Contents

Chapter 3: Linear Equations

Chapter 4: Introduction to Functions

Foreword

Math Mammoth Grade 8 comprises a complete math curriculum for the eighth grade mathematics studies. The curriculum meets the Common Core standards.

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics.

The main areas of study in Math Mammoth Grade 8 are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Scatter plots/bivariate data.

We start with a study of exponent laws, using both numerical and algebraic expressions. The first chapter also covers scientific notation (both with large and small numbers), significant digits, and calculations with numbers given in scientific notations.

In chapter 2, students learn about geometric transformations (translations, reflections, rotations, dilations), common angle relationships, and volume of prisms, cylinders, spheres, and cones.

Next, in chapter 3, our focus is on linear equations. Students both review and learn more about solving linear equations, including equations whose solutions require the usage of the distributive property and equations where the variable is on both sides.

Chapter 4 presents an introduction to functions. Students construct functions to model linear relationships, learn to use the rate of change and initial value of the function, and they describe functions qualitatively based on their graphs.

In part 8-B, students graph linear equations, learn about irrational numbers and the Pythagorean Theorem, solve systems of linear equations, and investigate patterns of association in bivariate data (scatter plots).

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

Chapter 1: Exponents and Scientific Notation
Introduction

The first chapter of Math Mammoth Grade 8 starts out with a study of basic exponent laws and scientific notation.

We begin with a review of the concept of an exponent and of the order of operations. The next lesson first reviews multiplication of integers, and then focuses on powers with negative bases, such as $(-5)^3$.

Then we get to the “meat” of the chapter: the various laws of exponents. The first lesson on that topic allows students to explore and to find for themselves the product law and the quotient law of exponents. After that, students find out the logical way to define negative and zero exponent by looking at patterns. They practice simplifying various expressions with exponents, both with numerical values and with variables.

The lesson “More on Negative Exponents” focuses on expressions with a negative exponent in the numerator, such as $7/(a^{-4})$. This is to prepare students for calculations that ask them to find how many times bigger one number is than another, when the numbers are written in scientific notation.

Next, in the lesson “Laws of Exponents, Part 2”, students practice applying the power of a power law: $(a^n)^m = a^{nm}$.

Then the chapter has one more lesson on the laws of exponents (“Laws of Exponents, Part 3”), which summarizes the laws and gives more practice. This lesson is not absolutely essential if you're following Common Core Standards. It is presented here to give a summary, to give practice on all exponent laws, including the power of a quotient law which was not dealt with a lot in the previous lessons. This lesson also allows the book to meet the Florida B.E.S.T. standards for 8th grade.

Then we turn our attention to scientific notation, first learning how it is used with large numbers and then with small numbers. The lesson on significant digits follows, helping students to know how to round final answers in calculations with measurements.

The last topic of the chapter is calculations with numbers given in scientific notations. These calculations, naturally, involve many scientific topics such as the atomic world or astronomy.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Powers and the Order of Operations	13	*3 pages*	1 day	
Powers with Negative Bases	16	*3 pages*	1 day	
Laws of Exponents, Part 1	19	*3 pages*	1 day	
Zero and Negative Exponents	22	*3 pages*	1 day	
More on Negative Exponents	25	*2 pages*	1 day	
Laws of Exponents, Part 2	27	*3 pages*	1 day	
Laws of Exponents, Part 3	30	*2 pages*	1 day	

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Scientific Notation: Large Numbers	32	*3 pages*	1 day	
Scientific Notation: Small Numbers	35	*2 pages*	1 day	
Significant Digits	37	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 1	40	*3 pages*	1 day	
Using Scientific Notation in Calculations, Part 2	43	*3 pages*	1 day	
Chapter 1 Review	46	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*35 pages*	13 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

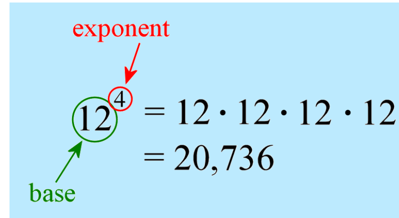
We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch1

Scan me

Powers and the Order of Operations

You will recall that we use **exponents** as a shorthand for writing repeated multiplications by the same number. For example, $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ is written 7^5.



The tiny raised number is called the **exponent**. It tells us how many times the **base** number is multiplied by itself.

The entire expression, 7^5, is a **power.** We read it as "seven to the fifth power," "seven to the fifth," or "seven raised to the fifth power." Similarly, 0.5^8 is read as "five tenths to the eighth power" or "zero point five to the eighth."

The "powers of 8" are the various expressions where 8 is raised to some power: for example, 8^3, 8^4, 8^{45}, and 8^{99} are powers of 8.

The expression 9^1 equals simply 9. In general, $a^1 = a$.

Powers of 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the <u>area of a square</u> with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the <u>volume of a cube</u> with edges 1.5 units long.

A calculator is not needed for the exercises of this lesson.

1. Evaluate.

a. four cubed　　**b.** 2^4　　**c.** 5^3

d. 0.2^3　　**e.** 1^{60}　　**f.** 100 squared

2. **a.** Which is more, 4^2 or 2^4?　　**b.** Which is more, 2^5 or 5^2?

3. Complete the patterns.

a.	**b.**	**c.**
$10^1 =$	$2^1 =$	$0.1^1 =$
$10^2 =$	$2^2 =$	$0.1^2 =$
$10^3 =$	$2^3 =$	$0.1^3 =$
$10^4 =$	$2^4 =$	$0.1^4 =$
$10^5 =$	$2^5 =$	$0.1^5 =$
$10^6 =$	$2^6 =$	$0.1^6 =$
$10^7 =$	$2^7 =$	$0.1^7 =$

The order of operations dictates that powers (expressions with exponents) are solved before multiplication, division, addition, and subtraction.

Example 1. Find the value of $5 \cdot 0.1^3 + 0.2^2$.

First the powers: $0.1^3 = 0.1 \cdot 0.1 \cdot 0.1 = 0.001$, and $0.2^2 = 0.2 \cdot 0.2 = 0.04$.

The expression becomes
$5 \cdot 0.001 + 0.04 = 0.005 + 0.04 = \underline{0.045}$.

<u>The Order of Operations (PEMDAS)</u>
("*Please Excuse My Dear Aunt Sally*")

1) Solve what is within parentheses **(P)**.

2) Solve exponents **(E)**.

3) Solve multiplication **(M)** and division **(D)** from left to right.

4) Solve addition **(A)** and subtraction **(S)** from left to right.

4. Find the value of the expressions.

a. $4 \cdot 10^3 - 5 \cdot 10^2$	**b.** $4(5^2 - 2^3)$	**c.** $\dfrac{3}{1^8} + \dfrac{5}{3^2}$
d. $7 \cdot 10^3 - 5(800 - 10^2)$	**e.** $500 - \dfrac{3 \cdot 8}{2^3} + 2 \cdot 8^2$	**f.** $\dfrac{2 \cdot 17 + 2^4}{7 \cdot 7 - 3^2} + 20$

5. Find the value of the expressions.

a. $0.5^2 - 0.2^2 - 0.1^2$	**b.** $3(0.1^2 - 0.2^3)$	**c.** $0.6^2 + 2(1 - 0.3^2)$

6. The table on the right shows a list of powers of 4.

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 256$
$4^5 = 1{,}024$
$4^6 = 4{,}096$

a. Find the value of 4^7 using the value for 4^6. (Do not use a calculator.)

b. Which power of 4 is equal to 65,536? Use estimation and the table, not a calculator.

c. Use the table to check whether $4^2 + 4^3 = 4^5$.

d. Use the table to check whether $4^2 \cdot 4^3 = 4^5$.

7. **a.** Find a power of 3 that is greater than seven squared.

b. Find a power of 5 that is greater than ten cubed.

c. Find a power of 1 that is greater than three squared.

8. **a.** If $3^6 = 729$, find the value of 3^8.

b. If $2^8 = 256$, find the value of 2^{11}.

9. Find the missing exponents.

a. $10^4 = 100^{\square}$	**b.** $2^6 = 4^{\square}$	**c.** $9^2 = 3^{\square}$	**d.** $0 = 0^{\square}$
e. $0.1^{\square} = 0.0001$	**f.** $0.2^{\square} = 0.00032$	**g.** $625 = 5^{\square}$	**h.** $128 = 2^{\square}$

10. Find the value of these powers.

a. $\left(\frac{1}{6}\right)^2 =$	**b.** $\left(\frac{3}{10}\right)^3 =$	**c.** $\left(\frac{2}{3}\right)^4 =$	**d.** $\left(\frac{3}{4}\right)^3 =$

Example 2. Simplify $3 \cdot s \cdot s \cdot s \cdot 3 \cdot t \cdot s \cdot t \cdot t$.

We can multiply in any order, so let's reorganize the expression as $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t$.

The variable *s* is multiplied by itself four times, and *t* three times. Naturally, $3 \cdot 3$ is 9.

So, we get $3 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t \cdot t = \mathbf{9s^4t^3}$.

11. Simplify.

a. $2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot 7$	**b.** $4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot 9 \cdot x \cdot y \cdot x$
c. $5 \cdot a \cdot b \cdot b \cdot a \cdot a \cdot 2 \cdot b \cdot 6$	**d.** $0.3 \cdot p \cdot r \cdot p \cdot r \cdot r \cdot 0.2 \cdot r \cdot 10$

12. **a.** Find the value of the expression $10a^4b^2$ when $a = 2$ and $b = 3$.

b. Find the value of the expression $14x^3y^5$ when $x = 2$ and $y = 0$.

13. When you fold a sheet of paper in half, its area is naturally now only 1/2 of the area of the original paper. Let's say you repeat this process, and fold that paper again in half, and again, and again. How many times do you need to fold a sheet of paper in order for the area of the folded piece to be 1/64 of the area of the original?

Puzzle Corner What is the simple value of $\frac{9^6}{9^5}$? There is no need for actual calculations!

(This page intentionally left blank.)

Using Scientific Notation in Calculations, Part 2

1. Some students get confused with the rules of exponents when *adding* numbers in scientific notation. Compare the problems carefully, and solve. Give your answers in decimal notation. Do not use a calculator.

a. $(2 \cdot 10^6) \cdot (3 \cdot 10^4)$

b. $2 \cdot 10^6 + 3 \cdot 10^4$

c. $8 \cdot 10^3 + 7 \cdot 10^5$

d. $(8 \cdot 10^3) \cdot (7 \cdot 10^5)$

It is easy to add or subtract numbers in scientific notation IF they have the same power of ten: you can simply add or subtract their decimal parts.

Example 1. Add $2.81 \cdot 10^{13} + 5.2 \cdot 10^{12}$.

We will write $2.81 \cdot 10^{13}$ with 10^{12} instead of 10^{13}: $2.81 \cdot 10^{13} = 28.1 \cdot 10^{12}$. Now, the problem becomes $28.1 \cdot 10^{12} + 5.2 \cdot 10^{12}$. We can simply add $28.1 + 5.2 = 33.3$. The final sum is $33.3 \cdot 10^{12}$.

Another possibility is to simply use decimal notation to add the numbers, like you probably did in question #1. This works if the absolute values of the exponents are not very large.

2. Solve. Give your answer in scientific notation.

a. $4.8 \cdot 10^8 + 5 \cdot 10^7$	**b.** $9.3 \cdot 10^6 + 8 \cdot 10^7$	**c.** $5 \cdot 10^7 - 7 \cdot 10^5$	**d.** $8.4 \cdot 10^9 - 4.7 \cdot 10^8$

3. Jeremy and Mia were working on the problem $5 \cdot 10^{-3} + 2 \cdot 10^{-4}$. One of them got the answer $7 \cdot 10^{-7}$ and the other got $5.2 \cdot 10^{-4}$. Is either answer correct? If not, find the correct answer (without a calculator).

4. Solve.

a. $8 \cdot 10^{-2} + 6 \cdot 10^{-3}$	**b.** $3 \cdot 10^{-6} + 5 \cdot 10^{-5}$
c. $2 \cdot 10^{-4} - 7 \cdot 10^{-6}$	**d.** $5.4 \cdot 10^{-3} - 7 \cdot 10^{-4}$

5. Compare the volumes of different planets, the moon, and the sun that are given in the table, in cubic kilometers.

Celestial Body	Volume
Earth	$1.0832 \cdot 10^{12}$ km^3
Moon	$2.1968 \cdot 10^{10}$ km^3
Mars	$1.6318 \cdot 10^{11}$ km^3
Jupiter	$1.4313 \cdot 10^{15}$ km^3
Sun	$1.4093 \cdot 10^{18}$ km^3

a. How many Jupiters would "fit" in the sun?

b. How much bigger is the volume of the earth than of Mars?

c. What is the combined volume of the earth and the moon?

6. Count how many breaths you take in a minute (at rest), and from that, estimate how many breaths in total you would take in a 70-year lifespan. Then choose the closest estimate from the options below.

The number of breaths a person takes in a lifetime is about: **a.** $5 \cdot 10^6$ **b.** $5 \cdot 10^8$ **c.** $5 \cdot 10^{10}$ **d.** $5 \cdot 10^{12}$

7. A 70-kg male body contains approximately $7 \cdot 10^{27}$ atoms. Of these, approximately $4.22 \cdot 10^{27}$ are hydrogen atoms, $1.61 \cdot 10^{27}$ are oxygen atoms, $8.03 \cdot 10^{26}$ are carbon atoms, and $3.9 \cdot 10^{25}$ are nitrogen atoms.

a. About what percentage of the atoms in a human body are hydrogen atoms?

b. About how many more times oxygen atoms does the human body have than nitrogen atoms?

c. About how many more oxygen atoms does the human body have than carbon atoms?

8. It is estimated that there are about 10^{15} ants on this planet, and that the average mass of each ant is about 1 mg.

a. Find the total mass of the ants living on this planet. Give your answer in a sensible unit.

b. Now find the total mass of humans living in Asia, in kilograms. Use 60 kg for the average weight of humans in Asia, and 4,800 million for the population of Asia (or check the current population at https://www.worldometers.info/world-population/asia-population/).

c. Which have a larger mass, all the ants on the planet, or the people living in Asia?

9. The water volume in Lake Victoria is approximately 2,750 km^3.

a. Convert this to cubic meters and write the resulting number in scientific notation.
Hint: the unit “km^3” means “1000 meters, cubed”, or $(1000\text{ m})^3$.

b. Now calculate how many bucketfuls of water there are in Lake Victoria. Use 20 liters for the volume of one bucket (consider it accurate to two significant digits). (1 m^3 = 1000 liters.)

Chapter 2: Geometry
Introduction

The second chapter of Math Mammoth Grade 8 covers geometric transformations, angle relationships, and the volume of prisms, cylinders, pyramids, cones, and spheres.

The chapter starts out with the basics of congruent transformations: translations, reflections, rotations. Students use transparent paper to perform several of these transformations hands-on, so as to gain an understanding of the attributes that are preserved in these transformations.

Next we practice these same transformations in the coordinate grid. Students learn how the coordinates of the points change when a figure is translated or reflected in the *x* or *y*-axis. They also explore rotating figures in the coordinate grid; here we limit the rotations to 90°, 180°, or 270° degrees.

Then it is time to study sequences of transformations, which enable us to describe more complex transformations. The key idea here is to understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of transformations.

All of this work has related to congruent transformations, which means the size of the figure has not changed. Now we turn our attention to dilations. In a dilation, the figure is transformed so that its size changes but its shape does not. Such figures are called similar figures. Yet another term describing the same process is scaling a figure.

Next, we study angle relationships. The first lesson in this section reviews certain angle relationships from 7th grade (complementary, supplementary, and vertical angles). Then students learn about angles formed when a transversal crosses two parallel lines: corresponding angles, alternate interior angles, and alternate exterior angles. They also investigate angle relationships related to triangles and learn how these relationships allow us to deduce angle measurements of other angles.

In all of this work, students are guided to reason using mathematical facts they have learned, and to justify their reasoning, thus becoming familiar with the process of mathematical proof.

The last major topic of the chapter is volume of various three-dimensional figures. Students solve a variety of real-world and mathematical problems involving multiple three-dimensional shapes.

Pacing Suggestion for Chapter 2

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Geometric Transformations and Congruence, Part 1	51	*4 pages*	1 day	
Geometric Transformations and Congruence, Part 2	55	*3 pages*	1 day	
Translations in the Coordinate Grid	58	*3 pages*	1 day	
Reflections in the Coordinate Grid	61	*3 pages*	1 day	
Translations and Reflections	64	*3 pages*	1 day	
Rotations in the Coordinate Grid	67	*4 pages*	1 day	
Sequences of Transformations	71	*3 pages*	1 day	
Sequences of Transformations, Part 2	74	*2 pages*	1 day	
Dilations	76	*3 pages*	1 day	
Dilations in the Coordinate Grid	79	*3 pages*	1 day	

The Lessons in Chapter 2	page	span	suggested pacing	your pacing
Similar Figures, Part 1	82	*3 pages*	1 day	
Similar Figures, Part 2	85	*2 pages*	1 day	
Similar Figures: More Practice	87	*3 pages*	1 day	
Review: Angle Relationships	90	*3 pages*	1 day	
Corresponding Angles	93	*2 pages*	1 day	
More Angle Relationships with Parallel Lines	95	*2 pages*	1 day	
The Angle Sum of a Triangle	97	*3 pages*	1 day	
Exterior Angles of a Triangle	100	*3 pages*	1 day	
Angles in Similar Triangles, Part 1	103	*2 pages*	1 day	
Angles in Similar Triangles, Part 2	105	*2 pages*	1 day	
Volume of Prisms and Cylinders	107	*2 pages*	1 day	
Volume of Pyramids and Cones	109	*3 pages*	1 day	
Volume of Spheres	112	*2 pages*	1 day	
Volume Problems	114	*2 pages*	1 day	
Chapter 2 Mixed Review	116	*2 pages*	1 day	
Chapter 2 Review	118	*5 pages*	2 days	
Chapter 2 Test (optional)				
TOTALS		*72 pages*	27 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch2

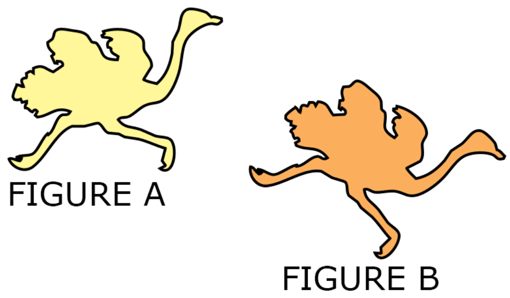
Scan me

Geometric Transformations and Congruence, Part 1

Two figures are congruent when they are, you might say, identical in the sense that they have the same shape and size (but may be of different color). We can define congruency as follows:

Two figures are **congruent** if they perfectly match, when one is placed on top of the other.

The figures don't have to be in the same position or orientation. For example, these two figures are congruent — if you rotate and move figure A, you can place it exactly on top of figure B.

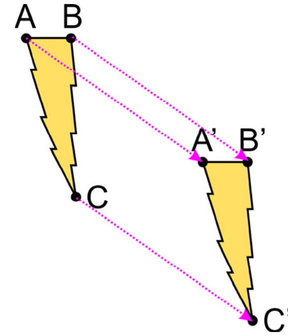


We will now study three geometric **transformations**, or basic ways to move a point, or by extension, a figure, since a figure can be considered to consist of many points.

1. A **translation** of a figure means sliding or moving it a certain distance in a certain direction, without turning or rotating it. The arrows show how three individual points of the figure were moved.

   We say the translation maps point A onto point A' (read "A prime"), point B onto point B', and point C onto point C'.

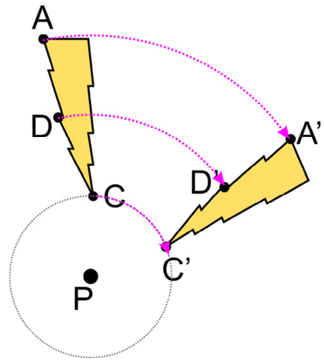
   We also say that point A' is the image of point A under the translation.



2. A **rotation** means turning a figure around a certain point.

   Here, the lightning figure is rotated around point P. Each point of the figure moves in a **circular arc around point P**.

   A rotation is measured in degrees, just like angles are. In this example, the lightning figure was rotated 67 degrees clockwise around point P.

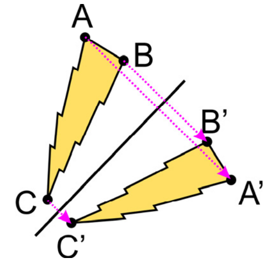


3. A **reflection** across a line means mirroring the figure in that line. You could also say the figure was "flipped".

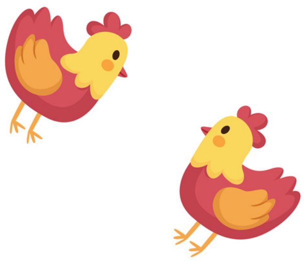
   In a reflection, the distance from each point to the reflection line and the distance of its image to the line are equal (measured along a line segment that is perpendicular to the line).

   For example, the distance from point C to the line equals the distance from point C' to the line.

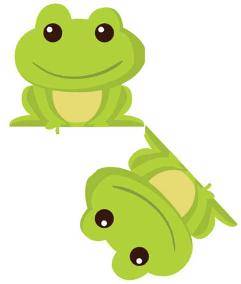
   A reflected figure is congruent to the original.



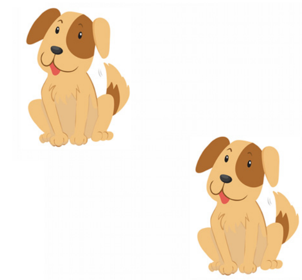
1. Name the transformation that was used to transform the figure on the left to the figure on the right.



a.



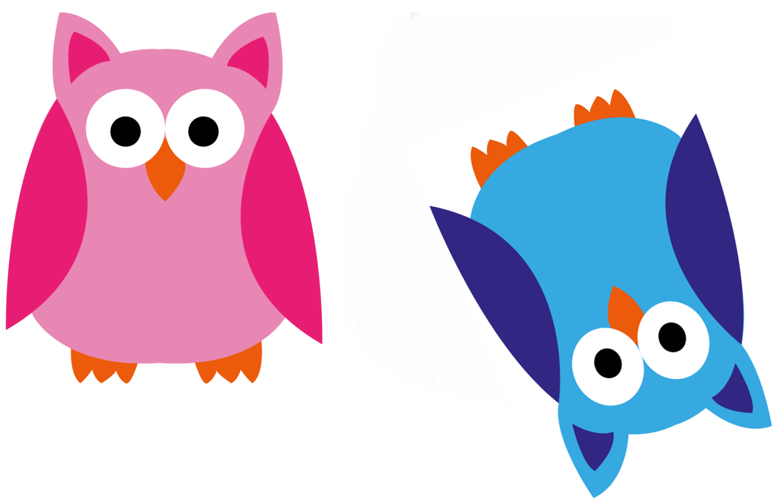
b.



c.

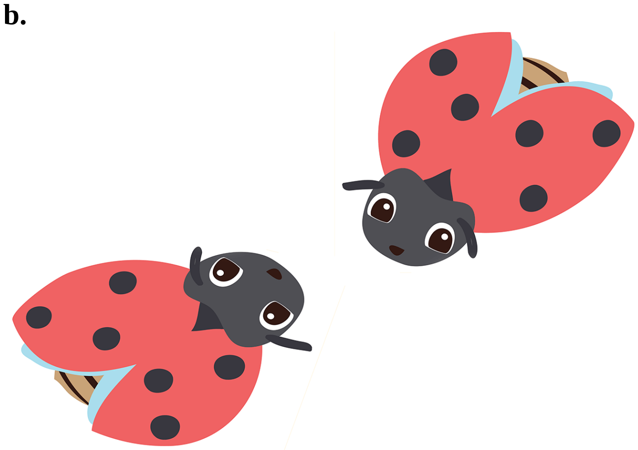
In continuation, we will explore geometric transformations and how they relate to congruence with the help of tracing paper (patty paper) or a transparency.

2. Use tracing paper to determine whether the two figures are congruent. You may move, turn, and/or flip the tracing paper. First, copy the outline of **one** figure to the tracing paper.
(Note: when checking for congruency, we ignore the colors.)

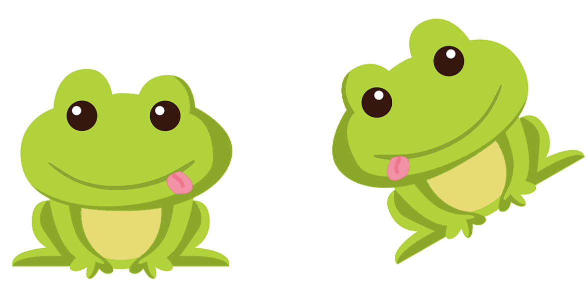


a.

b.



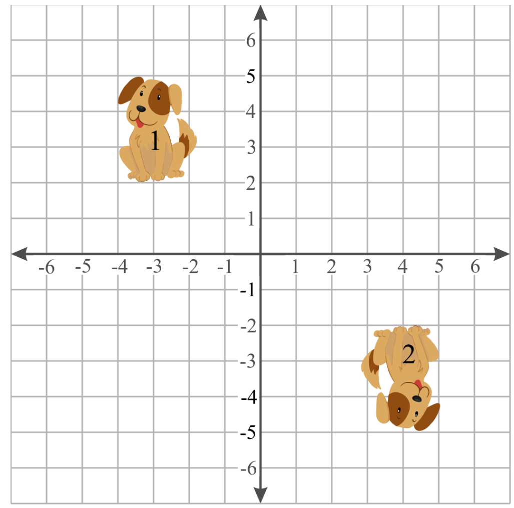
c.



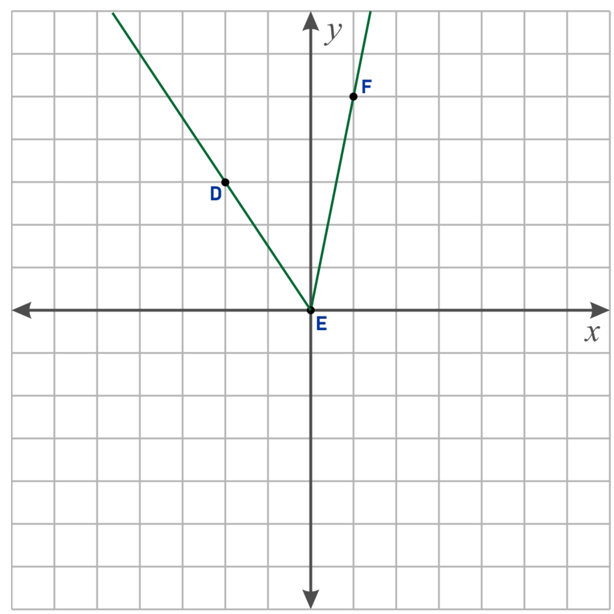
(This page intentionally left blank.)

Chapter 2 Review

1. Describe a sequence of transformations that can map figure 1 to figure 2.

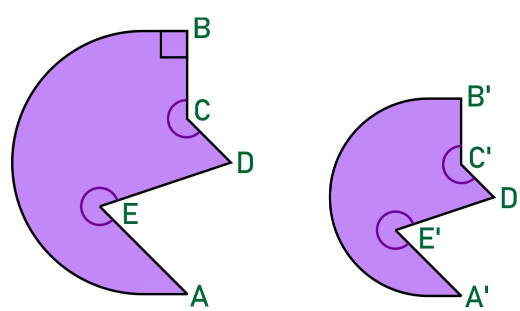


2. Rotate angle DEF both 90 degrees and also 180 degrees clockwise around the origin.



3. A quadrilateral was first reflected in the *y*-axis, and then rotated around the origin clockwise 90 degrees. Its vertices are now at points (3, −5), (5, −2), (4, −1), and (1, −4). What were the coordinates of its vertices before these transformations?

4. Figure A'B'C'D'E' is a dilation of figure ABCDE with scale factor 3/4. Angle B is a right angle. Check all the statements that are true.



a. Angle B' is a right angle.

b. The measure of ∠CDE is 3/4 of the measure of ∠C'D'E'.

c. ∠E is equal to ∠E'.

d. If CD = 1 inch, then C'D' = 3/4 inch.

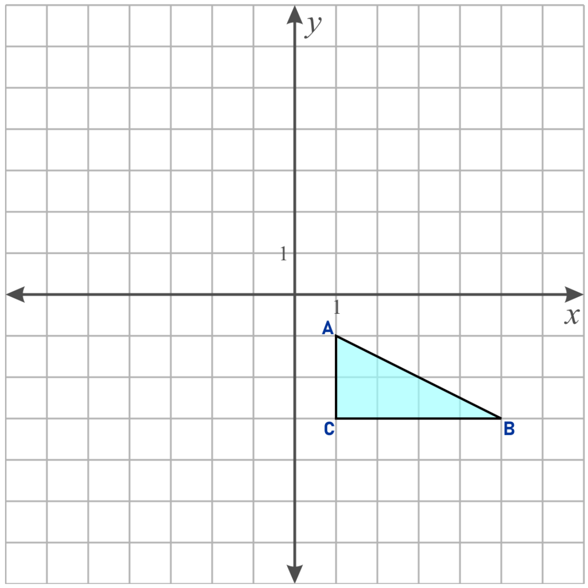
e. ∠D is equal to ∠E'.

f. If the perimeter of figure ABCDE is 20 inches, then the perimeter of A'B'C'D'E is 12 inches.

5. **a.** Perform the following sequence of transformations to triangle ABC:

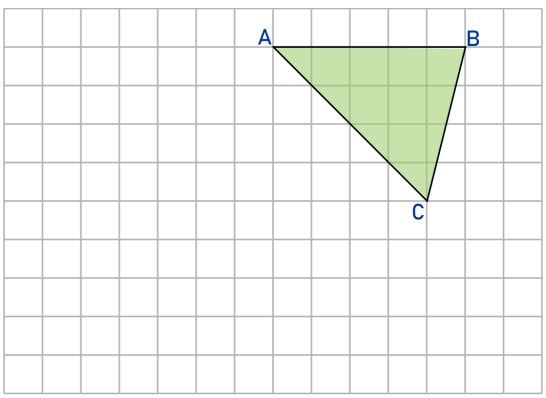
First rotate it counterclockwise around point C 90 degrees.
Then reflect it in the vertical line at $x = -1$.
Lastly, translate it two units to the right and three down.

b. Find another, different sequence of transformations that does the same as the sequence in (a), and starts with a reflection.

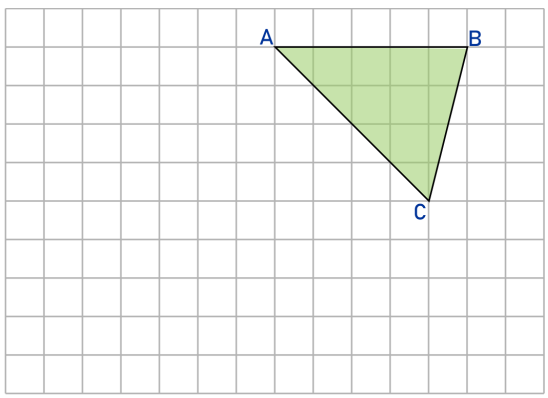


6. Draw a dilation of triangle ABC...

a. from point C and scale factor 1/2.



b. from point B and scale factor 2.



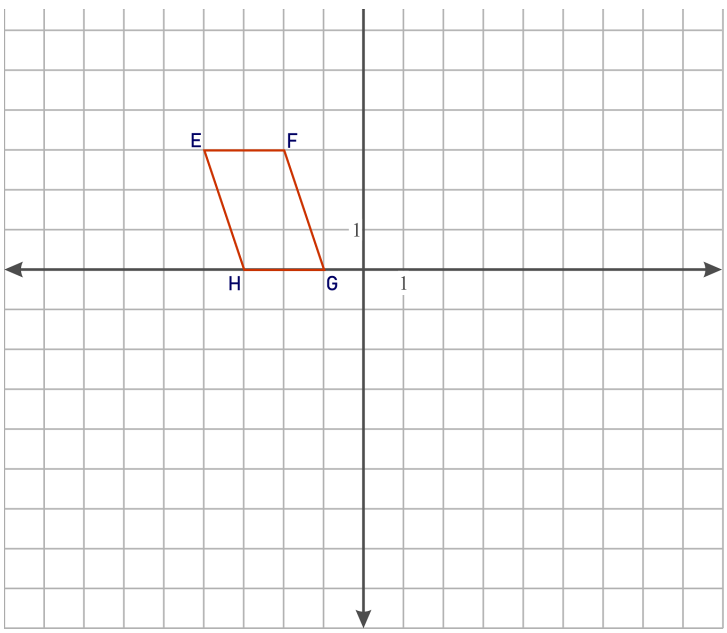
7. Parallelogram EFGH underwent the following transformations :

1. Reflection in the vertical line at $x = -0.5$.

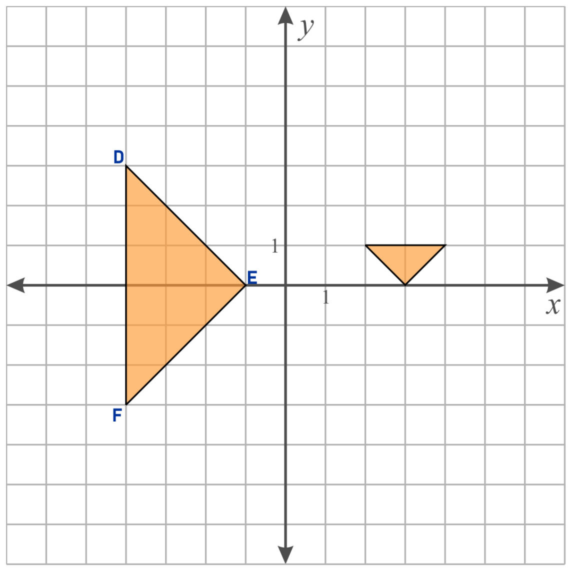
2. Translation 3 units to the left and 4 units down.

3. Dilation from point E" with scale factor 2.

What are the coordinates of the image of point F after all these transformations?

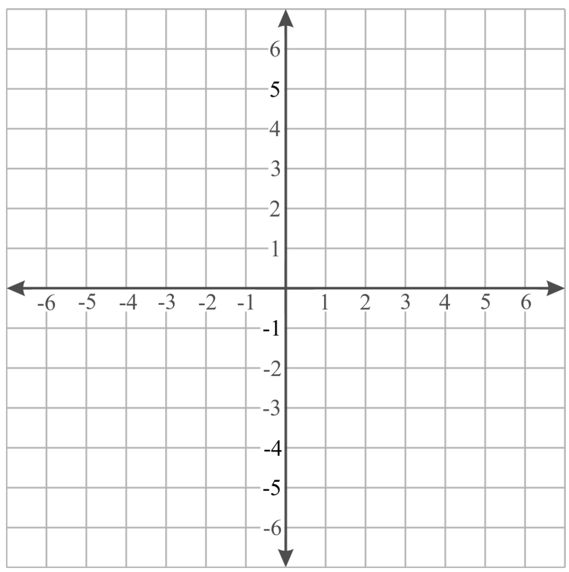


8. Show that the two triangles are similar by describing a sequence of transformations that could map △DEF to the smaller triangle



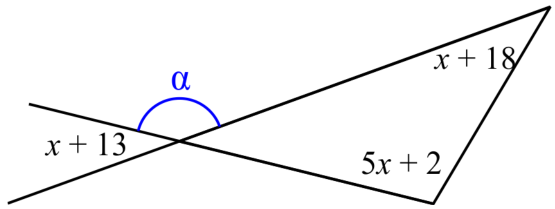
9. Figure PQRS underwent a dilation, then a rotation. Study the coordinates to find out the details about each transformation, then fill in the missing coordinates.

Original figure	**Dilation**	**Rotation**
P(−5, 3)	P'(−6, 5)	P"(___ , ___)
Q(0, 3)	Q'(4, 5)	Q"(___ , ___)
R(−1, 1)	R'(___ , ___)	R"(−4 , −5)
S(−4, 1)	S'(−4, 1)	S"(−4, 1)

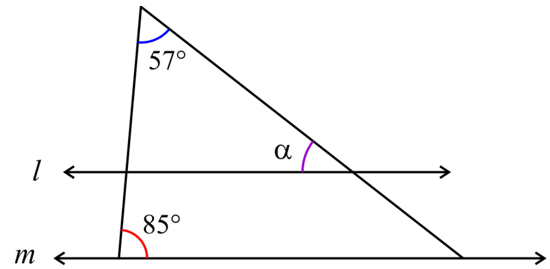


10. **a.** Find the value of *x*.

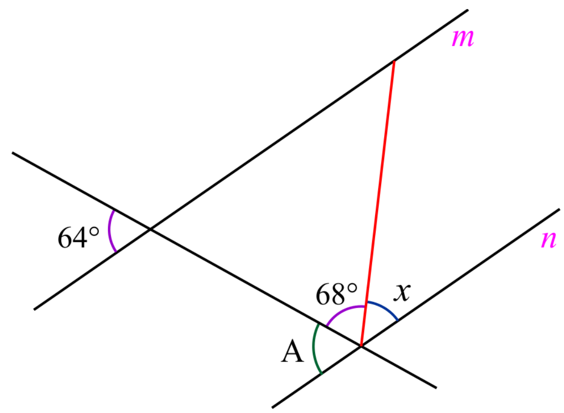
b. Find the value of α.



11. Lines *l* and *m* are parallel. Figure out the measure of the angle α. (You may need to mark more angles in the diagram.)



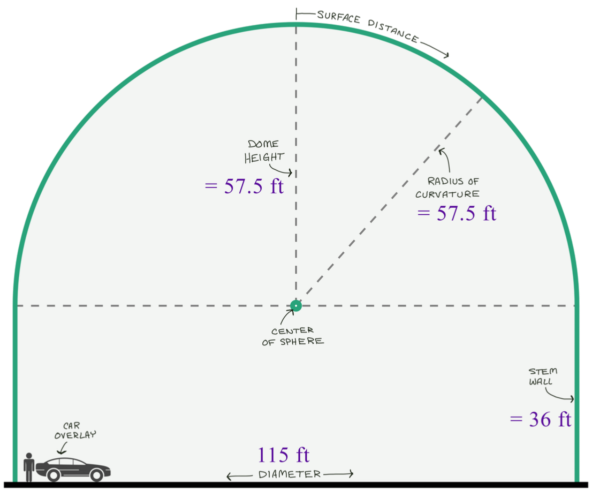
12. Lines *m* and *n* are parallel. Find the measure of angle *x*, and prove why it is what you find it to be. In other words, explain and justify your reasoning. You may need to mark more angles in the diagram.

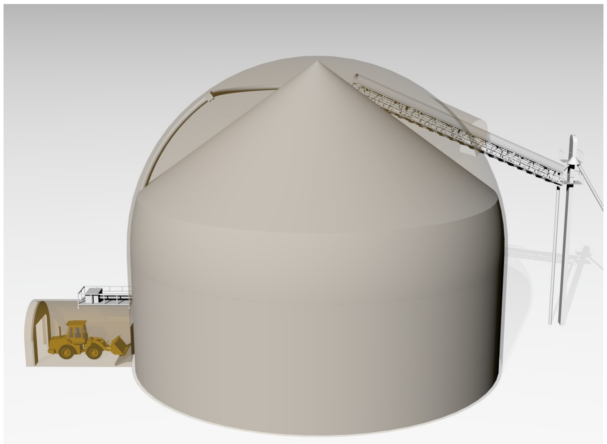


13. A shampoo bottle is in the shape of a circular cylinder. It says it contains 473 ml of shampoo. Its inner diameter is 6.0 cm and its height is 17 cm. What percent of the bottle does the shampoo take up?

14. Compare a sphere with radius 5 cm with a cone with the same radius. What is the height of the cone, given the two have the same volume?

15. Elkhart Ammonium Nitrate Storage in Elkhart, Texas, is a large building consisting of a half sphere on top of a circular cylinder. The stem wall is 36 ft high, and the diameter of the circle (which is also the diameter of the sphere) is 115 ft. When the storage is filled with ammonium nitrate, the top part of it (the part inside the half-sphere) forms a cone.





Images courtesy of Monolithic Dome Institute, www.monolithic.com

Find the volume of the ammonium nitrate mound when the cone reaches the top of the structure, to the nearest thousand cubic feet.

Chapter 3: Linear Equations
Introduction

The third chapter of Math Mammoth Grade 8 focuses both on the mechanics of solving linear equations and on problem solving.

The chapter starts with a vocabulary reference sheet. The first actual lesson is a review of integer addition and subtraction, which you can omit at your discretion. The next several lessons after that review simple equations of the form $px + q = r$ and $p(x + q) = r$ and and the distributive property from 7th grade.

The next step towards solving more complex equations is the lesson *Combining Like Terms*. Students add and subtract like terms, including with decimal or fractional coefficients, and solve equations where like terms need combined first.

Having learned this, students then tackle some typical algebraic word problems in the following lesson.

Then it is time to learn to solve equations where the variable is on both sides. There are often several possible solution pathways. Students also learn about the common error of adding or subtracting "across the sides."

The lesson *Simplifying Linear Expressions* focuses on how to remove parentheses after a minus sign, such as in the expression $2(3 + 2y) - 7(3 - 5y)$. After that, it is time for more practice and word problems, including age and coin word problems.

Then we turn our attention to equations with fractions, and the student learns to multiply both sides of the equation by a common multiple of the denominators. In the lessons on formulas, the student both solves various formulas for a variable in it, and uses formulas to solve a variety of word problems.

The lesson *More on Equations* deals with equations that have an infinite number of solutions (identities) or no solutions.

The chapter ends with two more lessons on word problems (percent word problems and miscellaneous problems).

Pacing Suggestion for Chapter 3

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 3	page	span	suggested pacing	your pacing
Algebra Terms (For Reference)	125	*(1 page)*		
Review: Integer Addition and Subtraction	126	*3 pages*	1 day	
Equations Review, Part 1	129	*4 pages*	1 day	
The Distributive Property	133	*3 pages*	1 day	
Equations Review, Part 2	136	*4 pages*	1 day	
Equations Review, Part 3	140	*4 pages*	1 day	
Combining Like Terms	144	*3 pages*	1 day	
Word Problems	147	*4 pages*	1 day	
A Variable on Both Sides	151	*4 pages*	1 day	

The Lessons in Chapter 3	page	span	suggested pacing	your pacing
Word Problems and More Practice	155	*3 pages*	1 day	
Simplifying Linear Expressions	158	*3 pages*	1 day	
More Practice	161	*3 pages*	1 day	
Age and Coin Word Problems	164	*3 pages*	1 day	
Equations with Fractions 1	167	*2 pages*	1 day	
Equations with Fractions 2	170	*3 pages*	1 day	
Formulas, Part 1	173	*2 pages*	1 day	
Formulas, Part 2	175	*2 pages*	1 day	
More on Equations	177	*3 pages*	1 day	
Percent Word Problems	180	*2 pages*	1 day	
Miscellaneous Problems	182	*2 pages*	1 day	
Chapter 3 Mixed Review	184	*3 pages*	1 day	
Chapter 3 Review	186	*3 pages*	1 day	
Chapter 3 Test (optional)				
TOTALS		*63 pages*	21 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch3

Scan me

Algebra Terms
For Reference

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as *x*, *y*, *a*, T, and so on.

Note: Expressions do *not* have an equals sign!

Examples of expressions:

$\frac{3}{5}x^2 - 3x + 5$ $\quad$ 5 $\quad$ $\left(\frac{3x}{y^2}\right)^2$

$T - 29$ $\quad$ $2^x - 5^y$

An **equation** has two expressions separated by an equals sign:

(expression 1) = **(expression 2)**

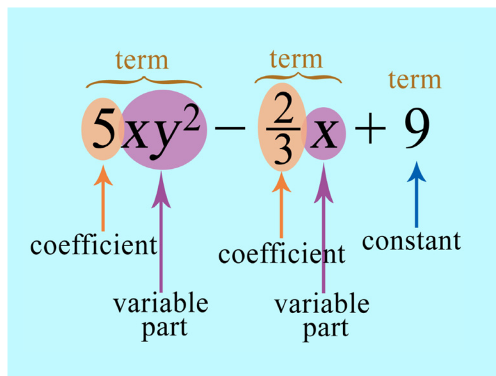
Examples of equations:

$0 = 0$ $\quad$ $2(z - 9) = -z^2$

$9 = -8$ (a false equation) $\quad$ $\frac{x + 3}{2} = -1.5$

A **term** is an expression that consists of numbers and/or variables that are *multiplied*. For example, $7x$ is a term and so is $0.6mn^2$.

A single number or a single variable is also a term. If the term is a single number, such as 4.5 or ¾, we call it a **constant**.



In the expression on the right, we have three terms: $5xy^2$, $\frac{2}{3}x$, and 9, that are separated by subtraction and addition.

If a term is not a single number, then it has a **variable part** and a **coefficient**.

- The coefficient is the single number by which the variable or variables are multiplied.
- The variable part consists of the variables and their exponents.

For example, in $4.3ab$, 4.3 is the coefficient, and *ab* is the variable part.

Note: a term that consists of variables only still has a coefficient: it is one. For example, the coefficient of the term x^3 is one, because you can write x^3 as $1 \cdot x^3$.

Example. Is $s - 5$ a term? No, it is not since it contains subtraction. Instead, $s - 5$ is an expression consisting of two terms, *s* and 5, separated by subtraction.

1. Write the expression based on the clues.

- It has four terms.
- The constant term is the square of the third smallest prime.
- The variable parts of the variable terms are *ab*, a^2, and *a*, respectively.
- The coefficients of the variable terms are the three consecutive integers with a sum of 21.
- The first two terms are separated by subtraction, the rest by addition.

Review: Integer Addition and Subtraction

Integers consist of the counting numbers (1, 2, 3, 4, ...), zero, and the negative counterparts of the counting numbers (−1, −2, −3, −4, ...). So, the set of integers is {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.

An **absolute value** of an integer is its distance from zero, and is marked with two vertical lines. For example, $|2| = 2$ and $|-18| = 18$.

We obtain the **opposite** or **negation** of an integer by changing its sign from positive to negative, or vice versa. For example, the opposite or negation of 17 is −17. The opposite of −4 is 4.
We can use the negative sign "–" to signify this: −(−5) means the opposite of −5, which is 5.

To **add several negative integers**, simply add their absolute values and write the answer as negative.	**Example 1.** To find the sum −8 + (−3) + (−7) + (−11), add 8 + 3 + 7 + 11 = 29. The value of the original sum is −29.

To **add a negative and a positive integer**, find the difference in their absolute values. The integer with the bigger absolute value determines the sign of the final answer.

Example 2. In the sum −9 + 11, the absolute values of the two integers are 9 and 11. Their difference is 11 − 9 = 2. This means the answer is either 2 or −2. To determine which, check the sign of the integer with the larger absolute value. In our case it is 11 (which is positive), so the answer is 2 (and not −2).

Example 3. In the sum 7 + (−12), the absolute values of the two integers are 7 and 12. Their difference is 12 − 7 = 5. This means the answer is either 5 or −5. To determine which, check the sign of the integer with the larger absolute value. Here it is −12 which is negative, so the answer is −5 (and not 5).

So, this is the mechanical rule, but you don't have to use it if you have learned other methods, such as visualizing a number line.

To **add several integers** where some are negative, some positive, first calculate the partial sums of all the negative integers and of all the positive ones. Lastly add those sums.	**Example 4.** −8 + 12 + (−9) + (−1) + 5 + (−6) = ? Positives: 12 + 5 = 17 Negatives: −8 + (−9) + (−1) + (−6) = −24 Total: 17 + (−24) = **−7**

1. Add.

a. (−4) + 8 =	**b.** 15 + (−25) =	**c.** −12 + 6 =	**d.** −11 + (−32) =

e. −12 + (−2) + (−5) =	**f.** 6 + (−1) + (−5) + 2 =	**g.** −7 + 10 + (−6) + 1 =

h. −11 + (−2) + 7 + (−5) + 4 + (−3) =	**i.** −6 + (−5) + 8 + (−12) + 24 + 1 =

(This page intentionally left blank.)

Word Problems

Example 1. The width of a rectangle is three times its length, and its perimeter is 28 m.
What are the width and the length of the rectangle?

First, check *what quantity* is unknown, and choose a variable for it.

We don't know the length nor the width of this rectangle. However, since we know that the width is three times the length, we can write the width as an expression of the length. In other words, we can let x be the length, and then naturally, the width is $3x$. We don't need another variable for the width.

In your solution, **you need to specify what *x* denotes.** You can write: "Let x be the length of the rectangle." After that, write something to the effect: "Then, $3x$ is the width." This is to ensure that a reader understands your thought process.

Continuing on with the solution, we know the perimeter is the sum of all four sides, and equals 28 m. From that we can write the equation: $x + 3x + x + 3x = 28$. Solving that will be easy!

1. Solve the equation in example 1. Lastly, answer the actual question (specify what the width and the height of the rectangle actually are).

2. The length of a certain rectangle is 12 cm more than its width. The perimeter is 136 cm. How much do the sides of the rectangle measure? Write an equation.

   Reminder: Note clearly what your chosen variable denotes in the problem.

Example 2. Susan and Henry divide a task of washing 170 windows in a large building in a ratio of 3:2. How many windows will Henry wash?

You have learned to solve this type of problem using a bar model. We can also use an equation (which will exactly match the basic idea in the bar model).

Let $3x$ be the amount of windows that Susan washes, and $2x$ the amount that Henry washes. This way, the windows they washed are in the ratio of 3:2. Since we know they washed a total of 170 windows, we can write the equation $3x + 2x = 170$.

3. Solve the equation in example 2. How many windows will Henry wash?

4. An inheritance of $354,000 was divided between three heirs in the ratio of 1:6:5.
How much did each heir get?

5. The two sides of a rectangle are in a ratio of 3:5 and its perimeter is 416 cm.
What are the dimensions of the rectangle?

Example 3. Another type of problem you have solved previously using a bar model is where the total is known, and the parts making up the total differ by a known amount.

Eric and Jeremy worked last week for a total of 99 hours, and Eric worked 5 more hours than Jeremy. How many hours did Eric work?

Let x be Jeremy's working hours. Then, Eric worked for $x + 5$ hours. Knowing the total, we can easily write an equation for this situation.

6. Write an equation for Example 3, and solve it. Lastly, answer the actual question that was asked.

7. Anna has a flock of chickens and a flock of ducks. She has 17 more chickens than ducks, and in total she has 135 birds. How many chickens does she have?

8. Hans has to cross a bridge and pay a $6 bridge toll when going to work (coming back, he doesn't have to pay it). Some days he carpools with two other people, and they share the toll fee equally.

   a. Hans noticed that in the last two weeks (which had 10 workdays), he had paid a total of $36 in bridge tolls. How many days of those 10 did he carpool?

   b. In a month (which had 22 work days), he had paid a total of $96 in bridge tolls. How many days of those did he *not* carpool?

9. The pet store had three different leashes for sale. The price of one was $5.40 more than the price of the cheapest, and the price of another was $11.60 more than the cheapest. If you bought all three, your bill came to $62.60. How much did the most expensive leash cost?

10. The sum of three consecutive whole numbers is 360. What are the numbers?

Hint: Let *x* be the first of the three consecutive whole numbers. What are the other two, in terms of *x*?

11. The sum of three consecutive odd numbers is 1,971. What are the numbers?

Hint: Let *x* be the first of the three consecutive odd numbers. What is the next odd number, in terms of *x*?

12. The sum of four consecutive multiples of 5 is 1,570. What are the numbers?

Puzzle Corner

It is possible to write a set of equations for the following problems, but you haven't studied those types of equations (quadratic) yet. So, use **guess and check**. It can work equally efficiently!

a. The sum of two numbers is 35, and their product is 300. What are the numbers?

b. The sum of two numbers is 220, and their product is 9,600. What are the numbers?

A Variable on Both Sides

Example 1. Solve $2x + 8 = -5x$.

Notice that the unknown appears on both sides of the equation. This is not a problem; we can still use the principle of doing the same operation to both sides in order to isolate the unknown on one side. In this case, we can either subtract $2x$ from both sides or add $5x$ to both sides. See both options below.

First subtract $2x$:

$$
\begin{aligned}
2x + 8 &= -5x && \mid -2x \\
8 &= -7x && \text{(Switch sides.)} \\
-7x &= 8 && \mid \div -7 \\
x &= -8/7
\end{aligned}
$$

First add 5x:

$$
\begin{aligned}
2x + 8 &= -5x && \mid +5x \\
7x + 8 &= 0 && \mid -8 \\
7x &= -8 && \mid \div 7 \\
x &= -8/7
\end{aligned}
$$

Check:

$$
\begin{aligned}
2 \cdot (-8/7) + 8 &\stackrel{?}{=} -5 \cdot (-8/7) \\
-16/7 + 8 &\stackrel{?}{=} 40/7 \\
-2\ 2/7 + 8 &\stackrel{?}{=} 5\ 5/7 \\
5\ 5/7 &= 5\ 5/7 \checkmark
\end{aligned}
$$

1. Solve the equation in two ways, as instructed.

First add $2s$:

$10 - 2s = 4s + 9 \quad \mid +2s$

First subtract $4s$:

$10 - 2s = 4s + 9 \quad \mid -4s$

2. Solve. Check your solutions (as always!).

a. $3x + 2 = 2x - 7$

b. $9y - 2 = 7y + 5$

3. A common student error is to add or subtract "across the sides," instead of carefully adding or subtracting the same quantity to/from both sides.

Here is an example of it: the student added $7w$ and $2w$, and wrote $9w$ on the next line. Correct the error and solve the equation.

$$7w + 8 = 2w - 5$$
$$9w + 8 = -5$$

4. Solve. Check your solutions (as always!).

a. $-2y - 6 = 20 + 6y$	**b.** $8x - 12 = -1 - 3x$	**c.** $6z - 5 = 9 - 2z$

5. Fred is contemplating two different job offers. In one, he gets paid \$19.50 per hour plus he will receive a bonus based on the sales he brings in, which he estimates to be about \$150 per week. In another job, he will earn \$21 per hour (no bonuses).

a. Write an expression for the weekly earnings in each job, for *m* hours of work.

Job 1: Job 2:

b. In which job would he earn more, if he worked 20 hours per week?

c. For what amount of work hours would both jobs provide him the same wages?

(This page intentionally left blank.)

Chapter 4: Introduction to Functions
Introduction

The fourth chapter of Math Mammoth Grade 8 covers various introductory topics from the theory of functions. These topics prepare students for studying functions in great detail in high school math, and even include preparatory ideas for calculus (rate of change).

The first lesson focuses on the basic definition of a function, as a relationship between two sets that assigns exactly one output for each input. It also briefly explains the range and domain of a function, even though those terms are not required in the CCS.

Next, we study the rate of change in the context of linear functions. Students calculate the rate of change from functions given as a table of values or from their graphs. They also encounter nonlinear functions and calculate the rate of change for those in specific intervals.

Then, students learn about the initial value of a function (its value when the input is zero), and learn that the equation $y = mx + b$ defines a linear function. They write and plot equations of that form to model linear relationships. We also spend one lesson looking at linear versus nonlinear relationships.

The following major topic is describing functions. Students analyze a graph and tell whether a function is increasing, decreasing, or constant; linear or nonlinear. They sketch a graph matching a given verbal description, and interpret given graphs of nonlinear functions in a variety of the real-life contexts.

Lastly, students compare properties of two functions represented in different ways (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, distance as a function of time is given as an equation for one airplane, and as a graph for another, and students answer questions concerning the speed and distance of the two airplanes.

Pacing Suggestion for Chapter 4

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 4	page	span	suggested pacing	your pacing
Functions	191	*4 pages*	1 day	
Linear Functions and the Rate of Change 1	195	*4 pages*	1 day	
Linear Functions and the Rate of Change 2	199	*3 pages*	1 day	
Linear Functions as Equations	202	*3 pages*	1 day	
Linear versus Nonlinear Functions	205	*3 pages*	1 day	
Modeling Linear Relationships	208	*4 pages*	1 day	
Describing Functions 1	212	*3 pages*	1 day	
Describing Functions 2	215	*3 pages*	1 day	
Describing Functions 3	218	*4 pages*	1 day	
Comparing Functions 1	222	*3 pages*	1 day	
Comparing Functions 2	225	*2 pages*	1 day	
Chapter 4 Mixed Review	227	*3 pages*	1 day	
Chapter 4 Review	230	*4 pages*	2 days	
Chapter 4 Test (optional)				
TOTALS		*43 pages*	14 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch4

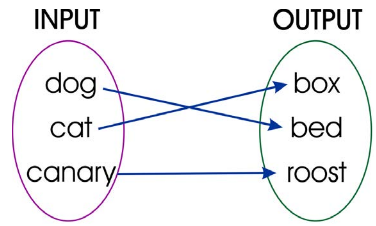
Scan me

Functions

A **function** is a rule or a relationship between two sets that assigns **exactly one output for each input.** We also use the word **mapping** for a function.

Example 1. The illustration below shows a simple function that maps each animal to its favorite sleeping place.

Each animal has a sleeping place, and only one, so this is a function.

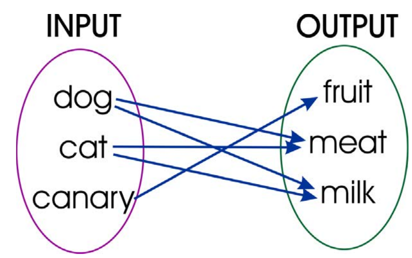


Example 2. The table lists the name of seven children, and the month when each child has their birthday. Notice that several of them have their birthday in December. Is this a function?

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	September	December	December	June	August	December	February

Yes. The definition only requires that there has to be exactly one output for each input; **the outputs don't have to be unique.**

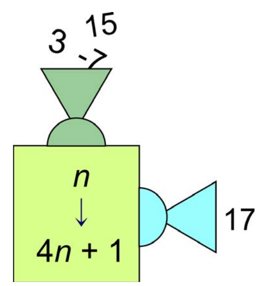
1. The relationship shown on the right is *not* a function. Why?



2. A function machine "ingests" a number (the input) and "spits out" another (the output) based on some rule. This function machine turns any number n into $4n + 1$.

   a. Number −7 is just going in. What will be the output?

   b. Number 17 just came out. What was the input?



3. Potatoes costs $3 per kilogram. Fill in the tables #1 and #2.

   Does each table represent a function? Explain.

#1

(Input) **Weight**	(Output) **Cost**
1 kg	$3
2 kg	
3 kg	
5 kg	
12 kg	

#2

(Input) **Cost**	(Output) **Weight**
$12	
$30	
$48	
$72	
$90	

4. The table lists seven children, and each child's favorite color.

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	pink and blue	blue	gray	yellow	blue and red	?	purple

Is this a function? If not, change it in some manner(s) so it *is* a function.

5. T is a function that maps the name of a month to the number of days in it.

a. Create a depiction of T using a diagram like in example 1.

b. If you reverse the inputs and outputs, is the resulting relationship a function? Explain.

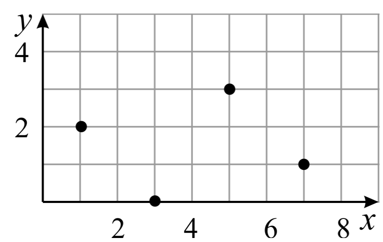
If the inputs and outputs are numbers, we can plot **a graph of the function** in the coordinate grid. Each input-output pair is viewed as an ordered pair (a single point).

We also use the terms "independent variable" for the input, and "dependent variable" for the output.

Example 3. Let F be the function (1, 2), (3, 0), (5, 3), (7, 1).

Note: A function *can* be given as a list of ordered pairs.

The image on the right is the plot of F; yet the plot is *not* F. The function F is the specific list of inputs and outputs, or the relationship itself.



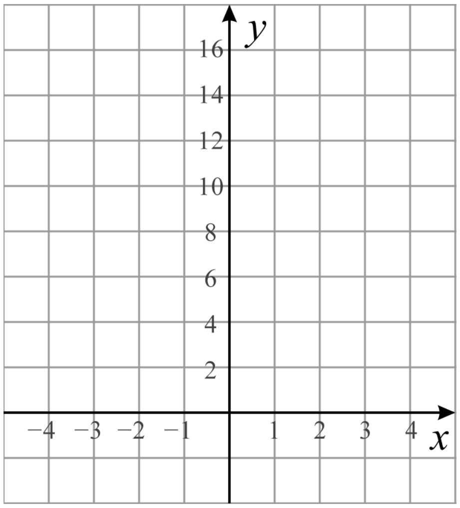
6. Let G be the function that maps each integer from −4 to 4 to its square minus one.

a. Fill in the table, listing the ordered pairs of G.

Input (x)	−4	−3	−2						
Output (y)	15								

b. Make a plot of G.

c. If you reversed the inputs and the outputs, would the relationship still be a function? Explain.



Example 4. Mary bicycled from her home to a friend's house. The table shows the distance (d) Mary had covered at specific amounts of time (t).

Input (t)	5 min	10 min	12 min	13 min	15 min	18 min	20 min	22 min
Output (d)	0.8 km	1.5 km	1.9 km	1.9 km	1.9 km	2.4 km	2.7 km	3 km

We say that **distance is a function of time.** The output variable, or the dependent variable, is always said to be a function of the input (or independent) variable. This means that for each moment of time (input) there is a specific distance she has traveled (output).

Is it true in reverse? Is *time* a function of *distance*?

This means we consider distance as the input, and time as the output. If yes, then for each distance (input), there is exactly one time (output). Is that so in this case?

7. Is Age a function of Name? Explain.

Is Name a function of Age? Explain.

Name	Age
FenFen	14
Larry	15
Pierre	13
Sam	12
Amy	14

Age	Name
14	FenFen
15	Larry
13	Pierre
12	Sam
14	Amy

8. Choose the relationships that are functions.

(1)

Rainfall (mm)	2	0	0	5	0	13	0
Day of month	6	7	8	9	10	11	12

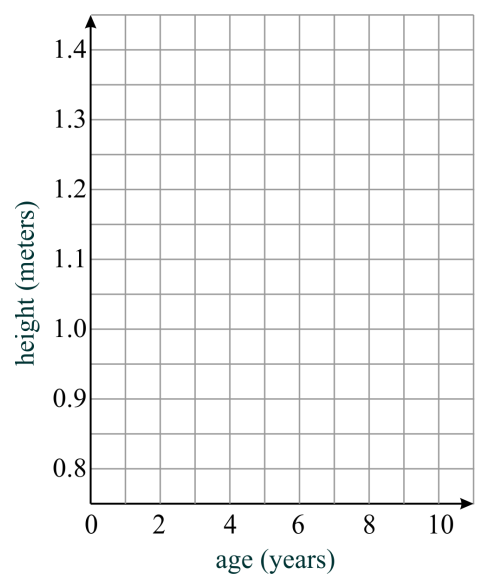
(2) Let S be a rule that takes any number x as input, and gives $4x + 1$ as output.

(3) Input is a zip code, output is a person that lives there.

(4) Input is a person's first name, output is their bank account number.

9. Plot the following points that give the age (in years) and the height (in meters) of various children.

(2, 0.8) (5, 1.05) (10, 1.40) (9, 1.31) (6, 1.17) (5, 1.09)



a. Is this a function? Explain.

b. What is the independent variable? The dependent variable?

(Optional content; beyond the CSS)

The **domain** of a function is the set of inputs. The **range** of a function is the set of outputs.

Let's go back to example 3, where we had kindergartners and their birthday months.

Input	Allie	Julie	Danny	Juan	Pete	Bob	Samantha
Output	September	December	December	June	August	December	February

The domain of this function is the list of the children's names. To write it as a set, we enclose the items of the set in curly brackets: {Allie, Julie, Danny, Juan, Pete, Bob, Samantha}.

The range of this function is {September, December, June, August, February}.

10. **a.** Change some thing(s) in this table so it is a function.

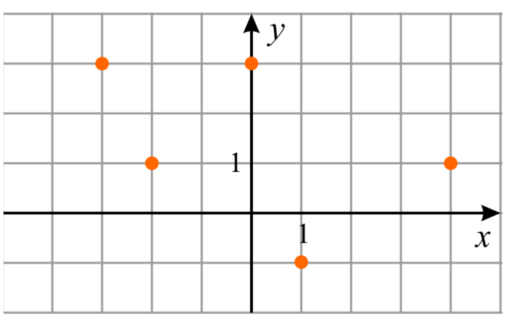
b. Give the domain of the function.

c. Give the range of the function.

Input	Output
Name	**Grade level**
Jenny	8
Pedro	7
Ann	8
Marsha	
Rob	9
Ann	6

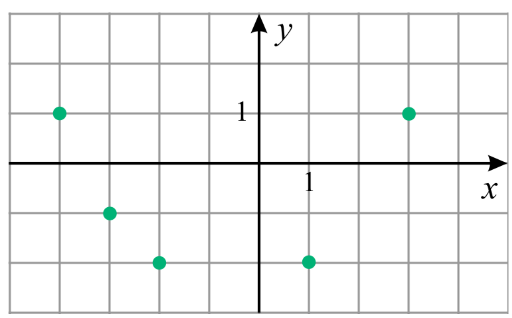
11. Let F be the function that maps a number x to $2x + 1$.
Let the set {0, 1, 2, 3, 4, 5} be its domain.
What is its range?

12. Give the domain and range of each function.

a.


Domain:

Range:

b.


Domain:

Range:

13. Let S be the function that allows any word from this sentence as the input, and the output is the number of letters in it. What is the range of this function?

14. G is a function that maps a number x to $x - 5$.
If the set {0, 5, 10, 15, 20} is its range, what is its domain?

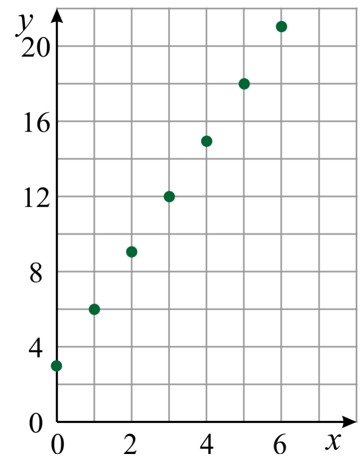
Linear Functions and the Rate of Change 1

If the graph of a function consists of points that fall on a single line, it is a **linear function**.

We will define a linear function in a different manner later, but for now, this is sufficient, so let's look at some examples.

Example 1. The input and output values in the table below define a function. Notice the patterns: the x-values increase by ones, and the y-values increase by 3s.

Input (x)	0	1	2	3	4	5	6
Output (y)	3	6	9	12	15	18	21



The graph shows that the points fall on a line. This is a linear function.

The **rate of change** of a function is the rate at which the output values change as compared to the change in the input values.

We calculate it as the ratio of $\frac{\text{change in output values}}{\text{change in input values}}$.

In the context of this graph, **rate of change =** $\frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}$.

In this case, each time the x-values increase by 1, the y-values increase by 3. **The rate of change is 3/1 = <u>3</u>**.

Example 2. The price of bananas is a function of their weight. What is the rate of change?

Weight in kg (input)	0	2	5	10	12	15
Price in $ (output)	0	5	12.50	25	30	37.50

Check how much the output (price) changes for a certain change in the input (the weight). For example, when the weight increases from 0 to 2 kg, the price increases from $0 to $5, or by $5. This happens also when the weight increases from 10 to 12 kg: the price increases $5 (from $25 to $30).

$$\text{Rate of change} = \frac{\$5}{2\text{ kg}} = \$2.50/\text{kg}$$

Note that if the independent and dependent variables have units, **we include the units in the rate of change**.

This rate of change tells us that for each one-kilogram increase in weight, the price increases by $2.50.

1. **a.** Calculate the rate of change in example 2, using the increase in weight from 5 to 10 kg, and the corresponding increase in price. Do you get the same rate of change as calculated in the example?

   b. Do the same using the input values 10 kg and 15 kg.

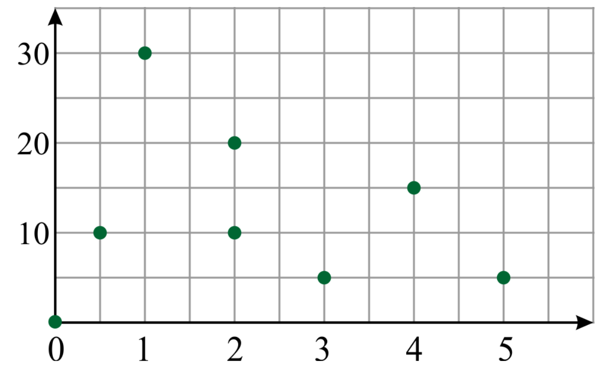
(This page intentionally left blank.)

Chapter 4 Review

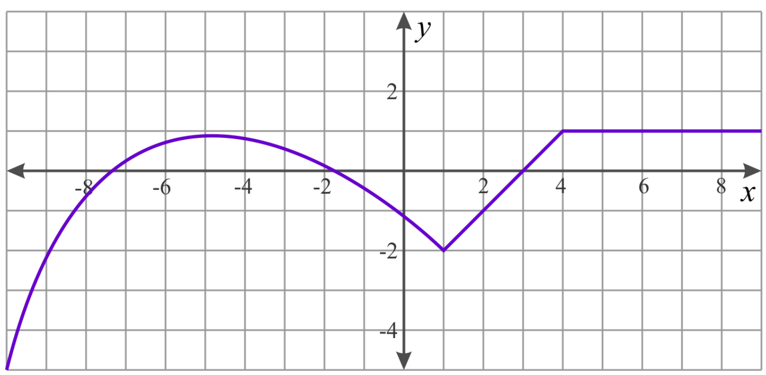
1. Change some thing(s) in this table so it represents a function.

Input	Output
Name	**Age**
Fifi	2
Bella	5
Max	
Luna	2
Charlie	6
Luna	3

2. Why is the relationship depicted by the graph *not* a function?



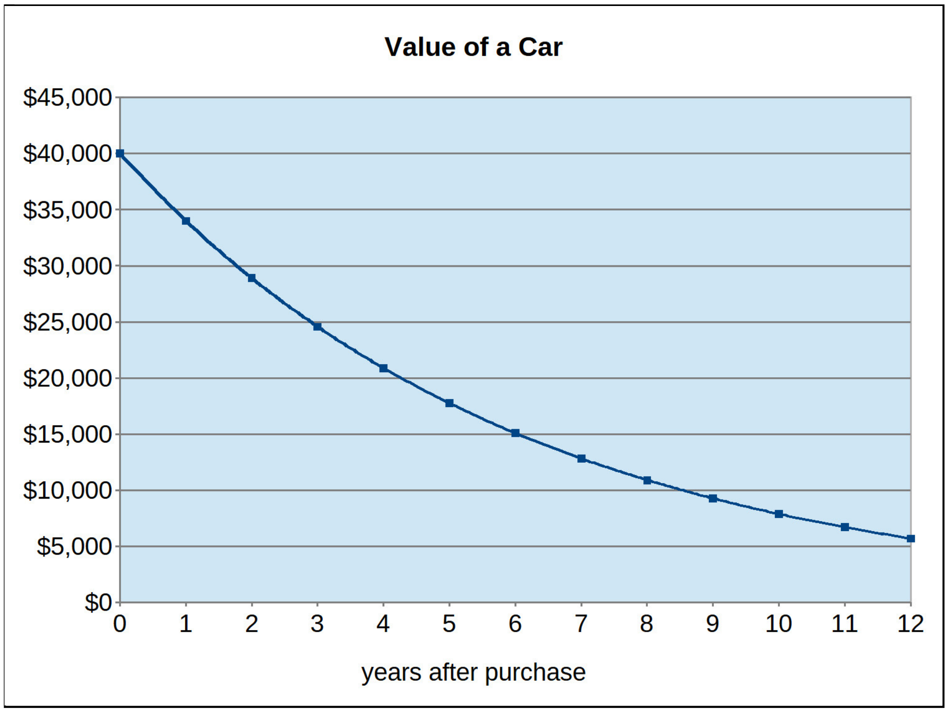
3. Describe this function by intervals where it is increasing, decreasing, or constant. Include also whether it is linear or nonlinear in those intervals.



4. Give an example where the cost of something is a function of time, and where the function is *not* linear. Use a table of values to give the function.

Time (hours)	0	1	2	3	4	5	6	7	8	9	10	11	12
Cost ($)													

5. The graph shows the value of a car over time.



a. Is this a linear function?
How can you tell?

Use approximate values that you can read from the graph, and find the rate of change:

b. from 1 to 2 years after purchase

c. from 6 to 7 years after purchase

6. Jayden and his sister race on bicycles from school back home, a route that is 1.5 miles long. The equation $d = 0.22t$ represents the distance (d, in miles) that Jayden has ridden, as a function of time (t, in minutes). The table below shows the distance his sister has bicycled, at various points in time.

Function 1 — Jayden:

$d = 0.22t$

Function 2 — his sister:

time (minutes)	0	1	2	3	4	5	6
Distance (mi)	0	0.2	0.45	0.68	0.9	1.12	1.24

a. Which function has a greater rate of change from $t = 2$ to $t = 4$ minutes?
What does that represent in terms of real life?

b. Classify each function as either linear or nonlinear.

c. Assume his sister continues with the same speed till the end as what she is riding between 5 and 6 minutes. Who will reach home first?

7. The table below shows the depth of the snow as a function of time from the beginning of a blizzard.

time (hours)	0	1	2	3	4	5	6
depth (inches)	17	20	23	26	29	32	35

a. What is the rate of change?

What does it mean in this situation?

b. What is the initial value?

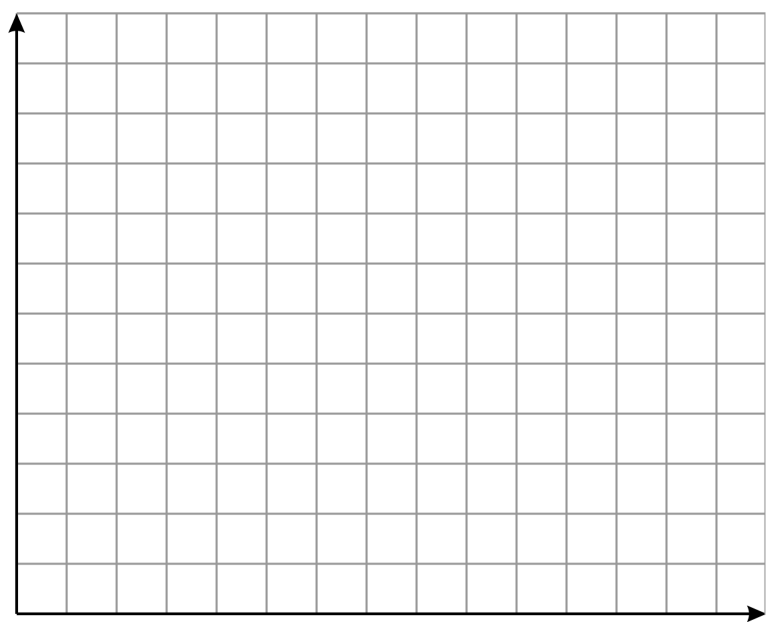
What does it mean in this situation?

c. Write an equation to represent the relationship between the amount of snow and time in hours.

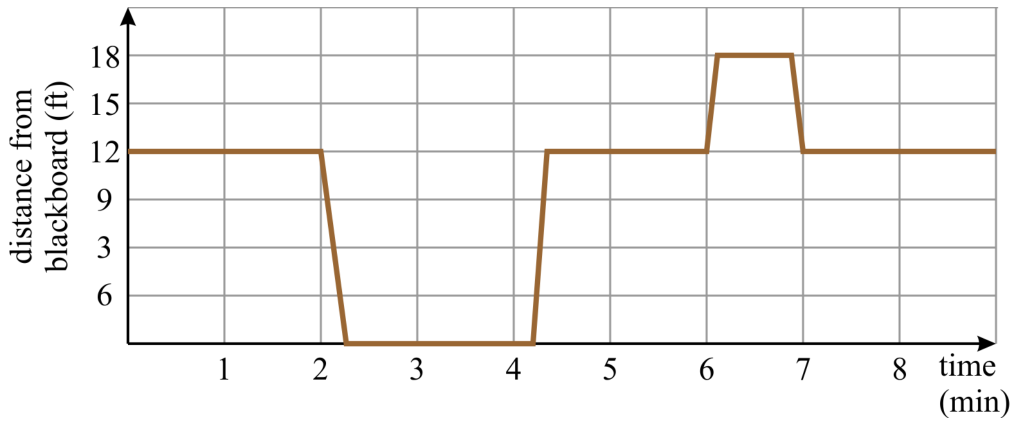
d. If the blizzard started at 2:30 PM, at what time was the snow 26.5 inches deep?

e. How deep will the snow be at 9 PM?

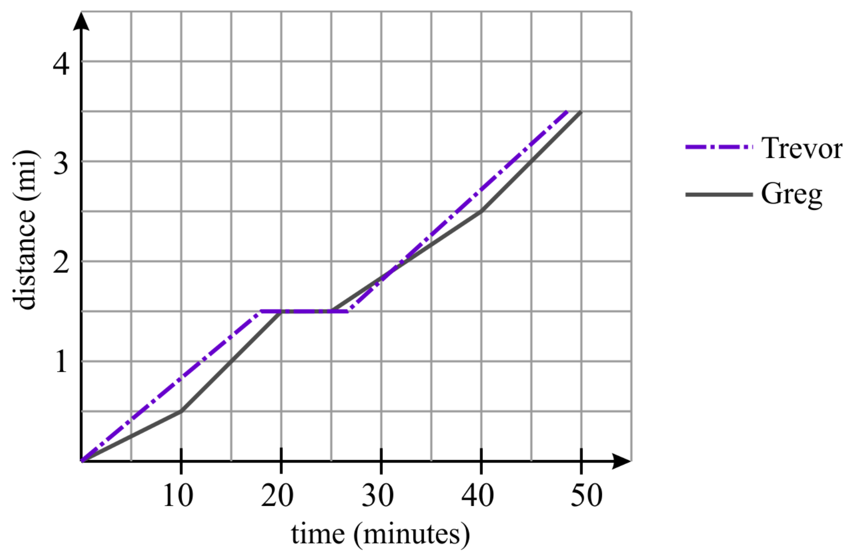
f. Plot the equation you wrote in (c).



8. Marcia is in math class (in a classroom). The graph shows the distance between her and the blackboard, as a function of time. Make up a story that matches the graph.



9. Greg is running along his usual running route, and his son Trevor goes with him riding a bicycle, but they don't exactly stay together. The two graphs show the total distance each has covered, as a function of time.



a. From 10-20 minutes, who is going faster?

b. Find two points in time where the two meet. What distance have they traveled at those times?

c. Who finishes the route first? About how much quicker than the other (estimate from the graph)?

(This page intentionally left blank.)

Contents

Chapter 7: Systems of Linear Equations

Chapter 8: Bivariate Data

Foreword

Math Mammoth Grade 8 comprises a complete math curriculum for the eighth grade mathematics studies. The curriculum meets the Common Core standards.

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics.

The main areas of study in Math Mammoth Grade 8 are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Bivariate data.

This book, 8-B, covers the topic of graphing linear equations. The focus is on the concept of slope.

In chapter 6, our focus is on square roots, cube roots, the concept of irrational numbers, and the Pythagorean Theorem and its applications.

Next, in chapter 7, students solve systems of linear equations, using both graphing and algebraic techniques. There are also lots of word problems that are solved using a pair of linear equations.

The last chapter then delves into bivariate data. First, we study scatter plots, which are based on numerical data of two variables. Then we look at two-way tables, which are built from categorical bivariate data.

Part 8-A covers exponent laws, scientific notation, geometry, linear equations, and an introduction to functions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

(This page intentionally left blank.)

Chapter 5: Graphing Linear Equations
Introduction

This chapter focuses on how to graph linear equations, and in particular, on the concept of slope in that context.

We start by graphing and comparing proportional relationships, which have the equation of the form $y = mx$. Students are already familiar with these, and know that m is the constant of proportionality. In this chapter, they learn that m is also the slope of the line, which is a measure of its steepness.

Then we go on to study slope in detail, its definition as the ratio of the change in y-values and the change in x-values. Students learn that it doesn't matter which two points on a line you use to calculate the slope, and study a geometric proof of this fact. They practice drawing a line with a given slope and that goes through a given point, and determine if three given points fall on the same line.

Then it is time to study the slope-intercept equation of a line, and connect the idea of an initial value of a function (chapter 4) with the concept of y-intercept in the context of graphing. Students graph lines given in the slope-intercept form, and write equations of lines from their graphs.

Next, we study horizontal and vertical lines and their simple equations. The standard form of a linear equation follows next. The last major topic is how the slope reveals to us whether two lines are parallel or perpendicular to each other.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 5	page	span	suggested pacing	your pacing
Graphing Proportional Relationships 1	13	*3 pages*	1 day	
Graphing Proportional Relationships 2	16	*3 pages*	1 day	
Comparing Proportional Relationships	19	*4 pages*	1 day	
Slope, Part 1	23	*4 pages*	1 day	
Slope, Part 2	27	*3 pages*	1 day	
Slope, Part 3	30	*5 pages*	2 days	
Slope-Intercept Equation 1	35	*4 pages*	1 day	
Slope-Intercept Equation 2	39	*3 pages*	1 day	
Write the Slope-Intercept Equation	42	*3 pages*	1 day	
Horizontal and Vertical Lines	45	*3 pages*	1 day	
The Standard Form	48	*3 pages*	1 day	
More Practice (optional)	51	*(2 pages)*	(1 day)	
Parallel and Perpendicular Lines	53	*3 pages*	1 day	
Mixed Review Chapter 5	56	*3 pages*	1 day	
Chapter 5 Review	59	*4 pages*	1 day	
Chapter 5 Test (optional)				
TOTALS		*48 pages*	15 days	
with optional content		*(50 pages)*	(16 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch5

Scan me

Graphing Proportional Relationships 1

We will now review what it means when two variables are **in direct variation** or **in proportion**. The basic idea is that whenever one variable changes, the other varies (changes) proportionally or at the same rate.

Example 1. The wholesaler posted the following table for the price of potatoes:

weight (kg)	5	10	15	20	25	30
cost	$5.50	$11.00	$16.50	$22.00	$27.50	$33.00

Each pair of cost and weight forms a rate — and so does each pair of weight and cost. However, it is more common to look at the rate "cost over weight", such as $27.50/(25 kg), than vice versa.

If all of the rates in the table are equivalent, then the weight and the cost *are* proportional.

To check for that, we have several means. One is to calculate **the unit rate** (the rate for 1 kg) from each of these rates, and check whether you get the same unit rate.

In this case, that is so. The unit rate is $1.10/kg, no matter which rate from the table we'd use to calculate it.

One other way to check is, if one quantity doubles (or triples), will the other double (or triple) also?
This is especially useful for noticing if the quantities are *not* in direct variation.

Example 2. Here, when the weight doubles from 5 kg to 10 kg, the price also doubles. But what happens with the price when the weight doubles from 10 kg to 20 kg?

weight (kg)	5	10	15	20	25	30
cost	$6	$12	$18	$22	$26	$30

The price does not double! So, the quantities are not in proportion.
The seller is giving you some discount if you purchase higher quantities.

Also, if you calculate the unit rate from $6/(5 kg) and from $22/(20 kg), they are not equal. (Verify this.)

1. Are the quantities in a proportional relationship? If yes, list the unit rate.

a.

time (hr)	0	1	2	3	4	5
distance (km)	0	50	90	140	190	240

b.

time (hr)	0	1	2	3	4	5
distance (km)	0	45	90	135	180	225

c.

age (days)	0	1	2	3	4	5	6	7
height (in)	0	0	0	1	2	3	4	5

d.

length (m)	0	0.5	1	1.5	2	4	5	10
cost ($)	0	3	6	9	12	24	30	60

2. Now consider the tables of values in #1 as functions, where the variable listed on top is the independent variable. For the ones where the quantities were in proportion, calculate the rate of change.

   What is its relationship to the unit rate?

When two quantities are in a proportional relationship, or in direct variation (the two are synonyms):

(1) Each rate formed by the quantities is equivalent to any other rate of the quantities.

(2) The equation relating the two quantities is of the form $y = mx$, where y and x are the variables, and m is a constant. The constant m is called the **constant of proportionality** and is also the unit rate.

(3) When plotted, the graph is a straight line that goes through the origin.

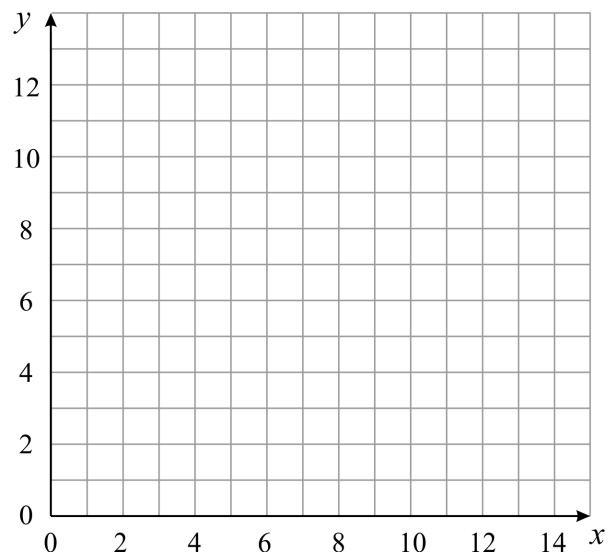
3. Choose an equation from below where the variables x and y are in direct variation (proportional):

$y = \frac{3}{x}$ $\quad\quad$ $y = 3x$

$xy = 3$ $\quad\quad$ $y = x^3$

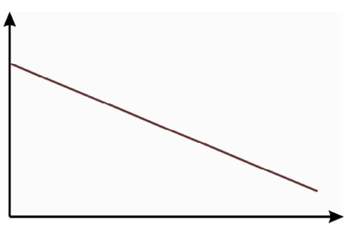
Then graph that equation in the grid.

Hint: The point (0, 0) is always included in direct variation. All you need to do is plot one other point, and then draw a line through the origin and that point.

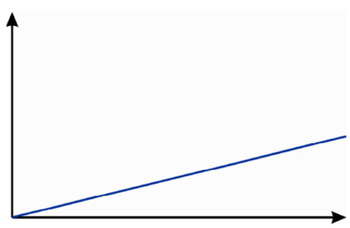


4. Choose the representations that show a proportional relationship.

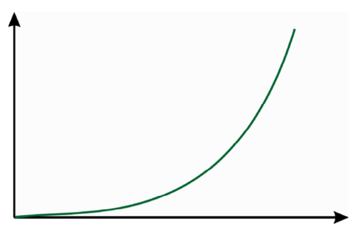
a.



b.



c.



d.

x	0	1	2	3	4	5
y	15	17	19	21	23	25

e. $y = 2x + 9$

f. $y = (3/4)x$

g.

x	0	4	8	12	16	20
y	0	3	6	9	12	15

5. Two of the above representations are the exact same relationship. Which ones?

Example 2. In a direct variation, *y* = 9 when *x* = 12. Write an equation for the relationship.

Since this is direct variation (proportional relationship), the equation is of the form *y* = *mx*, where *m* is the constant of proportionality.

The constant of proportionality is the ratio **(dependent variable)/(independent variable)**, so in this case it is *y*/*x* = 9/12, or 3/4. So, the equation is *y* = (3/4)*x*.

At this point, it is good to check that the point (12, 9) satisfies the equation, to check for errors:
Is it true that 9 = (3/4) · 12 ? Yes, it is.

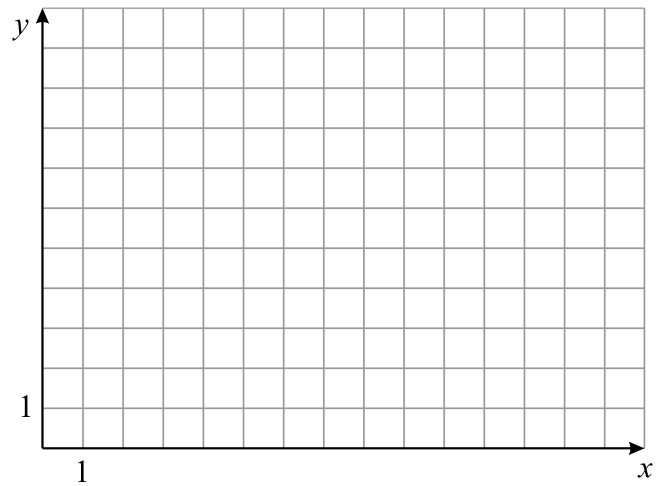
To graph the equation, we could simply plot the point (12, 9), and draw a line through it and the origin.

6. In a direct variation, when *x* is 14, *y* is 10.

a. Write an equation for this proportional relationship.

b. Graph a line for this relationship in the grid.

c. What is *x* when *y* = 40?

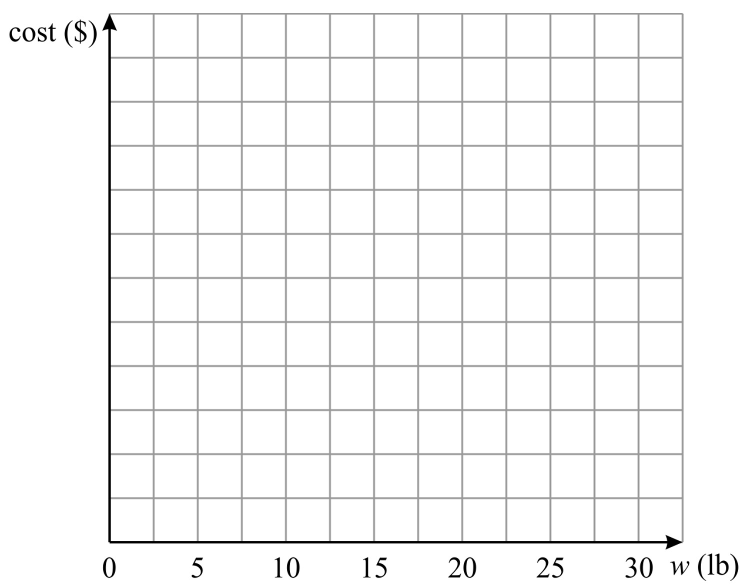


7. Organic rolled oats cost $20 for 8 lb.

a. Write an equation for this proportional relationship, using the variables C for cost and *w* for the amount (weight) of oats.

b. Graph the equation in the grid. Design the scaling on the cost-axis so that the point corresponding to 30 pounds fits on the grid.

c. How much do 36 lb of the oats cost?



8. If *y* is 120 when *x* is 400 in a direct variation, then what is *y* when *x* is 80?

Graphing Proportional Relationships 2

Example 1. The graph for the cost of apples as a function of their weight is a line through the origin, which means the cost and the weight are proportional.

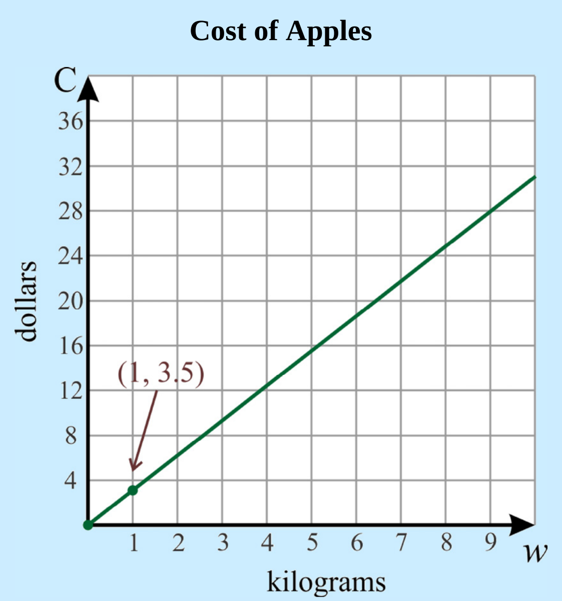
The equation for the graph is C = 3.5*w*. So, the constant of proportionality is 3.5, which is also the rate of change of this function.

The unit rate is $3.50/kg. This corresponds to the point (1, 3.5) on the graph.

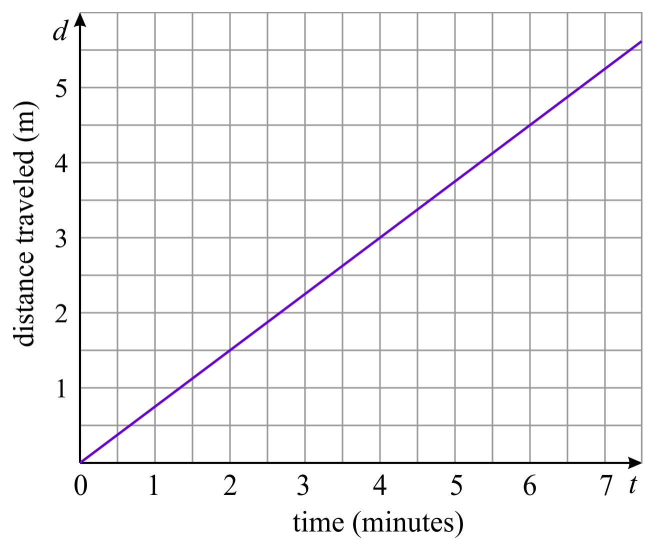
We also talk about **the slope of the line**, which is a measure of the steepness of the line, or how quickly it rises upwards (or slopes downwards).

It is the same idea as the rate of change, but in the context of graphing. Here, for each 1-kg increase in weight, the cost increases by $3.50. This means the slope of this line is 3.5 — the same as the rate of change, and the unit rate.

We will look at slope in more detail in other lessons.



1. The graph shows the distance a caterpillar has crawled over time.



a. Are the quantities *time* and *distance* proportional?

How can you tell?

b. What is the caterpillar's speed? Use the same units as in the graph.

What is the unit rate?

What is the slope of the line?

c. Write an equation for the graph.

d. Continuing with the same speed, how long will the caterpillar take to travel 9.5 meters?

2. Another caterpillar crawls 4.5 meters in 5 minutes. Draw another line in the grid for question #2, for the distance that this second caterpillar crawls over time, going with the same speed. Is this second caterpillar faster than the first?

(This page intentionally left blank.)

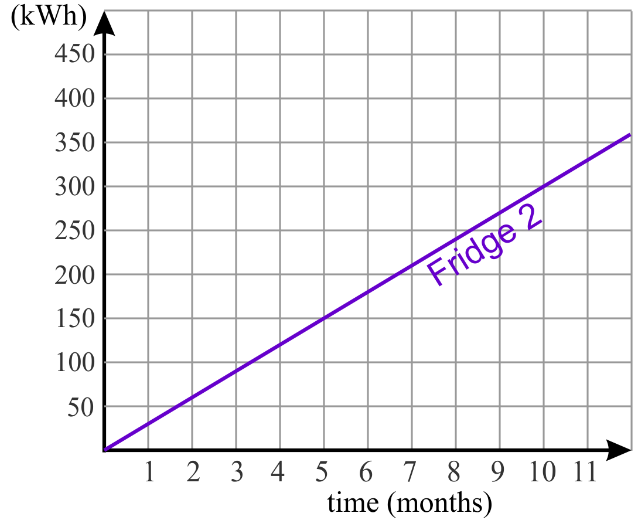
Chapter 5 Review

1. Refrigerator companies make estimates of how much energy their fridges consume in typical usage. The table shows how many kilowatt-hours (kWh) of energy fridge 1 consumed over time, and the graph shows the same for fridge 2.

Fridge 1

time (mo)	energy (kWh)
2	75
4	150
6	225
8	300
10	375
12	450

Fridge 2



a. Which fridge consumes more electricity in a month?

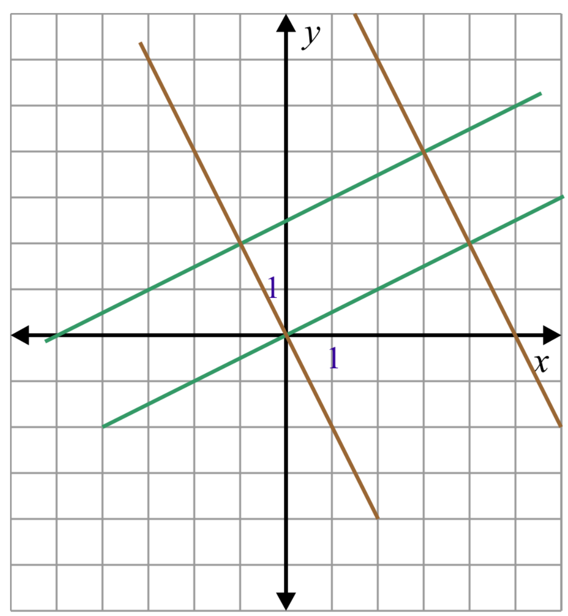
How much more?

b. Write an equation for each fridge, relating the energy (E, in kWh) and the time (t, in months).

c. Plot the equation for Fridge 1 in the grid.

d. Plot the point corresponding to the unit rate, for Fridge 1.

2. **a.** Find the equations of the four lines, in slope intercept form.



b. (optional) Find the area of the rectangle.

3. Find the equation of each line, in slope-intercept form:

a. has slope 3/4 and passes through (−2, 3)

b. is horizontal and passes through (9, −10)

4. Find the slope of the lines.
Notice the scaling.

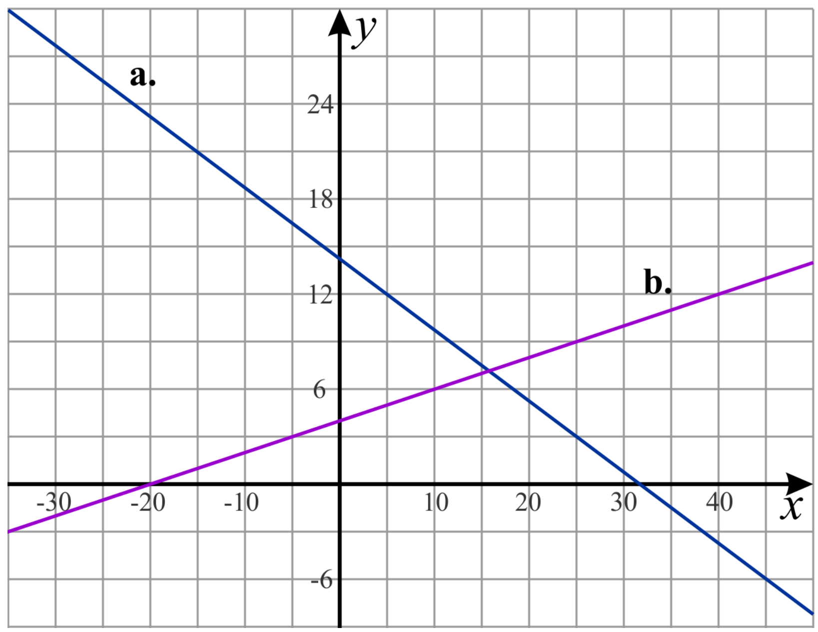
a.

b.

Now find the equations for the lines.

a.

b.



5. Do the three points fall on one line? Explain your reasoning.

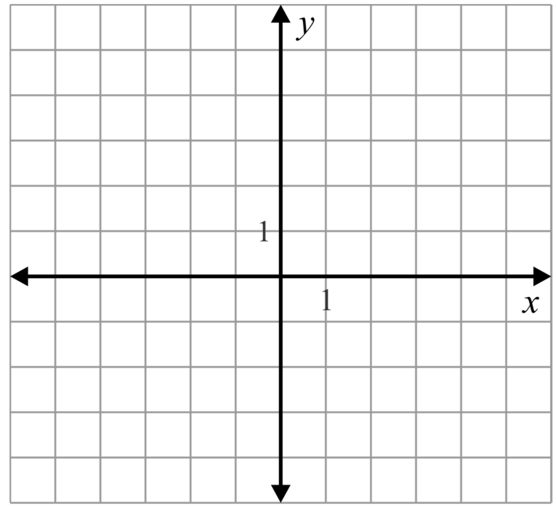
(−3, 1), (−1, −4), (1, −8)

6. Find *s* so that the point (*s*, 12) will fall on the same line as the points (3, 9) and (15, 18).

7. Line S passes through (−5, −2) and (0, 4). Line T is perpendicular to Line S, and passes through (1, 1).

a. Find the equation of line T, in slope-intercept form.

b. Write the equation also in the standard form.



8. Mr. Henson runs a garbage pick-up business, with 12 garbage trucks. To run one truck costs him $1,500 per month in maintenance costs, plus $110 a day for fuel.

Consider the cost of running one truck as a function of time, in days (during one month only). Is this a linear relationship, a proportional relationship, or neither?

Write an equation for it.

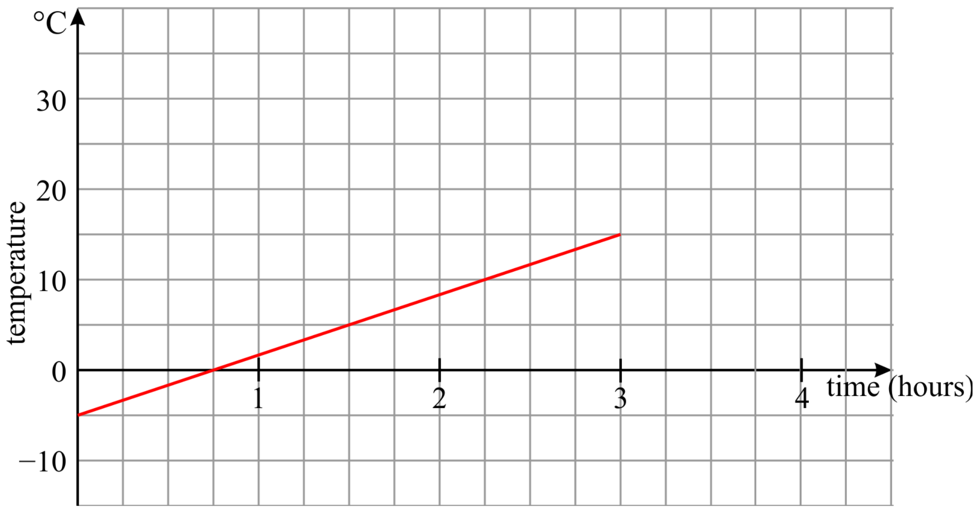
9. Match the descriptions and the equations.

$y = (-4/3)x - 7$	Is parallel to $x = 9$ and passes through (2, 7)
$3x - y = -21$	Has y-intercept −4 and is perpendicular to $y = -2x$.
$y = -4$	Passes through (−5, 6) and has slope 3.
$x - 2y = 8$	Passes through (−9, 5) and (−3, −3)
$x = 2$	Passes through (−3, 0) and (0, 9)
$y = 3x + 9$	Has y-intercept −4 and is parallel to $y = -2$.

10. Transform each equation of a line to the standard form, and then list its x and y-intercepts.

a. $y - 6 = 2(x + 2)$	**b.** $-\frac{1}{3}x - \frac{3}{2}y = 1$

11. A heater was turned on at 10 AM in a cold, uninhabited house, to prepare it for people later that day. The graph shows the temperature of the house. The count of hours starts at 10 AM.



a. Write an equation for the line.

b. If the temperature continues to rise in the same fashion, what will the temperature be at 2:30 PM?

c. When will the temperature reach 22°C?

d. Let's say the heater is turned off at 1:45. What is the temperature at that time?

e. If the house had started out at a temperature of −12°C instead, and the heating process worked in the same fashion (the temperature rose at the same rate), at what time would the house reach a temperature of 22°C?

Chapter 6: Irrational Numbers and the Pythagorean Theorem

Introduction

We start out this chapter by studying the concept of a square root, as the opposite operation to squaring a number. In the next lesson, on irrational numbers, students find values of square roots by hand. They make a guess and then square the guess, and based on how close the square of their guess is to the radicand, they refine their guess until desired accuracy is reached. This will help solidify the concept of a square root, while also showing how most square roots are nonending decimal numbers, and how in real life, we need to use approximations of them to do calculations. Students also practice placing irrational numbers on the number line, using mental math to find their approximate location.

Next, the chapter has a review lesson on how to convert fractions to decimals. The following lesson has to do with writing decimals as fractions, and teaches a method for converting repeating decimals to fractions.

Then it is time to learn to solve simple equations that involve taking a square or cube root, over the course of two lessons. After learning to solve such equations, students are now fully ready to study the Pythagorean Theorem and apply it.

The Pythagorean Theorem is introduced in the lesson by that name. Students learn to verify that a triangle is a right triangle by checking whether it fulfills the Pythagorean Theorem. They apply their knowledge about square roots and solving equations to solve for an unknown side in a right triangle when two of the sides are given.

Next, students solve a variety of geometric and real-life problems that require the Pythagorean Theorem. This theorem is extremely important in many practical situations. Students should show their work for these word problems to include the equation that results from applying the Pythagorean Theorem to the problem and its solution.

There are literally hundreds of proofs for the Pythagorean Theorem. In this book, we present one easy proof based on geometry (not algebra). As an exercise, students are asked to supply the steps of reasoning to another geometric proof of the theorem. Students also study a proof for the converse of the theorem, which says that if the sides of a triangle fulfill the equation $a^2 + b^2 = c^2$ then the triangle is a right triangle.

Our last topic is distance between points in the coordinate grid, as this is another simple application of the Pythagorean Theorem.

Pacing Suggestion for Chapter 6

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Square Roots	65	*4 pages*	1 day	
Irrational Numbers	69	*4 pages*	1 day	
Cube Roots and Approximations of Irrational Numbers ...	73	*4 pages*	1 day	
Fractions to Decimals (optional)...	77	*(2 pages)*	(1 day)	
Decimals to Fractions	79	*3 pages*	1 day	
Square and Cube Roots as Solutions to Equations	82	*3 pages*	1 day	
More Equations that Involve Roots	85	*3 pages*	1 day	
The Pythagorean Theorem	88	*5 pages*	2 days	
Applications of the Pythagorean Theorem 1	93	*3 pages*	1 day	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch6

Scan me

(This page intentionally left blank.)

Cube Roots and Approximations of Irrational Numbers

Similarly to the square root, we can take a **cube root** of a number.

Recall that the **cube of a number** is that number multiplied by itself three times. For example, two cubed = $2^3 = 2 \cdot 2 \cdot 2 = 8$. This gives us the volume of a cube with edges 2 units long.

The cube root of 8 is 2. We write it as $\sqrt[3]{8} = 2$. Notice the little "3" that is added to the radical sign to signify a cube root.

Example 1. Since $(-3)(-3)(-3) = -27$, then $\sqrt[3]{-27} = -3$.

Like square roots, most cube roots are irrational numbers. When it comes to integers, only the cube roots of perfect cubes are rational; the rest are irrational.

1. Find the cube roots without a calculator.

a. $\sqrt[3]{27}$	**b.** $\sqrt[3]{125}$	**c.** $\sqrt[3]{64}$	**d.** $\sqrt[3]{1{,}000}$
e. $\sqrt[3]{1}$	**f.** $\sqrt[3]{216}$	**g.** $\sqrt[3]{27{,}000}$	**h.** $\sqrt[3]{-8}$
i. $\sqrt[3]{-1}$	**j.** $\sqrt[3]{-125}$	**k.** $\sqrt[3]{0}$	**l.** $\sqrt[3]{-8{,}000}$

2. **a.** The volume of a cube is 216 cm^3. How long is its edge?

b. What is $(\sqrt[3]{4})^3$?

c. If the edge of a cube measures 50 cm, find its volume.

d. If the volume of a cube is 729 in^3, find its surface area.

3. (optional) Find the cube roots of these fractions and decimals, without a calculator.

a. $\sqrt[3]{0.008}$	**b.** $\sqrt[3]{0.125}$	**c.** $\sqrt[3]{-0.027}$
d. $\sqrt[3]{\frac{8}{125}}$	**e.** $\sqrt[3]{\frac{64}{27}}$	**f.** $\sqrt[3]{-\frac{1}{8}}$

Example 2. We can know that $\sqrt{98}$ lies between 9 and 10, because $9 = \sqrt{81} < \sqrt{98} < \sqrt{100} = 10$.

We can even tell it is much closer to 10 than to 9, since 98 is much closer to 100 than to 81.

From that, we can estimate that $2\sqrt{98}$ is slightly less than 20, and that $\sqrt{98} + 4$ is slightly less than 24.

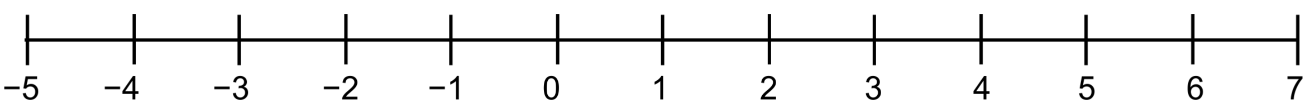
Example 3. The opposite of $\sqrt{2}$ is $-\sqrt{2}$. Since $\sqrt{2}$ is approximately 1.41, then $-\sqrt{2} \approx -1.41$.

4. Find between which two whole numbers the root lies. Notice some of them are cube roots.

a. _____ $< \sqrt{31} <$ ______	**b.** _____ $< \sqrt{65} <$ ______	**c.** _____ $< \sqrt{87} <$ ______
d. _____ $< -\sqrt{5} <$ ______	**e.** _____ $< -\sqrt{44} <$ ______	**f.** _____ $< -\sqrt{50} <$ ______
g. _____ $< \sqrt[3]{7} <$ ______	**h.** _____ $< \sqrt[3]{37} <$ ______	**i.** _____ $< \sqrt[3]{101} <$ ______

5. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

$\sqrt{15}$ $\sqrt{47}/2$ $\sqrt[3]{9}$ $-\sqrt[3]{27}$ $-\sqrt{10}$ $\sqrt{66}/2$ π $\sqrt{18} + 1$



6. Compare, writing >, <, or = between the numbers. Think between which two whole numbers the root lies, using mental math.

a. 5 ☐ $\sqrt{27}$	**b.** $\sqrt{48}$ ☐ 7	**c.** $\sqrt{18}$ ☐ 4	**d.** $\sqrt[3]{9}$ ☐ 2
e. 2 ☐ $\sqrt{2} + 1$	**f.** $\sqrt{32} + 1$ ☐ 6	**g.** $\sqrt{43} + 5$ ☐ 10	**h.** $\sqrt{88} - 3$ ☐ 7

7. **a.** Between which two whole numbers does $\sqrt{30}$ lie? And $\sqrt{60}$?

b. Use your answers to (a) to determine whether $2\sqrt{30}$ is equal to $\sqrt{2 \cdot 30}$.

8. Is $\frac{\sqrt{50}}{2}$ equal to $\sqrt{\frac{50}{2}}$? Explain your reasoning.

9. Use the decimal approximations of common irrational numbers on the right to estimate the value of the expressions below, to one decimal digit. Use mental math and paper-and-pencil calculations, not a calculator.

$\pi \approx 3.14$

$\sqrt{2} \approx 1.41$

$\sqrt{5} \approx 2.24$

a. $5\sqrt{2}$ **b.** π^2 **c.** $\sqrt{5} - \sqrt{2}$ **d.** $2\sqrt{5} - 5\sqrt{2}$

10. **a.** Find an approximation to $\sqrt{11}$ to one decimal digit, without using the square root function of a calculator.

b. Use the approximation you found to estimate the values of $\sqrt{11} - \sqrt{2}$ and $3\sqrt{11}$.

11. Sarah has used the method of squaring her guesses to find out that $\sqrt{45}$ is between 6.7 and 6.8.
How can she continue from this point to get a better approximation? Do it for her, to two decimal digits.

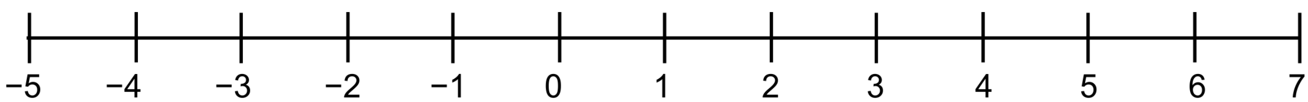
Use these exercises for additional practice.

12. Order the numbers from smallest to greatest. Estimate the value of the roots, thinking between which two whole numbers each square root lies, using mental math.

$\sqrt{5} - 1$ $\sqrt[3]{1}$ $\sqrt{19}/2$ $\sqrt[3]{100}$ $\sqrt[3]{8}$ $\sqrt{13}$ $\sqrt{9}$ 2π $\sqrt{22} + 1$

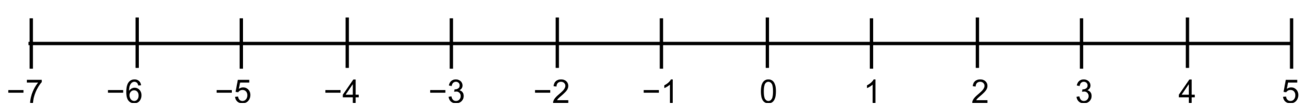
13. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two whole numbers the root lies, and whether it is close to one of those whole numbers.

a. $-2\sqrt{2}$ **b.** $\sqrt{80}/3$ **c.** $\sqrt{27} - 1$ **d.** $-\sqrt{5} + 7$



14. Plot the following numbers *approximately* on the number line.

a. $-\sqrt{2} - 3$ **b.** $-\pi$ **c.** $-\sqrt[3]{9}$ **d.** $-\sqrt{36} + 9$ **e.** $-\sqrt{26}/2$



Fractions to Decimals

(This lesson is review, and optional.)

Each fraction is a rational number (by definition!). Each fraction can be written as a decimal. It will either be a terminating decimal, or a non-terminating repeating decimal.

It is easy to rewrite a fraction as a decimal when the denominator is a power of ten. However, when it is not (which is most of the time), simply treat the fraction as a division and divide. You will get either a **terminating decimal** or a non-terminating **repeating decimal**. See the examples below.

1. The denominator is a power of ten. In this case, writing the fraction as a decimal is straightforward. Simply write out the numerator. Then add the decimal point based on the fact that the number of zeros in the power of ten tells you the number of decimal digits.

Examples 1. $\frac{7809}{100} = 78.09$ $\frac{1458}{1000} = 1.458$ $\frac{506}{100{,}000} = 0.00506$

2. The denominator is a factor of a power of ten. Convert the fraction into one with a denominator that is a power of ten. Then do as in case (1) above.

Examples 2. $\frac{9}{20} = \frac{45}{100} = 0.45$ $\frac{2}{125} = \frac{16}{1000} = 0.016$ $\frac{33}{30} = \frac{11}{10} = 1.1$

3. Use division (long division or with a calculator). This method works in all cases, even if the denominator happens to be a power of ten or a factor of a power of ten.

Example 3. Write $\frac{31}{40}$ as a decimal.

This division terminates (comes out even) after just three decimal digits.

We get $\frac{31}{40} = 0.775$. This is a **terminating decimal**.

(The fact the division was even means that the denominator 40 is a factor of some power of ten, and so we could have used method 2 from above. In this case, 1000 = 40 · 25.)

```
      0 0.7 7 5
    ___________
 40 ) 3 1.0 0 0 0
    - 2 8 0
        3 0 0
     -  2 8 0
          2 0 0
        - 2 0 0
              0
```

Example 4. Write $\frac{18}{11}$ as a decimal.

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient, and the remainders 40 and 70, start repeating.

So $\frac{18}{11} = 1.\overline{63}$.

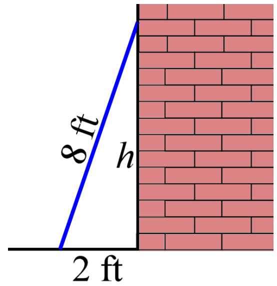
The fraction 18/11 equals $1.\overline{63}$, which is a **repeating decimal**.

```
      0 1.6 3 6 3
    _____________
 11 ) 1 8.0 0 0 0
     -1 1
        7 0
      - 6 6
          4 0
        - 3 3
            7 0
          - 6 6
              4 0
            - 3 3
                7
```

(This page intentionally left blank.)

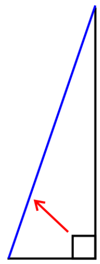
Applications of the Pythagorean Theorem 1

Example 1. An eight-foot ladder is placed against a wall so that the base of the ladder is 2 ft away from the wall. What height does the top of the ladder reach?



Since the ladder, the wall, and the ground form a right triangle, this problem is easily solved by using the Pythagorean Theorem. Let *h* be the unknown height. From the Pythagorean Theorem, we get:

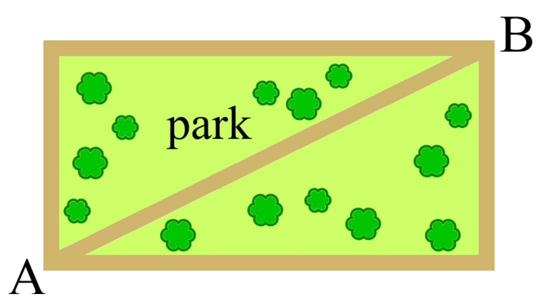
$$\begin{aligned} 2^2 + h^2 &= 8^2 \\ 4 + h^2 &= 64 \\ h^2 &= 60 \\ h &= \sqrt{60} \\ h &\approx 7.75 \end{aligned}$$



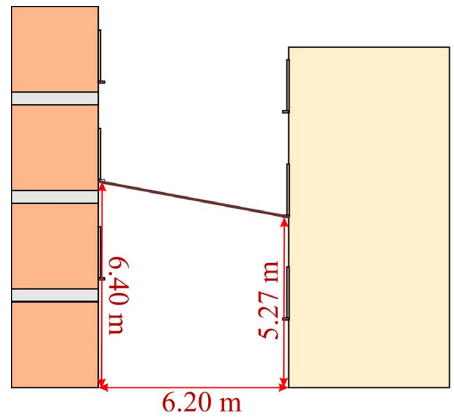
Our answer, 7.75, is in feet. This means the ladder reaches to about 7 3/4 ft = 7 ft 9 in. high.

1. The area of a square is 100 m^2. How long is the diagonal of the square?

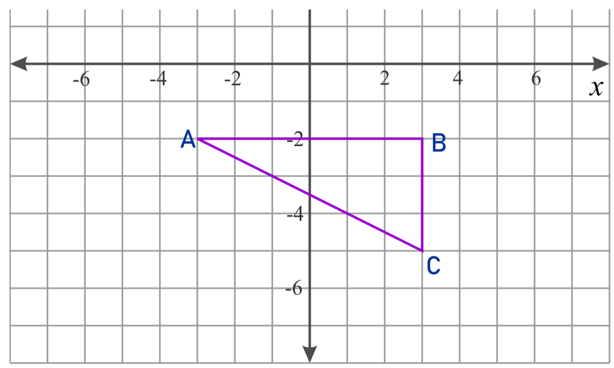
2. A park is in the shape of a rectangle and measures 48 m by 30 m. How much longer is it to walk from A to B around the park than to walk through the park along the diagonal path?



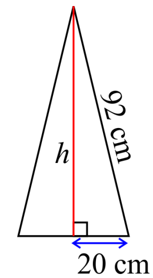
3. A clothesline is suspended between two apartment buildings. Calculate its length, assuming it is straight and doesn't sag any.



4. Find the perimeter of triangle ABC to the nearest tenth of a unit.



Example 2. Find the area of an isosceles triangle with sides 92 cm, 92 cm, and 40 cm.



Solution: To calculate the area of any triangle, we need to know its altitude. When we draw the altitude, we get a right triangle:

The next step is to apply the Pythagorean Theorem to solve for the altitude *h*, and after that calculate the actual area.

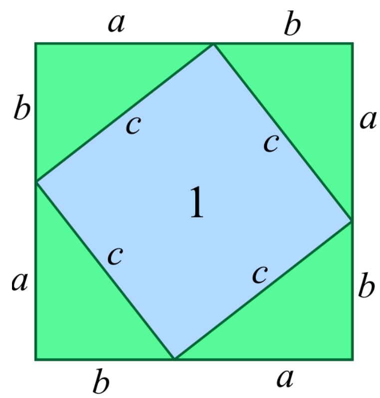
5. Calculate the area of the isosceles triangle in the example above to the nearest ten square centimeters.

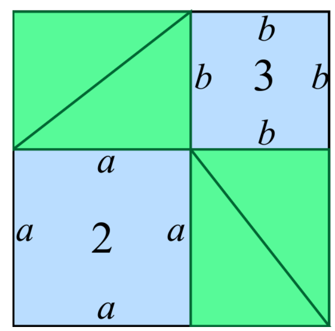
6. Calculate the area of an equilateral triangle with 24-cm sides to the nearest square centimeter.
Don't forget to draw a sketch.

A Proof of the Pythagorean Theorem and of Its Converse

There exist hundreds of different proofs for the Pythagorean Theorem. In this lesson, we will look at two geometric proofs.

Proof.



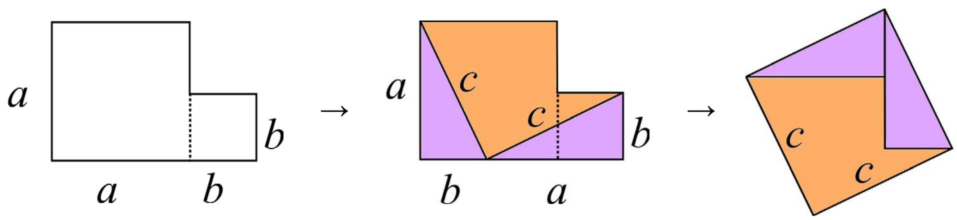


The figure above has four right triangles, each with sides *a*, *b* and *c*. The sides of the outside square are $a + b$. The triangles enclose a square with sides *c* units long.

Here the sides of the large square are still $a + b$, but the four right triangles have been rearranged to form two smaller squares, with sides *a* and *b*.

Since the areas of both large squares are equal, and the areas of the four right triangles are equal, it follows that the remaining (blue) areas are also equal. In other words, the area of square 1, which is c^2, equals the area of square 2 (which is a^2) plus the area of square 3 (which is b^2). In symbols, $c^2 = a^2 + b^2$. ☺

1. Figure out how this proof of the Pythagorean Theorem works.



2. Study one of the proofs enough so you can explain it to someone else. Then do so.

(This page intentionally left blank.)

Chapter 7: Systems of Linear Equations
Introduction

This chapter covers how to analyze and solve pairs of simultaneous linear equations. (The equations studied contain only two variables.)

The first lesson, *Equations with Two Variables*, is optional. It reinforces the idea that a point on a line satisfies the equation of the line, and thus prepares the way for the main topics in the book.

First, students learn to solve systems of linear equations by graphing. Since each equation is an equation of a line, this is a simple technique, but it has its limitations, thus, algebraic solution methods will be taught also, in later lessons.

In the next lesson, we look at the number of solutions that a system of two linear equations can have. The three possible situations are easy to see based on the graphs of the equations: either one solution (the lines intersect in one point), no solutions (lines are parallel), or an infinite number of solutions (the lines are the same).

In the next lesson, students learn the algebraic method of solving systems of equations by substitution. This is a straightforward technique that many students will grasp easily. However, one has to be careful not to make simple mistakes. The lesson has some practice problems where students practice finding errors in solutions. Instruct the student(s) to check their solutions each time, as that is the best way to catch errors.

As for me (Maria, the author), as I wrote the answer key, I immediately checked my solution for each system of equations, and several times found an error that way. (The funniest errors were when I had switched from x to y in the middle of the solution!) So, checking the solution is important. To save space the answer key does not include the checks, but the student should always do that, whether with mental math or with a calculator.

The following lesson, *Applications, Part 1*, has a variety of word problems that students can now solve using a system of equations.

After that, students learn another algebraic method for solving systems of equations: the addition or elimination method. This is useful when the coefficients of the variables are such that you can easily find their least common multiple. Students also practice solving more complex systems, where the equations first have to be transformed and simplified, or include fractions.

Then it is time for more word problems, in the lesson *Applications, Part 2*. One lesson is devoted to problems about speed, time, and distance, and another for mixtures and comparisons. Making a chart is very helpful in these situations.

Pacing Suggestion for Chapter 7

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 7	page	span	suggested pacing	your pacing
Equations with Two Variables	119	*4 pages*	1 day	
Solving Systems of Equations by Graphing	123	*5 pages*	1-2 days	
Number of Solutions	128	*4 pages*	1 day	
Solving Systems of Equations by Substitution	132	*7 pages*	2 days	
Applications, Part 1	139	*4 pages*	1 day	
The Addition Method, Part 1	143	*5 pages*	1 day	
The Addition Method, Part 2	148	*5 pages*	1 day	
More Practice	153	*4 pages*	1 day	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch7

Scan me

Equations with Two Variables

(This lesson is optional.)

The equation $2x + 3y = 16$ has **two variables**, x and y. One solution to the equation is $x = 2$ and $y = 4$, because when we substitute those values to the equation, it checks, or is a true equation:

$$2(2) + 3(4) = 16$$

But it also has the solution $x = 0.5$ and $y = 5$:

$$2(0.5) + 3(5) = 16$$

In fact, we can choose any number we like for the value of x, and then *calculate* the value of y, and thus find another solution to the equation.

For example, if we choose $x = -1$, then we get

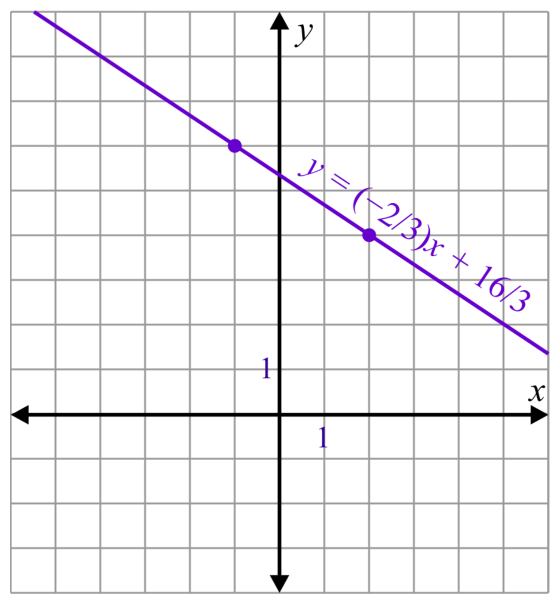
$$2(-1) + 3y = 16$$

from which $y = (16 + 2)/3 = 6$. So, $x = -1$, $y = 6$ is yet another solution.

All of these solutions, having both x and y values, are **number pairs**, and can be considered as **points on the coordinate plane.**

We can make a table of some of the possible (x, y) values (solutions):

x	y
−1	6
0	16/3
0.5	5
2	4



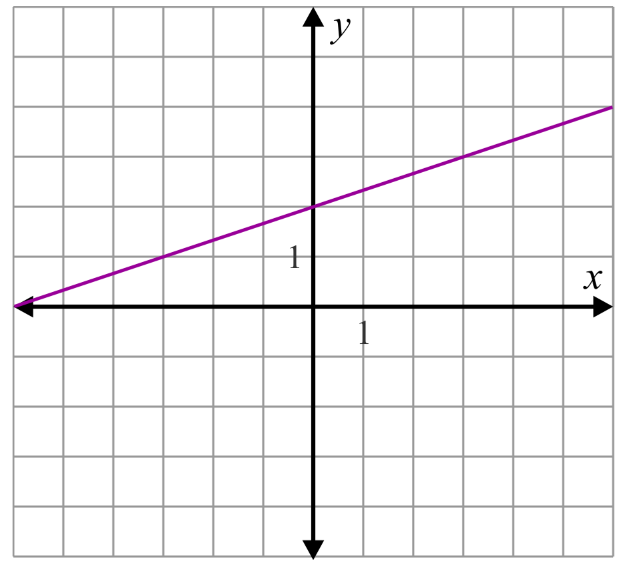
...and there are many more. When plotted, **these points fall on a line** — and you can probably guess, the equation of that line is $2x + 3y = 16$!
(Or, in slope-intercept form, $y = (-2/3)x + 16/3$.)

A line in the coordinate plane represents all the solutions to the equation that is the equation of the line. In other words, **each point on the line is a solution to the equation.**

1. Find three solutions to the equation $5x + 2y = 32$.

2. Find three solutions to the equation $-4x + y = -6$.

3. **a.** What is the equation if its solution set is represented by this line?



b. List two distinct integer number pairs that are solutions to the equation.

4. A certain linear equation with two variables has as solutions (0, −5), (2, 3) and (4, 11). Find the equation.

5. A certain linear equation with two variables has as solutions (−1, −5) and (2, 8). Find the equation.

6. Party hats cost \$2 apiece and party whistles cost \$3 apiece. Randy buys x hats and y whistles.

a. Write an expression depicting his total cost (C).

b. Now write an equation stating that his total cost is \$48.

How many hats and how many whistles could Randy have bought?

c. Find two other solutions to your equation.

7. Recall the formula tying together distance (*d*), constant speed (*v*), and time (*t*): ***d* = *vt*.**
Sarah jogs at the speed of 6 miles per hour, and she rides her bicycle at the speed of 12 miles per hour.

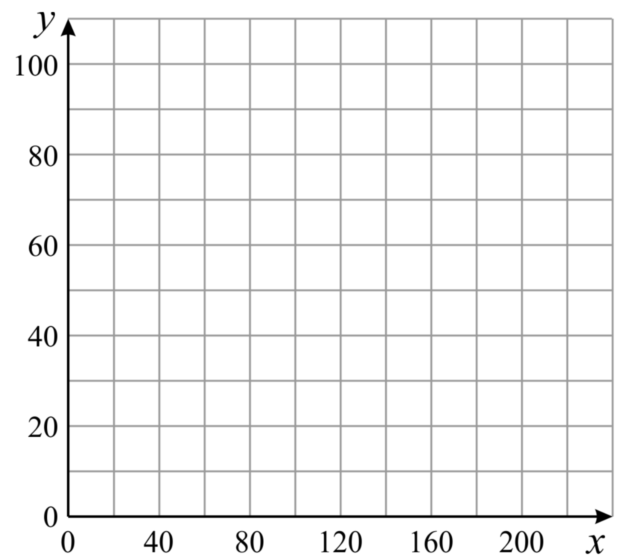
a. Convert these speeds to miles per minute.

b. Write an expression for the total distance (*d*) Sarah covers in *x* minutes of jogging plus *y* minutes of bicycling.

c. What distance does Sarah cover if she jogs for 20 minutes and bicycles for 10 minutes?

d. Let's say the distance Sarah covers, jogging and bicycling, is 20 miles. Write an equation stating this. How many minutes could she have jogged/bicycled? Find three possible solutions.

e. Write the equation in slope-intercept form and plot it.



8. General admission to a gardening seminar was \$15 but seniors paid only \$10. If the total of the admission fees was \$900, give three possibilities as to how many non-seniors and how many seniors could have attended.

9. A mystery basket contains a mixture of adult cats and kittens (it could even contain zero adults or zero kittens).
Each cat weighs 4 kg and each kitten weighs 0.5 kg.
The total weight of the cats and kittens is 20 kg.

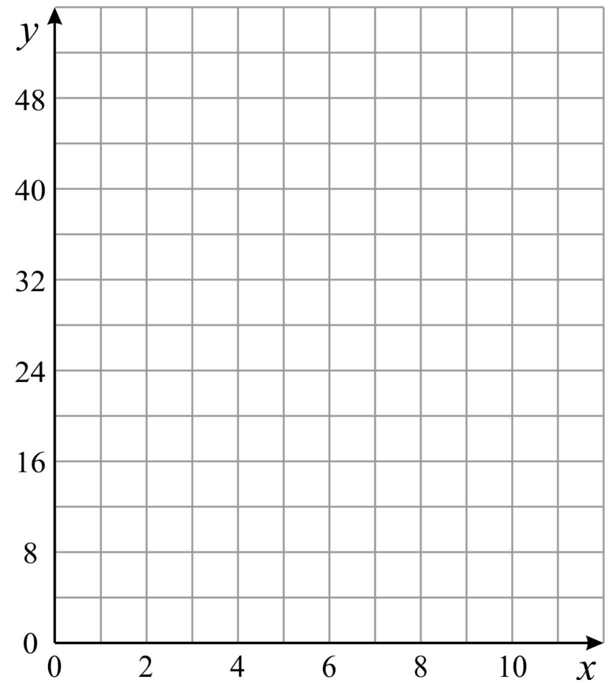
a. If there are x cats and y kittens, write an equation to match the situation.

b. How many adult cats and how many kittens could there be? Find at least three different solutions.

c. Plot your equation from (a).

d. If $x = 1.5$, what is y?
Why is this not a valid solution?

Plot the individual points on the graph that *are* valid solutions.



10. Ava and her family went to stay in a resort for a few nights. Each night cost $120 (for the whole family). The resort offered horse rides for $20 per person.

a. If the family stayed for x nights and did y horse rides in total, write an expression for the total cost of these two things.

b. In total, Ava's family spent $760 on the horse rides plus the nights they stayed.
How many nights and how many horse rides could they have paid for?

11. The equation $2x^2 - 6x - y = 5$ is a quadratic equation because the variable x is squared. If $x = 0$, then $y = -5$, so $(0, -5)$ is one solution to the equation. Find two other solutions to it.

Solving Systems of Equations by Graphing

A system of equations consists of several equations that have the same variables.

A **solution** to a system of equations is a list of values of the variables that satisfy *all* the equations in the system. For two equations, this is an ordered pair.

Example 1. This system of equations consists of two equations. We signify the system with a bracket.

$$\begin{cases} 5x + 4y = 12 \\ y = -x + 2 \end{cases}$$

The solution to the above system is the ordered pair (4, −2), because those values make both equations true: 5(4) + 4(−2) does equal 12, and −2 does equal −4 + 2.

Example 2. The equation $y = (3/2)x - 4$ has an infinite number of solutions, and we can represent those solutions with a line drawn in the coordinate plane.

Similarly, the equation $y = -2x + 3$ has infinitely many solutions.

Here is a system of equations consisting of both:

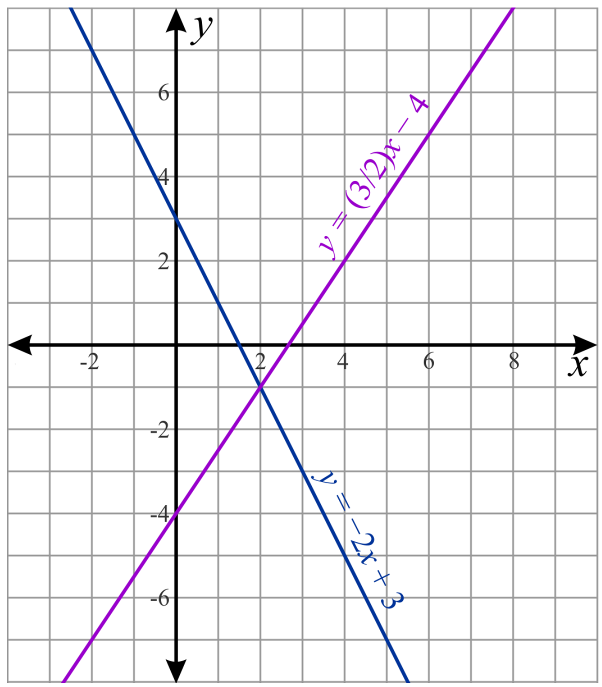
$$\begin{cases} y = (3/2)x - 4 \\ y = -2x + 3 \end{cases}$$

Since the solutions to the first equation form a line, and the solutions to the second also form a line, what would the point of intersection (2, −1) signify?

____

____

(The answer is found at the end of the lesson.)



1. Solve each system of equations using the image. The lines are already plotted in it.

a. $\begin{cases} y = -7x - 23 \\ y = (1/3)x - 1 \end{cases}$

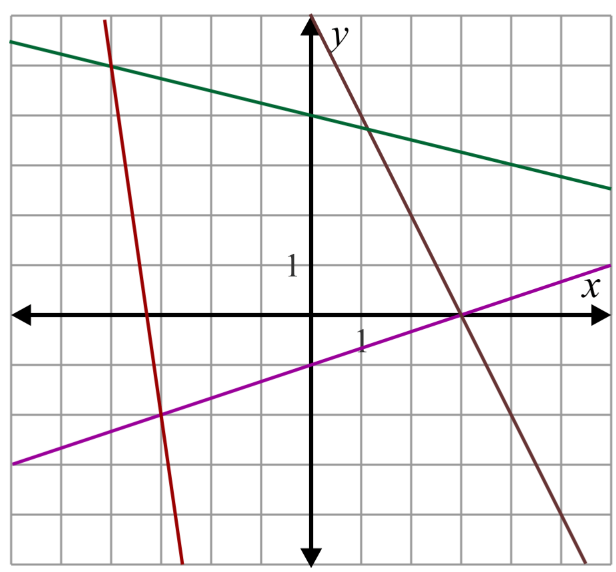
Solution: (______, ______)

b. $\begin{cases} y = -(1/4)x + 4 \\ y = -2x + 6 \end{cases}$

Solution: (______, ______)

c. $\begin{cases} -(1/3)x + y = -1 \\ 2x + y = 6 \end{cases}$

Solution: (______, ______)



(This page intentionally left blank.)

Speed, Time, and Distance Problems

There are many jokes about algebra word problems where a train leaves a station at a certain hour. You can now solve these types of problems with your knowledge of systems of equations. One of the most effective ways to do so is to first build a chart.

Example 1. A train leaves a station at 9:00 AM and travels with a constant speed of 90 km/h. Another train leaves the same station 10 minutes later, traveling to the same direction at the speed of 100 km/h. At what time will the second train reach the first?

We will be using the formula $d = vt$ extensively in these problems. Let's build a chart. The goal is to have TWO, not three or more, variables present in the chart. The formula $d = vt$ has three variables, and since the speed, distance, and time can be different for each train, theoretically we could have six variables. However, invariably, the problem gives information for one or some of these variables, and something about the situation means that the distance or the time or the speed is the same for both trains.

To get started, we gather some information in the chart. The distance that train 1 and train 2 travel until they meet is the same, so that is why we use the same variable, d, for it.

	distance	***velocity***	***time***
Train 1	d	90 km/h	t_1
Train 2	d	100 km/h	t_2

The times (t_1 and t_2) are different, but we do know that they differ by 10 minutes, so, actually we will get by using only *one* variable for time, like this:

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 10$

The chart now contains only two variables. However, we have one more thing to change. The speed is in km/h, whereas the 10 has to do with minutes. For our equation to work, the time units need to be the same, so we will change the 10 to 1/6 (in hours).

	distance	***velocity***	***time***
Train 1	d	90 km/h	t
Train 2	d	100 km/h	$t - 1/6$

The equations always follow the same formula: $d = vt$, and we use that same formula for both Train 1 and Train 2. So, the two equations we get are:

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

The quickest way to solve this system is to set $90t$ equal to $100(t - 1/6)$ and solve for t.

1. Solve the system of equations from example 1 and answer the question: At what time will the second train reach the first? Is the answer surprising?

$$\begin{cases} d = 90t \\ d = 100(t - 1/6) \end{cases}$$

2. Your friend starts walking at a speed of 6 km/h from your home to his. Exactly 15 minutes later, you decide you want to join him so you take your bicycle and start after him, with a speed of 18 km/h. How far are you when you reach your friend?

	distance	***velocity***	***time***
Your friend			
You			

3. A tortoise and hare race a distance of 100 m. The hare gives the tortoise a 10-minute lead time. Then he quickly runs the 100 meters and wins the race. After the hare has finished, the tortoise takes an *additional* 6 minutes to reach the finish line. If the speed of the hare is 15 m/s, find the time the tortoise takes to finish the race and the tortoise's speed.

Hint: since the speed is in meters per second, and the distance is in meters, the time unit will be seconds.

	distance	***velocity***	***time***
Tortoise			
Hare			

Example 2. A train leaves Turin (Italy), heading for Milan (Italy), a distance of 125 km, at 1 PM and travels with a constant speed of 90 km/h. Another train leaves Milan, heading for Turin and traveling at a constant speed, at the same time. They meet 45 minutes later. (We hope they don't crash!) What is the speed of the second train? What distance has the second train traveled by that time?

We fill our chart again. The time, 45 minutes, is 3/4 hour. The speed of the second train, v_2, is unknown.

There is a relationship between the two distances, because $d_1 + d_2 = 125$ km. So, we can get by with just one variable for the distance:

	distance	*velocity*	*time*
Train 1	d	90 km/h	3/4
Train 2	$125 - d$	v_2	3/4

Our equations are: $\begin{cases} d = 90(3/4) \\ 125 - d = (3/4)v_2 \end{cases}$

Here, it is handy to use the substitution method, since we have an expression for d. So, we substitute 90(3/4), which equals 67.5, in place of d in the second equation:

$$\begin{aligned} \text{(2)}\quad 125 - \mathbf{67.5} &= (3/4)v_2 \\ 57.5 &= (3/4)v_2 \quad \Big|\cdot 4 \\ 230 &= 3v_2 \\ v_2 &= 76.\overline{6} \end{aligned}$$

So, the speed of the second train is $76.\overline{6}$ km/h. The problem also asked what distance the second train has traveled by that time. To find that, we use the formula $d = vt$: $d = 76.\overline{6}$ km/h · (3/4 h) = 57.5 km.

4. Two trains leave the same station at the same time, one traveling due east and the other traveling due west. Train 1 travels at a speed of 120 km/h and Train 2 at the speed of 100 km/h. When are the trains 50 km apart from each other?

	distance	*velocity*	*time*
Train 1			
Train 2			

5. Train 1 leaves the station heading due south and Train 2 leaves the same station at the same time, heading due north. Train 1 travels at the speed of 70 mph. After 30 minutes, the trains are 75 miles apart. How fast is Train 2 traveling?

	distance	***velocity***	***time***
Train 1			
Train 2			

6. **a.** Two horses, Ranger and Chip, start racing at the same time. Ranger runs at a steady speed of 16 m/s. After 100 seconds, they are 600 m apart from each other, Ranger leading. How fast is Chip running?

	distance	***velocity***	***time***
Ranger			
Chip			

b. What would Chip's speed need to be, so that after 100 seconds, he would only be 50 m behind Ranger?

Example 3. A motorboat travels downstream on the river a distance of 5 miles, in 20 minutes. Doing the same trip upstream takes it 4 minutes longer. How fast is the river flowing? What is the speed of the boat in still water?

Our chart method will still work. We are dealing with two speeds: that of the boat (v_b) (in still water), and that of the water (v_w) in the river. Going downstream, the boat's actual speed is its own speed PLUS the speed of the water. Going upstream, it has to fight the current and its actual speed is $v_b - v_w$.

This is what the chart looks like. Using these quantities, the speeds will end up being in miles per hour.

	distance	***velocity***	***time***
Downstream	5 mi	$v_b + v_w$	20 min
Upstream	5 mi	$v_b - v_w$	24 min

Our equations are: $\begin{cases} 5 = 20(v_b + v_w) \\ 5 = 24(v_b - v_w) \end{cases}$

7. Solve the problem in example 3.

8. An airplane travels from City A to City B in 2 hours 30 minutes, flying with the wind, and in 2 hours 45 minutes flying against the wind. If the speed of the airplane in still air is 900 km/h, find the distance between the cities to the nearest 10 km.

9. With a tailwind, an airplane can fly from City 1 to City 2, a distance of 650 km, in 40 minutes. If the speed of the wind is 30 km/h, find the time the plane takes to fly the same distance against the wind.

10. You swim downstream, from a dock to a certain rock in the middle of the river, in 43 seconds. Swimming back (upstream) takes you 10 seconds longer. If your swimming speed in still water is 1.0 mph, what is the speed of the water in the river?

Puzzle Corner

The following are "trick" problems. Have fun!

(1) Train 1 leaves Jackson at 1:30 PM, traveling at 95 km/h towards Atlanta, a distance of 560 km, and Train 2 leaves Atlanta, heading towards Jackson at the same time, traveling at 105 km/h. When they meet, which train is closer to Atlanta?

(2) At 6:30 PM, you board a train in Dallas, heading south, and 10 minutes later, your friend boards a train at Kansas City, heading north. If both trains travel at 70 mph, when do you pass each other?

(3) Two trains leave a station at the same time, one heading east, the other heading west. After 15 minutes, they are 40 miles apart. Which train is traveling faster?

(This page intentionally left blank.)

Chapter 8: Bivariate Data
Introduction

The last chapter of grade 8 covers statistical topics that have to do with bivariate data, or data involving two variables.

The first lesson introduces scatter plots. Students analyze the data in a variety of scatter plots, and determine visually whether there is an association between the variables. Next, they learn about basic patterns we often see in scatter plots, such as positive and negative association, linear association, clusters, and outliers. They also make scatter plots from given data, describe any special features in the plot, and answer a variety of questions related to the data.

In the following lesson, students fit a line (informally) to the data points displayed in a scatter plot. Mathematicians have developed several algorithms for finding a line of best fit, such as linear regression, but we are not using those here. Students use the basic idea of trying to leave close to an equal number of points on each side of the line, and also judging the fit by the closeness of the points to the line. This resembles the thought behind the linear regression algorithm, which finds the line of best fit by minimizing the squares of the distances of the data points to the line.

The last topic relating to scatter plots is the equation of the trend line. Students use the equation of the trend line to solve problems in the context of the data, interpreting the slope and intercept of the equation.

Then we turn our attention to categorical bivariate data, that is, data involving two variables that may or may not be numerical, but is divided into categories. Students learn that bivariate categorical data can be summarized in a two-way table, and if there is a pattern of association between the variables, it can be seen in the table.

Students construct and interpret two-way tables summarizing data on two categorical variables. In the last lesson, they calculate relative frequencies for rows or columns, and use those to describe the possible association between the two variables.

Pacing Suggestion for Chapter 8

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 8	**page**	**span**	**suggested pacing**	**your pacing**
Scatter Plots	183	*3 pages*	1 day	
Scatter Plot Features and Patterns	186	*4 pages*	1 day	
Fitting a Line	190	*4 pages*	1 day	
Equation of the Trend Line	194	*5 pages*	1-2 days	
Two-Way Tables	199	*3 pages*	1 day	
Relative Frequencies	202	*5 pages*	2 days	
Mixed Review Chapter 8	207	*5 pages*	2 days	
Chapter 8 Review	212	*3 pages*	1 day	
Chapter 8 Test (optional)				
TOTALS		*32 pages*	10-11 days	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

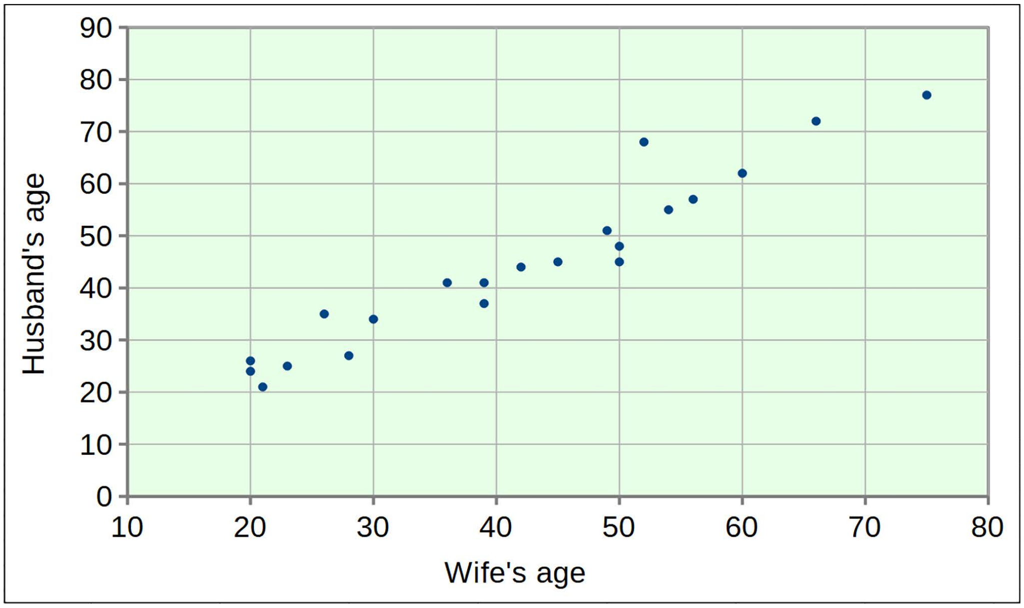
We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch8

Scan me

Scatter Plots

A **scatter plot** depicts **bivariate data**, meaning that the data involves **two variables**. In the scatter plot below, the variables are the husband's age and the wife's age. Each dot in this scatter plot represents a husband-wife couple. In other words, the coordinates of the dot give us the ages of the husband and the wife.



1. Refer to the scatter plot above.

a. Locate the dot with coordinates (36, 41). What does it signify?

b. Find two couples where the wife is the same age in both cases. Estimate the ages of their husbands.

c. Find the couple with the third oldest husband in this data set. How old is his wife?

d. Is it true that the youngest wife is married to the youngest husband? Explain.

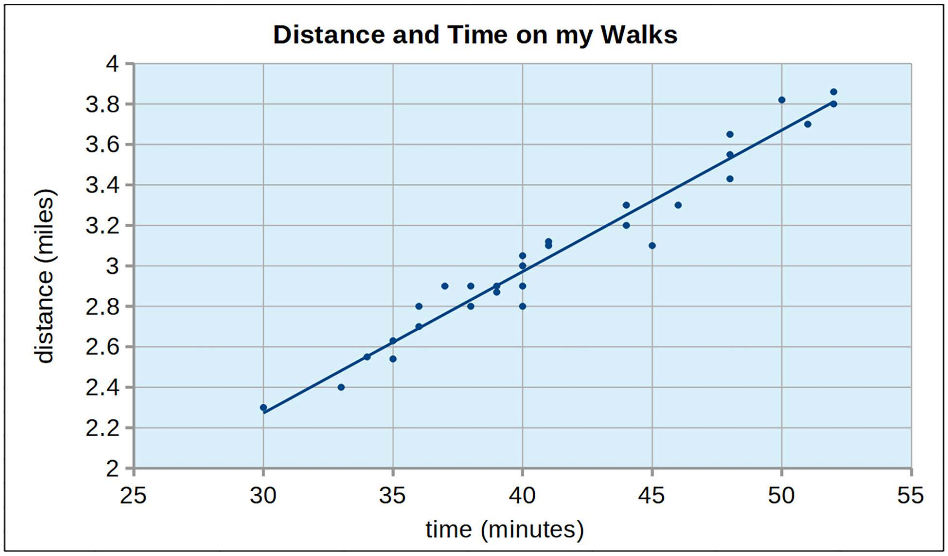
e. Is it true that the oldest wife is married to the oldest husband? Explain.

f. Do you notice a relationship between the two variables? Explain what you see.

(This page intentionally left blank.)

Equation of the Trend Line

Going back to the Sofia's walks, the graph shows a line that is fitted to the data points. Its equation is $d = 0.07t + 0.175$, where d is the distance in miles and t is the time in minutes. The equation was calculated by a spreadsheet program. The line we drew in the previous lesson by using the ellipse method is remarkably close to this line calculated with an algorithm.



The slope of the line, 0.07, tells us that for each one-minute increment, the distance increases by 0.07 miles. In other words, Sofia tends to walk at a speed of about 0.07 miles per minute.

The d-intercept, 0.175 miles, means that the equation predicts that at zero minutes, Sofia would have walked 0.175 miles. This doesn't make sense. Keep in mind that the equation is calculated using data that was limited to between 30 and 52 minutes. Estimating values outside of that range is called extrapolating, and we need to be careful with that. The equation is a model, and it may not be valid outside the original range of values.

1. Use the equation, $d = 0.07t + 0.175$ as a model for Sofia's walks. Choose all the correct statements.

- Each 1-minute increment in time is associated with a 0.175-mile increment in distance.
- Each 1-minute increment in time is associated with a 0.07-mile increment in distance.
- The average distance she walks is 0.175 miles.
- The equation predicts a distance of 0.07 miles when the time is 0.175 minutes.

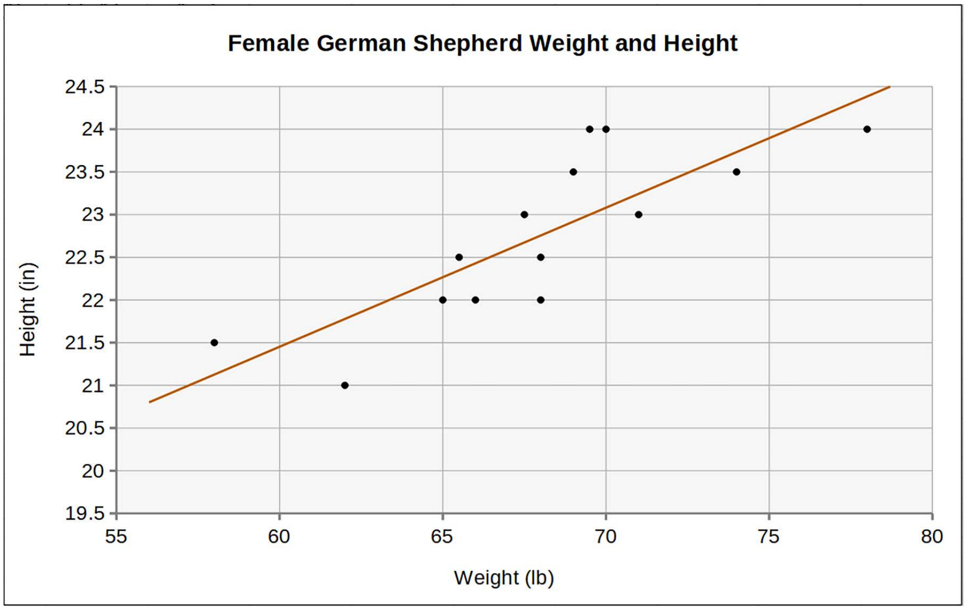
2. Concerning Sofia's walks again, if we tell the spreadsheet program to force the line to go through (0, 0), the program calculates the equation as $d = 0.074t$. This line is a slightly worse fit considering the data points, but it matches reality in the sense that when $t = 0$, the distance is zero also.

a. Using this equation, predict how many miles Sofia would walk in 32 minutes.

b. How much time would you expect Sofia to take for a 3.3-mile walk?

c. Use the equation $d = 0.074t$ and fill in. Each five minutes added to the walk time is associated with a ______-mile increase in her walking distance.

3. The scatter plot below shows the weight and height of various adult female German shepherds. (It does not have to do with weight gain/loss of an individual dog — each dot signifies a different dog.) The equation for the trend line is $h = 0.16w + 11.68$, where h is the height in inches and w is the weight in pounds.



a. Which statements are correct?

- Each 0.16-lb increase in weight is associated with a 1-inch increase in height.

- Each 1-lb increase in weight is associated with a 0.16-inch increase in height.

- Heavier dogs tend to be taller; and for each 5-lb increase in the weight, the dogs tend to be 0.8 inches taller.

- The model predicts a height of 11.68 inches for a dog weighing zero pounds.

- The model predicts a weight of 11.68 lb for a dog that is zero inches tall.

- We should be careful in using this model to extrapolate the heights of dogs less than 55 pounds.

b. Use the equation to predict the weight of a dog that is 22.5 inches tall, to the nearest pound.

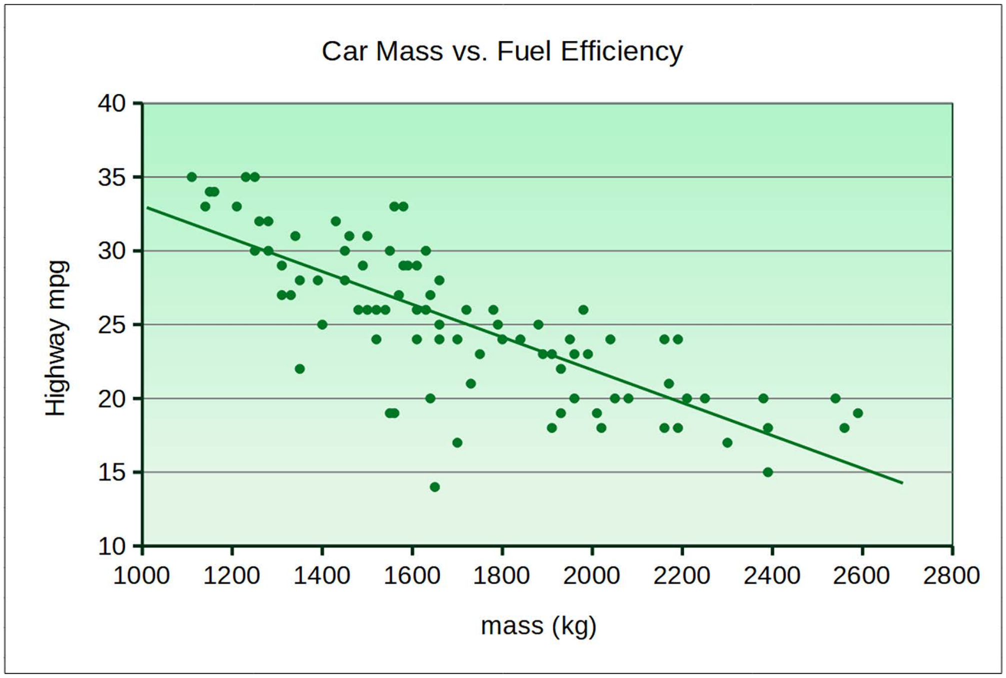
c. Use the equation to predict how tall a 63-lb dog would be.

d. Would a dog that weighs 60 lb and is 21 inches tall be considered an outlier?

e. What is the difference between the predicted height of a 75-lb dog and its real height, if in reality it is 24 1/4 inches tall?

4. The line fitted to the data about the mass (*m*) and fuel efficiency (MPG) of cars has the equation

$$\text{MPG} = -0.011115m + 44.156$$



a. What does the slope of −0.011115 signify in this context?

b. For each 100-kg increase in a car's mass, how much would you expect the fuel efficiency to change?

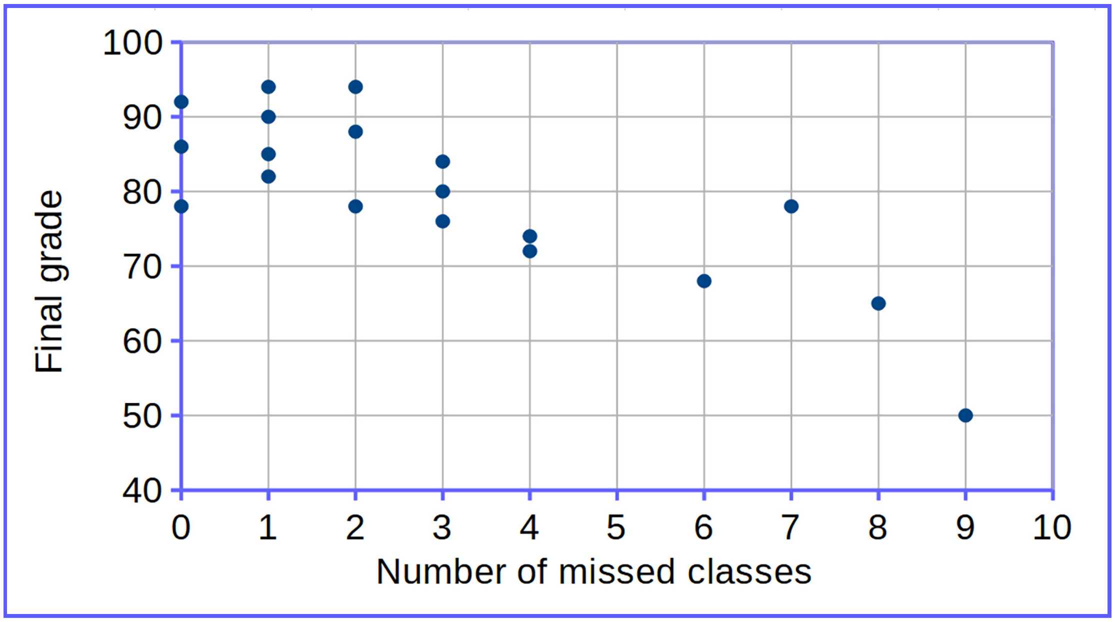
c. Predict the fuel efficiency of a 2500-kg car.

d. If a car gets 27.5 mpg in highway driving, what would you expect its mass to be?

e. Find the dot in the graph for Lamborghini Murcielago A-S6, which weighs 1650 kg and gets only 14 mpg in highway driving. Based on the equation of trend line, what would you expect this car's fuel efficiency to be?

f. Let's say you want to buy a car that gets at least 27 mpg in highway driving. Based on the scatter plot, how much would you expect your car to weigh, at a maximum?

5. **a.** Draw a line to fit the trend in the scatter plot below.



b. Find the (approximate) equation of your line.
Hint: find the y-intercept and the slope.

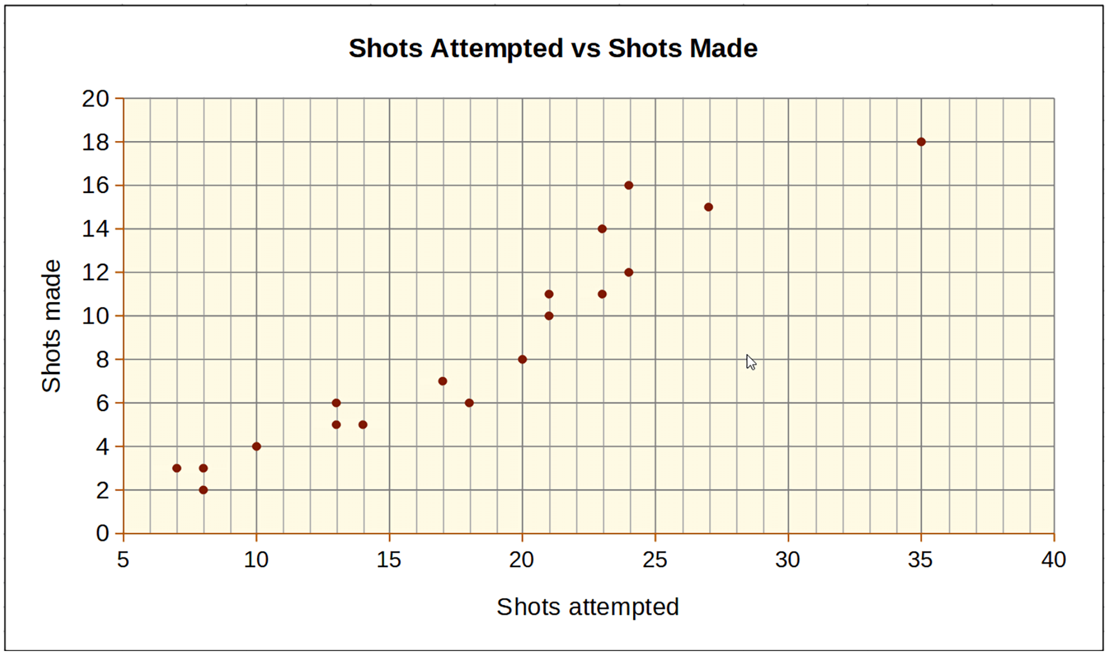
c. What does the slope of your equation signify in this context?

d. What does the *y*-intercept of your equation signify in this context?

e. Using your equation, predict what the final grade would be for a student that missed five classes.

f. Tanya calculated the equation of the trend line as G = 3*m* + 89, where G is the grade and *m* is the number of the missed classes. Explain why this cannot possibly fit the data.

6. The data shows shots the attempted shots and the shots made of 18 basketball players.



a. Draw a line that fits the trend in the data.

b. Find the equation of your line.

c. What does the slope in the equation signify in this context?

d. And the *y*-intercept?

e. If a player attempts 30 shots, how many shots does your model predict they would make?

f. What prediction does your model make about the number of shots attempted by a player if they made 8 shots?

Two-Way Tables

We have been studying bivariate data, in other words, data that involves two variables. So far, that data has all been numerical: we have had two numerical values for each data item, and therefore, we have been able to plot the data as points in the coordinate plane, the two coordinates being the values of the two variables.

Now we will look at bivariate data that is organized into categories, and the values may or may not be numerical.

The **two-way table** on the right shows how many students in each grade of an elementary school can swim, and how many cannot. It also shows the row and column totals.

It is called a two-way table because it records the information from *two* variables. In this case, the two variables are the student's grade level and whether the student can swim or not.

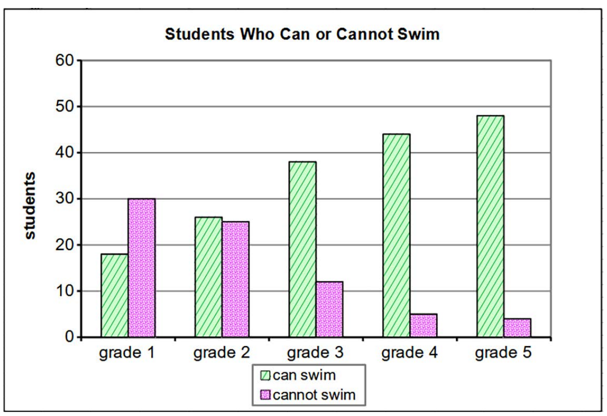
The first variable takes numerical values from 1 to 5. The table has a row corresponding for each of those values. The second takes the values "Yes" and "No", and those correspond to the two columns labeled "can swim" and "cannot swim".

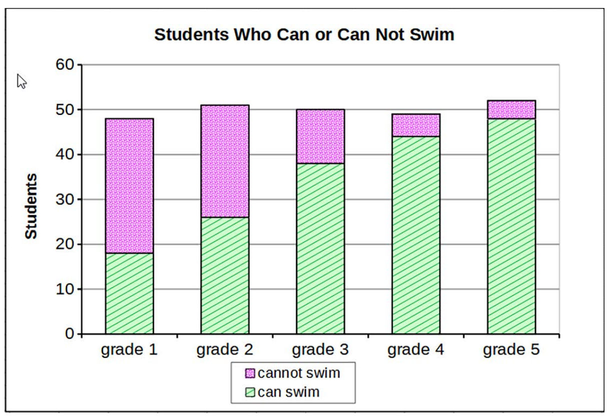
Grade	can swim	cannot swim	Total
1	18	30	48
2	26	25	51
3	38	12	50
4	44	5	49
5	48	4	52
TOTAL	**174**	**76**	**250**

The original data may look like this: (1, no), (1, yes), (2, no), (2, no), (3, no), (5, yes), etcetera, each pair of data depicting one student. But we cannot analyze or summarize the data when it is in that format. A two-way table allows us to **categorize and tally up the data**, and then also to analyze it to see if there is any **association between the two variables**.

In this case, we can see that with advancing grade level, there are many more students who can swim than who cannot. In 5th grade, nearly all students can swim. So, there *is* an association between the variables.

The data from a two-way table can be presented as a double-bar graph (left), or a stacked bar graph (right):





A stacked bar graph is more common. From the graphs, it is easy to see that as the grade level advances, more and more students are able to swim.

1. A community college tracked how many of their students in a given year completed a certain course that was offered both as an in-person course and as an online version.

	completed the course	**did not complete the course**	**Total**
In person	57	8	65
Online	23	25	48
TOTAL	80	33	113

a. Looking only at those who took the online course, what percentage of them completed the course? What percentage did not?

b. Looking only at those who took the in-person course, what percentage of them completed the course? What percentage did not?

c. Is there a relationship or association between the variables? Explain.

2. Jordan asked a bunch of people at a local gym as to their opinion on increasing the membership price in return for some improved facilities. He categorized the people as 20-30 year olds and as 31+ year olds. Here are some highlights of Jordan's research:

- Of the seventy-seven 20-30 year olds, 21 were in favor of the plan.
- In total, there were 79 people for and 70 people against this plan.

a. Fill in the two-way table below from this data. Use the two age groups as rows. Fill in all the numbers.

	for	**against**	**Total**
20-30 years old			
31+ years old			
TOTAL			

b. Overall, are most of the 20-30 year olds for or against this plan?

What about of the 31+ year olds?

c. Now look at the column totals. Are there more people in general that are for the price increase or that are against it?

d. Do you feel there is an association between the variables? Explain.

e. Why do you think so many of the younger people are against this plan?

3. At a family reunion, Ashley asked all her relatives whether at the next year's reunion they should pay a fee and gather at the local amusement park grounds. Here are the answers she got.

- Adults: (18) no, no, no, yes, no, no, yes, no, no, yes, yes, no, no, no, no, no, yes, yes
- Teens: (8) yes, yes, yes, yes, yes, yes, no, no
- Children (16): yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes, yes

a. Create a two-way table from the results.

b. Based on the results, are there more people in favor or against gathering at the amusement park next year?

c. Does the answer change if children are ignored?

d. What percentage of the adults are in favor of this?

What percentage of the teens? Of the children? Of everyone?

e. The two variables are: age group (adult/teen/child) and opinion (yes/no). Is there a relationship or an association between the variables? What kind?

Puzzle Corner

Is there an association between the variables? Explain.