MATH MAMMOTH

Grade 4-A

Complete Worktext

- Addition, subtraction, graphs, and money
- Big numbers & place value
- Multi-digit multipication
- Time and measuring

By Maria Miller

www.MathMammoth.com

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Foreword

Math Mammoth Grade 4-A and Grade 4-B worktexts comprise a complete math curriculum for the fourth grade mathematics studies.

In the fourth grade, students focus on multidigit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and get started in studying fractions and decimals. This is of course accompanied by studies in geometry and measuring.

The year starts out with review of addition and subtraction, graphs, and money. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations, and statistical graphs.

Next come large numbers -- up to millions, and the place value concept. At first the student reviews thousands and some mental math with them. Next are presented numbers till one million, calculations with them, place value concept and comparing. In the end of the chapter we find more about millions and an introduction to multiples of 10, 100, and 1000.

The third chapter is all about multiplication. After briefly reviewing the concept and the times tables, the focus is on learning multidigit multiplication (multiplication algorithm). The children also learn why it works when they work on multiplying in parts. We also study the order of operations again, touch on proportional reasoning, and do more money and change related word problems.

The last chapter in part A is about time, temperature, length, weight, and volume. Students will learn to solve more complex problems using various measuring units and to convert between measuring units.

In part B, we first study division. The focus is on learning long division and using division in word problems. The geometry chapter introduces students to measuring angles, and we do lots of drawing of different shapes and circles. Area and perimeter are other important topics in geometry.

Fractions and decimals are presented last in the school year. These two chapters practice only some of the basic operations with fractions and decimals. The focus is still on the conceptual understanding, building a good foundation towards 5th grade math, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a “framework”, but you do have some liberty in organizing the study schedule. Chapters 1, 2, and 3 should be studied in this order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry) and schedule them somewhat earlier or later if you so wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year; however finding parts with division should naturally be studied only after mastering division.

This product also includes an HTML page that you can use to make extra practice worksheets for computation.

I wish you success in your math teaching!

Maria Miller, the author

Concerning Challenging Word Problems

I would heartily recommend supplementing this program with regular practice of challenging word problems and puzzles. You could do that once a week to once every two weeks. The goal of challenging story problems and puzzles is to simply **develop children's logical and abstract thinking and mental discipline**. Fourth grade is a good place to start such a practice because students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging people from doing such in earlier grades, either.

I have made lots of word problems for the Math Mammoth curriculum. Those are for the most part multi-step word problems. I have included several lessons that utilize the bar model for solving problems and tried to vary the problems.

Even so, the problems I've created are usually tied to a specific concept or concepts. I feel children can also benefit from problem solving practice where the problems require "out of the box" thinking, or are puzzle-type in nature, or are just different from the ones I have made. Additionally, I feel others are more capable of making very different, very challenging problems.

So I'd like for you to use one or several of the resources below for some different problems and puzzles. Choose something that fits your budget (most of these are free) and that you will like using.

Math Kangaroo Problem Database
Easily made worksheets of challenging math problems based on actual past Math Kangaroo competition problems.
http://www.kangurusa.com/clark/pdb/

Primary Grade Challenge Math by Edward Zaccaro
The book is organized into chapters, with each chapter presenting a type of problem and the ways to think about that problem. And then there is a series of related story problems to solve, divided into 4 levels.
$25, **ISBN** 978-0967991535
You can find this at Amazon.com or various other bookstores.
http://www.amazon.com/Primary-Grade-Challenge-Edward-Zaccaro/dp/0967991536/

Problem Solving Decks from North Carolina public schools
Includes a deck of problem cards for grades 1-8, student sheets, and solutions. Many of these problems are best solved with calculators. All of these problems lend themselves to students telling and writing about their thinking.
http://community.learnnc.org/dpi/math/archives/2005/06/problem_solving.php

Math Stars Problem Solving Newsletter (grades 1-8)
These newsletters are a fantastic, printable resource for problems to solve and their solutions.
http://community.learnnc.org/dpi/math/archives/2005/06/math_stars_news.php

Mathematics Enrichment - nrich.maths.org
Open-ended, investigative math challenges for all levels from the UK. Find the past issues box down in the left sidebar. Use Stage 2, 1-star or 2-star problems for 4th grade.
http://nrich.maths.org/public/
http://nrich.maths.org/public/themes.php lets you find problems organized by mathematical themes.

Figure This! Math Challenges for Families
Word problems related to real life. They don't always have all the information but you have to estimate and think. For each problem, there is a hint, other related problems, and interesting trivia. Website supported by National Council of Teachers of Mathematics.
http://www.figurethis.org/

MathStories.com
Over 12,000 interactive and non-interactive NCTM compliant math word problems, available in both English and Spanish. Helps elementary and middle school children boost their math problem solving and critical-thinking skills. A membership site.
http://www.mathstories.com/

"Problem of the Week" (POWs)
Problem of the week contests are excellent for finding challenging problems and for motivation. There exist several:

- **Math Forum: Problem of the Week**
  Five weekly problem projects for various levels of math. Mentoring available.
  http://mathforum.org/pow/
- **Math Contest at Columbus State University**
  Elementary, middle, algebra, and "general" levels.
  http://www.colstate.edu/mathcontest/
- **Aunty Math**
  Math challenges in a form of short stories for K-5 learners posted bi-weekly. Parent/Teacher Tips for the current challenge explains what kind of reasoning the problem requires and how to possibly help children solve it.
  http://www.auntymath.com/
- **Grace Church School's ABACUS International Math Challenge**
  This is open to any child in three different age groups.
  http://www.gcschool.org/pages/program/Abacus.html
- **MathCounts Problem of the Week Archive**
  Browse the archives to find problems to solve. You can find the link to the current problem on the home page.
  http://mathcounts.org/Page.aspx?pid=355
- **Math League's Homeschool Contests**
  Challenge your children with the same interesting math contests used in schools. Contests for grades 4, 5, 6, 7, 8, Algebra Course 1, and High School are available in a non-competitive format for the homeschoolers. The goal is to encourage student interest and confidence in mathematics through solving worthwhile problems and build important critical thinking skills. By subscription only.
  http://www.mathleague.com/homeschool.htm

Chapter 1: Addition, Subtraction, Graphs and Money

Introduction

The first chapter of *Math Mammoth Grade 4-A Complete Worktext* covers addition and subtraction topics, word problems, graphs, and money problems.

At first, we review the "technical aspects" of adding and subtracting: mental math techniques plus adding and subtracting in columns. If these are fairly easy for your student(s), you can choose to skip some problems.

Going beyond those, the chapter includes lessons in addition and subtraction terminology. These lessons are already preparing your child for algebraic thinking.

In the next lessons, the student reviews the addition/subtraction connection, and solves word problems with the help of bar models. Next, we solve simple missing addend equations using subtraction, such as $x + 20 = 60$. We use bar models to illustrate these and connect them with fact families.

The lesson on the order of operations contains some review but it goes beyond that. In many of the problems, the student builds the mathematical expression (calculation) needed for a certain real-life situation.

Going towards applications of math, the chapter contains lessons on bar graphs, line graphs, rounding, estimating, and money problems.

The Lessons in Chapter 1

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Estimate Addition Quiz
Scroll down the page to find this quiz plus some others. Fast loading.
http://www.quiz-tree.com/Math_Practice_main.html

Mental Addition and Subtraction
A factsheet, quiz, game, and worksheet about basic mental addition and subtraction.
http://www.bbc.co.uk/skillswise/numbers/wholenumbers/addsubtract/mental/

Shop 'Til You Drop
Get as many items as you can and be left with the least amount of change, and practices your addition skills. The prices are in English pounds and pennies.
http://www.channel4.com/learning/microsites/P/puzzlemaths/shop.shtml

Change Maker
Determine how many of each denomination you need to make the exact change. Good and clear pictures! Playable in US, Canadian, Mexican, UK, or Australian money.
http://www.funbrain.com/cashreg/index.html

Cash Out
Give correct change by clicking on the bills and coins.
http://www.mrnussbaum.com/cashd.htm

Piggy bank
When coins fall from the top of the screen, choose those that add up to the given amount, and the piggy bank fills.
http://fen.com/studentactivities/Piggybank/piggybank.html

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool.
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

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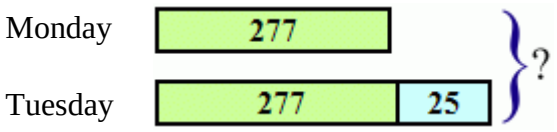
Word Problems and Bar Models

Bar models help you see how the numbers in a problem relate to each other. Whenever you get stumped by a word problem, try drawing a bar model.

On Monday, Dad drove 277 miles, and on Tuesday he drove 25 miles more than he did on Monday. How many miles did he drive in the two days?

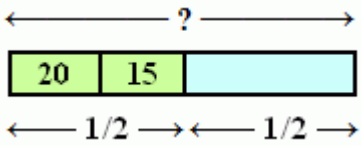
On Tuesday he drove 277 + 25 = 302 miles.
Altogether he drove 277 + 302 = 579 miles.



The bracket "}" means addition or the total of the two bars. We do not know the total or the sum of the two days' journey, so it is marked with a question mark.

After driving 20 miles, Dad says, "I still have 15 more miles to go to the half-way point." How long is the trip?

20 mi + 15 mi = 35 miles, and that is the first half of the trip. So, the total trip is 2 × 35 = 70 miles.



We do not know the total length, so it is marked with "?".

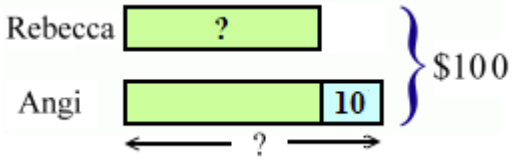
Mark the numbers given in the problem in the diagram. Mark what is asked with "?". Then solve the problem.

1. Jake worked for 56 days on a farm, and Ed worked for 14 days less. How many days did Ed work?

2. Of his paycheck, Dad paid $250 on taxes, and spent $660 on other bills and purchases. Then, half of his paycheck was gone. How much was his paycheck?

3. Dad bought two hammers. One cost $18 and the other cost $28 more. What was his total bill?

Angi and Rebecca split a $100 paycheck so that Angi got $10 more than Rebecca.
How much did each one get?



The bar diagram shows the situation. Angi got $10 more than Rebecca, and together they earned $100.

To solve it, you can think this way. If you took away (subtracted) the "additional" $10, then the total would be $90, and we would only have the two equal parts (the two green parts). So, $90 ÷ 2 = $45 gives us the amount Rebecca got, and then Angi got $45 + $10 = $55.

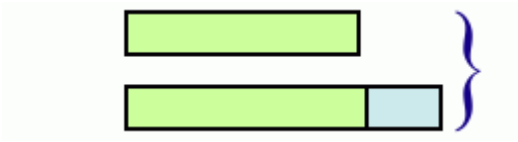
Here's another way of looking at the same situation.

We draw just one bar for the paycheck, and divide it into two halves in the middle (the dashed line). Then we draw half of the $10, or $5, on either side of that middle line.

We can then see Angi got $50 + $5 = $55 and Rebecca got $50 − $5 = $45.

Mark the numbers given in the problem in the diagram. Mark what is asked with "?".
Then solve the problem.

4. Mary and Luisa bought a $46 gift together.
Mary spent $6 more on it than Luisa.
How many dollars did each spend?



5. Henry bought two circular saws. One saw was $100 cheaper than the other.
His total bill was $590. What did each saw cost?

6. Eric and Angela did yard work together. They earned $80 and split it so that Eric got $12 more than Angela. How much did each one get?
Draw a bar diagram.

You can solve the rest of the problems any way you like best.

7. Mark bought four towels for $7 each, and a blanket for $17. He paid, and the clerk handed him back $5. What denomination was the bill Mark used to pay?

8. One plain yogurt costs $2.40, strawberry yogurt costs $0.15 less than plain yogurt, and plum yogurt costs $0.30 more than plain yogurt. What is your total bill if you buy all three?

9. Erica was 132 cm tall when she was 9 years old. In the next year, she grew 6 cm, and the next year 2 cm less than the previous year. How tall was she at the age of 11?

10. John's monthly phone service bill is $48. John said that with the money he earned on his summer job, he could pay his phone service for two months, spend $120 for a bike, and still have half his money left. How much did he earn?

11. Melissa found a nice shirt for $11.50, another for $2.55 less, and yet another for $2 less. If she buys all three, what will her total bill be?

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Chapter 2: Place Value
Introduction

The second chapter of *Math Mammoth Grade 4-A Complete Worktext* covers large numbers (up to 9 digits) and place value concepts with those.

The first lessons only deal with thousands or numbers with a maximum of four digits. These are for review and for deepening the student's understanding of place value. It is crucial that the student understands place value with these numbers before moving on to larger numbers. Yet again, these larger numbers can be very easy as long as the student understands the basics of how our place value system works.

Besides the concept of place value, the chapter contains lessons on comparing numbers, adding and subtracting in columns, mental math problems, and the idea of multiples.

The Lessons in Chapter 2

Helpful Resources on the Internet

Place Value Payoff
Match numbers written in standard form with numbers written in expanded form in this game.
http://www.quia.com/mc/279741.html

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Keep My Place
Fill in the big numbers to this cross-number puzzle.
http://www.mathsyear2000.org/magnet/kaleidoscope2/Crossnumber/index.html

Place value puzzler
Place value or rounding game. Click on the asked place value in a number, or type in the rounded version of the number.
http://www.funbrain.com/tens/index.html

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Can you say really big numbers?
Enter a really big number, try say it out loud, and see it written.
http://www.mathcats.com/explore/reallybignumbers.htm

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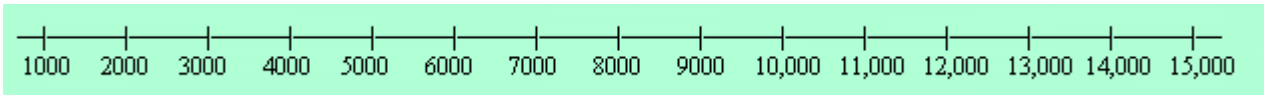
More Thousands



On this number line you see whole thousands from one thousand till fifteen thousand.

The colored digits are the "thousands period" and count as the whole thousands. Read the numbers is as if you say the word "thousand" for the comma.

We continue with whole thousands until reaching *a thousand* thousands.

That number has a new name: *one million.*

78,000	*Read:* 78 thousand
153,000	*Read:* 153 thousand
802,000	*Read:* 802 thousand
990,000	*Read:* 990 thousand
999,000	*Read:* 999 thousand
1,000,000	Thousand thousand = 1 million

The rest of the digits tell us our hundreds, tens, and ones just like you have learned.

17,544	*Read:* 17 thousand 544
609,230	*Read:* 609 thousand two hundred thirty
70,080	*Read:* seventy thousand eighty
902,005	*Read:* 902 thousand five

1. Place a comma into the number. Fill in missing parts.

a. **164000**	b. **92000**	c. **309000**	d. **34000**	e. **780000**
____ thousand	____ thousand	____ thousand	____ thousand	____ thousand

2. Place a comma into the number. Fill in missing parts. Read numbers aloud.

a. **164,453**	b. **92908**	c. **329033**	d. **14004**
164 thousand 453	____ thousand ____	____ thousand ____	____ thousand ____
e. 550053	**f. 72001**	**g. 800004**	**h. 30036**
____ thousand ____	____ thousand ____	____ thousand ____	____ thousand ____

3. Read these numbers aloud.

a. 456,098 **b.** 950,050 **c.** 23,090 **d.** 560,008

e. 78,304 **f.** 266,894 **g.** 219,513 **h.** 306,700

4. Think in whole thousands and add!

a. 30,000 + 5,000 =
think: 30 thousand + 5 thousand

b. 200,000 + 1,000 =

c. 400,000 + 30,000 =

d. 710,000 + 40,000 =

e. 300,000 + 600,000 =

f. 700,000 + 70,000 =

5. Add and subtract, thinking in whole thousands.

a. 35,000 + 5,000 =

b. 210,000 + 10,000 =

c. 420,000 + 30,000 =

d. 711,000 + 10,000 =

e. 300,000 – 60,000 =

f. 700,000 – 70,000 =

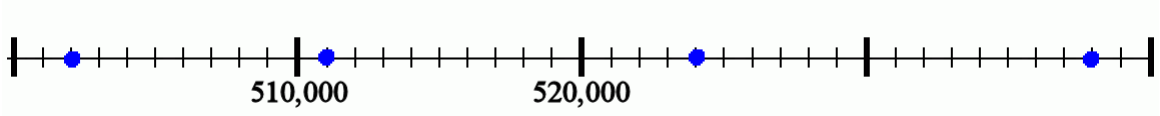
g. 30,000 – 5,000 =

h. 200,000 – 6,000 =

i. 723,000 – 400,000 =

j. 500,000 – 1,000 =

6. On the number line below, 510,000 and 520,000 are marked (at the "posts").
Write the numbers that correspond to the dots.



7. Make a number line from 320,000 to 340,000 with tick-marks at every whole thousand, similar to the one above. Then mark the following numbers on the number line:
323,000 328,000 335,000 329,000 330,000

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Chapter 3: Multiplication
Introduction

The third chapter of *Math Mammoth Grade 4-A Complete Worktext* covers multi-digit multiplication and some related topics.

While the first lessons briefly review the multiplication concept and the times tables, the focus in fourth grade is on multi-digit multiplication (also called algorithm of multiplication, or multiplying in columns).

We start out by multiplying by whole tens and hundreds. After this is mastered, comes the very important concept of **multiplying in parts**. This essentially means that 4×63 is done in two parts: 4×60 and 4×3, and the results are added.

The whole algorithm of multiplication is based on this principle, so it is important to master it. I don't want kids to multiply in columns "blindly", without understanding what is going on with that algorithm.

Before showing the traditional form of multiplying in columns, the lesson *Multiply in Columns - the Easy Way* shows a simplified form of the same, which is essentially just multiplying in parts. You may skip that lesson at your discretion or skim through it quickly if your child is ready to understand the standard form of the algorithm, which comes next.

Other lessons in this chapter practice estimation and the order of operations, and multiplying with money. Many kinds of word problems abound.

The lesson "*So Many of the Same Thing*" could be entitled "Proportional Reasoning" but I wanted to avoid scaring parents and children with such a high-sounding phrase. The idea in that lesson is really simple, but it does prepare for proportions as they are taught in 7th grade and in algebra.

After that, we multiply by whole hundreds in order to prepare for double-digit multiplier problems, and to understand the algorithm of multiplication with more digits.

The Lessons in Chapter 3

Helpful Resources on the Internet

Math Playground
Learn how to think algebraically with these clever weighing scales.
http://www.mathplayground.com/algebraic_reasoning.html

Thinking Blocks
Thinking Blocks is an engaging, interactive math tool that helps students learn how to solve multistep word problems. Scroll down to Multiplication and Division.
http://www.mathplayground.com/thinkingblocks.html

Rectangle Multiplication
An interactive tool that illustrates multiplying in parts using the area model. Choose the "common" option for multiplying in parts.
http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html

Interactive Pan Balance
Each of the four shapes is assigned a certain weight. Place shapes on either side of the pan balance and figure out their relationships.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=131

Scales Problems from Math Kangaroo Problem Database
http://www.kangurusa.com/clark/pdb/quiz.pl?
dir=./kangur/output&y1=2002&l1=0304&i1=10&y2=2004&l2=0304&i2=10&y3=2005&l3=02&i3=19&r

Multiplication Games
A list of times tables games and activities to practice multiplication facts.
http://www.homeschoolmath.net/math_resources_2.php#multiplication

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Multiplying by Whole Tens and Hundreds

1. **a.** Ten tens make a hundred. How about 20 tens or more?	**b.** Ten hundreds make a thousand. How about 20 hundreds or more?
10 tens = 10 × 10 = ____	10 hundreds = 10 × 100 = ____
13 tens = 13 × 10 = ____	12 hundreds = 12 × 100 = ____
20 tens = 20 × 10 = ____	15 hundreds = 15 × 100 = ____
21 tens = 21 × 10 = ____	18 hundreds = 18 × 100 = ____
37 tens = 37 × 10 = ____	20 hundreds = 20 × 100 = ____

56 × 10 is the same as 10 × 56. Both are 560.

92 × 100 is the same as 100 × 92. Both are 9,200.

To multiply a number by 10, just tag a zero in the end.
To multiply a number by 100, just tag two zeros in the end.

1**0** × 56 = 56**0**	1**00** × 47 = 47**00**
1**0** × 481 = 4,81**0**	1**00** × 2,043 = 204,3**00**

Note especially what happens when the number you multiply already ends in a zero. The rule works the same; you still have to tag a zero or two zeros.

1**0** × 60 = 60**0**	1**00** × 20 = 2,0**00**
1**0** × 500 = 5,00**0**	1**00** × 3,400 = 340,0**00**

2. Multiply.

a. 10 × 315 = ____	**b.** 100 × 62 = ____	**c.** 10 × 25,000 = ____
3,560 × 10 = ____	10 × 1,200 = ____	100 × 25,000 = ____
35 × 100 = ____	100 × 130 = ____	10 × 5,060 = ____

What is 20 × 14?

Imagine the problem without the zero. Then it becomes 2 × 14 = 28. Then, just tag a zero to the end result: 20 × 14 = 280.

Why does that work? It is based on the fact that 20 = 10 × 2. For example,

20 × 14 = 10 × 2 × 14

In that problem, first multiply 2 × 14 = 28. Then multiply by ten:

10 × (2 × 14) = 10 × 28 = 280.

What is 200 × 31?

Imagine the problem without the zeros. Then it becomes 2 × 31 = 62. Then, just tag *two* zeros to the result: 200 × 31 = 6,200.

Why does that work? It is based on the fact that 200 = 100 × 2. For example,

200 × 31 = 100 × 2 × 31

In that problem, you can multiply first 2 × 31 = 62. Then multiply by a hundred:

100 × (2 × 31) = 100 × 62 = 6,200.

3. Multiply by 20 and 200.

a. 20 × 8 = ____	**b.** 200 × 7 = ____	**c.** 20 × 12 = ____	**d.** 20 × 16 = ____
4 × 20 = ____	5 × 200 = ____	35 × 20 = ____	42 × 200 = ____
20 × 5 = ____	11 × 200 = ____	200 × 9 = ____	54 × 20 = ____

The same principle works if you multiply by 30, 40, 50, 60, 70, 80, or 90. You can imagine multiplying by 3, 4, 5, 6, 7, 8, or 9, and then tag a zero into the end result.

Similarly, if you multiply by some whole hundred, imagine multiplying without those two zeros, and tag the two zeros to the end result.

50 × 8 = 400	**90 × 11 = 990**	**300 × 8 = 2,400**	**12 × 800 = 9,600**

4. Multiply.

a. 40 × 3 = ____	**b.** 70 × 6 = ____	**c.** 80 × 9 = ____	**d.** 60 × 11 = ____
8 × 20 = ____	50 × 11 = ____	30 × 15 = ____	12 × 40 = ____
e. 200 × 9 = ____	**f.** 700 × 6 = ____	**g.** 200 × 12 = ____	**h.** 3 × 1100 = ____
7 × 400 = ____	600 × 11 = ____	15 × 300 = ____	8 × 900 = ____

It even works this way:

In a problem 40 × 70 you can just multiply

4 × 7, and tag two zeros to the result:

$$4\mathbf{0} \times 7\mathbf{0} = 2{,}8\mathbf{00}$$

In a problem 600 × 40 you can multiply 6 × 4, and tag three zeros to the result:

$$6\mathbf{00} \times 4\mathbf{0} = 24{,}\mathbf{000}$$

In a problem 700 × 800 you can multiply 7 × 8, and tag four zeros to the result.

$$7\mathbf{00} \times 8\mathbf{00} = 56\mathbf{0}{,}\mathbf{000}$$

5. Multiply.

a. 20 × 90 =

70 × 300 =

b. 60 × 80 =

30 × 900 =

c. 400 × 50 =

200 × 200 =

d. 80 × 800 =

200 × 500 =

e. 100 × 100 =

40 × 30 =

f. 800 × 300 =

90 × 1100 =

6. Write different factors for these products, using whole tens and whole hundreds.

Have you noticed? **7 × 80 = 560 and** **70 × 8 = 560 !!**	**a.** 6 × ___ = 420 and 60 × ___ = 420	**b.** ___ × ___ = 350 and ___ × ___ = 350
c. ___ × ___ = 280 and ___ × ___ = 280	**d.** ___ × ___ = 400 and ___ × ___ = 400	**e.** ___ × ___ = 990 and ___ × ___ = 990
Have you noticed? **6 × 400 = 2,400 and** **60 × 40 = 2,400 and** **600 × 4 = 2,400 !!**	**f.** 2 × ___ = 1,800 and 20 × ___ = 1,800 and 200 × ___ = 1,800	**g.** ___ × ____ = 5,400 and ___ × ____ = 5,400 and ____ × ___ = 5,400
h. ___ × ____ = 3,000 and ___ × ____ = 3,000 and ____ × ___ = 3,000	**i.** ___ × ____ = 3,600 and ___ × ____ = 3,600 and ____ × ___ = 3,600	**j.** ___ × ____ = 3,600 and ___ × ____ = 3,600 and ____ × ___ = 3,600

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Multiplying in Columns, Standard Way

Here we learn the standard algorithm of multiplication. It is based on the same principle of multiplying in parts: you simply multiply ones and tens separately, and add. In the standard algorithm the *adding* is done at the same time as multiplying. The calculation looks more compact and takes less space.

$$\begin{array}{r} {\scriptstyle 1} \\ 63 \\ \times\ \ 4 \\ \hline 2 \end{array}$$

Multiply the ones first.

$4 \times 3 = 12$

Place 2 under the line at the ones place, but the tens digit (1) is written above the tens column as a little memory note. This is called *carrying to tens*.

$$\begin{array}{r} {\scriptstyle 1} \\ 63 \\ \times\ \ 4 \\ \hline 252 \end{array}$$

Then multiply the tens, and *add* the 1 ten that was carried over.

$4 \times 6 \ + \ 1 = 25$

There is a total of 25 tens, which actually signifies 250. Write the 25 in front of the ones digit (2).

(In the calculation $4 \times 6 \ + \ 1 = 25$, the 6 and the 1 are actually tens. So in reality we calculate $4 \times 60 \ + \ 10 = 250$.)

$$\begin{array}{r} 63 \\ \times\ \ 4 \\ \hline 12 \\ +\ 240 \\ \hline 252 \end{array}$$

Compare to the method of "multiplying in parts" that you learned previously, where the adding is done separately.

Look at other examples. In each case, some tens are carried as a result of multiplying the ones.

$$\begin{array}{r} {\scriptstyle 2} \\ 27 \\ \times\ \ 4 \\ \hline 8 \end{array}$$

$4 \times 7 = 28$

$$\begin{array}{r} {\scriptstyle 2} \\ 27 \\ \times\ \ 4 \\ \hline 108 \end{array}$$

$4 \times 2 + 2 = 10$

$$\begin{array}{r} {\scriptstyle 6} \\ 69 \\ \times\ \ 7 \\ \hline 3 \end{array}$$

$7 \times 9 = 63$

$$\begin{array}{r} {\scriptstyle 6} \\ 69 \\ \times\ \ 7 \\ \hline 483 \end{array}$$

$7 \times 6 + 6 = 48$

$$\begin{array}{r} {\scriptstyle 2} \\ 54 \\ \times\ \ 6 \\ \hline 4 \end{array}$$

$6 \times 4 = 24$

$$\begin{array}{r} {\scriptstyle 2} \\ 54 \\ \times\ \ 6 \\ \hline 324 \end{array}$$

$5 \times 6 + 2 = 32$

$$\begin{array}{r} 83 \\ \times\ \ 9 \\ \hline \end{array}$$

$9 \times 3 =$

$$\begin{array}{r} 83 \\ \times\ \ 9 \\ \hline \end{array}$$

$9 \times 8 + 2 =$

$$\begin{array}{r} {\scriptstyle 4} \\ 77 \\ \times\ \ 7 \\ \hline \end{array}$$

$7 \times 7 =$

$$\begin{array}{r} {\scriptstyle 4} \\ 77 \\ \times\ \ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ \times\ \ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 38 \\ \times\ \ 5 \\ \hline \end{array}$$

Compare the earlier method with the one in this lesson:

```
  75
 ×  8
 ----
  40
+560
 ----
 600
```

OR

```
   4
  75
 ×  8
 ----
   0
```

5 × 8 = 40,
4 is carried.

```
   4
  75
 ×  8
 ----
 600
```

7 × 8 + 4 =
56 + 4 = 60

You can choose which one you use. Discuss it with your teacher.

1. Multiply. Be careful with the carrying.

a. 53 × 8

b. 51 × 6

c. 88 × 3

d. 79 × 3

e. 62 × 2

f. 46 × 7

g. 18 × 5

h. 19 × 3

i. 66 × 6

j. 39 × 9

k. 87 × 3

l. 67 × 2

m. 20 × 9

n. 54 × 8

o. 34 × 6

p. 46 × 2

q. 17 × 9

r. 43 × 4

s. 76 × 8

t. 35 × 9

With a 3-digit number you might have to carry twice, to tens and to hundreds.

Multiply the ones first.

$4 \times 8 = 32$

Place 2 under the line and carry the tens digit (3) to the tens' column.

Then multiply the tens' digit, and add the 3 tens that were carried over.

$4 \times 3 + 3 = 15$

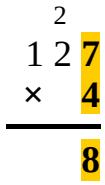
Place the 5 in the tens' place and carry the 1 into the hundreds' column.

Then multiply the hundreds digit, and add the 1 hundred that was carried over.

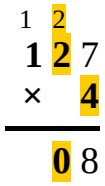
$4 \times 2 + 1 = 9$

Place the 9 in the hundreds' place.

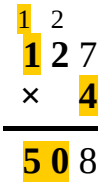
Look at other examples. Compare to the earlier method of multiplying in parts.



$4 \times 7 = 28$

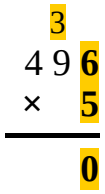


$4 \times 2 + 2 = 10$



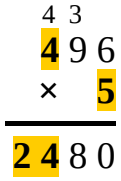
$4 \times 1 + 1 = 5$

```
  127
×   4
-----
   28
   80
+ 400
-----
  508
```



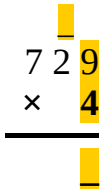
$5 \times 6 = 30$

$5 \times 9 + 3 = 48$

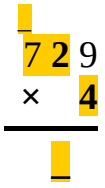


$5 \times 4 + 4 = 24$

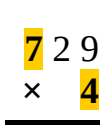
```
  496
×   5
-----
   30
  450
+2000
-----
 2480
```



multiply ones



multiply tens and add what was carried



multiply hundreds and add what was carried

```
  729
×   4
-----
   36
   80
+2800
-----
```

2. Multiply.

a.

	1	2	3
x			8

b.

	1	5	1
x			6

c.

	2	8	8
x			3

d.

	2	7	9
x			3

e.

	4	6	2
x			2

f.

	5	0	6
x			7

g.

	2	7	8
x			5

h.

	3	1	9
x			7

i.

	1	5	6
x			6

j.

	8	0	9
x			9

k.

	2	8	7
x			3

l.

	3	6	7
x			2

m.

	1	2	0
x			8

n.

	2	5	4
x			2

o.

	3	3	4
x			2

p.

	1	5	7
x			7

3. Solve the word problems. Write a number sentence for each one.

a. The school has 304 students. To go to the museum, they hired buses which can each seat 43 passengers. How many buses did they need?

b. The school also has 24 teachers. How many seats were left empty when all of the students and all of the teachers joined the trip?

c. Each package of paper contains 250 sheets. Marie needed 1300 sheets. How many packages did she need to buy?

d. Mick earned $345 from strawberry picking, and Jeanine earned three times as much. How much did they earn in all?

e. Emily solved 17 crossword puzzles, and Elaine solved three times as many. How many more did Elaine solve than Emily?

Puzzle Corner Find the missing numbers in these multiplications:

□ 1 □	1 □ 4	□ 3 □	3 □ 9
× 4	× □	× 7	× 3
4 6 8	8 7 0	9 □ 6	□ 5 □

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Chapter 4: Time and Measuring
Introduction

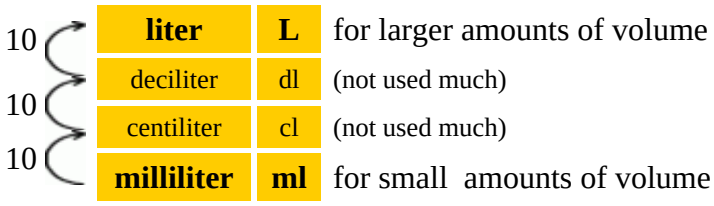
The fourth chapter of *Math Mammoth Grade 4-A Complete Worktext* includes time, temperature, length, weight, and volume related lessons.

The focus on fourth grade is no longer the actual act of measuring, but calculations that involve conversions between different measuring units.

In time lessons, the student gets to do fairly complex calculations concerning hours and minutes. In temperature, the student is introduced to negative numbers and gets to do a few simple calculations even.

The lessons concerning measuring units usually include a table that lists the units and the conversion factors.

For metric units, those tables always include all of the units, even when they are not in common usage. For example, when studying metric units of volume, the chart looks like this:



Only milliliters and liters are dealt with in the lesson. But the chart shows the two other units as well in order to get the student used to two basic ideas of metric measuring units:

1. How the units always differ by a factor of ten,
2. How the units are *named* consistently, with always the same prefixes such as milli-, centi-, deci-, deca-, hecto-, and kilo-. These prefixes and their meanings are NOT yet studied in detail in fourth grade; but I wanted to include the charts to familiarize the students with the terms and the ideas. You may, of course, at your discretion, explain it all to the student.

The Lessons in Chapter 4

Helpful Resources on the Internet

Calculating Time from BBC SkillsWise
Fact sheets, worksheets, and an online game to practice time calculations.
http://www.bbc.co.uk/skillswise/numbers/measuring/time/calculatingtime/

A Dictionary of Units of Measurement
Explains the common measuring systems and has lots of background info of their history.
http://www.unc.edu/~rowlett/units/

Measure It!
Practice measuring lines with either centimeters or inches. Multiple choice questions.
http://onlineintervention.funbrain.com/measure/index.html

Measures
Activities, revision bites, and quizzes about measuring time, weight, and capacity (in metric units).
http://www.bbc.co.uk/schools/ks2bitesize/maths/shape_space_measures.shtml

Measurements
Online lessons with interactive exercises on metric prefixes, symbols, number values, metric mass, length, volume, US length and volume, and temperature conversions.
http://www.aaamath.com/B/mea.htm

Units of Measurement Quizzes
Quizzes for area, distance, volume, and mass - both metric and English systems.
http://www.quiz-tree.com/Units_of_Measurement_main.html

Metric Measurement Matching Game
Match metric terms and prefixes with the correct match
http://www.quia.com/mc/4177.html

Reading a tape measure worksheets
Worksheet generator - you can choose to which accuracy to measure, inches, or inches & feet.
http://themathworksheetsite.com/read_tape.html

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Metric Units for Measuring Length

The basic unit in the metric system is the meter. All of the other metric units for measuring length have the word "meter" in them.

The conversion factors in the metric system are based on 10. That is why you will use either 10, 100, or 1,000 when changing one metric unit of length to another.

10 millimeters makes 1 centimeter.
10 centimeters makes 1 decimeter.
10 decimeters makes 1 meter. And so on.

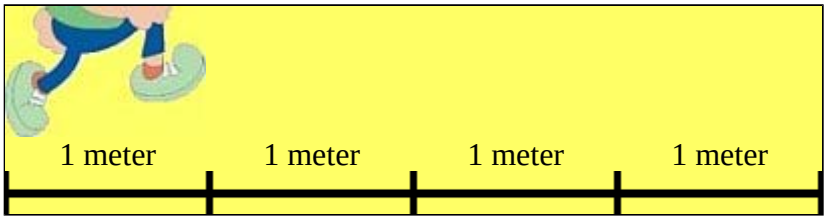
Units of length in the metric system

	kilometer	**km**	1,000 meters
10	hectometer	hm	(not used)
10	decameter	dam	(not used)
10	**meter**	**m**	the basic unit
10	decimeter	dm	(not used much)
10	**centimeter**	**cm**	look at your ruler!
10	**millimeter**	**mm**	look at your ruler!

Remember that 1 meter is very close to 1 yard. 1 meter is a tiny bit longer than 1 yard.

1. Outside, or in a long corridor or room, draw two lines that start at the same place.

a. Using a measuring tape, mark on the one line 1 m, 2 m, 3 m, and 4 m. Can you take "hops" 1 meter long?



b. Mark on the second line marks from 1 foot to 13 feet.
Make 1-yard hops.
Compare: do the two kinds of hops feel about the same?

2. Measure how tall you and other people are in centimeters. Write it also using whole meters and centimeters.

Name	How tall
	_____ cm = _1_ m ____ cm.

Conversions between units

Remember what millimeters look like on your ruler. 10 mm = 1 cm.
Decimeters aren't usually marked on rulers.10 centimeters make 1 decimeter.

10 decimeters end up being 100 centimeters, and that is 1 meter.

1 km = 1,000 m	**1 m = 100 cm**	**1 cm = 10 mm**

3. Convert between meters, centimeters, and millimeters.

a. 5 m = _____ cm 12 m = _____ cm 6 m 20 cm = _____ cm	**b.** 4 m 6 cm = _____ cm 10 m 80 cm = _____ cm 9 m 9 cm = _____ cm	**c.** 800 cm = _____ m 239 cm = ___ m ____ cm 407 cm = ___ m ____ cm
d. 58 mm = ___ cm ___ mm 78 cm = _____ mm 234 mm = ____cm ___ mm	**e.** 5 km = _____ m 57 km = _____ m 5,000 m = _____ km	**f.** 2 km 800 m = _____ m 6 km 50 m = _____ m 60,000 m = _____ km

4. Calculate. Give your answer using kilometers and meters.

a. 5 km 200 m + 8 km 900 m

b. 3 × 2 km 800 m

c. 1,500 m + 2 km

d. 6 × 700 m

5. Solve the problems.

a. How many millimeters are in a meter?

b. Mary can walk 1 km in 10 minutes. How far can she walk in 34 minutes?

c. John jogs through a track 1 km 800 m long twice a day, five days a week. How long a distance does he jog in a week?

d. A 10-meter wall is divided into five segments (not of equal length). Four of the segments are 1 m 20 cm each; how long is the fifth segment?

e. Kathy's wallpaper has butterflies that are 80 mm wide. She will put the wallpaper in her room. How many complete butterflies can she have on a wall 3 meters long?

MATH MAMMOTH

Grade 4-B

Complete Worktext

- Division, Long Division and Problem Solving
- Geometry
- Fractions
- Decimals

By Maria Miller

www.MathMammoth.com

Contents

Chapter 5: Division

Chapter 6: Geometry

Chapter 7: Fractions

Chapter 8: Decimals

Chapter 5: Division
Introduction

The fifth chapter of *Math Mammoth Grade 4-A Complete Worktext* includes lessons on division, long division, remainder, part problems, average, and problem solving. It is a long chapter, as division and long division are "in focus" in fourth grade.

We start out reviewing basic division by single-digit numbers. Then students study some basic division topics such as division terms, division by 1 and 0, and dividing by whole tens and hundreds.

The lesson Finding Parts with Division is very important. It shows an important relationship between fractions and division. For example, we can find 3/4 of a number by first finding 1/4 (divide by 4), then multiplying that result by 3.

The lesson on remainder is just before the first lesson on long division, because that is where the student needs to understand this concept very well.

Long division is taught in several small steps over many lessons. We start with the situation where each of the thousands, hundreds, tens, and ones can be divided evenly by the divisor. Then is introduced the remainder in the ones. Next comes the situation where we have a remainder in the tens. Finally, when we have a remainder in the hundreds, and so on.

All along the long division lessons, the process is explained so that the student can understand what it is based on. After the many lessons that practice and explain long division, we see a comparison between repeated subtraction and long division. The purpose of this lesson is to shed light in the basic idea of long division, and not to practice a new calculation method.

After long division is mastered, we study the concept of average. Next comes *Long Division with Remainder*, which also contains a section on packing problems

The following two lessons contain plenty of part-related problems to solve. These problems deal with fractional parts of a total, and include both dividing and multipying. I have included many diagrams and pictorial representations of these problems to help the student. Encourage him to draw a picture for those problems that don't have any.

The last two topics in this section are divisibility and a two-digit divisor in long division. These topics are introductory only, and we continue them in the fifth grade.

The Lessons in Chapter 5

Helpful Resources on the Internet

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Double-Division.org
Another form of long division algorithm - takes the guesswork away from estimating how many times the divisor goes into what needs to be divided. Also called 1-2-4-8 division.
http://www.doubledivision.org/

Factor Game
Interactive game to practice divisibility among numbers 1-100. Play against the computer or a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

The following games can be used to practice basic division facts, if the student hasn't mastered them.

A+ math games
Practice all four basic operations with math bingo (matho), hidden picture games, or concentration games.
http://www.aplusmath.com/games/

Math Magician games
Flashcard problems in all 4 operations. Answer 20 questions in 1 minute.
http://www.oswego.org/ocsd-web/games/Mathmagician/cathymath.html

Division Practice at AAAMath
Learn or practice basic division facts, and more.
http://www.aaastudy.com/div39hx3.htm

Cross the Swamp
Help Little Ron move from log to log across the swamp and practice multiplication/division or addition/subtraction.
http://www.bbc.co.uk/schools/starship/maths/crosstheswamp.shtml

Math Car Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations.
http://www.funbrain.com/osa/index.html

Arithmetic Game
Find numbers to fit an equation that may use all four operations.
http://www.primarygames.com/math/arithmeticgame/index.htm

Primary Games
A collection of games. The following links open the evaluation versions of some division-related games. The game collections themselves are sold at
http://www.primarygames.co.uk/

- **Eggs on Legs**
  http://www.primarygames.co.uk/PG5/Eggs/Div/eggsdiv.html
- **DiviPods**
  http://www.primarygames.co.uk/pg4/Divipods/divipods.html
- **Division Divers**
  http://www.primarygames.co.uk/pg3/ddivers/ddivers.html
- **Sum Sense - Division**
  http://www.primarygames.co.uk/pg2/sumsense/sumdiv.html

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Long Division 1

Divide hundreds, tens, and ones separately.

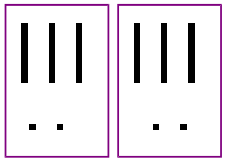
Write the dividend inside the long division "corner", and the quotient on top.

64 ÷ 2 = ?

Divide tens and ones separately:

6 tens ÷ 2 = 3 tens (t)

4 ones ÷ 2 = 2 ones (o)



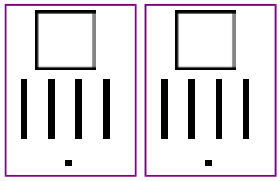
t o
3 2
2) 6 4

282 ÷ 2 = ?

2 hundreds ÷ 2 = 1 hundred (h)

8 tens ÷ 2 = 4 tens (t)

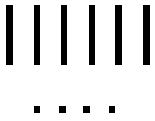
2 ÷ 2 = 1. (o)



h t o
1 4 1
2) 2 8 2

1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.

a. Make 2 groups

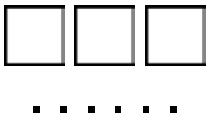


2) 6 2

b. Make 3 groups

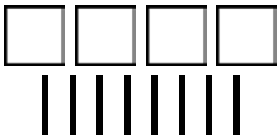
3)

c. Make 3 groups



3)

d. Make 4 groups



4)

2. Divide thousands, hundreds, tens, and ones separately.

a. 4) 8 4

b. 3) 3 9 3

c. 3) 6 6 0

d. 4) 8 0 4 0

e. 3) 6 6

f. 6) 6 0 3 6

g. 3) 3 3 0

h. 4) 4 8 0 4

h t o 0 4) 2 4 8 h t o 0 6 2 4) 2 4 8 4 does not go into 2. You can put zero in the quotient in the hundreds place or omit it. But 4 does go into 24, six times. Put 6 in the quotient.	th h t o 0 5) 3 5 0 5 th h t o 0 7 0 1 5) 3 5 0 5 5 does not go into 3. You can put zero in the quotient. But 5 does go into 35, seven times.
Explanation: The 2 of 248 is of course 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient won't have any whole hundreds. But then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result 6 tens goes as part of the quotient. Check the final answer: 4 × 62 = 248.	**Explanation:** 3,000 ÷ 5 will not give any whole thousands to the quotient because the answer is less than 1,000. But 3 thousands and 5 hundreds make 35 hundreds together. You can divide 3,500 ÷ 5 = 700, and place 7 as part of the quotient in the hundreds place. Check the final answer: 5 × 701 = 3,505.
If the divisor does not "go into" the first digit of the dividend, look at the first two digits of the dividend.	

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. $3\overline{)123}$

b. $4\overline{)284}$

c. $6\overline{)360}$

d. $8\overline{)248}$

e. $2\overline{)184}$

f. $7\overline{)427}$

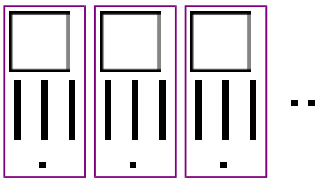
g. $3\overline{)1833}$ (quotient: 06)

h. $4\overline{)2404}$

i. $7\overline{)4970}$

j. $5\overline{)4505}$

Ones division is not even. There is a remainder.



395 ÷ 3 = 131 R2

h t o
1 3
3) 3 9 5

3 goes into 3 one time.
3 goes into 9 three times.

h t o
1 3 1 R2
3) 3 9 5

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

h t o
0 4 1 R1
4) 1 6 5

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

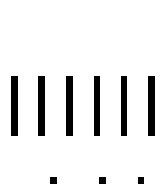
4 goes into 5 once, leaving a remainder of 1.

t h t o
0 4 0 0 R7
8) 3 2 0 7

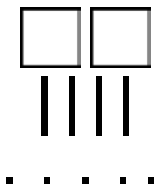
8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

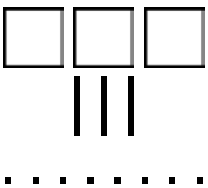
4. Divide into groups. Find the remainder.



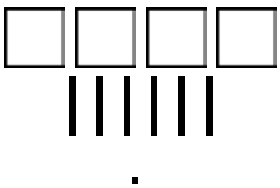
a. 2) 6 3



b. 2)



c. 3)



d. 2)

5. Divide. Indicate the remainder if any.

a. 4) 8 4 7

b. 2) 6 9

c. 3) 3 6 7

d. 4) 8 9

e. 2) 1 2 1

f. 6) 1 8 0 5

g. 7) 2 1 5

h. 8) 2 4 8 2

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

h t o 0 6 1 4) 2 4 7 − 4 3 When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247	th h t o 0 4 0 2 4) 1 6 0 9 − 8 1 When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1. Check: 4 × 402 + 1 = 1,609

6. Practice some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3\overline{)128}$

b. $3\overline{)95}$

c. $6\overline{)4267}$

d. $4\overline{)2845}$

e. $5\overline{)5507}$

f. $2\overline{)8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. 440 ÷ 4 = 820 ÷ 2 =	**b.** 3600 ÷ 400 = 369 ÷ 3 =	**c.** 824 ÷ 2 = 560 ÷ 90 =

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Part Problems

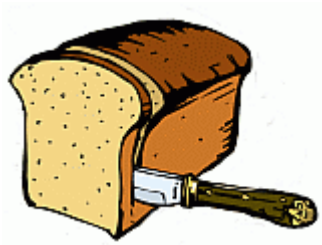
Drawing pictures can help you with part problems (if the picture is not yet given).

1. Mary divided her 285 buttons evenly into the five compartments.
   Find out how many buttons are

   a. in one compartment

   b. in three compartments

   c. in four compartments.



2. A loaf of bread weighs 400 g. It is cut into 20 slices.

   a. How much does one slice weigh?

   b. How much do three slices weigh?

   c. How much do 11 slices weigh?

3. A pie weighs 450 g. It is cut into six equal-size pieces.

   a. How much does 2/6 of the pie weigh?

   b. How much does 5/9 of the pie weigh?

4. If you need to calculate 5/9 of the number 729, first divide 729 by _____,

   then _____ the result by _____. Now, find 5/9 of 729.

5. James bought five computer mice for $36.50.
   Then he sold two of them to his friend.
   How much should he charge his friend?

6. Mary had already used $\frac{3}{4}$ of her $268.

a. How much money had she spent?

b. How much did she have left?

7. The plane had flown $\frac{2}{9}$ of the 12,600-mile trip when the passengers were served supper.

a. How far was there yet to fly?

b. How many miles had they already flown?

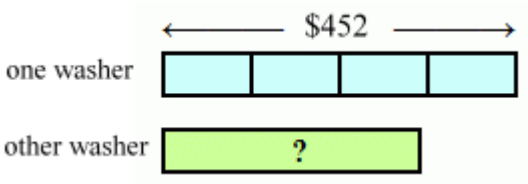
8. Twenty tons of flour arrived at the port.
3/10 of it was sent to New York.
How many pounds of flour was that?

9. A gallon of ice cream costs $12.96. Mom will pay for 3/4 of the price and you and your brother pay for 1/8 each.

a. Find each person's share of the cost.

b. Find each person's share of the ice cream in cups.

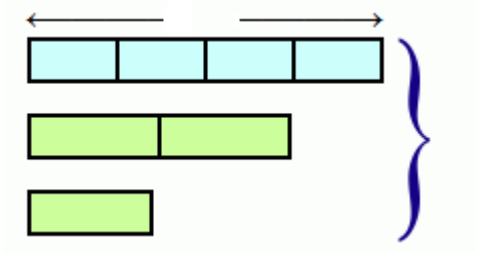
10. A washer costs $452 and another washer costs 3/4 as much.
Find the price of the other washer.



11. Jerry worked 56 hours on a project.
Edward worked 3/4 of that amount.
James worked 1/2 as many hours as Edward.

How many hours did the men work altogether?

Write the names and the 56 hours on the diagram. Place the question mark "?" to show what the problem asks to find.



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Chapter 6: Geometry
Introduction

In fourth grade, students are introduced to the concept of an angle, and learn about acute, right, obtuse, and straight angles. Students learn how to measure and draw angles with a protractor, and estimate some common angles.

After angles, we study triangles, and classify them according to the angles. Classifying triangles according to their sides (equilateral vs. isosceles triangles) is left for the 5th grade.

Next we study rectangles and parallelograms in more detail, and students learn to draw them, given either some side lengths or angle measures. The lesson on polygons concentrates on learning their names.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

The last topics in the geometry section are perimeter, area, and volume. These are topics that mostly involve calculations, and in many math books that is all you will find, but I have also included problems of drawing figures with a given area or perimeter. The concepts of area and perimeter are very important, and sometimes students confuse them. For that reason, I included a lesson where we compare both concepts in detail.

The study of geometry is full of strange-sounding words to learn. I encourage you to get the student(s) started with a *geometry notebook*, where they will write every new concept or term, and draw a picture or pictures and text to explain the term. This notebook will then be their own creation, and while working with it, the terms also will stick better in their memory. The students could also do the drawing exercises in this book, or just keep it as a terminology notebook, either way.

The Lessons in Chapter 6

Helpful Resources on the Internet

Turtle Pond
Guide a turtle to a pond using commands, which include turning him in certain angles, or moving him a specific distance.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Banana hunt at Primary Games
Help the monkey to find bananas and learn to estimate angles.
http://www.primarygames.co.uk/pg2/bhunt/bhunt.html

Ladybug Leaf
Guide the ladybug by giving her commands to turn 90° or 45°, right or left, or to move forward/backward.
http://nlvm.usu.edu/en/nav/frames_asid_287_g_2_t_3.html

LadyBug Mazes
Similar to the Ladybug Leaf, but this time you guide the ladybug through the maze.
http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html

Polygon Matching Game
http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary
A matching game.
http://www.quia.com/cc/2758.html

Shape explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Interactive Tangram Puzzle
Place the tangram pieces so they form the given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Tangram set
Cut out your Tangram set by folding paper
http://tangrams.ca/inner/foldtan.htm

Cubes
Fill a box with cubes, rows of cubes, or layers of cubes, and then fold in the sides of the box.
Illustrates the concept of volume.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

Cuboid Exploder and **Isometric Shape Exploder**
These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes, and then "explode" them to the unit cubes, illustrating volume.
www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html and
www.teacherled.com/resources/isoexplode/isoexplodeload.html

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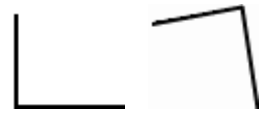
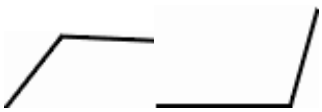
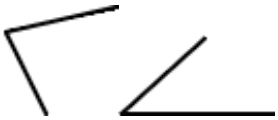
Triangles



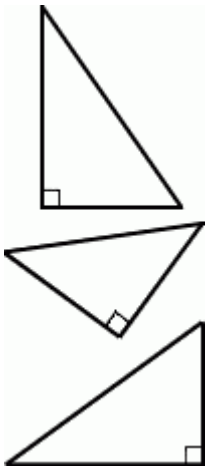
right angles
(exactly 90°)



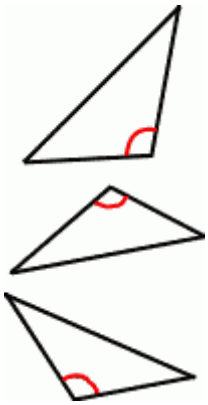
obtuse angles
(more than 90°, less than 180°)



acute angles
(less than 90°)



Right triangles have exactly one right angle.



Obtuse triangles have exactly one obtuse angle.

Acute triangles have three acute angles. In other words, ALL the angles are acute.

1. **a.** Draw a right angle.
   Then make it into a right triangle
   by drawing in the third side.

   b. Draw two more right triangles with
   different sizes or positions.
   You can use a notebook.

   c. Measure all the angles in
   all three triangles.
   Are those angles acute,
   right, or obtuse?

   d. What is the sum of the angles within each triangle?

A right triangle has one right angle. The other two angles are _____.

2. **a.** Draw an obtuse angle. Then make it into an obtuse triangle by drawing in the third side.

b. Draw two more obtuse triangles with different sizes or positions.

c. Then measure all the angles in all three triangles. Are those angles acute, right, or obtuse?

d. What is the sum of the angles within each triangle?

An obtuse triangle has one obtuse angle. The other two angles are _____.

3. **a.** Draw a triangle with one 60° angle and another 40° angle. You can decide the length of the sides. *Hint:* start out by drawing a 60° angle.

b. Measure the third angle. It is _____ degrees.

c. What kind of triangle did you get (acute, right, obtuse)?

4. **a.** Draw a triangle with one 55° angle and another 35° angle.

b. Measure the third angle. It is _____ degrees.

c. What kind of triangle did you get (acute, right, obtuse)?

5. **a.** Draw a triangle whose one angle is 85° and the other angle is 40°.

   b. Measure the third angle: it is _____ °.

   c. What kind of triangle did you get (acute, right, obtuse)?

6. **a.** Draw a triangle whose one angle is 125° and the other angle is 40°.

   b. Measure the third angle. It is _____ degrees.

   c. What kind of triangle did you get (acute, right, obtuse)?

7. Observe all you have done thus far in this lesson, and fill in.

Right triangles have exactly 1 _____ _____.

Obtuse triangles have exactly 1 _____ _____.

Acute triangles have ___ _____ angles. ALL the angles are _____.

8. Draw a rectangle, and one diagonal in it. What kind of triangles are formed?

New terms to remember:

- *an acute triangle*
- *a right triangle*
- *an obtuse triangle*

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Chapter 7: Fractions
Introduction

In the third grade, children studied the concept of a fraction, added and subtracted like fractions (with the same denominator), and compared some easy fractions. In fourth grade, it is time to slightly expand the fraction topics. We study

- mixed numbers
- addition and subtraction with like fractions and mixed numbers with like fractional parts
- comparing fractions
- equivalent fractions
- finding the fractional part of a quantity again. This topic has already been studied in the division chapter.

Then in fifth grade, students tackle *all* of the four operations of fractions. Our studies here are still laying groundwork for that, emphasizing the concepts and using visual models a lot.

These lessons are also important because they are the basis for understanding decimal numbers, the topic of the next chapter. Remember, all decimals are just another way of writing fractions with denominators 10, 100, 1,000 etc.

The topics in this chapter are first studied with the help of pictures in order to help to cement the concepts. Avoid presenting fraction math as a list of computational rules. Children easily confuse the various fraction rules, because there are so many, such as:

- a rule for converting a mixed number to a fraction, and vice versa
- a rule for adding like fractions
- a rule for finding a common denominator
- a rule for changing fractions to like fractions
- a rule for adding unlike fractions
- a rule for simplifying fractions
- a rule for finding equivalent fractions
- a rule for multiplying fractions
- a rule for dividing fractions
- a few rules for doing the four operations with mixed numbers

There is a place for the rules, as shortcuts for ideas that are already understood, but do not start with them. In fourth grade, there is no hurry to study all of these rules. Let the big ideas sink in conceptually first. Then, if a child understands the concept, notices a shortcut (a rule), and wants to use one, let him go ahead with the rule.

The Lessons in Chapter 7

Helpful Resources and Games on the Internet

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math
Conceptua Math has free, interactive fraction tools and activities that are very well made. The activities include identifying fractions, adding and subtracting, estimating, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions.
www.conceptuamath.com

Who Wants pizza?
Explains the concept of fraction, addition and multiplication with a pizza example, then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction Model
Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44

Clara Fraction's Ice Cream Shop
Convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors on the cone.
http://www.mrnussbaum.com/icecream/index.html

MathSplat
Click on the right answer to addition problems or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Fraction Worksheets: Addition and Subtraction
Create custom-made worksheets for fraction addition and subtraction. Choose "Like Fractions" for this level.
http://www.homeschoolmath.net/worksheets/fraction.php

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fractioncity
Make "fraction streets" and help kids with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

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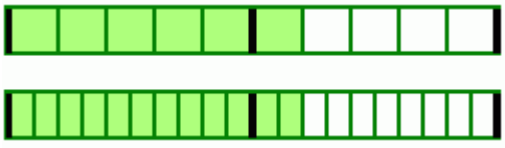
Equivalent Fractions



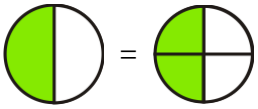
$1\frac{1}{5}$

$1\frac{2}{10}$

The two fraction strips show an equal amount. So we can write an equal sign between the two fractions:

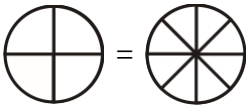
$$1\frac{1}{5} = 1\frac{2}{10}$$

If you eat half of a pizza, or 2/4 of a pizza, you have eaten the same amount.

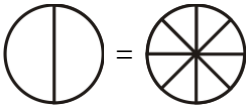


$$\frac{1}{2} = \frac{2}{4}$$

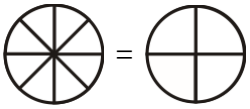
1. Shade the pie parts that the first fraction shows. Shade the same *amount of pie* in the second picture. Write the second fraction.



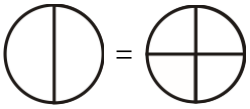
a. $\frac{1}{4} =$



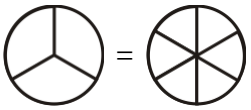
b. $\frac{1}{2} =$



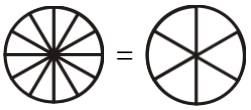
c. $\frac{6}{8} =$



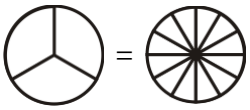
d. $\frac{1}{2} =$



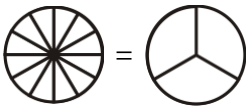
e. $\frac{2}{3} =$



f. $\frac{10}{12} =$

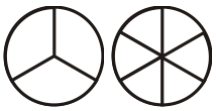


g. $\frac{1}{3} =$



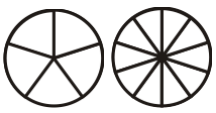
h. $\frac{8}{12} =$

2. Write the fractions that have thirds with sixths instead. You can imagine shading parts in the pictures.



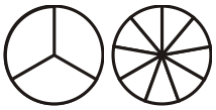
a. $\frac{1}{3} =$ **b.** $\frac{2}{3} =$ **c.** $2\frac{1}{3} =$ **d.** $\frac{4}{3} =$ **e.** $1\frac{2}{3} =$

3. Write the fractions that have fifths with tenths instead.



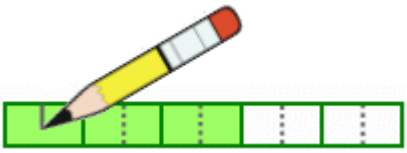
a. $\frac{1}{5} =$ **c.** $\frac{4}{5} =$ **c.** $1\frac{3}{5} =$ **d.** $\frac{2}{5} =$ **e.** $5\frac{1}{5} =$

4. Write the fractions that have thirds with ninths instead.



a. $\frac{1}{3} =$ **c.** $\frac{2}{3} =$ **c.** $1\frac{1}{3} =$ **d.** $5\frac{2}{3} =$ **e.** $\frac{5}{3} =$

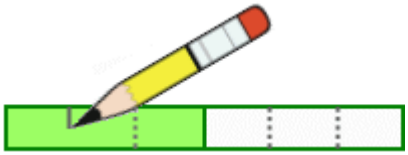
The fraction strip on the right illustrates $\frac{3}{5}$. If you split each piece (both the colored and white pieces) into *two* new pieces, what fraction do you get?



You get $\frac{6}{10}$ – six colored pieces, and ten pieces total.

You have *two* times as many colored pieces, and *two* times as many total pieces as before.

The fraction strip illustrates $\frac{1}{2}$. If you split each piece (both the colored and the white piece) into *three* new pieces, what fraction do you get?



You get $\frac{3}{6}$ – three colored pieces, and six pieces total.

You have *three* times as many colored pieces, and *three* times as many total pieces as before.

5. Split both the colored and white pieces as instructed. Write the fraction as it was and the fraction after you change it.

a. Split all the pieces into two new ones.  $\frac{1}{2} = \frac{\quad}{\quad}$	**b.** Split all the pieces into four new ones.  $\frac{1}{2} = \frac{\quad}{\quad}$	**c.** Split all the pieces into two new ones.  $\frac{1}{3} = \frac{\quad}{\quad}$
d. Split all the pieces into three new ones.  $\frac{1}{3} = \frac{\quad}{\quad}$	**e.** Split all the pieces into three new ones.  $\frac{1}{4} = \frac{\quad}{\quad}$	**f.** Split all the pieces into two new ones.  $\frac{3}{4} = \frac{\quad}{\quad}$
g. Split all the pieces into two new ones.  $\frac{4}{5} = \frac{\quad}{\quad}$	**h.** Split all the pieces into two new ones.  $\frac{5}{6} = \frac{\quad}{\quad}$	**i.** Split all the pieces into three new ones.  $\frac{2}{5} = \frac{\quad}{\quad}$

6. Split the pieces to show the fraction that is given.

 **a.** This is $\frac{3}{4}$. Make it $\frac{9}{12}$.	 **b.** This is $\frac{1}{3}$. Make it $\frac{4}{12}$.	 **c.** This is $\frac{1}{2}$. Make it $\frac{5}{10}$.
 **d.** This is $\frac{1}{4}$. Make it $\frac{4}{16}$.	 **e.** This is $\frac{2}{3}$. Make it $\frac{10}{15}$.	 **f.** This is $\frac{2}{3}$. Make it $\frac{8}{12}$.

7. Now write the equivalent fraction yourself. You can imagine a picture in your mind.

a. Split all the pieces into two new ones. $\frac{1}{2} = \frac{\quad}{\quad}$	**b.** Split all the pieces into three new ones. $\frac{1}{2} = \frac{\quad}{\quad}$	**c.** Split all the pieces into four new ones. $\frac{1}{2} = \frac{\quad}{\quad}$	**d.** Split all the pieces into five new ones. $\frac{1}{2} = \frac{\quad}{\quad}$
e. Split all the pieces into two new ones. $\frac{1}{3} = \frac{\quad}{\quad}$	**f.** Split all the pieces into two new ones. $\frac{2}{3} = \frac{\quad}{\quad}$	**g.** Split all the pieces into two new ones. $\frac{1}{4} = \frac{\quad}{\quad}$	**h.** Split all the pieces into two new ones. $\frac{3}{4} = \frac{\quad}{\quad}$
i. Split all the pieces into three new ones. $\frac{1}{3} = \frac{\quad}{\quad}$	**j.** Split all the pieces into three new ones. $\frac{2}{3} = \frac{\quad}{\quad}$	**k.** Split all the pieces into three new ones. $\frac{1}{4} = \frac{\quad}{\quad}$	**l.** Split all the pieces into three new ones. $\frac{3}{4} = \frac{\quad}{\quad}$

8. Connect the equivalent fractions with a line.

a.	$\frac{2}{3}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{10}$	$\frac{2}{8}$
	$\frac{2}{6}$	$\frac{6}{9}$

b.	$\frac{1}{2}$	$\frac{2}{10}$
	$\frac{3}{4}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{6}{12}$
	$\frac{4}{12}$	$\frac{9}{12}$

c.	$\frac{3}{6}$	$\frac{3}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$

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Chapter 8: Decimals
Introduction

In fourth grade, we study the concept of decimal numbers with one or two decimal digits, adding and subtracting them, and multiplying decimals by a whole number. It is important that the student grasps these simple topics well, because we are laying a groundwork towards fifth and sixth grade, when decimal operations and using decimals take more of a "center stage".

Right now, the focus is first of all grasping the fact that decimals are nothing more than fractions with a denominator 10 or 100. I have only included decimals with one or two decimal digits to keep from confusion, to keep it simple enough so that the concepts can be well understood.

With that in mind (decimals are fractions), we study adding and subtracting them. The important ideas to grasp are:

- In problems of the type 0.5 + 0.9, we get 14 tenths, which is *more* than one whole. The answer is NOT 0.14, but 1.4. If the student has problems, have him compare it with fraction addition.
- In a problem such as 0.5 + 0.11, the answer is NOT 0.16. We cannot add the decimal parts as if they were "whole numbers". Instead, we rewrite 0.5 as 0.50, and the problem becomes 0.50 + 0.11 = 0.61.

Then we multiply decimals by whole numbers. This is essentially just repeated addition. For example, $3 \times 0.8 = 0.8 + 0.8 + 0.8 = 2.4$. Note how the answer has one decimal digit, because we kept adding 0.8, which also has one decimal digit (tenths).

After this idea is understood, the student can just multiply 3×8 using the knowledge of multiplication tables, and remember to place the decimal point in the answer.

In the lesson Using Decimal Numbers, we do some conversions between metric measuring units. This topic will be studied further in 5th and 6th grades.

In general, decimal numbers will be studied in a lot more detail in grades 5 and 6.

The Lessons in Chapter 8

Helpful Resources on the Internet

Mathematical Interactivities
http://mathematics.hellam.net/
Find several games related to fractions and decimals in the **Number Puzzles** section, including:

- **Decimal Challenge** - Guess the decimal number between 0 and 10. Each time feedback tells whether your guess was too high or too low.
  http://www.interactivestuff.org/sums4fun/decchall.html
- **Switch** - Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
  http://www.interactivestuff.org/sums4fun/switch.html
- **Scales** - Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
  http://www.interactivestuff.org/sums4fun/scales.html

Smaller and smaller maze
Practice ordering decimal numbers and find your way through the maze.
http://www.mathsyear2000.org/magnet/kaleidoscope/smaller/index.html

A Decimal Puzzle
Make every circle add up to 3.
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=category_g_2_t_1.html

Fraction/Decimal Worksheets
Change fractions to decimal numbers or decimal numbers to fractions.
http://www.homeschoolmath.net/worksheets/fraction-decimal.php

Decimal and Square Root Gizmos from Explorelearning.com
Interactive "Gizmos" illustrating comparing and ordering decimals, multiplying with decimals, sums and differences with decimals, solving equations with decimals, and exploring square roots. Exploration guide and assessment questions included. By subscription, but has Free 30-day trial account. *Excellent resources.*
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211

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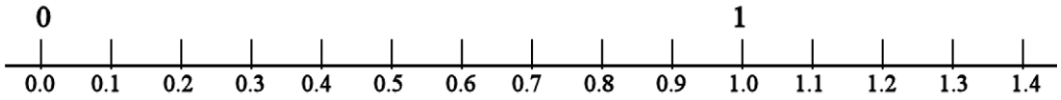
Adding with Tenths

You *already* know how to add or subtract decimals with tenths. They are just fractions with a denominator of 10. Compare these additions that are written with decimals or fractions.	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
There is one tricky part though: 0.6 + 0.7 *is* ***NOT*** 0.13 !! To see why, add the fractions. Notice that six tenths and seven tenths makes more than one whole!	$0.6 + 0.7 = \ ? \ = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each "number line jump".



a. You're at 0.7, and you jump *five tenths* to the right.

b. You're at 0.6, and you jump *eight tenths* to the right.

c. You're at 1.1, and you jump *eight tenths* to the left.

d. You're at 1.3, and you jump *four tenths* to the left.

2. Add.

a. 0.9 + 0.2 =	**b.** 0.5 + 0.7 =	**c.** 0.8 + 0.7 =	**d.** 0.8 + 0.9 =
1.9 + 0.2 =	3.5 + 0.7 =	0.8 + 2.7 =	0.8 + 3.9 =

3. Add a decimal so the sum is the next whole number.

a. 2.1 + ____ = 3 **b.** 4.5 + ____ = **c.** 8.9 + ____ = **d.** 5.3 + ____ =

4. Find the missing addend.

a. ____ + 0.5 = 3 **b.** 0.2 + ____ = 8 **c.** 0.4 + ____ = 1.2 **d.** 0.7 + ____ = 1.4

5. Add.

a. 2.3 + 0.9 =	**b.** 1.5 + 0.7 =	**c.** 2.2 + 3.4 =	**d.** 4.6 + 0.7 =
2.3 + 0.8 =	1.8 + 0.4 =	2.2 + 5.6 =	6.6 + 0.5 =

6. Write the numbers.

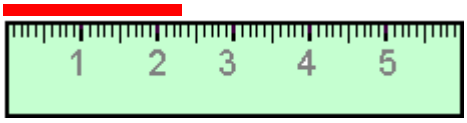
a. 3 tenths, 5 ones **b.** 7 tens, 8 ones, 4 tenths **c.** 4 tenths, 3 ones, 6 tens	**d.** Write the numbers in order. 9 8.9 9.1 9.0 9.9 1.9

7. Continue the patterns.

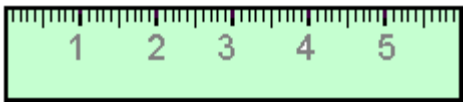
a. 0.1	**b.** 1.1	**c.** 2.5	**d.** 6.0
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____
+ 0.2 = ____	+ 0.5 = ____	+ 0.3 = ____	+ 0.4 = ____

8. Remember? 1 millimeter is one-tenth of a centimeter. Or, 1 mm = 0.1 cm.

a. Measure the line in centimeters. Use a decimal.



b. Draw a line that is 4.7 cm long.



9. Convert and add.

a. 0.5 cm = _____ mm

1.2 cm = _____ mm

b. 7 mm = _____ cm

35 mm = _____ cm

c. 5 mm + 0.9 cm =

4 cm + 3.4 cm =

10. The two sides of a rectangle measure 6.5 cm and 3.6 cm.
Draw the rectangle.
What is its perimeter?