Contents

Chapter 8: Geometry

Chapter 9: Probability

Chapter 10: Statistics

Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. It follows the Common Core Mathematics Standards (CCS) for 7th grade. Those standards are so constructed that students can continue to a traditional algebra 1 curriculum after studying this. However, you also have the option of following this course with Math Mammoth Grade 8, which provides a gentler and slower transition to high school math.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-B, covers ratios and proportions (chapter 6), percent (chapter 7), geometry (chapter 8), probability (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 7-A worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

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Ratios and Rates

A **ratio** is a comparison of two numbers, or quantities, using division.

For example, to compare the hearts to the stars in the picture, we say that the ratio of hearts to stars is 5:10 (read "five to ten").

The two numbers in the ratio are called the **first term** and the **second term** of the ratio. The order in which these terms are mentioned does matter! For example, the ratio of stars to hearts is *not* the same as the ratio of hearts to stars. The former is 10:5 and the latter is 5:10.

We can write this ratio in several different ways:

- The ratio of hearts to stars is 5:10.
- The ratio of hearts to stars is 5 to 10.
- The ratio of hearts to stars is $\frac{5}{10}$.
- For every five hearts, there are ten stars.

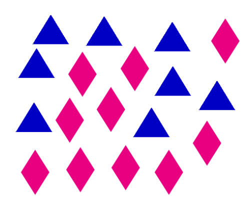
Note that we are not comparing two numbers to determine which one is greater (as in 5 < 10). The comparison is relative as in a multiplication problem. For example, the ratio 5:10 can be simplified to 1:2, and it indicates to us that there are twice as many stars as there are hearts.

We **simplify ratios** in exactly the same way we simplify fractions.

Example 1. In the picture at the right, the ratio of hearts to stars is 12:16. We can simplify that ratio to 6:8 and even further to 3:4. These three ratios (12:16, 6:8, and 3:4) are called **equivalent ratios**.

The ratio that is simplified to lowest terms, 3:4, tells us that for every three hearts, there are four stars.

1. Write the ratio and then simplify it to lowest terms.



The ratio of triangles to diamonds is _____ : _____ = _____ : _____ .

In this picture, there are ______ triangles to every _______ diamonds.

2. **a.** Draw a picture with pentagons and circles so that the ratio of pentagons to the total of all the shapes is 7:9.

b. What is the ratio of circles to pentagons?

3. **a.** Draw a picture in which (1) there are three diamonds for every five triangles, and (2) there is a total of 9 diamonds.

b. Write the ratio of all the diamonds to all the triangles, and simplify this ratio to lowest terms.

4. Write the equivalent ratios.

a. 5 to 45 = 1 to ________	**b.** 3 : _____ = 9 : 60	**c.** 280 : 420 = 2 : _____	**d.** $\frac{5}{13} = \frac{\boxed{}}{65}$

We can also form **ratios using quantities that have units**. If the units are the same, they cancel.

Example 2. Simplify the ratio 250 g : 1.5 kg.

First we convert 1.5 kg to grams and then simplify: $\frac{250\text{ g}}{1.5\text{ kg}} = \frac{250\text{ g}}{1{,}500\text{ g}} = \frac{250}{1{,}500} = \frac{1}{6}$.

5. Use a fraction line to write ratios of the given quantities as in the example. Then simplify the ratios.

a. 5 kg and 800 g $\frac{5\text{ kg}}{800\text{ g}} =$	**b.** 600 cm and 2.4 m
c. 1 gallon and 3 quarts	**d.** 3 ft 4 in and 1 ft 4 in

We can generally **convert** ratios with decimals or fractions **into ratios of whole numbers**.

Example 3. Because we can multiply both terms of the ratio by 10, $\frac{1.5\text{ km}}{2\text{ km}} = \frac{15\text{ km}}{20\text{ km}}$.

Then: $\frac{15\text{ km}}{20\text{ km}} = \frac{15}{20} = \frac{3}{4}$. So the ratio 1.5 km : 2 km is equal to 3:4.

You can also see that the ratio is 3:4 by noticing that both 1.5 km and 2 km are evenly divisible by 500 m.

Example 4. Simplify the ratio ¼ mile to 5 miles.

First, the units cancel: ¼ mi : 5 mi = ¼ : 5. Multiplying both terms of the ratio by 4, we get ¼ : 5 = 1:20.

6. Use a fraction line to write ratios of the given quantities. Then simplify the ratios to whole numbers.

a. 5.6 km and 3.2 km	**b.** 0.02 m and 0.5 m
c. 1.25 m and 0.5 m	**d.** 1/2 L and 7 1/2 L
e. 1/4 cup and 3 1/2 cups	**f.** 2/3 mi and 1 mi

If the two terms in a ratio have *different* units, then the ratio is also called a **rate.**

Example 5. The ratio "8 km to 40 minutes" is a rate that compares the quantities "8 km" and "40 minutes," perhaps for the purpose of giving us the speed at which a person is running.

We can write this rate as 8 km : 40 minutes or $\frac{8\text{ km}}{40\text{ minutes}}$ or 8 km *per* 40 minutes.

The word "per" in a rate signifies the same thing as a colon or a fraction line.

This rate can be simplified: $\frac{8\text{ km}}{40\text{ minutes}} = \frac{1\text{ km}}{5\text{ minutes}}$. The person runs 1 km in 5 minutes.

Example 6. Simplify the rate "15 pencils per 100¢." Solution: $\frac{15\text{ pencils}}{100¢} = \frac{3\text{ pencils}}{20¢}$.

7. Write each rate using a colon, the word "per," or a fraction line. Then simplify it.

a. Jeff swims at a constant speed of 400 meters : 15 minutes.

b. A car can travel 54 miles on 3 gallons of gasoline.

8. Fill in the missing terms to form equivalent rates.

a. $\frac{1/2\text{ cm}}{30\text{ min}} = \frac{\quad}{1\text{ h}} = \frac{\quad}{15\text{ min}}$

b. $\frac{\$88.40}{8\text{ hr}} = \frac{\quad}{2\text{ hr}} = \frac{\quad}{10\text{ hr}}$

9. Simplify these rates. Don't forget to write the units.

a. 280 km per 7 hours

b. 2.5 inches : 1.5 minutes

10. A car is traveling at a constant speed of 72 km/hour. Fill in the table of equivalent rates: each pair of numbers in the table (distance/time) forms a rate that is equivalent to the rate 72 km/hour.

Distance (km)							
Time (min)	10	30	40	50	60	90	100

11. Eight pairs of socks cost $20. Fill in the table of equivalent rates.

Cost ($)								
Pairs of socks	1	2	4	6	7	8	9	10

Solving Problems Using Equivalent Rates

Example 1. It took Liam 1 ½ hours to paint 8 meters of fence. Painting at the same speed, how long will it take him to paint the rest of the fence, which is 28 meters long?

In this problem, we see a rate of 8 m per 1 ½ hours. There is another rate, too: 28 m per an unknown amount of time. These two are equivalent rates. We can use a table of equivalent rates to solve the problem.

Amount of fence (m)	8	4	28
Time (minutes)	90	45	315

(1) We figure that Liam can paint 4 m of fence in 45 minutes (by dividing the terms in the original rate by 2).

(2) Next we multiply both terms in the rate of 4 m/45 min by seven to get the rate 28 m/315 min.

It will take Liam 315 minutes, or 5 hours 15 minutes, to paint the rest of the fence.

Example 2. Sofia rides her bike 20 miles in 45 minutes. Riding at the same speed, how far will she go in 1 hour?

$$\frac{20 \text{ miles}}{45 \text{ minutes}} \underset{\div 3}{\overset{\div 3}{=}} \frac{\boxed{} \text{ miles}}{15 \text{ minutes}} \underset{\times 4}{\overset{\times 4}{=}} \frac{\boxed{} \text{ miles}}{60 \text{ minutes}}$$

We can <u>multiply or divide both terms of a rate by the same number</u> to form another, equivalent rate. (You have used this same idea in the past with equivalent fractions.)

It's not easy to go directly from 45 minutes to 60, but we can use 15 as a "stepping stone" in between.

Recall that 20 ÷ 3 is easy to solve when you think of it as a fraction: 20/3 = 6 2/3. Sofia can ride 6 2/3 miles in 15 minutes.

Then, we multiply both terms of that rate by 4. Again, don't be intimidated by the fraction: 4 · (6 2/3) = 4 · (20/3) = 80/3 = 26 2/3. So, Sofia can ride 26 2/3 miles in 1 hour.

1. Fill in the tables of equivalent rates.

a.

Distance	15 km			
Time	3 hr	1 hr	15 min	45 min

b.

Pay	$15			
Time	45 min	15 min	1 hr	1 hr 45 min

2. Fill in the missing terms in these equivalent rates.

a. $\frac{3 \text{ pies}}{8 \text{ boys}} = \frac{\underline{\qquad}}{2 \text{ boys}} = \frac{\underline{\qquad}}{12 \text{ boys}} = \frac{\underline{\qquad}}{20 \text{ boys}}$

b. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\underline{\qquad}}{1 \text{ min}} = \frac{\underline{\qquad}}{3 \text{ min}}$

3. Aiden can ride his bicycle 8 miles in 28 minutes. At the same constant speed, how long will he take to go 36 miles?

$$\frac{8 \text{ miles}}{28 \text{ minutes}} = \frac{4 \text{ miles}}{\boxed{} \text{ minutes}} = \frac{\boxed{} \text{ miles}}{\boxed{} \text{ minutes}}$$

Example 3. You get 20 erasers for \$3.80. How much would 22 erasers cost (at the same rate)?

One way to solve this is to calculate the cost for ONE eraser, or **the unit rate**, and then multiply that by 22:

$$\frac{\$3.80}{20\text{ erasers}} \overset{\div 20}{=} \frac{\$0.19}{1\text{ eraser}} \overset{\times 22}{=} \frac{\$4.18}{22\text{ erasers}}$$

Another way is to find the cost for two erasers first.

Price	\$3.80	\$0.38	\$4.18
Erasers	20	2	22

Twenty erasers cost \$3.80, so 2 erasers cost 1/10 that much, or \$0.38. Lastly, the cost of 22 erasers is the cost of 20 + 2 erasers, or \$3.80 + \$0.38 = \$4.18.

4. If 15 muffins cost \$10.80, how much would 40 muffins cost?

5. See Lucas's and Avery's solutions to a problem about pencils below. One of them must be in error. Check their work and find the error.

A set of 30 pencils costs \$4.50. Is that equal to the rate of 50 pencils for \$7.25? If not, which pencils are cheaper?

Lucas: I will calculate the unit rates.

1st set: \$4.50/30 = \$0.15 per pencil

2nd set: \$7.25/50 = \$0.155 per pencil

The pencils in the first set are cheaper per pencil.

Avery: I will calculate the price for 150 pencils using both rates.

1st set:

Price	\$4.50	\$22.50
Pencils	30	150

2nd set:

Price	\$7.25	\$21.75
Pencils	50	150

The pencils in the second set are cheaper.

6. **a.** A train travels at a constant speed of 111 km per hour. How far will it go in 140 minutes?

   b. Is this equal to the rate of traveling 90 km in 40 minutes?

7. Mason earns \$220 for eight hours of work. In how many (whole) hours will he earn at least \$600?

Example 4. For each hour of computer work, you are supposed to do simple exercises for 5 minutes.

Since 1 hour is 60 minutes, we can write this rate as 5 minutes : 60 minutes.

Notice how the rate has the *same* units in both terms. Those units can therefore be canceled, and the rate (really, a ratio) is equivalent to the unitless ratio 5 : 60 = 1 : 12. This means you are supposed to spend 1/12 of the time doing exercises when doing computer work.

8. Refer to Example 4. How much time should you spend doing exercises for 4 1/3 hours of computer work?

9. A certain medicine for dogs is administered at the rate of 5 mg per each kilogram of body weight.

a. You have 125 mg of that medicine. What is the maximum weight for a dog you could treat with it?

b. Another medicine for the same problem is administered at the rate of 20 mg per each 5 kg of body weight. If your dog weighs 13 kg, which of the two medicines would you need more of?

10. A car uses 3.9 liters of gasoline to travel 45 km. How many liters of gasoline would the car need for a trip of 60 km?

11. In a poll that interviewed 1,000 people about their favorite color, 640 people said they liked blue.

a. Simplify this ratio to lowest terms.

b. Assuming the same ratio holds true in another group of 125 people, how many of those people can we expect to like blue?

12. Jane and Stacy ran for 30 seconds. Afterward each girl checked her heartbeat. Jane counted that her heart beat 38 times in 15 seconds, and Stacy counted that her heart beat 52 times in 20 seconds. Which girl had a faster heart rate, measured in bpm (beats per minute)? How much faster?

Unit Rates

Remember that a rate is a ratio where the two terms have different units, such as 2 kg/$0.45 and 600 km/5 hr.

In a **unit rate, the second term of the rate is one** (of some unit).

For example, 55 mi/1 hr and $4.95/1 lb are unit rates. The number "1" is nearly always omitted so those rates are usually written as 55 mi/hr and $4.95/lb.

To convert a rate into an equivalent unit rate simply divide the numbers in the rate.
This means you divide the first term of the rate by the second term of the rate.

Example 1. Mark rides his bike 35 km in 1 ½ hours. What is the unit rate?

We are given the rate 35 km : 1 ½ hr. To find the unit rate, we *divide* 35 km by 1 ½ hr.

Note how the units "km" and "hours" are divided, too, and become "km per hour" or "km/hour."

$$\frac{35 \text{ km}}{1\ 1/2 \text{ h}} = 35 \div \frac{3}{2} \text{ km/hr} = 35 \cdot \frac{2}{3} \text{ km/hr}$$

$$= \frac{70}{3} \text{ km/hr} = 23\tfrac{1}{3} \text{ km/hr.}$$

We could also use decimal division:
35 km ÷ 1.5 hr = 23.333... km/hr.

So, the unit rate is 23 ⅓ km per hour.

Example 2. A snail can slide through the mud 5 cm in 20 minutes. What is the unit rate?

We simply divide 5 cm ÷ 20 min. Notice how the units cm and min also get divided:

$$\frac{5 \text{ cm}}{20 \text{ min}} = \frac{5}{20} \text{ cm/min} = \frac{1}{4} \text{ cm/min}$$

As a decimal, this is 0.25 cm/min.

Here, the unit rate is the snail's speed, and considering we got such a small number, it is more practical to express it in centimeters per *hour*.

To do that, we can multiply the 1/4 cm/min by 60. Or, we can multiply both terms of the *original* rate by 3 to get the equivalent rate of 15 cm : 60 min, or 15 cm per hour. He's not going very fast!

1. Find the unit rate.

a. $125 for 5 packages

b. $6 for 30 envelopes

c. $1.37 for ½ hour

d. 2 ½ inches per 4 minutes

e. 24 m^2 per 3/4 gallon

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Proportional Relationship or Not?

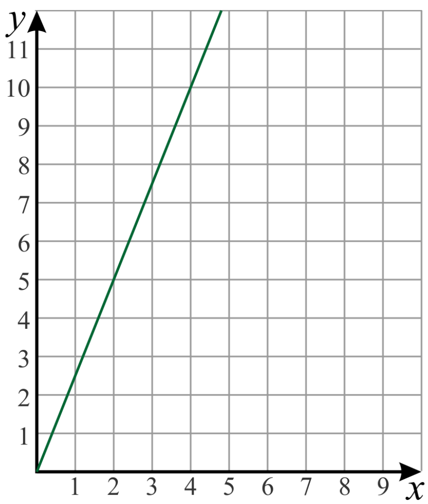
You have learned that if two variables are in a proportional relationship, then the rates formed by the values of the variables are equivalent. This means that if the value of one doubles, the value of the other doubles also. If one quantity decreases to 1/5 of its value, the other does the same. In fact, if one value is multiplied or divided by any number, the same happens to the other (with the exception of the point (0, 0)).

Now we will learn something about the graph of an equation that depicts a proportional relationship.

1. Find the two situations below that show the variables to be in a proportional relationship.
   How do the graphs of those equations differ from the graphs depicting a *non*-proportional relationship?

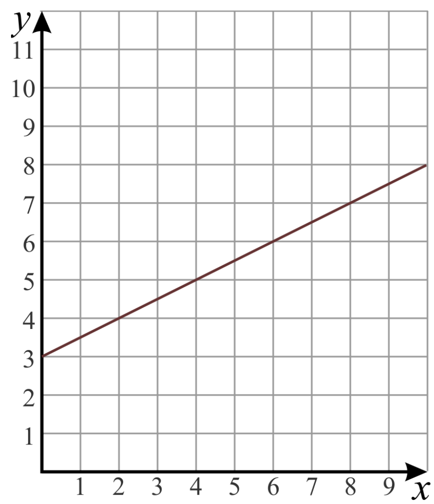
(i)
$y = 2.5x$

y	0	2.5	5	7.5	10	12.5
x	0	1	2	3	4	5



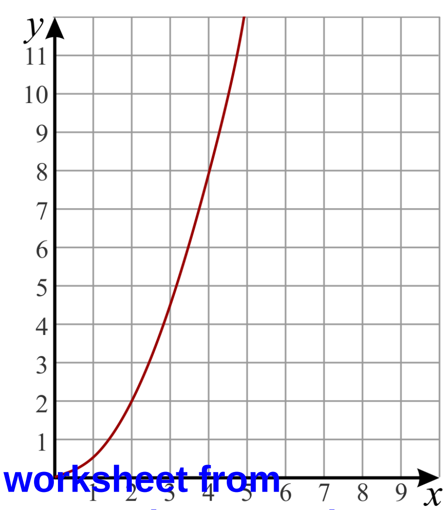
(ii)
$y = 0.5x + 3$

y	3	3.5	4	4.5	5	5.5
x	0	1	2	3	4	5



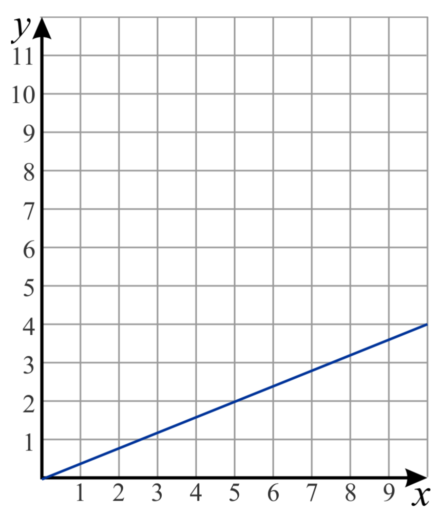
(iii)
$y = 0.5x^2$

y	0	0.5	2	4.5	8	12.5
x	0	1	2	3	4	5



(iv)
$y = 0.4x$

y	0	0.4	0.8	1.2	1.6	2
x	0	1	2	3	4	5



You can check to see if two variables are in direct variation in several different ways.

(1) Check if the <u>values</u> of the variables are in direct variation (if the rates are equivalent).
If you double the value of one, does the value of the other double also? Or, maybe one rate is $42/6 kg and the other is $35/5 kg. Those are equivalent rates, both being equal to the unit rate of $7/kg.

(2) When two variables are proportional, the equation relating the two is of the form $y = mx$, where y and x are the variables, and m is a constant. If the equation is of some other form, such as $y = 2x^2$ or $y = 1/x$ or $y = mx + b$, the variables are not proportional.

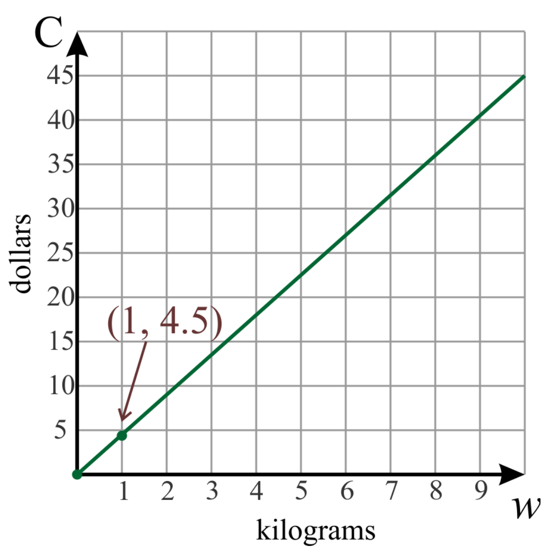
(3) The graph of such an equation is always a line through the origin.

Example. Apples cost $4.50 per kg. The equation relating the cost (C) and the weight (w) of apples is $C = 4.5w$, which is of the form $y = mx$. The quantities are proportional.

Note the point (1, 4.5) that corresponds to the unit rate. The other special point on the graph is the origin at (0, 0). That point is always on the graph of a proportional relationship.

It is possible to look at the situation just the opposite way, and consider how the weight of the apples depends on the cost of the apples. In that case, we would write the equation $w = \frac{1}{4.5}C$, which is approximately $w = 0.22C$, and plot the values of C on the horizontal axis. This way is not as common as observing how the cost depends on the weight.

Cost of Apples



You might wonder, "Why does the line have to go through the origin if the quantities are in proportion?"

Consider this principle governing direct variation: if one quantity is cut in half, then the other is cut in half. Let's say you start with certain values of the two quantities, such as 6 meters per 2 minutes. Now cut both in half to get 3 meters per 1 minute. Do it again to get 1.5 meters per 1/2 minute, then 0.75 m per 1/4 minute. Do it again, and again.

Notice that both numbers get smaller and smaller yet—they approach zero. This would happen no matter what values of the two quantities you started with. So the point (0, 0) has to be included in the graph of quantities that are in direct variation.

2. Determine whether the two variables are proportional. If so, write an equation relating them.

a.

b	2	3	4	5	6
a	0	1	2	3	4

b.

y	0	4	8	12	16
x	0	1	2	3	4

c.

t	0	1/3	2/3	1	4/3
s	0	1	2	3	4

d.

w	20	15	10	5	0
v	0	1	2	3	4

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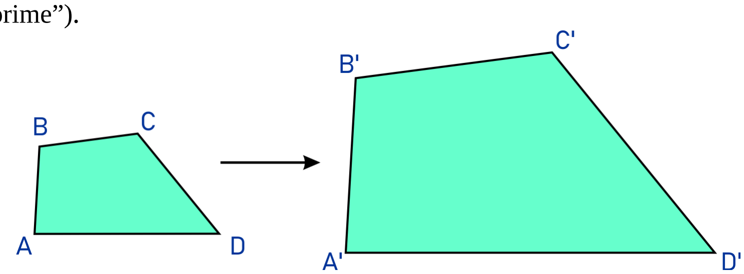
Scaling Figures

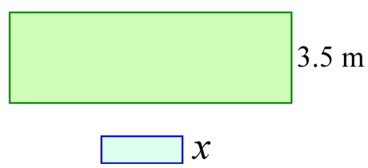
Example 1. Trapezoid ABCD has been enlarged or **scaled** proportionally to become trapezoid A'B'C'D'. (Note: A' is read as "A prime").



The two trapezoids have the same basic shape, but one is bigger than the other.
We say such figures are **similar figures**, and that the one is a **scaled image** of the other.

When we **scale a figure**, we enlarge or shrink it while maintaining its shape. In this process, all the dimensions of the figure are multiplied by the same number called the **scale factor** or just the **scale**.

Example 2. The bigger rectangle was shrunk using a scale factor of 2/7. Find the length of the side marked x.

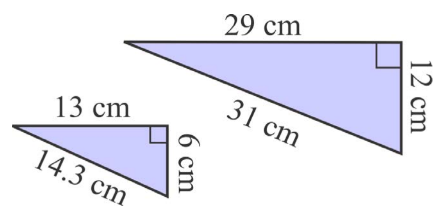


We simply multiply the given length 3.5 m by the scale factor:

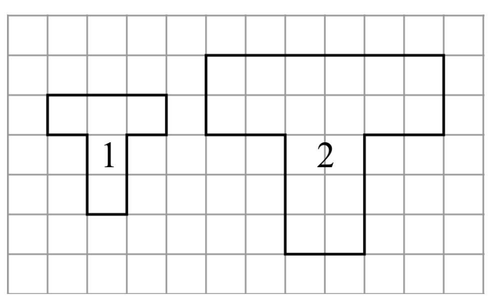
$(2/7) \cdot 3.5 \text{ m} = 2 \cdot 3.5 \text{ m} / 7 = \underline{\ 1 \text{ m}\ }$

1. Find the scale factor used in Example 1 to enlarge trapezoid ABCD. Do so by measuring the sides of the trapezoids with a millimeter-ruler.

2. Are the two triangles similar? Why or why not? (Note: the images are not to scale.)

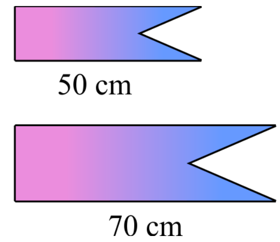


3. Is Figure 2 a scaled image of figure 1? Explain.



Scale factor and similarity ratio

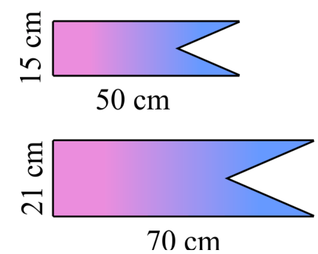
Example 3. The smaller figure was enlarged. What is the **scale factor**?



Since the 50 cm-side became 70 cm long, the scale factor from the smaller figure to the larger one is 70/50 = 7/5 = 1.4.
So, each side of the figure became 1.4 times as long as before.

(Note that the scale factor is *not* 5/7, because that is less than 1, and would signify that the figure shrunk.)

We can also consider the **similarity ratio** (or ratio of magnification): it is the ratio of any side of the figure and the corresponding side in the scaled figure.



Using the 50-cm and 70-cm sides, we get that the similarity ratio of the smaller shape to the bigger shape is 50:70 = **5:7**.

If we use the other marked sides, we get the same: 15:21 = **5:7**.
This ratio is the same, no matter which side or dimension we use.

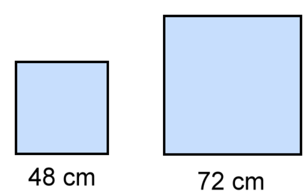
If we want the similarity ratio of the bigger shape to the smaller shape, we list the bigger shape's side length first. The similarity ratio is then 70:50 = 7:5.

So, the scale factor was 7/5 or 1.4, and the similarity ratio was 5:7 or 7:5, depending on whether you list it as of the smaller shape to the bigger shape or vice versa.

Note that the scale *factor* is a single **number** (such as 3), but the similarity *ratio* is a **ratio** of two numbers (such as 3:1). Always be careful to note which way you use the scale factor or the similarity ratio. If the figure gets bigger, then multiply by a number that is more than 1. If it gets smaller, multiply by a number that is less than 1.

4. The smaller square was enlarged to become the larger square.
   What is the scale factor?

   The similarity ratio?



5. A rectangle with 20 ft and 48 ft sides is shrunk proportionally so that its shorter side becomes 15 ft.

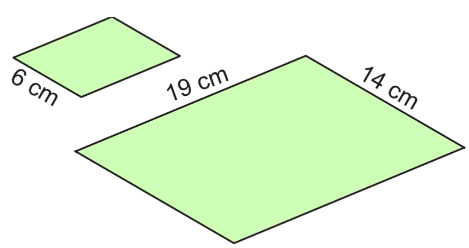
   a. What is the similarity ratio?

   b. What is the scale factor?

   c. How long is the longer side in the smaller rectangle?

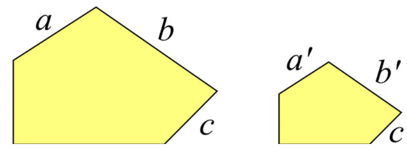
6. **a.** Find the scale *factor* when the smaller parallelogram is enlarged to become the bigger one.

   b. What is the similarity *ratio*?



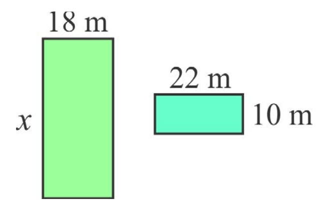
When two figures are similar, the **corresponding dimensions of the original image and the scaled image are proportional**. In other words, they are in the same ratio — and this ratio is the similarity ratio.

The illustration on the right shows two similar pentagons, with some side lengths marked. The ratio $a : a'$ equals the ratio $b : b'$, and both of them equal the ratio $c : c'$ — and the same would be true for the other corresponding sides.



As an equation, $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

Example 4. Sometimes you need to look carefully to find the corresponding sides. The two rectangles at the right are similar. Notice that the “top” sides of 18 m and 22 m do *not* correspond. Instead, the 18 m side corresponds to the 10 m side, because they are the *shorter* sides of the rectangles.



The ratio of 18 m : 10 m = 9:5 is the similarity ratio. It is equal to the ratio x : 22 m we can write using the other corresponding sides.

This means we can use a proportion to solve for x (on the right).

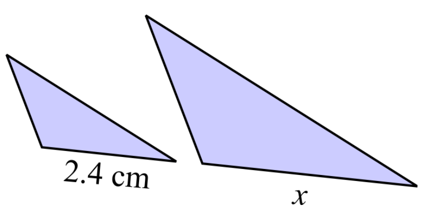
$$\frac{x}{22\text{ m}} = \frac{9}{5}$$

$$198\text{ m} = 5x$$

$$x = 39.6\text{ m}$$

7. The figures are similar. Find the length of the side labeled x.

a. Similarity ratio 3:5.



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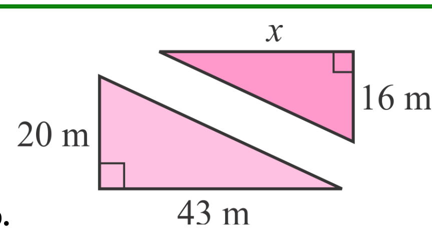
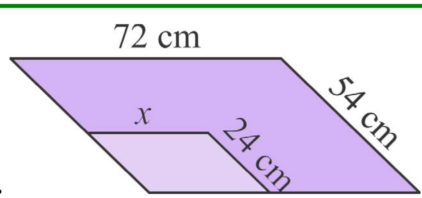


c.



8. The sides of two similar triangles are in a ratio of 3:4. If the sides of the larger triangle are 4.8 cm, 6.0 cm, and 3.6 cm, what are the sides of the smaller triangle?
   Hint: Here you don't need a proportion since the numbers are easy. You can reason logically.

9. The sides of a rectangle measure 12 cm and 18 cm. The shorter side of another, similar rectangle is 4 cm.

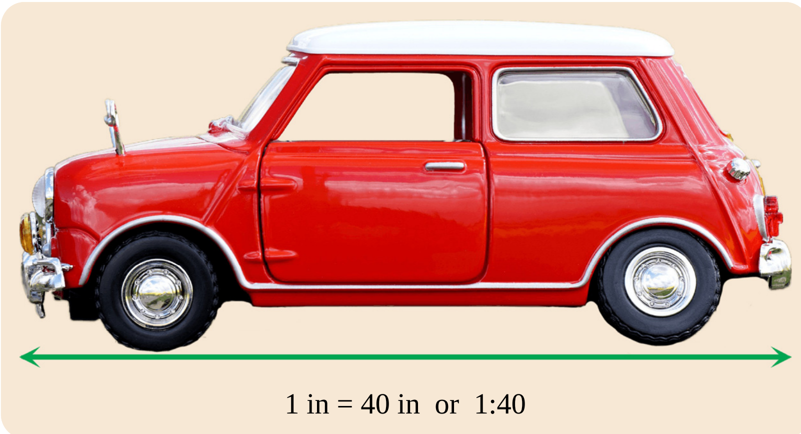
   a. In what ratio are the sides of the two rectangles?

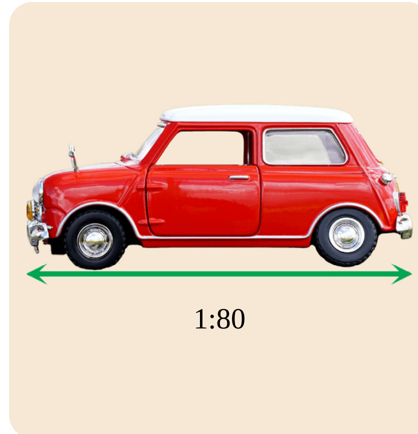
   b. Calculate the areas of both rectangles.

   c. (optional) In what ratio are their areas?

10. (optional) Draw any triangle on blank paper or below. Then draw another, bigger triangle using the scale ratio of 2:5. You will probably need to measure some angles from your triangle to be able to draw it. Note that **corresponding *angles* in the two triangles will be equal**. Only the side lengths change — not the angles.

Scale Drawings 1





Example 1. Here you see two scaled images of a car, with two different scales. The left image has the scale of **1:40**, which means that 1 inch in the picture corresponds to 40 inches in reality.

In fact, the scale is also written as "**1 in = 40 in**". This doesn't mean that 1 inch really is as long as 40 inches; that wouldn't make sense. It means that 1 inch *in the image* equals 40 inches *in reality*.

The image on the right has the scale of 1:80. One unit in the image corresponds to 80 units in reality.

Notice how, for the picture on the right, the number in the scale got *bigger*, but the picture got *smaller*. How much smaller did it get?

Find how long the car is in reality. Use a ruler.

Do you get the same answer using either picture and its scale?

You can use a calculator for all the problems in the lesson.

1. **a.** Let's say you are to draw a third picture of the car in Example 1, with a scale of 1:160. How can you use the scale of 1:80 and the new scale of 1:160 to figure out the dimensions (width and length) of the new drawing, without figuring it from the dimensions in reality?

   b. How long will the car be if drawn to the scale of 1:160? How tall?

2. This rectangle shows a plot of land, drawn here at the scale of 1:2000.

   a. Measure its dimensions, and then redraw it at the scale of 1:500.
   (Will it be bigger or smaller than the image above?)

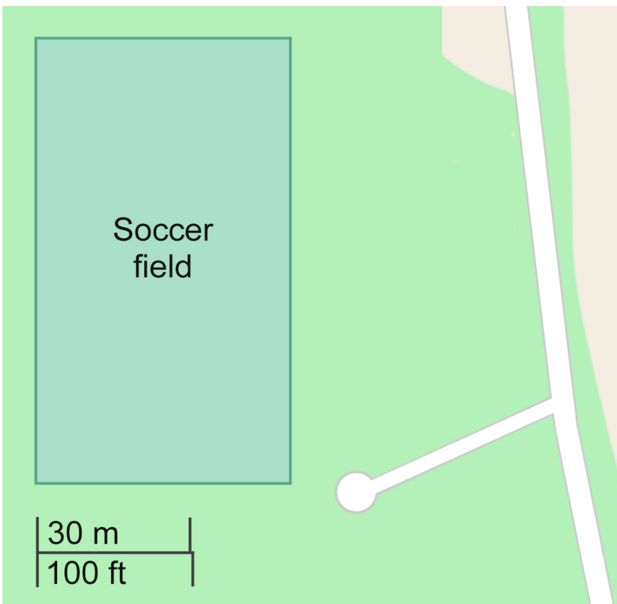
   b. Find its dimensions and its area in reality.

3. **a.** Look at the scale on the map of the soccer field below. The line indicating 30 m on the map is 2 cm long.*
Rewrite this scale of **2 cm = 30 m** as a unitless ratio, in the form 1: (some number).
(Hint: First make sure both measurements are in the same unit.)
*It is 2 cm long if the page was printed at 100%.

b. If the soccer field was redrawn with the scale 1 cm = 30 m, would it be bigger or smaller than the picture here?

By how much?

c. Draw another scale drawing of the soccer field on the right, using the scale 1 cm = 20 m.



Draw the soccer field here, using the scale 1 cm = 20 m.

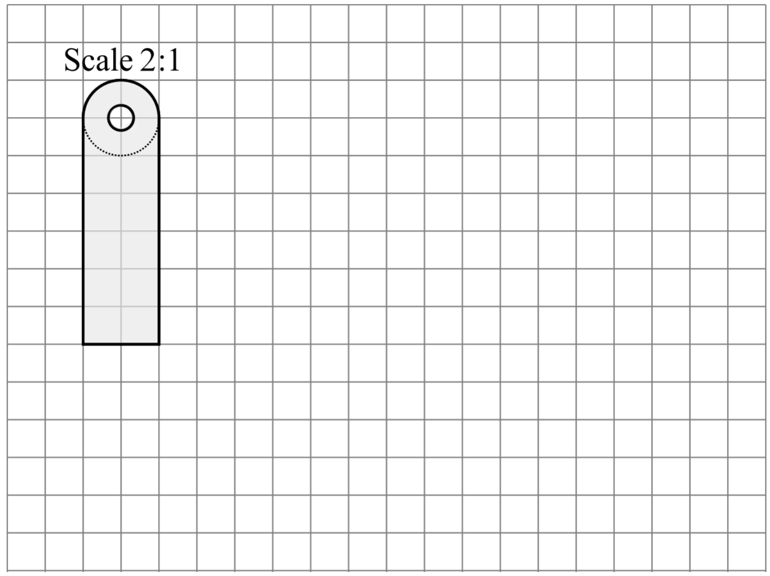
d. Find the length, width, perimeter, and area of the soccer field in reality. Use meters and square meters.

4. Your family is purchasing an air conditioner. In the store, you take photos of the AC units on a phone. One unit is 32 inches wide by 12 inches tall in reality. On the phone, in the picture, the unit is 6 in by 2 1/4 in. What is the scale? What is the scale *factor* (from the picture to reality)?

5. The picture shows a metal part designed for a machine.

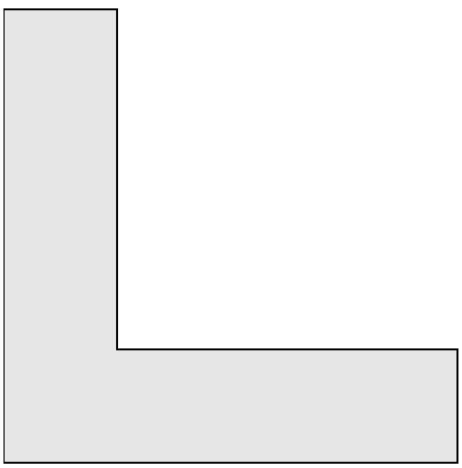
   a. Redraw the part in the scale 1:1.

   b. Redraw the part in the scale 3:1.



6. The L-shape depicts a driveway to Shaun's house. It is drawn here at the scale of 1:300.

   a. Draw another scale drawing of the driveway at the scale of 1:400.



   b. Measure the necessary dimensions of the driveway, and find its actual area in square meters.

   c. Gravel is sold by cubic yard, and it will be spread one foot deep, so the calculation for the volume of the driveway needs to be done in cubic feet, then converted to cubic yards.

   (i) Convert your answer from (b) to square feet using this factor: 1 m^2 = 10.7639 square feet.

   (ii) The gravel will be applied 1 foot deep. Now find the volume of the needed gravel in cubic feet.

   (iii) Convert this to cubic yards, using 1 cubic yard = 27 cubic feet.
   Round your answer *up* to the nearest half cubic yard.

   (iv) Gravel costs $65 per cubic yard. Now calculate the cost of the gravel for the driveway.

Floor Plans

Floor plans are drawn using a **scale**, which is a ratio relating the distances in the plan to the distances in reality. For example, a scale of 1 cm : 2 m means that 1 cm in the drawing corresponds to 2 m in reality.

Example 1. A room measures 1 ¾″ by 2 ½″ in a plan with a scale of 1 in: 10 ft. How big is it in reality?

Since 1 inch corresponds to 10 ft, we simply need to multiply the length and width given in inches by 10 to get the dimensions in feet.

Using decimals, the dimensions are 1.75 in by 2.5 in. So, the dimensions of the room in actual size are 1.75 · 10 = 17.5 ft and 2.5 · 10 = 25 ft.

However, we're really not just multiplying by the number 10 but by the ratio 10 ft/1 in. That is how we keep track of the units to make sure that our final answer ends up with the correct units (feet and not inches). This is what's really happening in the calculation:

$$1.75\ \cancel{\text{in}} \cdot \frac{10\text{ ft}}{1\ \cancel{\text{in}}} = 17.5\text{ ft} \quad \text{and} \quad 2.5\ \cancel{\text{in}} \cdot \frac{10\text{ ft}}{1\ \cancel{\text{in}}} = 25\text{ ft}$$

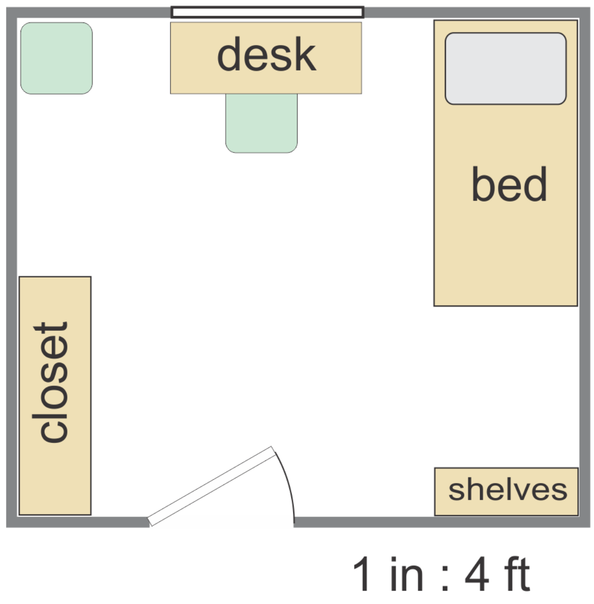
Why not multiply by 1 in/10 ft (or 1 in : 10 ft) as the ratio is stated in the problem? Then the inches ("in") in the dimension wouldn't cancel the inches in the conversion factor, and we would end up "in^2/ft" as our unit of length, instead of "ft."

1. This room is drawn at a scale of 1 in : 4 ft. This means that 1 in on the plan corresponds to 4 ft in reality. It also means you can simply multiply or divide by 4 to convert dimensions between the plan and reality and vice versa.

   Measure the dimensions for the bed and the desk from the picture. Then calculate the actual (real) dimensions.

   a. the bed

   b. the desk



2. What is the area of this room in reality?

3. In the middle of the plan for the room, draw a table that in reality measures 3.5 ft × 2.5 ft.

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Review: Percent

Percent (or **per cent**) means *per hundred* or “divided by a hundred.” (The word “cent” means one hundred.) So, percent means the rate per hundred, or a hundredth part.

To convert percentages into fractions, simply read the “per cent” as “per 100.” Thinking of hundredths, you can also easily write them as decimals.

Therefore, 8% = 8 per cent = 8 per 100 = 8/100 = 0.08.

Similarly, 167% = 167 per 100 = 167/100 = 1.67.

$\frac{5}{100}$ five per cent = 5%

1. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
	0.07	
52%		
		$\frac{59}{100}$

Percent	Decimal	Fraction
109%		
200%		
		$\frac{382}{100}$

A number with two decimal digits has hundredths, so it can easily be written as a percentage. For example, 0.56 = 56%. But we can write numbers with more decimal digits as percents, also.

Example 1. As a percentage, the number 0.5642 is 56.42%. Compare this to 0.56 = 56%. The digits “42” simply follow the digits “56”, and become the decimal digits for the percentage.

Decimal	Percent	Fraction
0.09	9%	$\frac{9}{100}$
0.091	9.1%	$\frac{91}{1000}$
0.09146	9.146%	$\frac{9146}{100{,}000}$

2. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
0.9%		
		$\frac{282}{1000}$
	0.8914	

Percent	Decimal	Fraction
		$\frac{91}{10{,}000}$
2.391%		
	0.94284	

Writing fractions as percentages

Example 2. Sometimes you can convert a fraction into an equivalent fraction with a denominator of 100, 1000, or some other power of 10. After that it is easy to write it as a decimal and then as a percentage.

$$\frac{46}{25} \overset{\cdot 4}{=} \frac{184}{100} = 1.84 = 184\%$$

Example 3. For most fractions, we need to use *division* to convert the fraction to a decimal first, and then to a percentage.

Simply treat the fraction line as a division symbol and divide (using long division or a calculator), to get a decimal. Then write it as a percentage.

$$\frac{8}{9} = 0.888... \approx 0.889 = 88.9\%$$

```
    0.8888
 9 )8.0000
   -7 2
      8 0
    - 7 2
        8 0
      - 7 2
          8 0
        - 7 2
            8
```

3. Fill in the table. First write each fraction as an equivalent fraction where the denominator is a power of ten.

Fraction	Fraction (denominator is a power of ten)	Decimal	Percent
$\frac{8}{25}$	$\frac{\square}{100}$		
$\frac{142}{200}$	$\frac{\square}{100}$		
$\frac{24}{20}$			
$\frac{31}{250}$			
$\frac{3}{8}$			

4. Write as percentages. Use long division. Round your answers to the nearest tenth of a percent.

a. 11/8	**b.** 11/24

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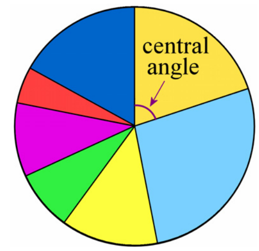
Circle Graphs

(This lesson is optional.)

A **circle graph** shows visually how a total is divided into parts (percentages). Each of the parts (pie slices) is a **sector**, and each sector has a **central angle**.

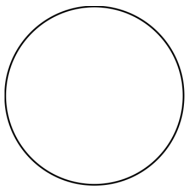


To make a circle graph, we need to calculate the measure of the central angle for each sector. For example, if a circle graph is supposed to show the percentages 25%, 13%, and 62%, we calculate those percentages of 360º (the full circle):

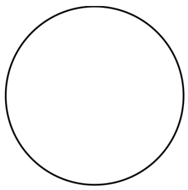
25% of the total corresponds to $0.25 \cdot 360^\circ = 90^\circ$.
13% of the total corresponds to $0.13 \cdot 360^\circ = 46.8^\circ$.
62% of the total corresponds to $0.62 \cdot 360^\circ = 223.2^\circ$.

1. Sketch a circle graph that shows...

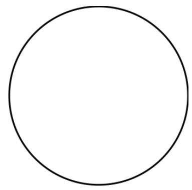
a. 50%, 25%, and 25%



b. 33.3%, 33.3%, 1/6, and 1/6

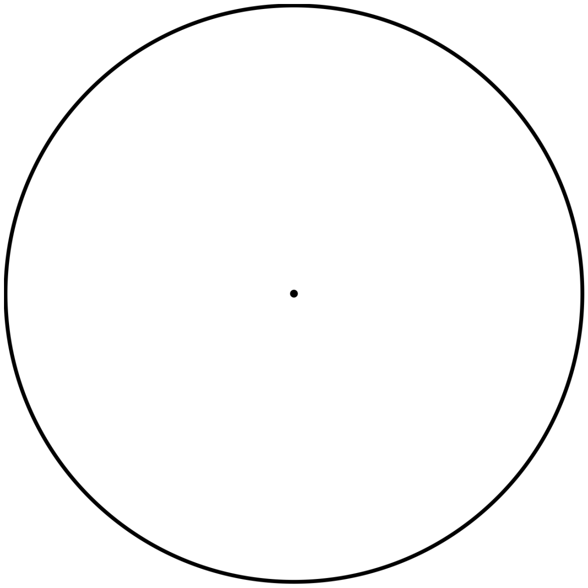


c. 20%, 20%, 10%, and 50%



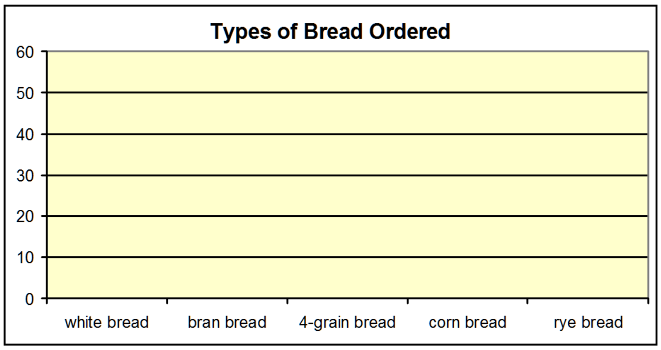
2. The table shows different kinds of specialty breads that a grocery store ordered. Fill in the table. Make a circle graph. (*Note:* You will need a protractor to draw the angles.)

Type	Quantity	Percentage	Central Angle
white bread	50		
bran bread	25		
rye bread	30		
corn bread	40		
4-grain bread	55		
TOTALS	**200**	**100%**	**360º**

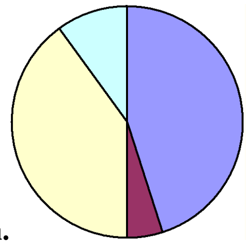


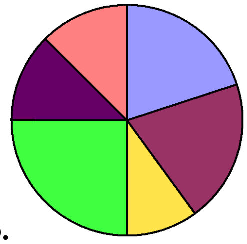
3. Make a bar graph of the quantities of each type of bread from the table above. →

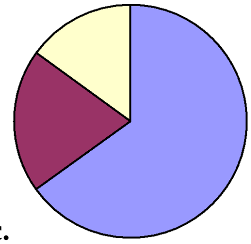
Does the bar graph show percentages?



4. Think of fractions. Estimate the percentages that the sectors of the circle represent.

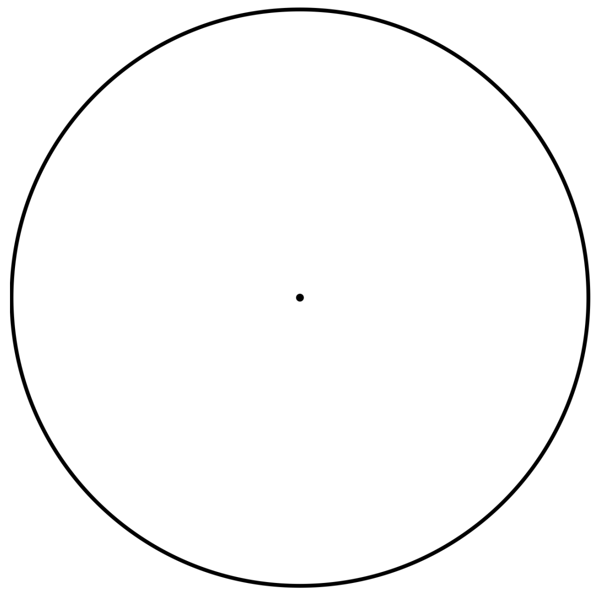

a.


b.


c.

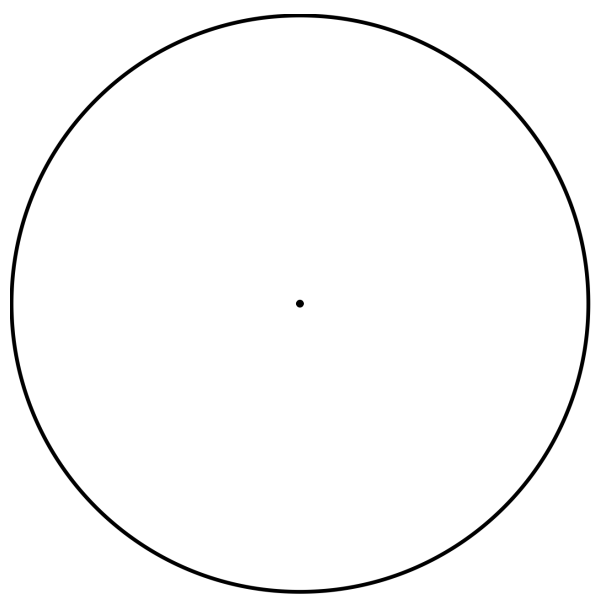
5. The table lists by flavor how many units of protein powder a company sold. Draw a circle graph showing the percentages. You will need a protractor and a calculator.

Flavor	Amount sold	Percentage of total	Central Angle
chocolate	67		
vanilla	34		
strawberry	16		
blueberry	26		
TOTALS		**100%**	**360°**



6. Mark polled some seventh graders about their favorite hobbies. Draw a circle graph to show the percentages. Round the angles to whole degrees. You will need a protractor.

Favorite hobby	Percentage	Central Angle
Reading	12.3%	
Watching TV	24.5%	
Computer games	21%	
Sports	22.3%	
Pets	7.1%	
Collecting	8.1%	
no hobby	4.7%	
TOTALS	**100%**	**360°**



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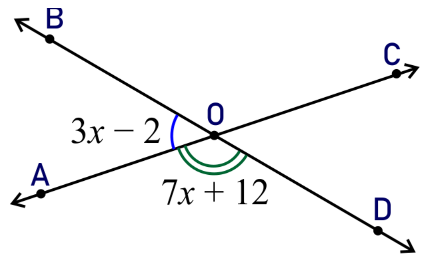
Angle Relationships 3

Example 1. Two lines intersect at O. Find the measure of ∠AOD.



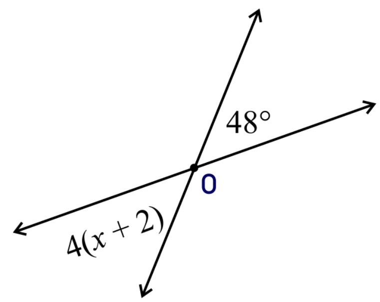
We can write an equation to solve for x, based on the fact that angles AOB and DOA are supplementary, thus sum to 180°:

$$\begin{aligned} (3x - 2) + (7x + 12) &= 180 \\ 3x - 2 + 7x + 12 &= 180 \\ 10x + 10 &= 180 \\ 10x &= 170 \\ x &= 17 \end{aligned}$$

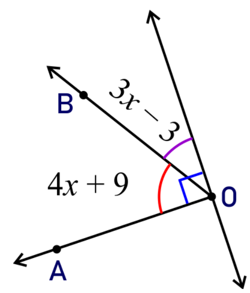
Now, 17 is not yet the final answer, because ∠AOD does not measure x°. It measures $7x + 12$ degrees!

We calculate: $7x + 12 = 7(17) + 12 = 131$. So, ∠AOD = 131°.

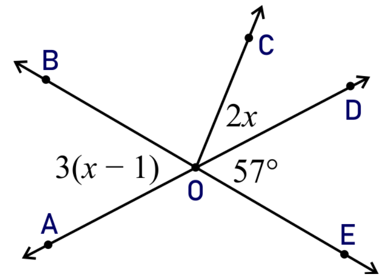
1. Two lines intersect at O. Find the value of x.



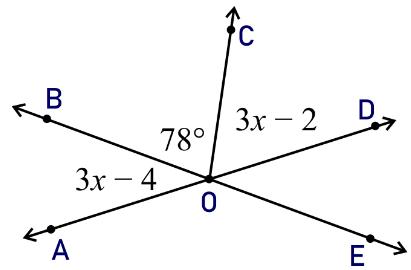
2. Find the measure of ∠AOB.



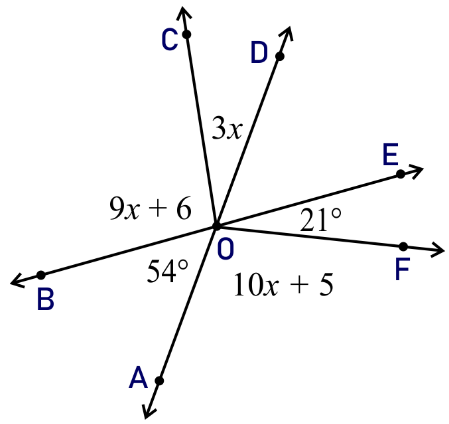
3. Lines AD and BE intersect at O. Find the measure of ∠COD.



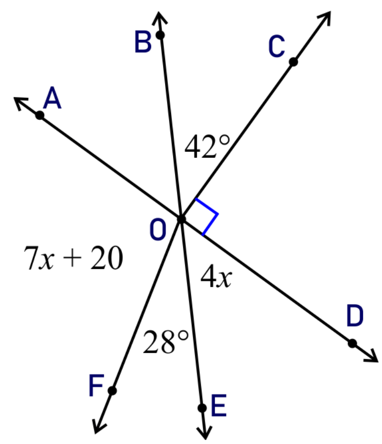
4. Lines AD and BE intersect at O. Find the measures of ∠AOE and ∠DOE.



5. Lines AD and BE intersect at point O which is also the starting point for rays OC and OF.
Find the measures of ∠BOC and ∠DOE.



6. Lines AD and BE intersect at point O which is also the starting point for rays OC and OF.
Find the measures of ∠AOB and ∠AOF.

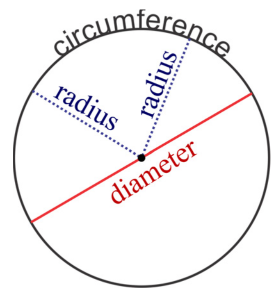


Puzzle Corner

In an isosceles right triangle, the top angle measures $2x + 5$ degrees. Find the value of x.

Circumference of a Circle

Circle terms

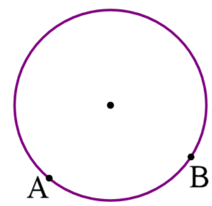


- The **circumference** of a circle is the perimeter, or outside curve, of the circle.
- The **radius** is any line segment from the center point to the circumference.

In fact, what makes a circle, a circle is the fact that all the points on the circumference are **at the same distance from the center point** of the circle. This distance is called the **radius** of the circle.

- The **diameter** is any line segment from circumference to circumference that goes through the center point of the circle.

You may use a calculator for every problem in this lesson.



1. **a.** Draw a radius for this circle from the point A and a diameter from the point B.

   b. What simple relationship exists between the diameter and the radius of any circle?

2. A circle is drawn on the ground. Your 15-foot jump rope is just enough to go around it. Match the approximate measurements and the terms.

diameter	about 15 ft
radius	about 2.5 ft
circumference	about 5 ft

3. There exists an amazing relationship between the circumference and the diameter of every circle! Let's study it now. Find at least **five circular objects**, such as a plate, a can, a glass, and so on. Measure the diameter (d) of each circle with a ruler. Measure the circumference (C) of each circle by placing a string around the object, and then measuring the length of the string. Record your results in the table.

   In the last column, divide the circumference by the diameter using a calculator (separately for each object). In other words, you will calculate the ***ratio* of the circumference to the diameter**.

Object	C	d	C ÷ d

What do you notice?

If you have measured accurately, for each object, the ratio of C to d should be a little over 3.

Probability

You *probably* already have an intuitive idea of what *probability* is. In this lesson we look at some simple examples in order to study probability from a mathematical point of view.

If we flip a coin, the chance, or **probability**, of getting "heads" is 1/2. The chance of getting "tails" is also 1/2. "Heads" and "tails" are the two possible **outcomes** when tossing a coin, and they are equally likely.

When rolling a six-sided number cube (a die), you have six possible **outcomes**: you can roll either 1, 2, 3, 4, 5, or 6. These are all equally likely (assuming the die is fair).

Thus the probability of rolling a five is 1/6. The probability of rolling a three is also 1/6. In fact, the probability of each of the six outcomes is 1/6.

The probability of rolling an even number is 3/6, or 1/2, because three of the six possible outcomes are even numbers.

Simple probability has to do with situations where each possible outcome is <u>equally likely</u>.

Then the **probability** of an event is the fraction $\dfrac{\textbf{number of favorable outcomes}}{\textbf{number of possible outcomes}}$

"Favorable outcomes" are those that make up the event you want. The examples will make this clear.

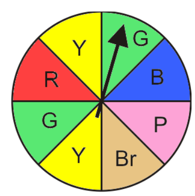
Example 1. What is the probability of getting a number that is less than 6 when tossing a fair die?

Count how many of the outcomes are "favorable" (less than 6). There are five: 1, 2, 3, 4, or 5.
And there are six possible outcomes in total.

Therefore, the probability is $\dfrac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \dfrac{5}{6}$.

In math notation we write "P" for probability and put the event in parentheses: **P(less than 6) = 5/6.**

Example 2. On this spinner the number of possible outcomes is eight, because the arrow is equally likely to land on any of the eight wedges. What is the probability of spinning yellow?



There are TWO favorable outcomes (yellow areas) out of EIGHT possible outcomes.

P(yellow) = 2/8 = 1/4.

(Because green and yellow each have two wedges, there are only six possible colors that can result. When we list the possible outcomes, we list the six colors. However, when we figure the probabilities, we must use the eight equal-sized wedges to find the probability.)

By convention, the probability of an event is always at least 0 and at most 1. In symbols: $0 \leq P(\text{event}) \leq 1$.

A probability of 0 means that the event does not occur; it is impossible. Probability of 1 means that the event is sure to occur; it is certain. A probability near 1 (such as 0.85) means that the event is likely to occur. A probability of 1/2 means that an event is neither likely nor unlikely.

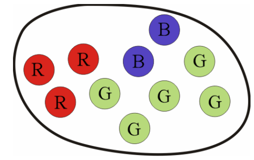
Example 3. What is the probability of rolling 8 on a standard six-sided die?

This is an impossible event, so its probability is zero: P(8) = 0.

Example 4. What is the probability of rolling a whole number on a die?

This is a sure event, so its probability is one. P(whole number) = 1.

1. There are three red marbles, two dark blue marbles, and five light green marbles in Michelle's bag. List all the possible outcomes if you choose one marble randomly from her bag.



2. Michelle chooses one marble at random from her bag. What is the probability that...

   a. the marble is blue?

   b. the marble is not red?

   c. the marble is neither blue nor green?

3. Make up an event with a probability of zero in this situation.

4. Suppose you choose one letter randomly from the word "PROBABILITY."

   a. List all the possible outcomes for this event.

   Now find the probabilities of these events:

   b. P(B)

   c. P(A or I)

   d. P(vowel)

   e. Make up an event for this situation that is likely to occur, yet not a sure event, and calculate its probability.

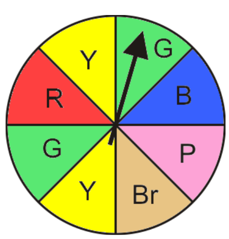
The complement of an event and the probability of "not"

The ***complement*** of any event A is the event that A does *not* occur.

If the probability of event A is a, then the probability of A not happening is simply $1 - a$.

5. The weatherman says that the chance of rain for tomorrow is 1/10. What is the probability of it not raining?

6. The spinner is spun once. Find the probabilities as simplified fractions.



   a. P(green)

   b. P(not green)

   c. P(not pink)

   d. P(not black)

   e. Make up an event for this situation that is not likely, yet not impossible either, and calculate its probability.

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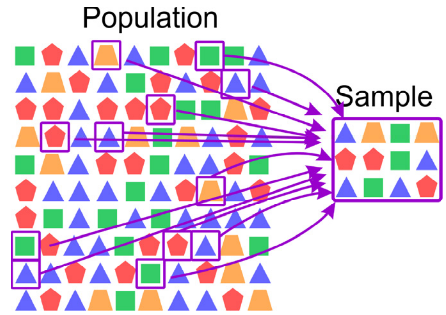
Random Sampling

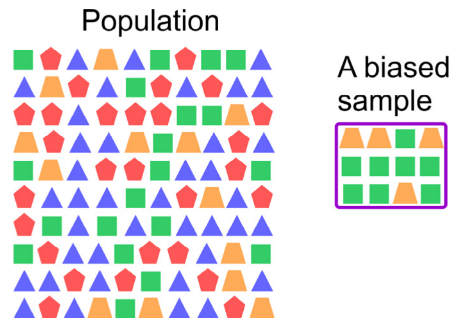
When researchers have a question concerning a large population, they obtain a **sample** (a part) of that population. That is because it is typically impossible to study the entire population.

For example, if you want to know how the citizens of France feel about climate change, you cannot just go and ask every person in France about it. You would choose for example 600 French citizens as your sample and ask them your question.



The way a sample is chosen is very important. Some methods of sampling may produce a sample that is *not* representative of the entire population. We call that a **biased sample**.

For example, if you are studying a student population of 630 in a school with close to an equal number of boys and girls, and you happen to choose a sample of 20 boys, then your sample is biased. It doesn't represent the entire population well.



We need to use **unbiased sampling methods** in order to get a sample that truly represents the population being studied. The best way to avoid biased samples is to select a **random sample.**

The main characteristics of a random sample are:

1. **Randomness:** each member of the population has an equal chance of being selected.

   Let's say a researcher is studying the types of cars Americans own. He decides to interview only people he finds at a local mall because that mall is close to where he lives, so it is convenient for him. His sample is biased because not every member of the US population even has a chance to be selected in his sample. Maybe the people at his local mall are predominantly rich people who own several cars per family, so in that respect those people would not be a good representation of the entire population of the US.

   We call this type of sample a **convenience sample** because it is convenient or easy to obtain.

2. **External selection:** respondents must be chosen by the researcher, not self-selected.

   If our researcher mails a questionnaire to various people across the US asking them to fill it out and return it, his sample is a **voluntary response sample**, which is a biased sample. Some people volunteer to return the questionnaire, but others don't. The people themselves decide whether or not to be a part of the sample.

   Why might this be a problem? Some of the people who would choose to take part may have an external reason to do so. They might want to show off how "good" they are in the particular aspect being studied, or they might just like to speak out about their opinions.

Our researcher could get a true random sample by choosing people randomly from a list of people living in the US and calling them. That way, each person has an equal chance of being selected in the sample (it is random), and the people cannot self-select to take part (the researcher chooses who takes part).

An unbiased sampling method is more likely to produce a representative sample.

1. You are studying whether students in a large college prefer to drink coffee black, with milk, with cream, or with sweetener, or whether they prefer not to drink coffee at all.

   a. Which of the six sampling methods listed below produce a voluntary response sample?

   b. Which methods don't give each member of the student population an equal chance to be selected for the sample?

   c. Which method is likely to produce a sample with only coffee drinkers, overlooking those who don't drink coffee?

   d. Which method will be the most likely to give you a representative (unbiased) sample?

   <u>Sampling Methods</u>

   (1) You interview 80 students in a cafe on the campus.

   (2) You interview 80 students who come in at the main door of the campus.

   (3) You interview the first 80 students you happen to meet on a certain day.

   (4) You choose 80 names randomly from a list of all the students. You call them to interview them.

   (5) You send an email to all the students in the college, asking them to fill in a form on a web page you have set up. You hope to get at least 80 responses.

   (6) You choose 80 names randomly from a list of all the students. You send them an email, asking them to fill in a form on a web page you have set up.

2. A recipe website posts a poll on their home page that any visitor to that website can take. In it, they ask if people are looking for a recipe for a dessert, a main dish, a side dish, bread, or salad. During the course of one Sunday, 4,600 people visit the page, and 252 of them fill in the poll. Explain why the poll results will be based on a biased sample.

Some common random sampling methods are:

1. **Simple random sampling.** Each individual in the sample is chosen randomly and entirely by chance, perhaps by using dice, through pulling names out of a hat, or with a random number generator.

2. **Systematic random sampling.** The individuals of the population are placed in some order, and then each individual at a certain specified interval is selected for the sample.

   For example, a supermarket might study the shopping habits of its customers by choosing every 15th customer who enters the store for the sample.

3. **Stratified random sampling.** The population is first divided into categories (strata) and then a random sample is obtained from each category.

   For example, to study how much sleep students in a particular school get, you might first divide the students into groups by grade levels (the stratification), then select a random sample from each of the grade levels.

3. A population to be studied doesn't have to be of people. A factory produces MP3 players. Out of the 500 units that the factory produces each day, a quality control inspector selects 25 for testing to study their quality and reliability. Which way should he choose those 25 so that his sample would best represent all the MP3 players that the factory produces?

   a. Choose the first 25 produced on a given day.

   b. First choose a number between 1 and 20 randomly. Select the player corresponding to that number, and after that, every 20th player, in the order they were produced that day.

   c. Choose 25 players that have just been finished around 1 PM when the inspector is touring the factory.

   d. Generate 25 random numbers between 1 and 500 and choose the corresponding 25 MP3 players in the order they were produced that day.

4. Ryan has two large fields planted with green beans. He wants to compare the bean plants in one field with the plants in the other. Design a practical sampling method for him to produce an unbiased sample.

Sometimes it is not obvious how a particular sampling method might be biased.

If you are studying students' homework habits in a particular school, it might initially make sense to interview the first 25 students who come into the school in the morning. However, there could be an underlying factor that makes that method biased. What if students who are diligent with their homework also tend to come to school early? In that case, students who are not diligent don't have an equal chance of being selected for your sample. A better method is to use systematic random sampling and to choose, say, every 10th student entering the school for the sample.

5. Heather is studying the effect of how the method of feeding affects the health of a baby during its first year of life. She has already determined that babies who are fed with infant formula get sick more often than babies who are fed with human milk, but she especially wants to find out how often babies who are fed both formula and breast milk get sick. Explain why interviewing mothers in the places below will produce a biased sample:

   a. A pediatrician's office.

   b. A breastfeeding class for new moms.

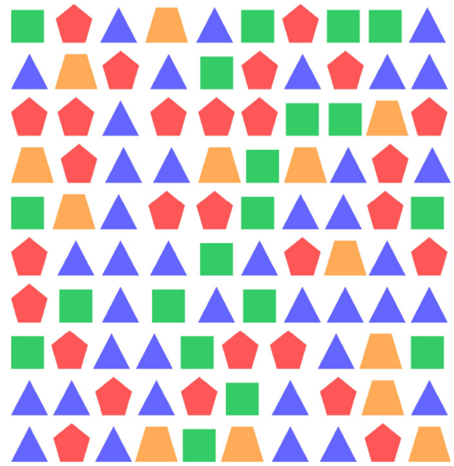
6. **a.** Devise a method that will produce a biased sample based on self-selection, and explain how that would happen, based on Heather's situation in Question 5.

   b. Design a sampling method for Heather that is most likely to produce a representative sample.

7. An organization that helps teenagers with drug problems has set up a telephone hot line for teens to call in to discuss their problems. After a few months of operations, the organization wants to evaluate the effectiveness of their service. Since they don't usually get as many calls on Tuesdays, they decide to choose a particular Tuesday to ask each teen at the end of the call to answer a few questions about how the service has helped. Is this a good method for selecting a sample? Explain.

Using Random Sampling

1. In this activity, you will make several samples of 10 from this population of shapes:



Since the shapes are in a 10 by 10 grid arrangement, you can easily assign a number from 1 to 100 for each shape. To obtain a random sample, you can use one of these ideas or come up with your own.

- Choose a random number between 1 and 10. Then, starting from that number, choose every 10th shape.
- Go to **https://www.random.org/integers/** and generate 10 random integers between 1 and 100. (If the set of numbers contains a duplicate, discard that set and make another.)

Here is an example sample (Sample 1) to help you get started: 

It is based on generating these random numbers at the website above: `76 17 51 63 88 29 95 73 40 69`

Generate at least five more samples. Count the number of each kind of shape in each of your samples, and fill in the table. Lastly, calculate the average number of triangles, the average number of squares, and so on.

	Sample 1	**Sample 2**	**Sample 3**	**Sample 4**	**Sample 5**	**Sample 6**	**Averages**
triangles	5						
squares	1						
pentagons	3						
trapezoids	1						
Total	10	10	10	10	10	10	—

2. Now imagine that you haven't seen the entire population of shapes but you try to infer (conclude) something about the entire population of shapes based on these six samples.

a. Which shape seems to be the most common?

b. Which shape seems to be the least common?

c. List the shapes in order, from the least common to the most common:

d. Based on the average number of the shapes in the six samples, *estimate* how many of each shape there are in the entire population of 100 shapes:

______ triangles ______ squares ______ pentagons ______ trapezoids

Here are some important points to realize and remember concerning random sampling. You probably noticed these facts while doing the activity:

1. **Random samples vary.** The differences occur simply because their members are indeed chosen from the population at random.
2. **A random sample is more likely to be unbiased, but that is not 100% certain.** In other words, it is possible for a random sample *not* to be representative of the population as a whole. However, the chances of a random sample being unbiased are greater than the chances of a non-random sample being unbiased.
3. **It is better to base inferences about the population on more than one sample.** However, if you cannot obtain more than one sample, then it is recommended to increase the sample size as much as possible, because a large sample represents the whole population better than a small one.

Example 1. Three people are running for mayor in a town with 20,000 voters. Two companies conducted separate polls of 350 people, asking who they would vote for in the final election. Here are the results:

	Smith	Harrison	Jones
Poll 1	63	220	67
Poll 2	53	238	59

Based on these results, what can we conclude about the results of the final election?

In both polls, Harrison is winning and by a large margin. In other words, far more people are claiming that they will vote for Harrison than for Smith or Jones. So we can fairly confidently conclude that <u>Harrison will be the winner of the actual election</u>.

Not only that, but in both polls Jones did better than Smith. So <u>it is likely, but not sure, that Jones will beat Smith in the actual election</u>. We cannot say that for sure because the differences are small: 4 and 6 votes.

We can also quantify the election results (use actual numbers). In Poll 1, 220 is more than three times 63 or 67. In Poll 2, 238 is more than four times 53 and about four times 59. Based on those, we can say that <u>Harrison will get roughly 3-4 times more votes than either of his opponents</u>.

Example 2. The data below presents the results of three different samples from a study about how students prefer to drink coffee.

	Black	With milk	With cream	Milk and sugar	Cream and sugar	Totals
Sample 1	12	21	24	36	37	130
Sample 2	9	23	22	37	39	130
Sample 3	14	18	20	36	42	130

What can we infer based on the data?

(1) Looking at the numbers carefully we can see that in each of the samples “Cream and sugar” was the winner and “Milk and sugar” came fairly close behind it.

(2) The two options “With milk” and “With cream” are also close to each other, but we cannot say for sure which of them is preferred, because in Samples 1 and 3, “With cream” beats “With milk”, whereas in sample 2 it is the opposite way.

(3) Drinking black coffee is the least popular option in all three samples.

(4) We can quantify the results. For example, “Cream and sugar” is the favorite of roughly 40/130 = 4/13 of the students, and 4/13 is almost 4/12 = 1/3. So we can state that <u>almost 1/3 of the students prefer to drink their coffee with cream and sugar</u>. You can make similar statements using approximate fractions for the other options.

What kinds of inferences can you make about the entire population based on random samples?

Based on what the data demonstrates, you may be able to…

- state which option is the most or least, the best or worst, the winner or loser, *etc*.
- compare two options as better or worse, more or less, *etc*.
- quantify the above statements with numbers, fractions, or percentages: *How much* more or less is one option than another?
- find trends: identify an increase or a decrease in some quantity as some other quantity, such as time, increases or decreases.

3. A large workplace conducted a survey of their employees' sleeping hours. They took two samples of 65 people, one week apart. What can you infer based on these results?

	< 5 h	5 h	6 h	7 h	8 h	> 8h	Totals
Sample 1	1	4	21	32	6	1	65
Sample 2	2	8	23	26	4	2	65

4. A music band wanted to find out which of their songs their audience likes best. They randomly chose some people to be interviewed after two of their concerts, asking them what their favorite song was. The results are in the table at the right.

   What conclusions can you draw from the data?

Songs	Concert 1 (Sample 1)	Concert 2 (Sample 2)
"Love You"	4	3
"My Best"	9	11
"Never Again"	7	5
"Sunshine"	5	6
Totals	25	25