. Find how many percent $0.15 is of the original price, $3. It is 15/300 = 5/100 = 5%. So, the percent of increase was 5%.

In finding the percent of increase (on the right), we work "backwards" compared to finding the price when the percent of increase is known (on the left).

Notice also that we used *cents* (¢) when calculating what part $0.15 (15¢) is of $3 (300¢)!

6. Fill in the missing information. Find the percent of increase.

a. A bouquet of flowers used to cost $15, but now it costs $20.
What is the percent of increase?

1. First we find out how much was added to $15 to get $20 (the DIFFERENCE).
It is $_________
2. Then we find how many percent $________ is of the original price, $15:

b. A chair used to cost $20, but now it costs $26.
What is the percent of increase?

1. First we find out how much was added to $20 to get $26 (the DIFFERENCE).
It is $_________
2. Then we find how many percent $________ is of the original price, $20:

7. Fill in the missing information and find the percent of increase or decrease.

a. A flashlight used to cost $9, but now it costs $8.10.
What is the discount percent?

1. The difference in price is $_________
2. Find how many percent that $________ is of the original price, $9:

b. A stove used to cost $160, but now it costs $200.
What is the percent of increase?

1. The difference in price is $_________
2. Find how many percent that $________ is of the original price, $________:

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Chapter 7: Geometry
Introduction

The main topics in this chapter include

- angle relationships
- classifying triangles and quadrilaterals
- angle sum of triangles and quadrilaterals
- congruent transformations, including some in the coordinate grid
- similar figures, including using ratios and proportions
- review of the area of all common polygons
- circumference of a circle (Pi)
- area of a circle
- conversions between units of area (both metric and customary)
- volume and surface area of common solids
- conversions between units of volume (both metric and customary)

This is a lot of topics, and includes a lot of geometry topics taught commonly in middle school, though not all. For example, Pythagorean Theorem, constructions, and the area of a sector of a circle are not covered here. Some topics are not covered to the same depth as you would expect in 7th or 8th grade, such as congruent transformations or angle relationships. After studying this chapter, most students should be fine to study the geometry sections of a typical pre-algebra course (8th grade).

We start out by studying some simple angle relationships such as vertical angles and corresponding angles, and angle problems that involve logical reasoning. Classifying triangles (according to angles and sides) is mostly review, as is the later lesson of classifying quadrilaterals. As new topics, students learn the angle sum of a triangle and any quadrilateral.

After some drawing problems and a simple lesson on similar figures, we study similar figures using a scale ratio. This lesson ties in with proportions and involves mostly calculations.

Next we turn our attention to congruent transformations (translation, rotation, reflection). These are studied also in the coordinate grid, which allows us to exactly describe the location of the original and of the transformed figure.

The next lessons have to do with area. We review the area of triangles, parallelograms, and polygons in two lessons. Then we study new material: the circumference and area of a circle, which both involve using *pi*.

Unit conversions (for area units) are a difficult topic for many. I have tried to take it step-by-step and emphasize the principle in the conversions. And lastly, we study the volume and surface area of solids.

I have striven to make connections between geometry and other areas of mathematics, such as using proportions and ratios with similar figures, including problems that involve the use of percent, coordinate grid, and even equations. Student will also use a calculator in some problems (those are marked with a small calculator symbol). There are still lots of drawing problems, though calculations are becoming more important than in earlier grades.

The Lessons in Chapter 7

	page	span
Angle Relationships	55	*4 pages*
Classify Triangles	59	*2 pages*
Angles in a Triangle	61	*2 pages*
Quadrilaterals Review	63	*2 pages*
Angles in Polygons	65	*2 pages*
Drawing Problems	67	*2 pages*
Congruent and Similar Figures	69	*2 pages*
Similar Figures and Scale Ratio	71	*3 pages*
Congruent Transformations	74	*2 pages*
Transformations in the Coordinate Grid	76	*4 pages*
Review: Area of Polygons 1	80	*3 pages*
Review: Area of Polygons 2	83	*2 pages*
Circumference of a Circle	85	*3 pages*
Area of a Circle	88	*3 pages*
Area and Perimeter Problems	91	*3 pages*
Converting Between Metric Area Units	94	*4 pages*
Converting Between Customary Area Units	98	*4 pages*
Volume of Prisms and Cylinders	102	*3 pages*
Volume of Pyramids and Cones	105	*3 pages*
Surface Area	108	*3 pages*
Converting Between Units of Volume	111	*2 pages*
Geometry Review	113	*5 pages*

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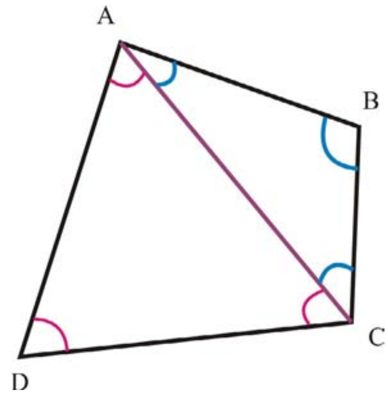
Angles in Polygons

<u>The angle sum in a quadrilateral is 360</u>°.

See if you can understand and fill in this proof about the angle sum in a quadrilateral!

PROOF. Look at the quadrilateral ABCD. We draw a diagonal into it. The diagonal divides the quadrilateral into two triangles, triangle ABC and triangle ACD.



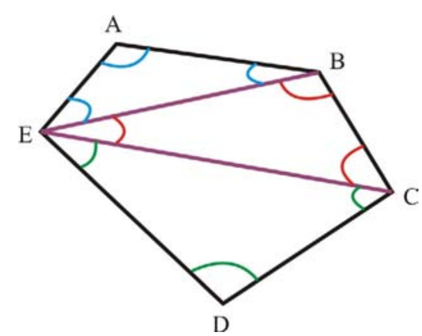
The angle B from triangle ABC is also an angle of the quadrilateral. The angle D from triangle ACD is also an angle of the quadrilateral.

Angle BCA and angle ACD are angles in the two triangles (the two angles with vertex C), but they also form together one angle of the quadrilateral. Similarly, angle CAB and angle DAC are angles in the two triangles (the two marked angles with vertex A), but they also form together another angle of the quadrilateral.
The angle sum of triangle ADC is _______ degrees, and the angle sum of triangle ABC is also _______ degrees,

It follows that the four angles in the quadrilaterals ABCD are formed of the angles of the two triangles. Thus, the angle sum of a quadrilateral is twice ______° , or ________ °.

1. The figure illustrates how we can find the angle sum in a pentagon. Use the reasoning above to find the angle sum of a pentagon.

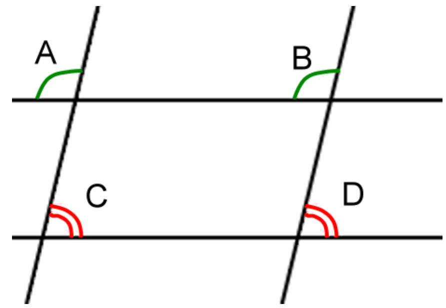
   The angle sum of a pentagon is ___________°.



2. Draw six points randomly. Connect them with a ruler so you get a hexagon. Then, divide it into triangles using diagonals. Use the reasoning above to find the angle sum of a hexagon.

   It is ___________°.

3. You have seen this picture earlier. It has two sets of parallel lines. We see lots of *vertical angles* and *corresponding angles*.



a. Angle A is 102°. Mark in the picture (using a single arc) all the other angles that are also 102°.

b. Mark in the picture (using a double arc) all the other angles that measure the same as angle C.

c. How many degrees is angle C? _______°

d. What quadrilateral is enclosed by the two sets of parallel lines? ______________________________

From this figure we can learn something special about the angles in a parallelogram:

In a parallelogram, the opposite angles are congruent.
Also, two "neighboring" angles have the angle sum of 180 degrees.
In total, the four angles of course add up to 360°, just like in any quadrilateral.

4. One angle in a parallelogram is 74°.
What are the measures of its other angles?

_______°, _______° and _______°.

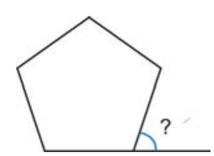
Now draw one such parallelogram.
You can choose the side lengths.

5. One angle of a rhombus is 115°.
What are the measures of its other angles?

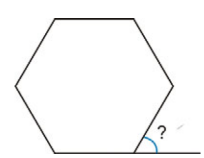
_______°, _______° and _______°.

Now draw one such rhombus. You can choose the side length. Just remember, in a rhombus, all sides are congruent.

Puzzle Corner



a. This is a regular pentagon. The angle marked with "?" is called an *exterior angle* of the pentagon. Figure out its angle measure.



b. This is a regular hexagon. Figure out the measure of the exterior angle (marked with "?").

c. How many degrees is the exterior angle of a regular nonagon?

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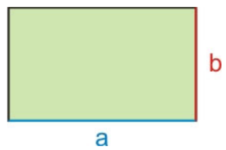
Review: Area of Polygons 1

1. Area of rectangle

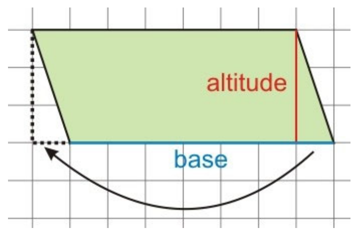


A = *ab*

The area is *side* × *side*. We can denote the side lengths with *a* and *b* or with other letters.

- Remember to use square units for the area! If you measure the sides in millimeters, the area will be in square millimeters.

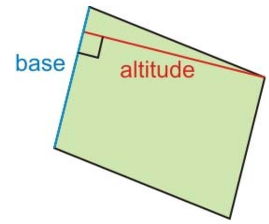
2. Area of parallelogram



A = *bh*

Any parallelogram can be transformed into a rectangle (see illustration) with the same area. That is why the area is *base* × *altitude*. The letter *b* stands for base, and *h* for height/altitude.

Here, *b* is 3 units, *h* is 7 units, and the area = 21 square units.



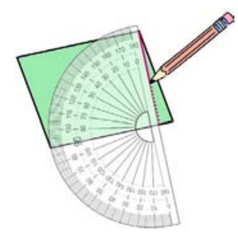
The altitude is always perpendicular to the base, and is drawn from between the base and the opposite side.

Any side of the parallelogram can be chosen to be the base.

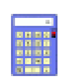
1. The area of Reynolds's rectangular plot is 750 m^2.
   If one side is 30 m, what is the other side?

2. The area of a rectangular stamp is 6.6 cm^2.
   If one side is 22 mm, what is the other side?

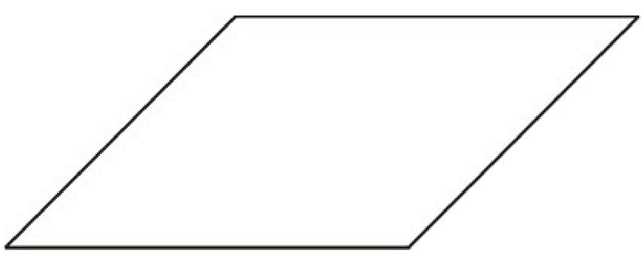
Use a protractor to draw the altitude so it is perpen-dicular to the base.



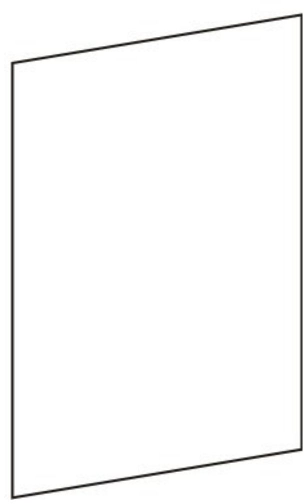
3. Measure what you need, and calculate the area of the parallelograms in square centimeters, rounding to the nearest square centimeter.

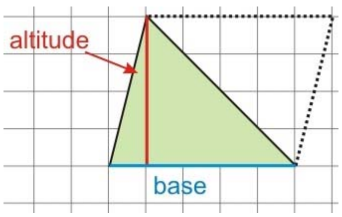


a.



b.





$A = \frac{1}{2}bh$

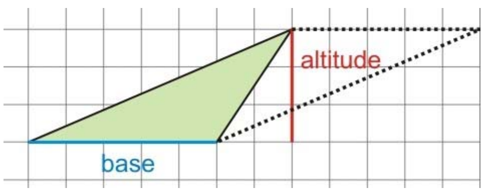
b is 5 units, *h* is 4 units,
A = 10 square units

3. Area of triangle

Any triangle is exactly half of a certain parallelogram.
The triangle and parallelogram share the same base and the same altitude. Therefore, the area of a triangle is half of the area of the corresponding parallelogram.

The altitude of the triangle is perpendicular to the base, and is drawn from the vertex that is opposite of the base.

The letter *b* stands for base, and *h* stands for height/altitude.



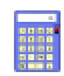
b is 5 units, *h* is 3 units,
A = 7.5 square units

Sometimes the altitude of a triangle falls outside the triangle itself. It still needs to be drawn from the vertex, and perpendicular to the base.

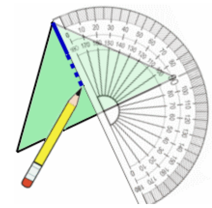
Again, you can choose which side of the triangle is the base. So, all triangles have three different altitude & base pairs.

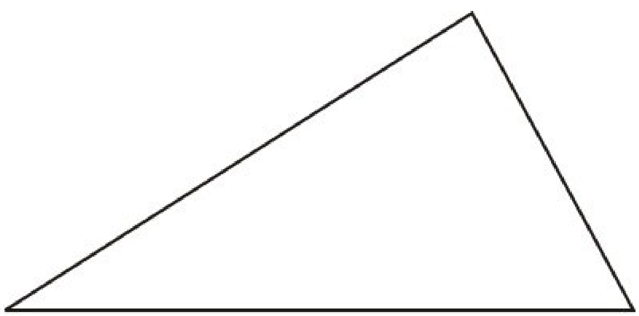
For *right triangles*, it is often easiest to use the two sides that are perpendicular to each other as the base and the altitude.

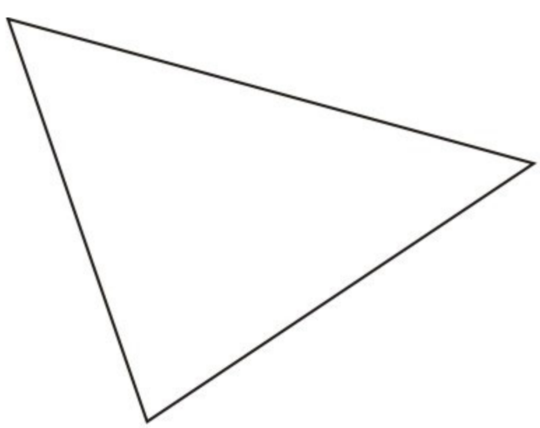
4. Draw an altitude to the triangles. Measure what you need, and calculate the area in square centimeters. Round to the nearest square centimeter.

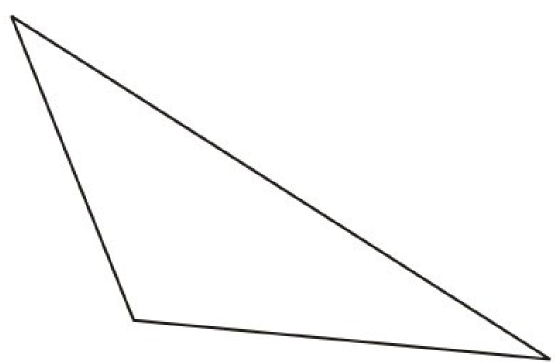


Use a protractor to draw the altitude so it is perpendicular to the base.









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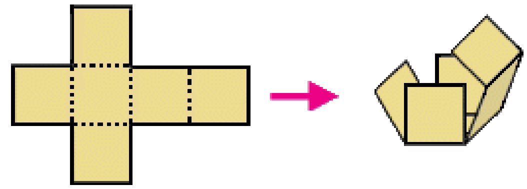
Surface Area

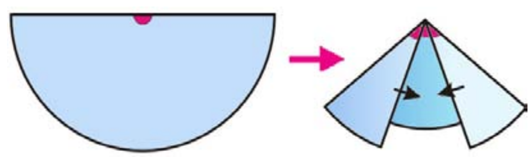
Surface area of a solid means the total area of all of its faces.

So, to calculate the surface area, simply find the area of each face, and add those.

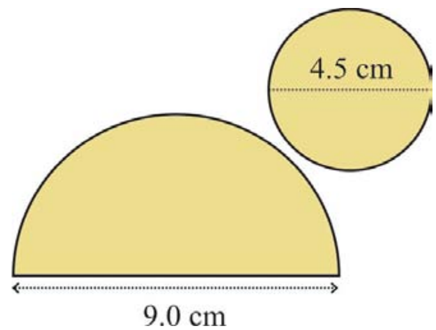
Sometimes the **net** of a solid helps us with that. The net shows all the faces of the solid drawn in a plane—as if on a flat paper—and you can build the solid from the net by folding.

The net of a circular right cone always has a circle as a base. Then, it has a partially drawn circle (sector of a circle) that is the **lateral** face, or the face that "wraps around" the base.


The net of a cube, and it being folded into a cube.


Folding the lateral face of the cone.

Example 1. Find the surface area of the cone with this net.

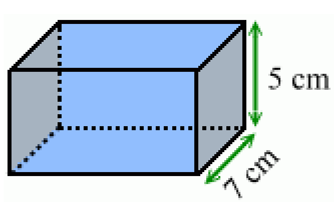


1. The bottom face is a circle with a radius of 2.25 cm. Its area is $\pi \times (2.25 \text{ cm})^2 \approx 15.89625 \text{ cm}^2$.
2. The lateral face is a half circle, with a radius of 4.5 cm. Its area is $0.5 \times \pi \times (4.5 \text{ cm})^2 \approx 31.7925 \text{ cm}^2$.

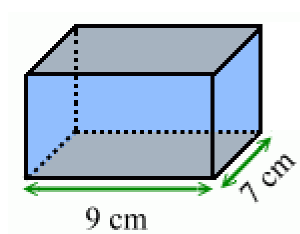
Lastly we add the two: $15.89625 \text{ cm}^2 + 31.7925 \text{ cm}^2 = 47.68875 \text{ cm}^2 \approx 48 \text{ cm}^2$.

Notice we kept many more decimals for the intermediate results than for our final answer. Don't round your intermediate results very much—do the rounding to the final accuracy only on the final answer.

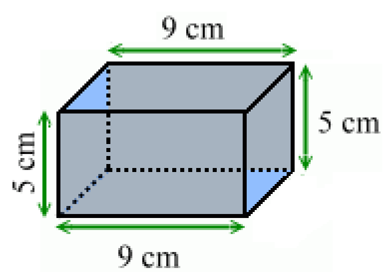
Example 2. Calculate the surface area of this rectangular prism.



The two faces on the ends are congruent (identical). Each of them has an area of $7 \text{ cm} \times 5 \text{ cm} = 35 \text{ cm}^2$.



The two faces on the top and bottom are congruent. Each has an area of $9 \text{ cm} \times 7 \text{ cm} = 63 \text{ cm}^2$.

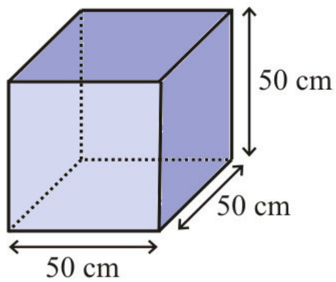


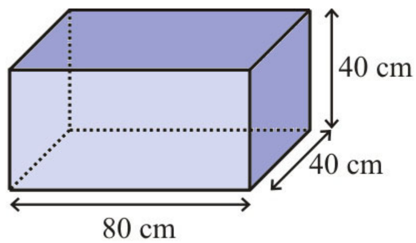
The two faces on the front and back are also congruent. Each has an area of $9 \text{ cm} \times 5 \text{ cm} = 45 \text{ cm}^2$.

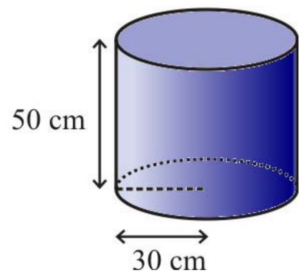
So the total surface area is:

$$2 \times 35 \text{ cm}^2 + 2 \times 63 \text{ cm}^2 + 2 \times 45 \text{ cm}^2$$
$$= 70 \text{ cm}^2 + 126 \text{ cm}^2 + 90 \text{ cm}^2 = 286 \text{ cm}^2.$$

1. Find the surface area of these water tanks.







a. ________________ cm^2 **b.** ________________ cm^2 **c.** ________________ cm^2

2. Note that 1 m^2 = (100 cm × 100 cm) = 10,000 cm^2. Use that to convert the surface areas of the water tanks into square meters.

a. ________________ m^2 **b.** ________________ m^2 **c.** ________________ m^2

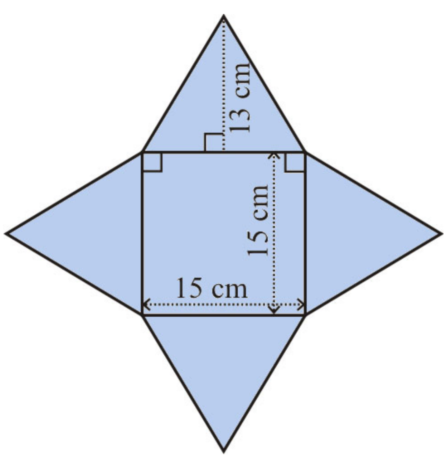
3. Find the volume of the water tanks in exercise #1.

a. ________________ cm^3 **b.** ________________ cm^3 **c.** ________________ cm^3

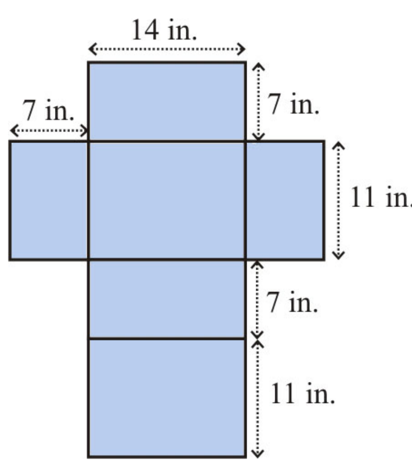
4. We know that 1,000 cm^3 = 1,000 ml = 1 liter. Use that to convert the volumes of the water tanks into liters.

a. ________________ l **b.** ________________ l **c.** ________________ l

5. Name the solids that can be built from these nets, and calculate their surface area.



a.



b.

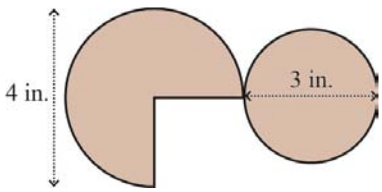
solid: ______________________________

surface area: ______________________________

solid: ______________________________

surface area: ______________________________

6. Find the surface areas.



a. A cone whose lateral face is 3/4 of a circle.

b. A triangular prism

7. A swimming pool is in the shape of a rectangular prism.
It is 12.5 m long, 6 m wide, and 2 m deep.

a. Find the surface area of the pool's bottom and sides
(not including the top, since it is not covered).

b. Tile costs $9.90 per square meter.
Calculate the cost of tiling the pool.

8. A gift box is in a shape of a cube with 20-cm sides.
Calculate its surface area.

9. One cube has a side of 1 unit, and another cube has a side of 2 units.
This means that their sides are in the ratio of 1:2.

a. In what ratio are their surface areas?

b. In what ratio are their volumes?

Puzzle Corner The surface area of a cube is 150 cm^2.
Calculate the volume of the cube.

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Chapter 8: Integers
Introduction

Chapter 8 covers all important integer topics in middle school math.

The first topic is integers themselves, presented on a number line and tied in with temperature. Next we move on to modeling additions and subtractions as movements on a number line. Another model used for addition is that of counters or chips. One lesson explains the shortcut for subtracting a negative integer using three different viewpoints (difference, counters, and number line movements). There is also a roundup lesson for addition and subtraction of integers.

Multiplication and division of integers is explained using counters, first of all, and then relying on the properties of multiplication and division. We use multiplication and division in the context of enlarging or shrinking geometric figures in the coordinate grid.

These lessons also include a few simple equations, problems with several operations, and fun riddles.

The last section of lessons in this chapter deals with the coordinate grid. Students move geometric figures up, down, to the right, and left. They reflect figures in the *x*-axis and in the *y*-axis. Lastly, students graph simple linear functions, first using a range of integer values only, and then without such limitation.

Note: The first few lessons in this chapter are very similar to the integer lessons in Math Mammoth Grade 5-B curriculum. They are similar, but the problems use different numbers. This is on purpose: If your student did study from Math Mammoth Grade 5-B, then these first lessons will be review, hopefully fairly easy. However, if your student didn't study Math Mammoth in 5th grade, then these lessons will provide a foundation for the concept of integers.

The Lessons in Chapter 8

	page	span
Integers	122	*2 pages*
Addition and Subtraction as Movements	124	*3 pages*
Adding Integers 1: Counters	127	*3 pages*
Adding Integers 2	130	*3 pages*
Subtracting a Negative Integer	133	*2 pages*
Add & Subtract Roundup	135	*3 pages*
Multiplying Integers	138	*3 pages*
Dividing Integers	141	*2 pages*
Multiply & Divide Roundup	143	*3 pages*
Coordinate Grid Practice	146	*4 pages*
Graphing Linear Functions	150	*4 pages*
Integers Review	154	*4 pages*

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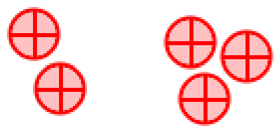
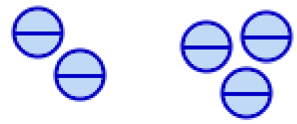
Adding Integers 1: Counters

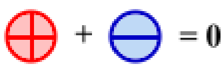
Addition of integers can be modeled using counters.
We'll use red counters with a "+" sign for positives and blue counters with a "–" sign for negatives.



This picture shows the addition, $2 + 3$. There is one group of 2 positives and another group of 3 positives. The sum is simply 5.



This picture shows the addition, $(-2) + (-3)$. We *add* negatives and negatives. In total there are five negatives, so the sum is -5.

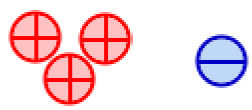


$1 + (-1) = 0$

One positive counter and one negative counter *cancel* each other.
In other words, their sum is zero!

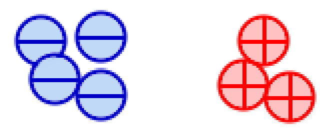
$2 + (-2) = 0$

Two negatives and two positives also cancel each other. Their sum is zero.



$3 + (-1) = 2$

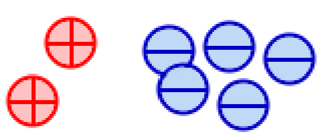
Here, one "positive-negative" pair is canceled, and we are left with 2 positives.



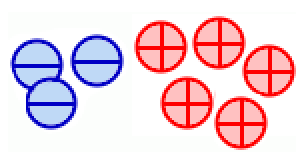
$(-4) + 3 = -1$

Now the negatives outweigh the positives. Pair up three of each, and there is still one negative left.

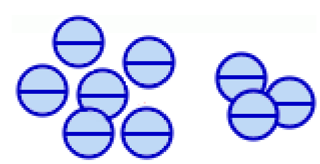
1. Refer to the pictures and add. Remember each "positive-negative" pair is canceled.



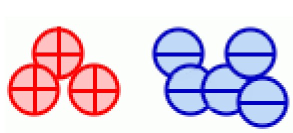
a. $2 + (-5) =$ ______



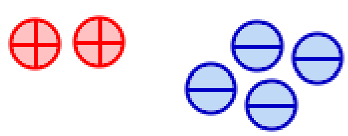
b. $(-3) + 5 =$ ______



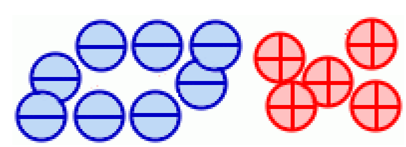
c. $(-6) + (-3) =$ ______



d. $3 + (-5) =$ ______

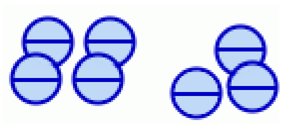
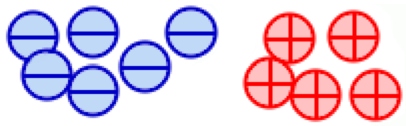
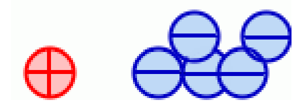
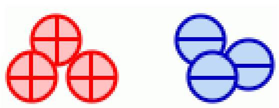
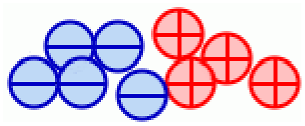
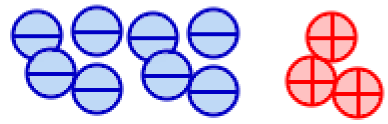


e. $2 + (-4) =$ ______



f. $(-8) + 5 =$ ______

2. Write addition sentences (equations) to match the pictures.

 **a.**	 **b.**	 **c.**
 **d.**	 **e.**	 **f.**

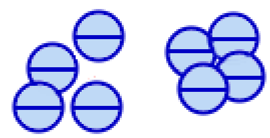
3. Rewrite these sentences using symbols, and solve the resulting addition problems.

a. The sum of seven positives and five negatives.

b. Add −3 and −11.

c. Positive 100 and negative 15 added together.

A note on notation



We can write an elevated minus sign to indicate a negative number: $^{-}4$.
Or we can write it with a minus sign and parentheses: (-4).
We can even write it without the parentheses if the meaning is clear: -4.

So $^{-}4 + {}^{-}4 = {}^{-}8$ is the same as $(-4) + (-4) = (-8)$, which is the same as $-4 + (-4) = -8$

You *should* write the parentheses if you have + and −, or two − signs, next to each other.
So don't write "8 + − 4"; write "8 + (−4)." And don't write "3 − −3"; write "3 − (−3)."

4. Think of the counters. Add.

a. $7 + (-8) =$ $(-7) + 8 =$	**b.** $(-7) + (-8) =$ $7 + 8 =$	**c.** $5 + (-7) =$ $7 + (-5) =$	**d.** $50 + (-20) =$ $10 + (-40) =$
e. $^{-}2 + {}^{-}4 =$ $^{-}6 + 6 =$	**f.** $10 + {}^{-}1 =$ $^{-}10 + {}^{-}1 =$	**g.** $^{-}8 + 2 =$ $^{-}8 + {}^{-}2 =$	**h.** $^{-}9 + {}^{-}1 =$ $9 + {}^{-}1 =$

5. Find the number that is missing from the equations.

a. (−3) + ______ = (−7)	**b.** (−3) + ______ = 3	**c.** 3 + ______ = (−7)

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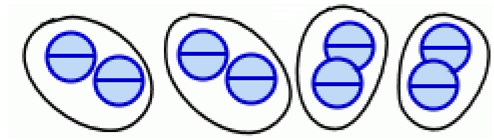
Dividing Integers

Divide a negative number by a positive



The image illustrates (−8) ÷ 4, or eight negatives divided into four groups. We can see the answer is −2.

Any time a negative integer is divided by a positive integer, we can illustrate it as so many negative counters divided into groups. The answer will be negative.

So each time you divide a negative integer by a positive integer, the answer is negative.

Divide a positive integer by a negative. For example, 24 ÷ (−8) = ?

Remember multiplication is the opposite operation of division. Let's write the answer of 24 ÷ (−8) as *s*. Then we write a multiplication equation using the division:

$24 \div (-8) = s \quad \Rightarrow \quad (-8)s = 24$

(You could use an empty line instead of *s*, if the variable *s* confuses you.)

The only number that fulfills the equation $(-8)s = 24$ is $s = -3$. Therefore, 24 ÷ (−8) = −3.

Similarly, each time you divide a positive integer by a negative integer, the answer is negative.

Divide a negative integer by a negative. For example, (−24) ÷ (−8) = ?

Again, let's mark the answer to −24 ÷ (−8) with *y*, and then write a multiplication sentence.

$-24 \div (-8) = y \quad \Rightarrow \quad (-8)y = -24$

The only number that fulfills the equation $(-8)y = -24$ is $y = 3$. Therefore, −24 ÷ (−8) = 3.

Similarly, each time you divide a negative integer by a negative integer, the answer is positive.

Summary. The symbols below show whether you get a positive or negative answer, when you multiply or divide integers. Notice that the rules for multiplication and division are the same!

Multiplication	Examples	Division	Examples
⊕ × ⊖ = ⊖	4 × (−5) = −20	⊕ ÷ ⊖ = ⊖	20 ÷ (−5) = −4
⊖ × ⊕ = ⊖	−4 × 5 = −20	⊖ ÷ ⊕ = ⊖	−20 ÷ 5 = −4
⊖ × ⊖ = ⊕	−4 × (−5) = 20	⊖ ÷ ⊖ = ⊕	−20 ÷ (−5) = 4
⊕ × ⊕ = ⊕	4 × 5 = 20	⊕ ÷ ⊕ = ⊕	20 ÷ 5 = 4

Here's a shortcut for *multiplication* and *division* (NOT addition or subtraction):

- If both numbers have the same sign (both are positive *or* negative), the answer is positive.
- Otherwise, the answer is negative.

1. Divide.

a. $-50 \div (-5) =$ ______ $-12 \div 2 =$ ______	**b.** $(-8) \div (-1) =$ ______ $14 \div (-2) =$ ______	**c.** $81 \div (-9) =$ ______ $-100 \div (-10) =$ ______

2. Multiply. Then write a division equation for each multiplication, using the same numbers.

a. $-5 \times (-5) =$ ______ ______ ÷ ______ = ______	**b.** $9 \times (-6) =$ ______ ______ ÷ ______ = ______	**c.** $-80 \times 8 =$ ______ ______ ÷ ______ = ______

3. Complete the patterns.

a.	**b.**	**c.**
$12 \div 4 =$ ______	______ $\div (-7) = -3$	$60 \div$ ______ $= 2$
$8 \div 4 =$ ______	______ $\div (-7) = -2$	$40 \div$ ______ $= 2$
$4 \div 4 =$ ______	______ $\div (-7) = -1$	$20 \div$ ______ $= 2$
$0 \div 4 =$ ______	______ $\div (-7) = 0$	$-20 \div$ ______ $= 2$
$(-4) \div 4 =$ ______	______ $\div (-7) = 1$	$-40 \div$ ______ $= 2$
$(-8) \div 4 =$ ______	______ $\div (-7) = 2$	$-60 \div$ ______ $= 2$
$(-12) \div 4 =$ ______	______ $\div (-7) = 3$	$-80 \div$ ______ $= 2$
$(-16) \div 4 =$ ______	______ $\div (-7) = 4$	$-100 \div$ ______ $= 2$

4. Here's a funny riddle. Solve the math problems to uncover the answer.

E ____ $\div (-8) = 2$	**N** $-12 \times (-5) =$ ______	**E** $(-144) \div 12 =$ _____
E $3 \times (-12) =$ ____	**H** ____ $\div 12 = 5$	**T** $-4 \times (-9) =$ ____
N $-15 \div$ ____ $= -5$	**E** ____ $\times (-6) = 0$	**V** $-45 \div$ ______ $= 5$
G $-1 \times (-9) =$ ______	**I** $-27 \div 9 =$ ______	**I** $-7 \times$ ____ $= -84$
S $-48 \div 6 =$ ______	**N** $3 \times$ _____ $= -24$	

Why is six afraid of seven? Because....

−8	−12	−9	−36	60		0	12	9	60	36		3	−3	−8	−16

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Chapter 9: Probability and Statistics
Introduction

In this chapter, probability is a totally new topic that we are tackling. In the past, probability was only taught in high school—for example, I personally encountered it for the first time in 12th grade. However, in recent years it has “crept” down the grade levels and many states require probability topics even in elementary school.

While the concepts we study here are not difficult, I have not felt any need for students to study probability in earlier grades. That way there is more time to focus on the more important topics in grades 1-5. But since probability IS so strongly present in state standards, these lessons allow students following Math Mammoth to learn the main ideas, for this level.

We start with the concept of simple (classic) probability, and then expand into probability involving two events. This is all that is needful to master at this point. The exercises involve tree diagrams, dice, flipping coins, picking marbles, spinning spinners, and probability involving statistics, which are the usual types of situations in the study of probability.

The last lessons then deal with statistical topics. We start out with a lesson on data analysis, which presents various types of graphs for students to read, and reviews some percent-related topics. We go on to mean, median, and mode—the three measures of central tendency— and a little bit on how to use them.

Students also learn how to make stem-and-leaf plots and histograms. Stem-and-leaf plots are simple plots that can be used with 15-100 data items. They are not often seen in media because you cannot use them with large amounts of data. Histograms, on the other hand, are very common graphs. They are just like bar graphs, just with the bars next to each other.

I decided to omit the topic of making box-and-whiskers plots (boxplots) though that is listed in some standards and is covered in some math curricula, because interpreting and using them is really beyond the knowledge of 6th graders. They are not common either.

Lastly, we study range as a simple measure of variance. There exist far better measures of variance, such as the interquartile range, standard deviation, and others, but I feel those are also advanced for 6th grade. Statistical measures is a vast area of study, and I feel it is not necessary to introduce to students all kinds of measures (such as interquartile range) if all that students could do with it is to calculate it, and not use it in a meaningful way in interpreting data. Interpreting data using statistical measures (such as mean, median, mode, standard deviation, and others) is a skill that requires more in-depth understanding of statistics than what can be covered here.

The Lessons in Chapter 9

	page	span
Simple Probability	163	*3 pages*
Probability Problems from Statistics	166	*2 pages*
Counting the Possibilities	168	*3 pages*
Compound Probability	171	*4 pages*
More Practice with Probability	175	*2 pages*
Data Analysis	177	*5 pages*
Mean, Median and Mode	182	*3 pages*
Using Mean, Median and Mode	185	*3 pages*
Stem-Lead-Plots	188	*2 pages*
Making Histograms	190	*2 pages*
Range	192	*2 pages*
Probability and Statistics Review	194	*3 pages*

Helpful Resources on the Internet

Mean, Median, Mode, Range, etc.

Using and Handling Data
Simple explanations for finding mean, median, or mode.
http://www.mathsisfun.com/data/index.html#stats

Math Goodies Interactive Statistics Lessons
Clear lessons with examples and interactive quiz questions.
http://www.mathgoodies.com/lessons/vol8/range.html
http://www.mathgoodies.com/lessons/vol8/mean.html
http://www.mathgoodies.com/lessons/vol8/median.html
http://www.mathgoodies.com/lessons/vol8/mode.html

Mean, Median, and Mode
Lesson on how to calculate mean, median, and mode for set of data given in different ways. Also has interactive exercises.
www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, mode and median lessons
Explanations with simple examples.
www.bbc.co.uk/schools/gcsebitesize/maths/data/measuresofaveragerev1.shtml

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Compound Probability

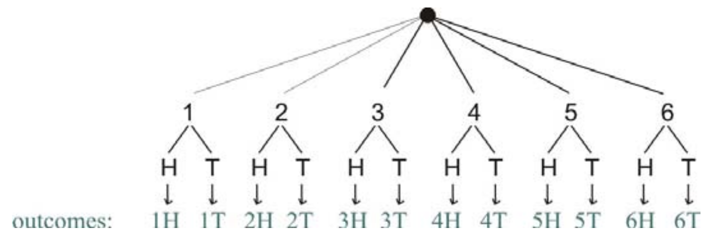
Compound probability means finding the probability where two events both occur. If the outcome of the one event does not affect the outcome of the other, they are said to be **independent.** In that case you can find the probability of two events occurring by multiplying the probabilities of the two events. Examples will make this clear.

Example 1. You toss a coin, and then you roll a die. What is the probability of getting 6 and heads?

P(6) is 1/6, and P(heads) is 1/2. Clearly, whether you get heads or tails on the coin does not affect what you get on the roll. The two events are *independent*. Therefore, we can multiply the two probabilities.

P(6 and heads) = $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

You can also see this probability by looking at the tree diagram, because in only one outcome out of the twelve possible ones do we have 6 and heads.

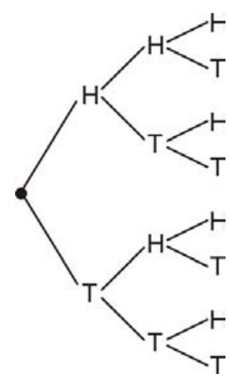


Example 2. You toss a coin three times. What is the probability of getting heads every time?

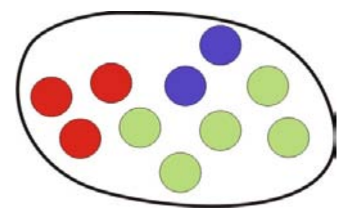
These three events—toss a coin, toss a coin, toss a coin— are independent. Getting heads on one toss doesn't affect whether you get heads or tails on the next.

P(heads) = 1/2. Therefore, P(heads and heads and heads) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

You can also see this from the tree diagram. There is only one outcome with “HHH”, and a total of 8 possible outcomes.



Example 3. The bag has three red marbles, two dark blue marbles, and five light green marbles. You take one marble, and put it back. Then you take a marble again, and put it back. What is the probability of getting first a red marble, then a blue one?



Again, we simply multiply the individual probabilities:

P(red, blue) = $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$.

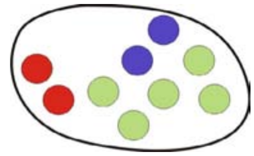
1. You toss a coin three times.

   a. What is the probability of getting tails, then heads, then tails?

   b. What is the probability that you get heads on your second toss?

   c. Use the tree diagram. What is the probability of getting two heads and one tails in three tosses? Note they can be in any order, such as THH or HTH.

2. You take a marble out of the bag and put it back. Then you take another marble. Find the probabilities.



a. P(red, then green)

b. P(green, then red)

c. P(not blue, not blue)

d. P(not red, not red)

3. You roll a six-sided die two times. Find the probabilities.

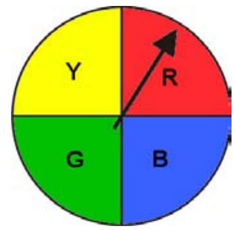
a. P(1; 5)

b. P(even; odd)

c. P(2; 5 or 6)

d. P(6; not 6)

4. The spinner is spun two times. Find the probabilities.



a. P(blue; blue)

b. P(green; not green)

c. P(not blue; yellow)

d. P(yellow or green; red or blue)

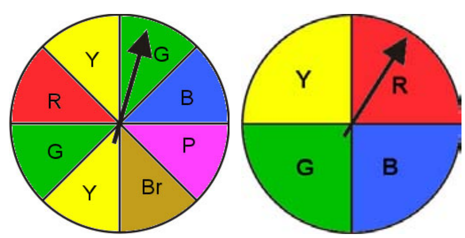
5. The weatherman says that the chance of rain is 20% for each of the next five days, and your birthday is in two days! You also know that the probability of your dad taking you to the amusement park on your birthday is 1/2.

a. What is the probability that you get to go the park, and it doesn't rain?

b. What is the probability that you get to go the park, and it rains?

Check: The sum of the probabilities in (a) and (b) should be 1/2.

6. The two spinners are spun. The first spinner has eight regions and the second spinner has four. Find the probabilities:



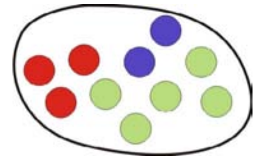
a. P(red, red)

b. P(blue, not blue)

c. P(yellow or green, yellow or green)

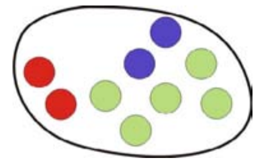
d. P(not red, red)

Taking an object without replacing it



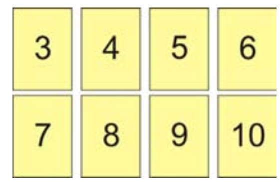
Let's say you choose a marble and don't put it back. Then you choose another marble. This means you have in effect chosen two marbles. What is the probability that both are red?

We first find the probability that the first marble is red. That is simply 3/10 since there are two red marbles and ten in all.



After you get a red marble, and don't put it back, the bag now has one red marble less. So, the probability of getting a red marble now is 2/9.

Thus, the probability of getting two red marbles is $\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} = \frac{1}{15}$.



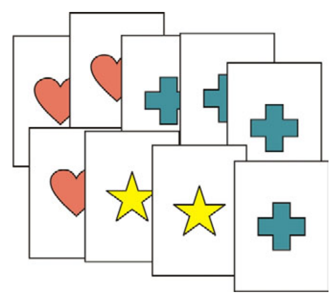
Example. You choose one card, without putting it back. Then you choose another. What is the probability that the first is an even number, and the second is 7?

There are four cards with an even number, and eight total cards. So, P(even) = 4/8 = 1/2.

After one card with an even number has been drawn, there are seven cards left, and one of them has number 7. So, P(7) is 1/7.

P(even, 7) = $\frac{1}{2} \times \frac{1}{7} = \frac{1}{14}$.

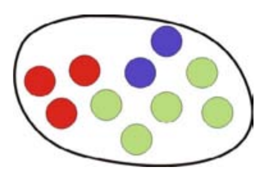
7. You choose a card randomly from this group of cards. Then you choose another card, without replacing the first. Find the probabilities.



a. P(heart, heart) **b.** P(star, cross)

c. P(not heart, not heart) **d.** P(star, not star)

8. You choose two marbles randomly from the bag, without replacement.



a. What is the probability that both are green?

b. What is the probability that the first is green, and the second is red?

c. What is the probability that the first is red, and the second is green?

d. Add the probabilities from (b) and (c) to get the probability that you get one red and one green marble when drawing two marbles.

9. You choose one card, without putting it back. Then you choose another. Find the probabilities. Give your answers as fractions and percents.

3	4	5	6
7	8	9	10

a. P(5; 6)

b. P(not 5; not 5)

c. P(9; even)

d. P(8 or 9; not 10)

10. A 6th grade classroom has 13 boys and 16 girls. The teacher randomly chooses two persons to be responsible for the cleanup after a bake sale. Give these probabilities to the tenth of a percent.

a. What is the probability that both are girls?

b. What is the probability that both are boys?

c. What is the probability that the first person chosen is a girl, and the second is a boy?

d. What is the probability that the first person chosen is a boy, and the second is a girl?

CHECK. The probabilities you get in (a), (b), (c), and (d) should total 100% because they are all the possible outcomes.

e. Add the probabilities in (c) and (d) to get the probability that one of the cleaners is a girl and one is a boy.

11. Michael has 10 white socks and 14 black socks mixed together in a drawer. He chooses one sock to wear randomly, and doesn't put it back. Then he chooses another sock. Find the probabilities:

a. P(white, white)

b. P(black, black)

c. P(black, white)

d. P(white, black)

CHECK. The four probabilities above should total 100%.

e. Add the probabilities in (a) and (b) to find the probability that Michael wears matching socks.

f. What is the probability Michael doesn't wear matching socks?

Puzzle Corner

Matthew has 8 white socks, 9 brown socks, and 10 black socks mixed together in a drawer. He chooses two socks randomly. Find the probability he gets to wear a matching pair. (That would be nice!)

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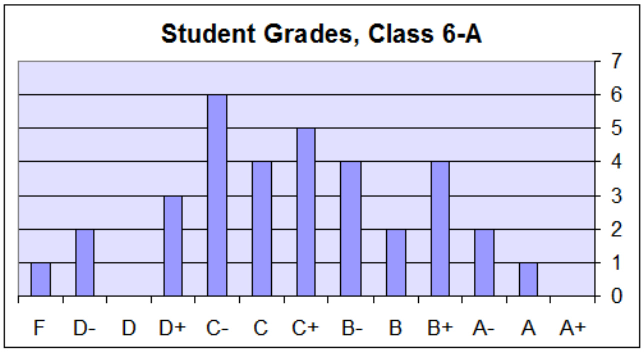
Using Mean, Median, and Mode

Example 1. The two bar graphs show the grades that two classes, 6-A and 6-B, got for science. Which class did better, generally speaking? Can you determine that just from the bar graphs?

You can probably figure out the answer just by looking at the graphs, but we can make sure by finding the *median* of both data sets. (We cannot find the mean because the data isn't numerical.)



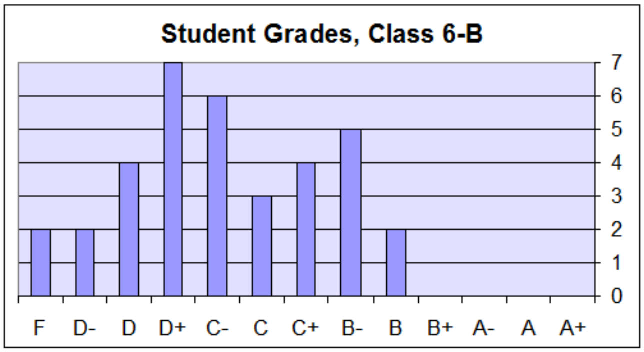
To find the median, we list the students' grades from smallest to the greatest, using the graph.
For class 6-A :

F, D-, D-, D+, D+, D+, C-, C-, C-, C-, C-, C-, C, C, C, C, C+, C+, C+, C+, C+, B-, B-, B-, B-, B, B, B+, B+, B+, B+, A-, A-, A.

Since there are 34 data entries, the exact middle one doesn't exist as such—both the 17th and 18th entries (C+ and C+) are equally "in the middle." In such a case, the median is the average of those two. And while we cannot calculate the average when the data entries are not numbers, clearly the "middle point" of C+ and C+ is C+. So the median is C+.

For class 6-B, we have these 35 grades:

F, F, D-, D-, D, D, D, D, D+, D+, D+, D+, D+, D+, D+, C-, C-, C-, C-, C-, C-, C, C, C, C+, C+, C+, C+, B-, B-, B-, B-, B-, B, B.



This time the middle item is the 18th, or C-.

Since the median for class 6-A is C+ and the median for class 6-B is C-, class 6-A did better on average. This can also be seen in the graphs: the bars in the graph for 6-B are more concentrated towards the left than in the graph for 6-A.

Example 2. Consider this data set: 3, 4, 4, 5, 5, 5, 5, 6, 8. Clearly, the median is 5, the mode is 5,

and the mean is $\dfrac{3+4+4+5+5+5+5+6+8}{9} = 5$.

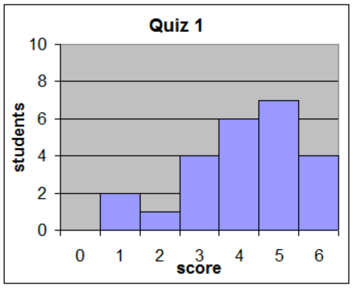
Now, let's say we add ONE more data item to the set (25). Perhaps this is a typing error, but it could also be a true data item, just very different from the others. Since it is very different from the other data points, it is called an **outlier**.

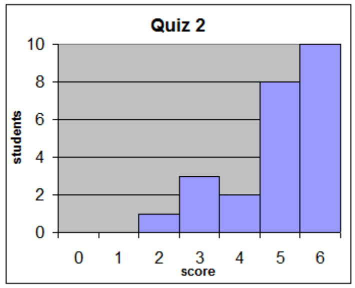
The data set is now 3, 4, 4, 5, 5, 5, 5, 6, 8, 25. How are mean, median, and mode affected by this one additional data item?

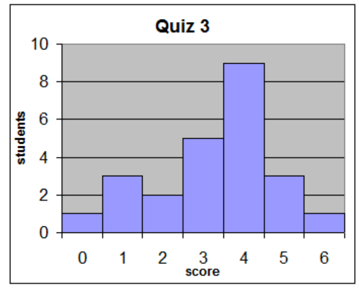
Mode is still 5. Median is still 5. But mean becomes $\dfrac{3+4+4+5+5+5+5+6+8+25}{10} = 7$.

In other words, mean was affected greatly by this outlier, whereas mode and median were not.

1. Mrs. Ross gave her students several quizzes in the calculus class. The graphs for the scores are below.







a. Mrs. Ross felt one of the quizzes turned out too easy (the students didn't!). Which one?

b. The mean scores for the three quizzes were: 3.29, 4.13, and 4.96.
Match each mean with the correct graph.

c. The median scores for the three quizzes were: 5, 4, and 4.
Match each median with the correct graph.

d. In which quiz did the students fare the worst?

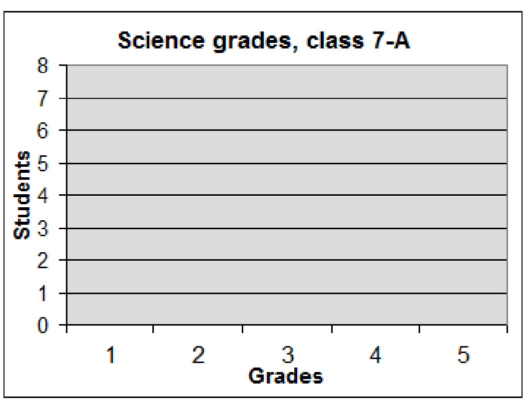
2. The following are the science grades of two 7th grade classes.

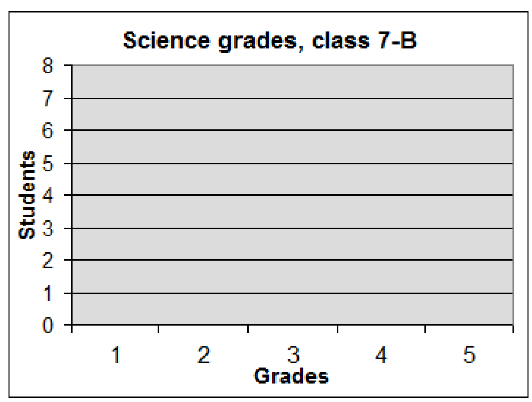
a. Make bar graphs from the data.

b. Find the mean, median, and mode for the grades of class A and class B.

Class 7-A	
Grades	**Students**
1	5
2	8
3	7
4	5
5	2

Class 7-B	
Grades	**Students**
1	3
2	6
3	7
4	7
5	4





Class A: mean _________ median _________ mode _________

Class B: mean _________ median _________ mode _________

c. Determine which class did better. Explain your reasoning.

3. **a.** The following data sets have an outlier. Find the mean, median, and mode for the data sets with and without the outlier.

8, 9, 11, 11, 12, 12, 12, 13, 13, 15, 18, 40	**with the outlier**	**without the outlier**
	mean ________ median ________ mode ________	mean ________ median ________ mode ________
−5, 2, 3, 3, 4, 4, 4, 4, 5, 7	**with the outlier**	**without the outlier**
	mean ________ median ________ mode ________	mean ________ median ________ mode ________

b. Which of the three measures for central tendency is easily affected by an outlier?

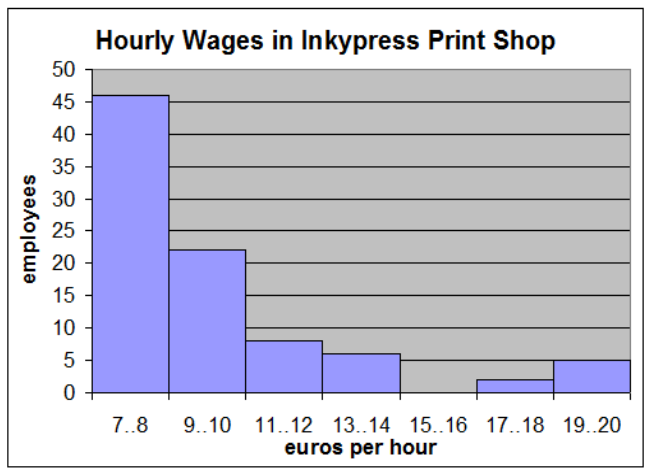
Which are “resistant”, in other words, not easily affected by an outlier?

4. Mean or median? This graph shows the hourly wages of the 89 employees in the Inkypress Print Shop, in euros per hour.

Notice that this graph does not have a “hill” shape, nor any “middle peak”. It's more like one side of a mountain. About half of the people there earn 7-8 euros/hour.

The mean is 9.66 euros/hour and the median is 8 euros/hour.

Is mean or median better in describing the majority's wages in this print shop?



5. Judith surveyed 55 teenagers about how much money they spent to purchase mother's day gifts. Her results are shown in the histogram.

a. Which of the numbers $11 and $9 is the mean? Which is the median?

b. Would mean or median better describe this data?

c. About how many percent of the teenagers spent less than $10 on a mother's day gift?