Contents

Chapter 7: Systems of Linear Equations

Chapter 8: Bivariate Data

Foreword

Math Mammoth Grade 8 comprises a complete math curriculum for the eighth grade mathematics studies. The curriculum meets the Common Core standards.

In 8th grade, students spend the majority of the time with algebraic topics, such as linear equations, functions, and systems of equations. The other major topics are geometry and statistics.

The main areas of study in Math Mammoth Grade 8 are:

- Exponents laws and scientific notation
- Square roots, cube roots, and irrational numbers
- Geometry: congruent transformations, dilations, angle relationships, volume of certain solids, and the Pythagorean Theorem
- Solving and graphing linear equations;
- Introduction to functions;
- Systems of linear equations;
- Bivariate data.

This book, 8-B, covers the topic of graphing linear equations. The focus is on the concept of slope.

In chapter 6, our focus is on square roots, cube roots, the concept of irrational numbers, and the Pythagorean Theorem and its applications.

Next, in chapter 7, students solve systems of linear equations, using both graphing and algebraic techniques. There are also lots of word problems that are solved using a pair of linear equations.

The last chapter then delves into bivariate data. First, we study scatter plots, which are based on numerical data of two variables. Then we look at two-way tables, which are built from categorical bivariate data.

Part 8-A covers exponent laws, scientific notation, geometry, linear equations, and an introduction to functions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have some liberty in planning your student's studies. In eighth grade, chapters 2 (geometry), 3 (linear equations) and chapter 4 (functions) should be studied before chapter 5 (graphing linear equations). Also, chapters 3, 4, and 5 should be studied before chapter 7 (systems of linear equations) and before chapter 8 (statistics). However, you still have some flexibility in scheduling the various chapters.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- Don't automatically assign all the exercises. Use your judgement, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- There are answer keys for everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for the student to do on their own.

Repeat this if the lesson has other blue teaching boxes.

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Worktext 8-A	
Chapter 1	13 days
Chapter 2	27 days
Chapter 3	21 days
Chapter 4	14 days
TOTAL	**75 days**

Worktext 8-B	
Chapter 5	15 days
Chapter 6	16 days
Chapter 7	17 days
Chapter 8	11 days
TOTAL	**59 days**

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 4 pages long, intended for one day. Some lessons are 5 pages and can be covered in two days.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 160-day (32-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A	84	8	214	76	2.8	14.1
8-B	76	8	189	68	2.8	13.9
Grade 8 total	160	16	403	144	2.8	14

The table below is for you to fill in. Allow several days for tests and additional review before tests — I suggest at least twice the number of chapters in the curriculum. Then, to get a count of "pages to study per day", **divide the number of lesson pages by the number of days for the student book**. Lastly, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Number of school days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
8-A			214			
8-B			189			
Grade 8 total			403			

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 8th graders might spend 45-75 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the printed version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
2. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
3. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
4. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
5. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for “brain puzzles for kids,” “logic puzzles for kids” or “brain teasers for kids.”

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 5: Graphing Linear Equations
Introduction

This chapter focuses on how to graph linear equations, and in particular, on the concept of slope in that context.

We start by graphing and comparing proportional relationships, which have the equation of the form $y = mx$. Students are already familiar with these, and know that m is the constant of proportionality. In this chapter, they learn that m is also the slope of the line, which is a measure of its steepness.

Then we go on to study slope in detail, its definition as the ratio of the change in y-values and the change in x-values. Students learn that it doesn't matter which two points on a line you use to calculate the slope, and study a geometric proof of this fact. They practise drawing a line with a given slope and that goes through a given point, and determine if three given points fall on the same line.

Then it is time to study the slope-intercept equation of a line, and connect the idea of an initial value of a function (chapter 4) with the concept of y-intercept in the context of graphing. Students graph lines given in the slope-intercept form, and write equations of lines from their graphs.

Next, we study horizontal and vertical lines and their simple equations. The standard form of a linear equation follows next. The last major topic is how the slope reveals to us whether two lines are parallel or perpendicular to each other.

Pacing Suggestion for Chapter 5

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing if you use the test.

The Lessons in Chapter 5	**page**	**span**	**suggested pacing**	**your pacing**
Graphing Proportional Relationships 1	13	*3 pages*	1 day	
Graphing Proportional Relationships 2	16	*3 pages*	1 day	
Comparing Proportional Relationships	19	*4 pages*	1 day	
Slope, Part 1	23	*4 pages*	1 day	
Slope, Part 2	27	*3 pages*	1 day	
Slope, Part 3	30	*5 pages*	2 days	
Slope-Intercept Equation 1	35	*4 pages*	1 day	
Slope-Intercept Equation 2	39	*3 pages*	1 day	
Write the Slope-Intercept Equation	42	*3 pages*	1 day	
Horizontal and Vertical Lines	45	*3 pages*	1 day	
The Standard Form	48	*3 pages*	1 day	
More Practice (optional)	51	*(2 pages)*	(1 day)	
Parallel and Perpendicular Lines	53	*3 pages*	1 day	
Mixed Review Chapter 5	56	*3 pages*	1 day	
Chapter 5 Review	59	*4 pages*	1 day	
Chapter 5 Test (optional)				
TOTALS		*48 pages*	15 days	
with optional content		*(50 pages)*	(16 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr8ch5

Scan me

Graphing Proportional Relationships 1

We will now review what it means when two variables are **in direct variation** or **in proportion**. The basic idea is that whenever one variable changes, the other varies (changes) proportionally or at the same rate.

Example 1. The wholesaler posted the following table for the price of potatoes:

weight (kg)	5	10	15	20	25	30
cost	$5.50	$11.00	$16.50	$22.00	$27.50	$33.00

Each pair of cost and weight forms a rate — and so does each pair of weight and cost. However, it is more common to look at the rate "cost over weight", such as $27.50/(25 kg), than vice versa.

If all of the rates in the table are equivalent, then the weight and the cost *are* proportional.

To check for that, we have several means. One is to calculate **the unit rate** (the rate for 1 kg) from each of these rates, and check whether you get the same unit rate.

In this case, that is so. The unit rate is $1.10/kg, no matter which rate from the table we'd use to calculate it.

One other way to check is, if one quantity doubles (or triples), will the other double (or triple) also?
This is especially useful for noticing if the quantities are *not* in direct variation.

Example 2. Here, when the weight doubles from 5 kg to 10 kg, the price also doubles. But what happens with the price when the weight doubles from 10 kg to 20 kg?

weight (kg)	5	10	15	20	25	30
cost	$6	$12	$18	$22	$26	$30

The price does not double! So, the quantities are not in proportion.
The seller is giving you some discount if you purchase higher quantities.

Also, if you calculate the unit rate from $6/(5 kg) and from $22/(20 kg), they are not equal. (Verify this.)

1. Are the quantities in a proportional relationship? If yes, list the unit rate.

a.

time (hr)	0	1	2	3	4	5
distance (km)	0	50	90	140	190	240

b.

time (hr)	0	1	2	3	4	5
distance (km)	0	45	90	135	180	225

c.

age (days)	0	1	2	3	4	5	6	7
height (cm)	0	0	0	2	4	6	8	10

d.

length (m)	0	0.5	1	1.5	2	4	5	10
cost ($)	0	3	6	9	12	24	30	60

2. Now consider the tables of values in #1 as functions, where the variable listed on top is the independent variable. For the ones where the quantities were in proportion, calculate the rate of change.

What is its relationship to the unit rate?

When two quantities are in a proportional relationship, or in direct variation (the two terms are synonymous):

(1) Each rate formed by the quantities is equivalent to any other rate of the quantities.

(2) The equation relating the two quantities is of the form *y* = *mx*, where *y* and *x* are the variables, and *m* is a constant. The constant *m* is called the **constant of proportionality** and is also the unit rate.

(3) When plotted, the graph is a straight line that goes through the origin.

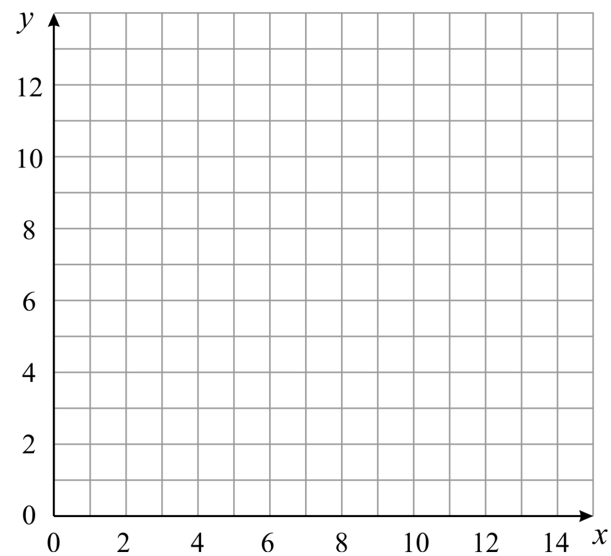
3. Choose an equation from below where the variables *x* and *y* are in direct variation (proportional):

$y = \frac{3}{x}$ $\quad\quad$ $y = 3x$

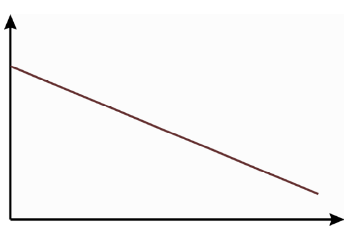
$xy = 3$ $\quad\quad$ $y = x^3$

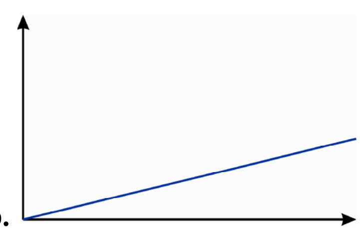
Then graph that equation in the grid.

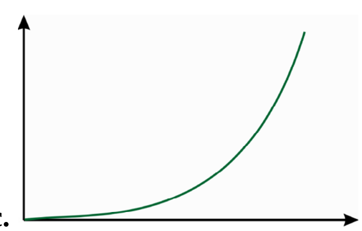
Hint: The point (0, 0) is always included in direct variation. All you need to do is plot one other point, and then draw a line through the origin and that point.



4. Choose the representations that show a proportional relationship.

a. 

b. 

c. 

d.

x	0	1	2	3	4	5
y	15	17	19	21	23	25

e. $y = 2x + 9$

f. $y = (3/4)x$

g.

x	0	4	8	12	16	20
y	0	3	6	9	12	15

5. Two of the above representations are the exact same relationship. Which ones?

Example 2. In a direct variation, $y = 9$ when $x = 12$. Write an equation for the relationship.

Since this is direct variation (proportional relationship), the equation is of the form $y = mx$, where m is the constant of proportionality.

The constant of proportionality is the ratio **(dependent variable)/(independent variable)**, so in this case it is $y/x = 9/12$, or 3/4. So, the equation is $y = (3/4)x$.

At this point, it is good to check that the point (12, 9) satisfies the equation, to check for errors: Is it true that $9 = (3/4) \cdot 12$? Yes, it is.

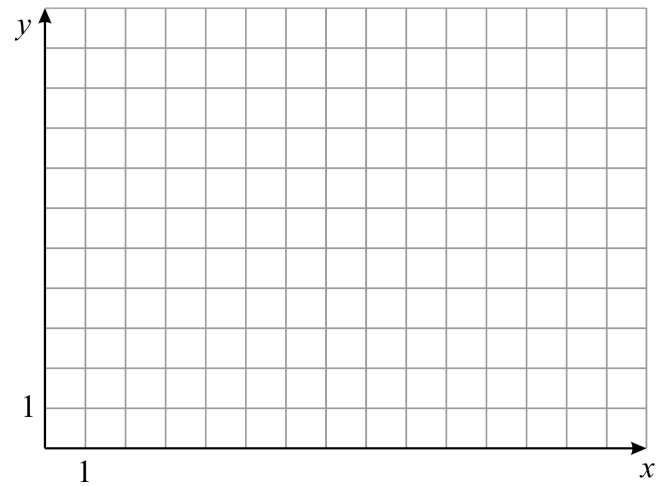
To graph the equation, we could simply plot the point (12, 9), and draw a line through it and the origin.

6. In a direct variation, when x is 14, y is 10.

   a. Write an equation for this proportional relationship.

   b. Graph a line for this relationship in the grid.

   c. What is x when $y = 40$?

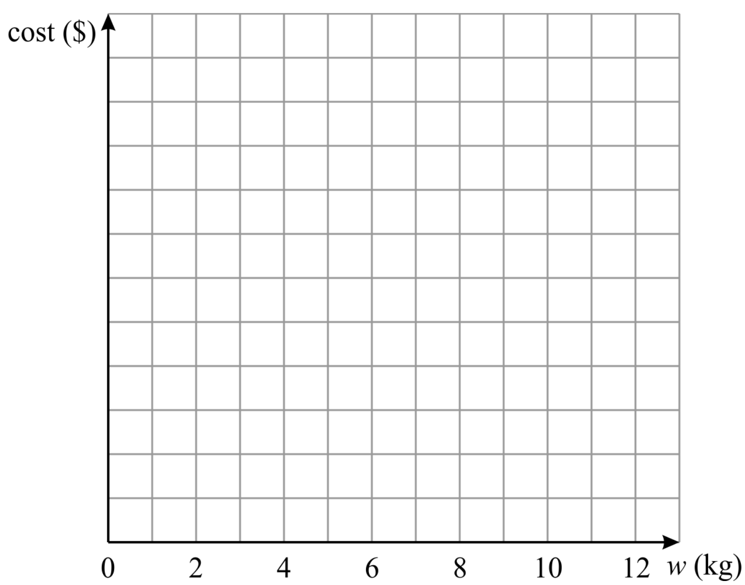


7. Organic rolled oats cost $34 for 4 kg.

   a. Write an equation for this proportional relationship, using the variables C for cost and w for the amount (weight) of oats.

   b. Graph the equation in the grid. Design the scaling on the cost-axis so that the point corresponding to 12 kg fits on the grid.

   c. How much do 15 kg of the oats cost?



8. If y is 120 when x is 400 in a direct variation, then what is y when x is 80?

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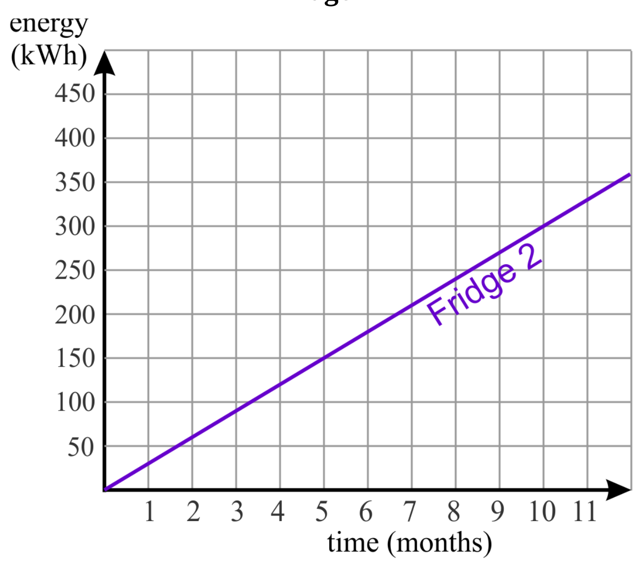
Chapter 5 Review

1. Refrigerator companies make estimates of how much energy their fridges consume in typical usage. The table shows how many kilowatt-hours (kWh) of energy fridge 1 consumed over time, and the graph shows the same for fridge 2.

Fridge 1

time (mo)	energy (kWh)
2	75
4	150
6	225
8	300
10	375
12	450

Fridge 2



a. Which fridge consumes more electricity in a month?

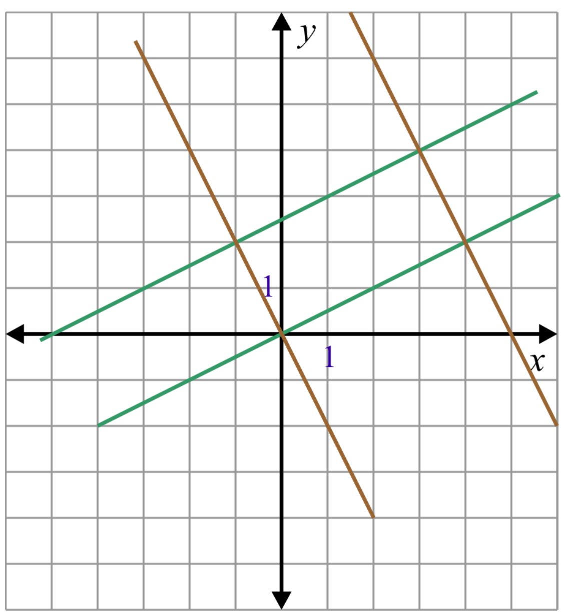
How much more?

b. Write an equation for each fridge, relating the energy (E, in kWh) and the time (t, in months).

c. Plot the equation for Fridge 1 in the grid.

d. Plot the point corresponding to the unit rate, for Fridge 1.

2. **a.** Find the equations of the four lines, in slope intercept form.



b. (optional) Find the area of the rectangle.

3. Find the equation of each line, in slope-intercept form:

a. has slope 3/4 and passes through (−2, 3)

b. is horizontal and passes through (9, −10)

4. Find the slope of the lines.
Notice the scaling.

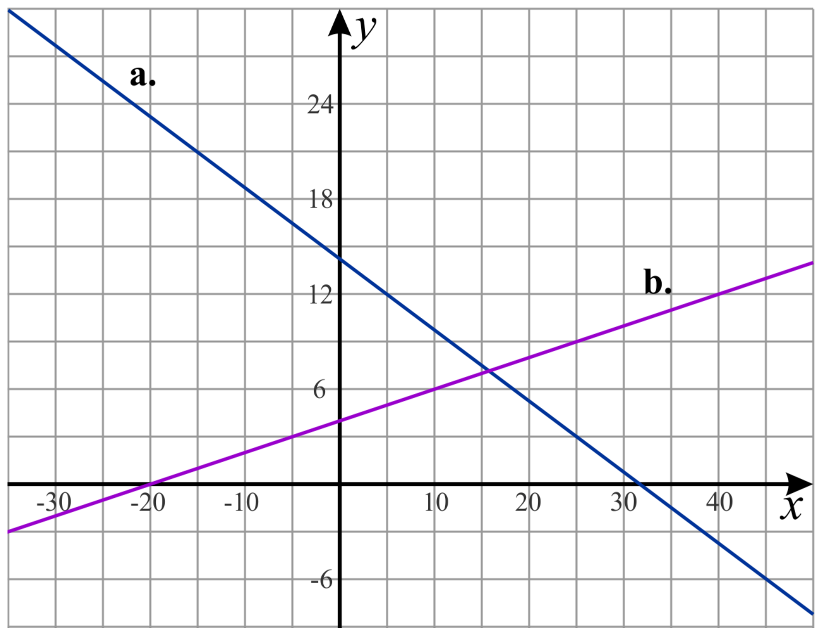
a.

b.

Now find the equations for the lines.

a.

b.



5. Do the three points fall on one line? Explain your reasoning.

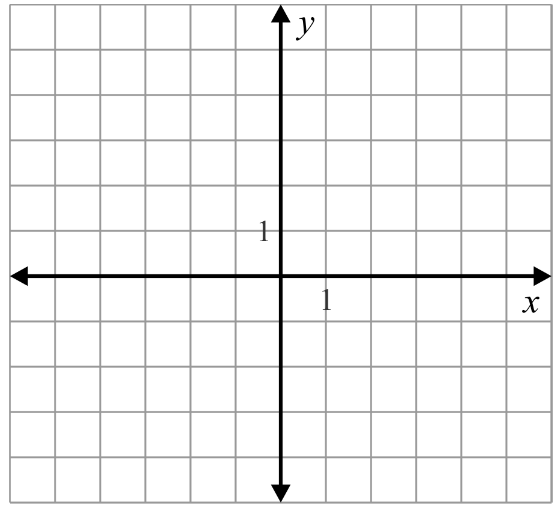
(−3, 1), (−1, −4), (1, −8)

6. Find s so that the point (s, 12) will fall on the same line as the points (3, 9) and (15, 18).

7. Line S passes through (−5, −2) and (0, 4). Line T is perpendicular to Line S, and passes through (1, 1).

   a. Find the equation of line T, in slope-intercept form.

   b. Write the equation also in the standard form.



8. Mr. Henson runs a garbage pick-up business, with 12 garbage trucks. To run one truck costs him \$2100 per month in maintenance costs, plus \$180 a day for fuel.

   Consider the cost of running one truck as a function of time, in days (during one month only). Is this a linear relationship, a proportional relationship, or neither?

   Write an equation for it.

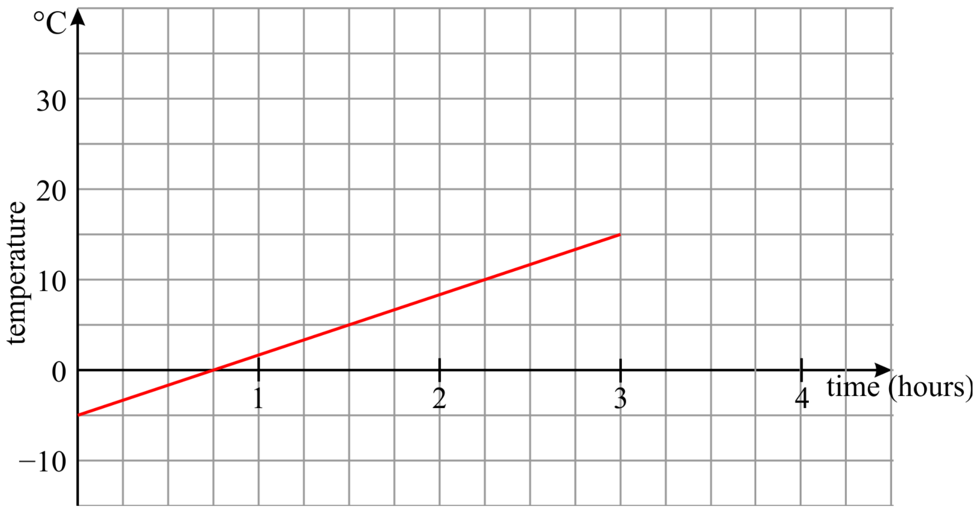
9. Match the descriptions and the equations.

$y = (-4/3)x - 7$	Is parallel to $x = 9$ and passes through (2, 7)
$3x - y = -21$	Has y-intercept −4 and is perpendicular to $y = -2x$.
$y = -4$	Passes through (−5, 6) and has slope 3.
$x - 2y = 8$	Passes through (−9, 5) and (−3, −3)
$x = 2$	Passes through (−3, 0) and (0, 9)
$y = 3x + 9$	Has y-intercept −4 and is parallel to $y = -2$.

10. Transform each equation of a line to the standard form, and then list its *x* and *y*-intercepts.

a. $y - 6 = 2(x + 2)$	**b.** $-\frac{1}{3}x - \frac{3}{2}y = 1$

11. A heater was turned on at 10 AM in a cold, uninhabited house, to prepare it for people later that day. The graph shows the temperature of the house. The count of hours starts at 10 AM.



a. Write an equation for the line.

b. If the temperature continues to rise in the same fashion, what will the temperature be at 2:30 PM?

c. When will the temperature reach 22°C?

d. Let's say the heater is turned off at 1:45. What is the temperature at that time?

e. If the house had started out at a temperature of −12°C instead, and the heating process worked in the same fashion (the temperature rose at the same rate), at what time would the house reach a temperature of 22°C?

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Cube Roots and Approximations of Irrational Numbers

Similarly to the square root, we can take a **cube root** of a number.

Recall that the **cube of a number** is that number multiplied by itself three times. For example, two cubed = $2^3 = 2 \cdot 2 \cdot 2 = 8$. This gives us the volume of a cube with edges 2 units long.

The cube root of 8 is 2. We write it as $\sqrt[3]{8} = 2$. Notice the little "3" that is added to the radical sign to signify a cube root.

Example 1. Since $(-3)(-3)(-3) = -27$, then $\sqrt[3]{-27} = -3$.

Like square roots, most cube roots are irrational numbers. When it comes to integers, only the cube roots of perfect cubes are rational; the rest are irrational.

1. Find the cube roots without a calculator.

a. $\sqrt[3]{27}$	**b.** $\sqrt[3]{125}$	**c.** $\sqrt[3]{64}$	**d.** $\sqrt[3]{1000}$
e. $\sqrt[3]{1}$	**f.** $\sqrt[3]{216}$	**g.** $\sqrt[3]{27\,000}$	**h.** $\sqrt[3]{-8}$
i. $\sqrt[3]{-1}$	**j.** $\sqrt[3]{-125}$	**k.** $\sqrt[3]{0}$	**l.** $\sqrt[3]{-8000}$

2. **a.** The volume of a cube is 216 cm^3. How long is its edge?

b. What is $(\sqrt[3]{4})^3$?

c. If the edge of a cube measures 50 cm, find its volume.

d. If the volume of a cube is 729 cm^3, find its surface area.

3. (optional) Find the cube roots of these fractions and decimals, without a calculator.

a. $\sqrt[3]{0.008}$	**b.** $\sqrt[3]{0.125}$	**c.** $\sqrt[3]{-0.027}$
d. $\sqrt[3]{\frac{8}{125}}$	**e.** $\sqrt[3]{\frac{64}{27}}$	**f.** $\sqrt[3]{-\frac{1}{8}}$

Example 2. We can know that $\sqrt{98}$ lies between 9 and 10, because $9 = \sqrt{81} < \sqrt{98} < \sqrt{100} = 10$.

We can even tell it is much closer to 10 than to 9, since 98 is much closer to 100 than to 81.

From that, we can estimate that $2\sqrt{98}$ is slightly less than 20, and that $\sqrt{98} + 4$ is slightly less than 24.

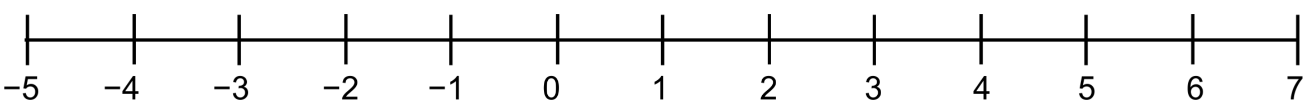
Example 3. The opposite of $\sqrt{2}$ is $-\sqrt{2}$. Since $\sqrt{2}$ is approximately 1.41, then $-\sqrt{2} \approx -1.41$.

4. Find between which two whole numbers the root lies. Notice some of them are cube roots.

a. _____ < $\sqrt{31}$ < ______	**b.** _____ < $\sqrt{65}$ < ______	**c.** _____ < $\sqrt{87}$ < ______
d. _____ < $-\sqrt{5}$ < ______	**e.** _____ < $-\sqrt{44}$ < ______	**f.** _____ < $-\sqrt{50}$ < ______
g. _____ < $\sqrt[3]{7}$ < ______	**h.** _____ < $\sqrt[3]{37}$ < ______	**i.** _____ < $\sqrt[3]{101}$ < ______

5. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two integers the root lies, and whether it is close to one of those integers.

$\sqrt{15}$ $\quad$ $\sqrt{47}/2$ $\quad$ $\sqrt[3]{9}$ $\quad$ $-\sqrt[3]{27}$ $\quad$ $-\sqrt{10}$ $\quad$ $\sqrt{66}/2$ $\quad$ π $\quad$ $\sqrt{18} + 1$



6. Compare, writing >, <, or = between the numbers. Think between which two whole numbers the root lies, using mental math.

a. 5 ☐ $\sqrt{27}$	**b.** $\sqrt{48}$ ☐ 7	**c.** $\sqrt{18}$ ☐ 4	**d.** $\sqrt[3]{9}$ ☐ 2
e. 2 ☐ $\sqrt{2} + 1$	**f.** $\sqrt{32} + 1$ ☐ 6	**g.** $\sqrt{43} + 5$ ☐ 10	**h.** $\sqrt{88} - 3$ ☐ 7

7. **a.** Between which two whole numbers does $\sqrt{30}$ lie? And $\sqrt{60}$?

b. Use your answers to (a) to determine whether $2\sqrt{30}$ is equal to $\sqrt{2 \cdot 30}$.

8. Is $\frac{\sqrt{50}}{2}$ equal to $\sqrt{\frac{50}{2}}$? Explain your reasoning.

9. Use the decimal approximations of common irrational numbers on the right to estimate the value of the expressions below, to one decimal digit. Use mental math and paper-and-pencil calculations, not a calculator.

$\pi \approx 3.14$

$\sqrt{2} \approx 1.41$

$\sqrt{5} \approx 2.24$

a. $5\sqrt{2}$ **b.** π^2 **c.** $\sqrt{5} - \sqrt{2}$ **d.** $2\sqrt{5} - 5\sqrt{2}$

10. **a.** Find an approximation to $\sqrt{11}$ to one decimal digit, without using the square root function of a calculator.

b. Use the approximation you found to estimate the values of $\sqrt{11} - \sqrt{2}$ and $3\sqrt{11}$.

11. Sarah has used the method of squaring her guesses to find out that $\sqrt{45}$ is between 6.7 and 6.8.
How can she continue from this point to get a better approximation? Do it for her, to two decimal digits.

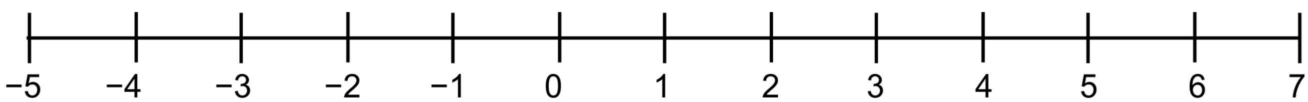
Use these exercises for additional practice.

12. Order the numbers from smallest to greatest. Estimate the value of the roots, thinking between which two whole numbers each square root lies, using mental math.

$\sqrt{5} - 1$ $\sqrt[3]{1}$ $\sqrt{19}/2$ $\sqrt[3]{100}$ $\sqrt[3]{8}$ $\sqrt{13}$ $\sqrt{9}$ 2π $\sqrt{22} + 1$

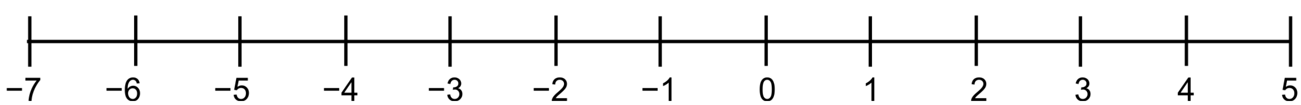
13. Plot the following numbers *approximately* on the number line. Do not use a calculator, but think about between which two integers the root lies, and whether it is close to one of those integers.

a. $-2\sqrt{2}$ **b.** $\sqrt{80}/3$ **c.** $\sqrt{27} - 1$ **d.** $-\sqrt{5} + 7$



14. Plot the following numbers *approximately* on the number line.

a. $-\sqrt{2} - 3$ **b.** $-\pi$ **c.** $-\sqrt[3]{9}$ **d.** $-\sqrt{36} + 9$ **e.** $-\sqrt{26}/2$



Fractions to Decimals

(This lesson is review, and optional.)

Each fraction is a rational number (by definition!). Each fraction can be written as a decimal. It will either be a terminating decimal, or a non-terminating repeating decimal.

It is easy to rewrite a fraction as a decimal when the denominator is a power of ten. However, when it is not (which is most of the time), simply treat the fraction as a division and divide. You will get either a **terminating decimal** or a non-terminating **repeating decimal**. See the examples below.

1. The denominator is a power of ten or the fraction can be simplified so that it is. In this case, writing the fraction as a decimal is straightforward. Simply write out the numerator. Then add the decimal point based on the fact that the number of zeros in the power of ten tells you the number of decimal digits.

Examples 1. $\frac{7809}{100} = 78.09$ $\frac{1458}{1000} = 1.458$ $\frac{506}{100\,000} = 0.00506$ $\frac{33}{30} = \frac{11}{10} = 1.1$

2. The denominator is a factor of a power of ten. Convert the fraction into one with a denominator that is a power of ten. Then do as in case (1) above.

Examples 2. $\frac{9}{20} = \frac{45}{100} = 0.45$ $\frac{2}{125} = \frac{16}{1000} = 0.016$ $\frac{9}{8} = \frac{1125}{1000} = 1.125$

3. Use division (long division or with a calculator). This method works in all cases, even if the denominator happens to be a power of ten or a factor of a power of ten.

Example 3. Write $\frac{31}{40}$ as a decimal.

$$\begin{array}{r} 0\,0.7\,7\,5 \\ 40\overline{)\,3\,1.0\,0\,0\,0} \\ -\underline{2\,8\,0} \\ 3\,0\,0 \\ -\underline{2\,8\,0} \\ 2\,0\,0 \\ -\underline{2\,0\,0} \\ 0 \end{array}$$

This division terminates (comes out even) after just three decimal digits.

We get $\frac{31}{40} = 0.775$. This is a **terminating decimal**.

(The fact the division was even means that the denominator 40 is a factor of some power of ten, and so we could have used method 2 from above. In this case, $1000 = 40 \cdot 25$.)

Example 4. Write $\frac{18}{11}$ as a decimal.

$$\begin{array}{r} 0\,1.6\,3\,6\,3 \\ 11\overline{)\,1\,8.0\,0\,0\,0} \\ -\underline{1\,1} \\ 7\,0 \\ -\underline{6\,6} \\ 4\,0 \\ -\underline{3\,3} \\ 7\,0 \\ -\underline{6\,6} \\ 4\,0 \\ -\underline{3\,3} \\ 7 \end{array}$$

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient, and the remainders 40 and 70, start repeating.

So $\frac{18}{11} = 1.\overline{63}$.

The fraction 18/11 equals $1.\overline{63}$, which is a **repeating decimal**.

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Square and Cube Roots as Solutions to Equations

Example 1. Solve $x^2 = 81$.

We can use mental math: one obvious solution is $x = 9$. However, there is also another solution! It is not only true that $9^2 = 81$, but $(-9)^2 = 81$ also, so $x = -9$ is a second solution to this equation.

Example 2. Solve $x^2 = 48$.

This time, we cannot solve the equation with mental math, but we will *take a square root of both sides of the equation*. This will undo the squaring, and isolate x, because taking a square root and squaring are opposite operations.

$$x^2 = 48$$

$$x = \sqrt{48} \approx 6.93$$

$$\text{or } x = -\sqrt{48} \approx -6.93$$

$\sqrt{\ }$ The radicand symbol signifies taking a square root of both sides of the equation.

Since taking a square root undoes the squaring, x is now left alone on the left side. Notice that there are two solutions: the square root of 48 and the negative square root of 48.

Notice that $-\sqrt{48}$ doesn't mean that we take a square root of a negative number. Instead, $-\sqrt{48}$ means we *first* take the square root of 48 (a positive number) and then take the opposite of that result.

1. Solve. Remember, there will be two solutions: one positive and one negative. When the two answers aren't integers, give them as square roots and also as decimals rounded to two decimal digits.

a. $x^2 = 25$	**b.** $y^2 = 3600$
c. $x^2 = 500$	**d.** $z^2 = 11$
e. $w^2 = 287$	**f.** $q^2 = 1\,000\,000$

The situation is similar with equations where the variable is cubed.

Example 3. Solve $x^3 = 125$.

Since 125 is a perfect cube, the solution is easy to find with mental math.

$$x^3 = 125 \quad \Big| \sqrt[3]{}$$
$$x = \sqrt[3]{125} = 5$$

With cube roots, **there is no other solution.** For example, in this case, $(-5)^3$ does not equal 125.

Example 4. Solve $x^3 = 35$.

We will take the cube root of both sides of the equation. This will undo the cubing and isolate *x*.

$$x^3 = 35 \quad \Big| \sqrt[3]{}$$
$$x = \sqrt[3]{35} \approx 3.27$$

Since 35 is not a perfect cube, $\sqrt[3]{35}$ is an irrational number. Depending on context, we might give the answer in root form, or with a decimal approximation.

Hint: If your calculator does not have a button for the cube root, you can instead use the button for exponentiation, with the exponent 1/3. For example, on my computer calculator, I enter $\sqrt[3]{23}$ this way:

[2] [3] [x^y] [(] [1] [/] [3] [)]

2. Solve. If the root is not a whole number, give it rounded to two decimal digits.

a. $x^3 = 64$	**b.** $n^3 = 216$	**c.** $z^3 = 27\ 000$
d. $x^3 = 7$	**e.** $b^3 = 109$	**f.** $a^3 = 18$

3. Below, the variable V signifies the volume of a cube. Find the edge of the cube (*s*) when the volume is given. Give the answer to the same amount of significant digits as the given volume.

a. $V = 510\text{ m}^3$ $s =$	**b.** $V = 24\ 500\text{ cm}^3$	**c.** $V = 5.83\text{ m}^3$

4. Find the surface area of a cube with a volume of 14.2 m^3.

Example 5.

$x^2 + 78 = 129$	We want to isolate the term x^2, so we first subtract 78 from both sides.
$x^2 = 51$	Now we take a square root of both sides.
$x = \sqrt{51}$ or $x = -\sqrt{51}$	There are two solutions, as usual.

If this is strictly a math problem and does not involve quantities with units, the answer can be left in the root form, which is exact. Otherwise, you should find its decimal approximation.

Here are the checks. Usually, it is enough to check only the positive root ($x = \sqrt{51}$), as the check for the negative root ($x = -\sqrt{51}$) is practically identical.

$$(\sqrt{51})^2 + 78 \stackrel{?}{=} 129$$
$$51 + 78 \stackrel{?}{=} 129$$
$$129 = 129 \checkmark$$

$$(-\sqrt{51})^2 + 78 \stackrel{?}{=} 129$$
$$51 + 78 \stackrel{?}{=} 129$$
$$129 = 129 \checkmark$$

5. Solve. Since these are pure mathematical problems, give the solutions in root form. Check your solutions.

a. $a^2 - 8 = 37$	**b.** $y^2 + 100 = 1000$
c. $b^2 + 1.5 = 6.4$	**d.** $x^2 - 26 = 709$

6. Solve. Give the solutions in exact form. Check your solutions.

a. $x^3 - 5 = 59$	**b.** $x^3 + 78 = 437$

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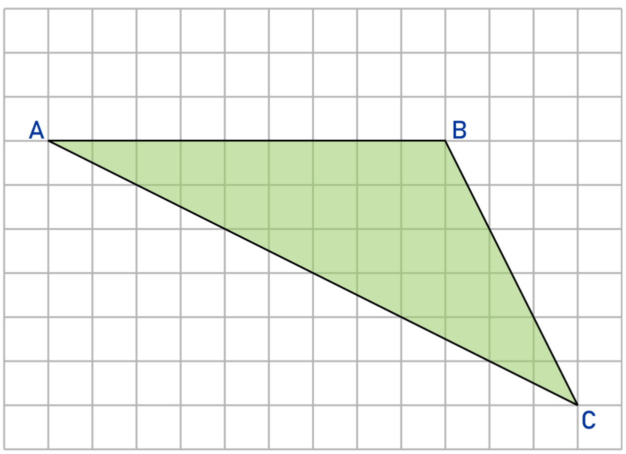
Mixed Review Chapter 6

1. Write an equivalent expression using the exponent laws, without negative exponents.

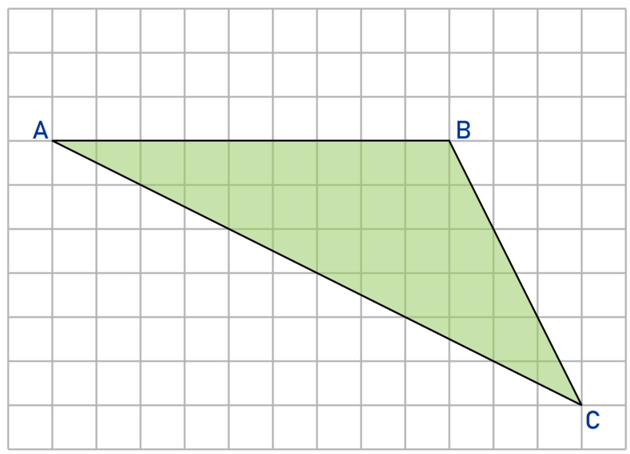
a. $3x^4y^5y^2 \cdot 6x^6 =$	**b.** $(3x)^{-3} =$	**c.** $(3yz)^2$	**d.** $(b^{-2})^4 =$
e. $\dfrac{8x^5}{28x^8} =$	**f.** $\dfrac{x^{-5}}{x^2} =$	**g.** $\left(\dfrac{-2}{5y}\right)^2 =$	**h.** $\left(\dfrac{3s}{t^2}\right)^4 =$

2. Draw a dilation of triangle ABC...

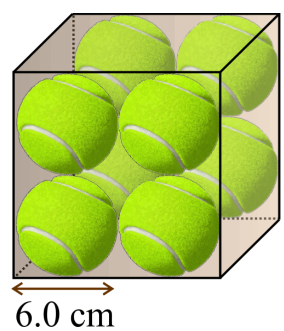
a. from point A with scale factor 1/3



b. from point C with scale factor 1/2



3. Eight tennis balls fit snugly in a cube-shaped container. Calculate what fraction of the total volume of the cube the tennis balls take up.
Hint: write this fraction using the formulas for the volumes, and simplify it.



4. Chloe bought 10 metres of material, five metres at the regular price of $5.95/m and the rest at some discounted price. Her total came to $53. At home, she started wondering how much the discount was.

   Write an equation to solve what the unknown discounted price was. Use p for the discounted price. Then solve your equation.

5. **a.** Make up two functions for the cost of renting a surfboard as a function of time. The first should be a proportional relationship, and the other nonlinear. Make your functions reasonable so that the cost of renting a surfboard for an entire day (8 hours) is $50 at a maximum.

   Give the linear function as an equation, and the nonlinear one as a table of values.

   <u>Function 1:</u>

   <u>Function 2:</u>

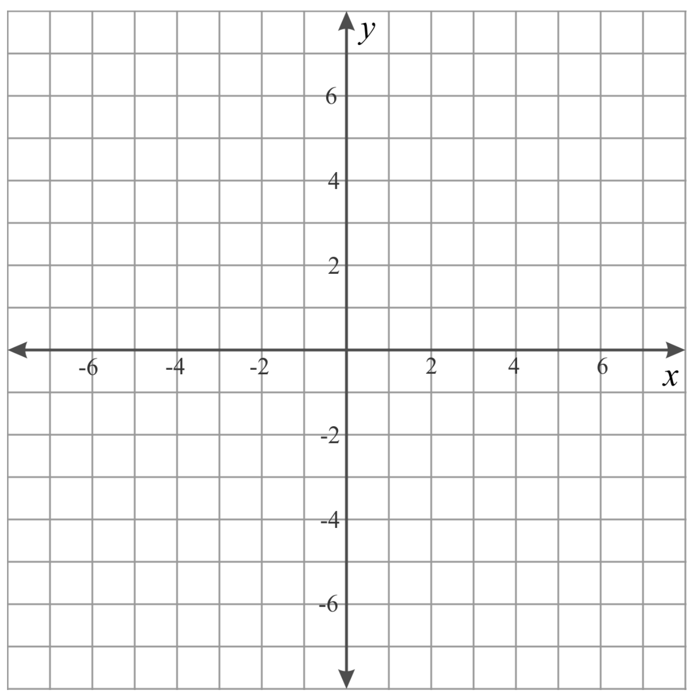
time (hours)	0	1	2	3	4	5	6	7	8
Cost ($)									

   b. Which function gives a better deal if you are renting a surfboard for 2 hours? For 6 hours?

6. Find the equation of each line, in slope-intercept form. Also graph the lines.

   a. has slope −2 and passes through (−2, 6)

   b. has slope 2/3 and passes through (4, −4)



7. **a.** What is the equation of a horizontal line that passes through (−3, −5)?

b. What is the equation of a vertical line that passes through (9, 8)?

c. What is the equation of a line that is parallel to $y = 5x + 2$ and passes through (1, 2)?

8. Convert, rounding your answer to the same number of significant digits as the measurement.

a. 71.0 cm = __________ in

b. 2 400 kg = _____________ lb

c. 235 ft = ___________ m

d. 83.5 lb = ___________ kg

e. 15.69 m = ____________ ft

f. 4.5 in = ___________ cm

1 inch = 2.54 cm

1 ft = 0.3048 m

1 kg = 2.2 lb

9. If 3 cm of rain falls over one square kilometre, how many raindrops fell? Give your answer in scientific notation, to three significant digits.

Besides the well-known conversion factors, here are some facts you may need:

- One cubic metre = 1000 litres.
- The size of raindrops varies but for this problem, use $2.65 \cdot 10^4$ raindrops per litre.

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Equations with Two Variables

(This lesson is optional.)

The equation $2x + 3y = 16$ has **two variables**, x and y. One solution to the equation is $x = 2$ and $y = 4$, because when we substitute those values to the equation, it checks, or is a true equation:

$$2(2) + 3(4) = 16$$

But it also has the solution $x = 0.5$ and $y = 5$:

$$2(0.5) + 3(5) = 16$$

In fact, we can choose any number we like for the value of x, and then *calculate* the value of y, and thus find another solution to the equation.

For example, if we choose $x = -1$, then we get

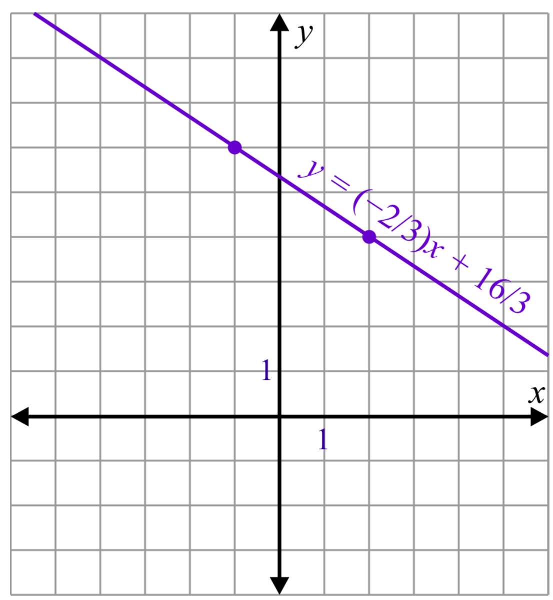
$$2(-1) + 3y = 16$$

from which $y = (16 + 2)/3 = 6$. So, $x = -1$, $y = 6$ is yet another solution.

All of these solutions, having both x and y values, are **number pairs**, and can be considered as **points on the coordinate plane.**

We can make a table of some of the possible (x, y) values (solutions):

x	y
−1	6
0	16/3
0.5	5
2	4



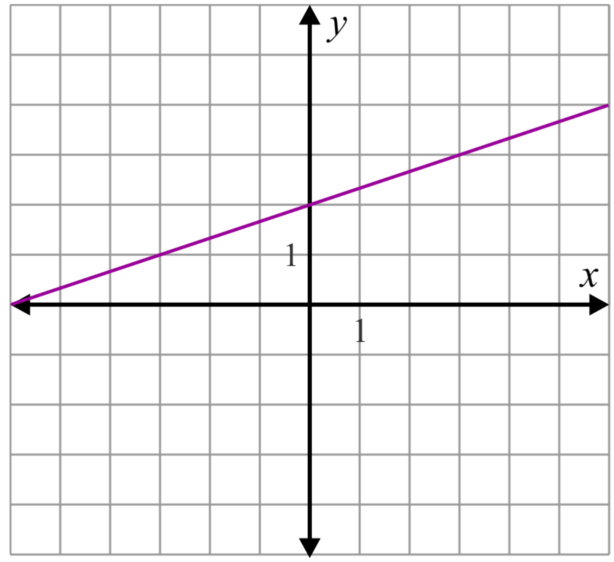
...and there are many more. When plotted, **these points fall on a line** — and you can probably guess, the equation of that line is $2x + 3y = 16$!
(Or, in slope-intercept form, $y = (-2/3)x + 16/3$.)

A line in the coordinate plane represents all the solutions to the equation that is the equation of the line. In other words, **each point on the line is a solution to the equation.**

1. Find three solutions to the equation $5x + 2y = 32$.

2. Find three solutions to the equation $-4x + y = -6$.

3. **a.** What is the equation if its solution set is represented by this line?



b. List two distinct integer number pairs that are solutions to the equation.

4. A certain linear equation with two variables has as solutions (0, −5), (2, 3) and (4, 11). Find the equation.

5. A certain linear equation with two variables has as solutions (−1, −5) and (2, 8). Find the equation.

6. Party hats cost $2 apiece and party whistles cost $3 apiece. Randy bought *x* hats and *y* whistles.

a. Write an expression depicting the total cost (C).

b. Now write an equation stating that the total cost is $48.

How many hats and how many whistles could Randy have bought?

c. Find two other solutions to your equation.

7. Recall the formula tying together distance (*d*), constant speed (*v*), and time (*t*): $d = vt$.
Sarah jogs at the speed of 9 km per hour, and she rides her bicycle at the speed of 18 km per hour.

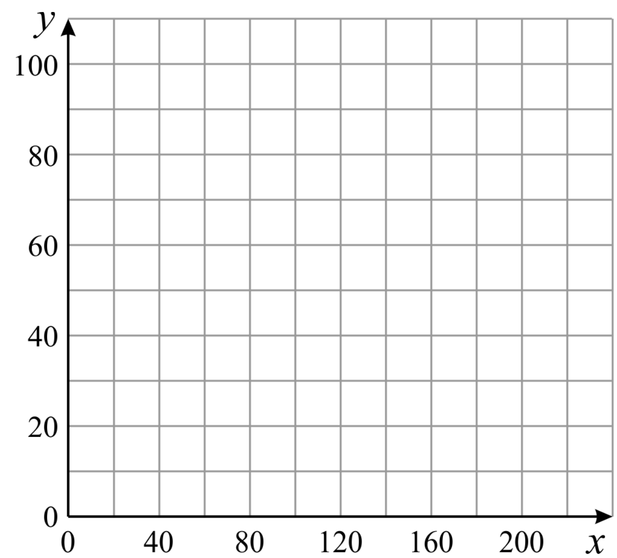
a. Convert these speeds to kilometres per minute.

b. Write an expression for the total distance (*d*) Sarah covers in *x* minutes of jogging plus *y* minutes of bicycling.

c. What distance does Sarah cover if she jogs for 20 minutes and bicycles for 10 minutes?

d. Let's say the distance Sarah covers, jogging and bicycling, is 30 km. Write an equation stating this. How many minutes could she have jogged/bicycled? Find three possible solutions.

e. Write the equation in slope-intercept form and plot it.



8. General admission to a gardening seminar was \$15 but seniors paid only \$10. If the total of the admission fees was \$900, give three possibilities as to how many non-seniors and how many seniors could have attended.

9. A mystery basket contains a mixture of adult cats and kittens (it could even contain zero adults or zero kittens).
Each cat weighs 4 kg and each kitten weighs 0.5 kg.
The total weight of the cats and kittens is 20 kg.

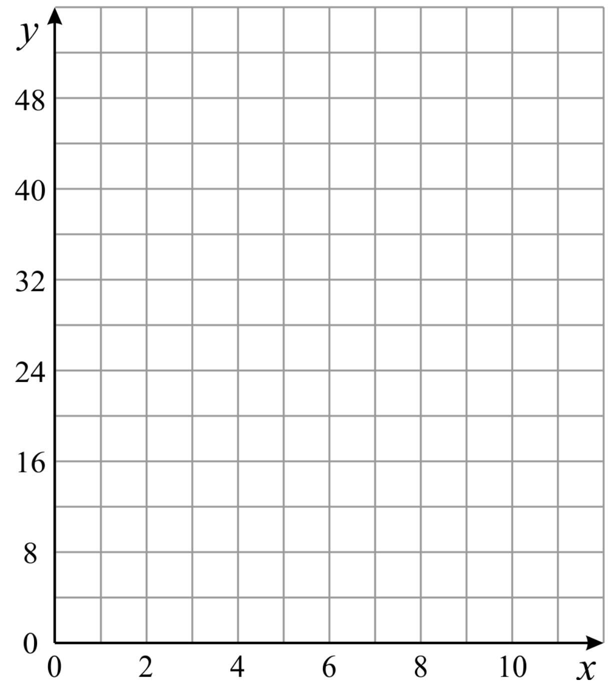
a. If there are x cats and y kittens, write an equation to match the situation.

b. How many adult cats and how many kittens could there be? Find at least three different solutions.

c. Plot your equation from (a).

d. If $x = 1.5$, what is y?
Why is this not a valid solution?

Plot the individual points on the graph that *are* valid solutions.



10. Ava and her family went to stay in a resort for a few nights. Each night cost \$120 (for the whole family). The resort offered horse rides for \$20 per person.

a. If the family stayed for x nights and did y horse rides in total, write an expression for the total cost of these two things.

b. In total, Ava's family spent \$760 on the horse rides plus the nights they stayed.
How many nights and how many horse rides could they have paid for?

11. The equation $2x^2 - 6x - y = 5$ is a quadratic equation because the variable x is squared. If $x = 0$, then $y = -5$, so $(0, -5)$ is one solution to the equation. Find two other solutions to it.

Solving Systems of Equations by Graphing

A system of equations consists of several equations that have the same variables.

A **solution** to a system of equations is a list of values of the variables that satisfy *all* the equations in the system. For two equations, this is an ordered pair.

Example 1. This system of equations consists of two equations. We signify the system with a bracket.

$$\begin{cases} 5x + 4y = 12 \\ y = -x + 2 \end{cases}$$

The solution to the above system is the ordered pair (4, −2), because those values make both equations true: 5(4) + 4(−2) does equal 12, and −2 does equal −4 + 2.

Example 2. The equation $y = (3/2)x - 4$ has an infinite number of solutions, and we can represent those solutions with a line drawn in the coordinate plane.

Similarly, the equation $y = -2x + 3$ has infinitely many solutions.

Here is a system of equations consisting of both:

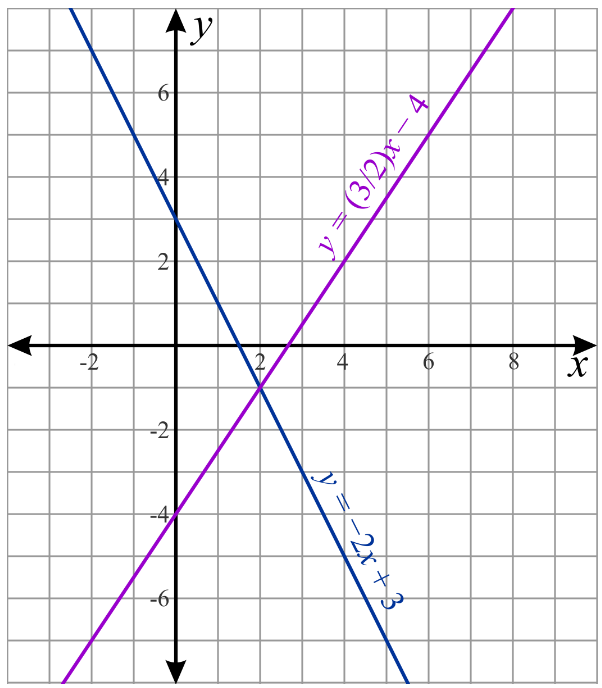
$$\begin{cases} y = (3/2)x - 4 \\ y = -2x + 3 \end{cases}$$

Since the solutions to the first equation form a line, and the solutions to the second also form a line, what would the point of intersection (2, −1) signify?

____

____

(The answer is found at the end of the lesson.)



1. Solve each system of equations using the image. The lines are already plotted in it.

a. $\begin{cases} y = -7x - 23 \\ y = (1/3)x - 1 \end{cases}$

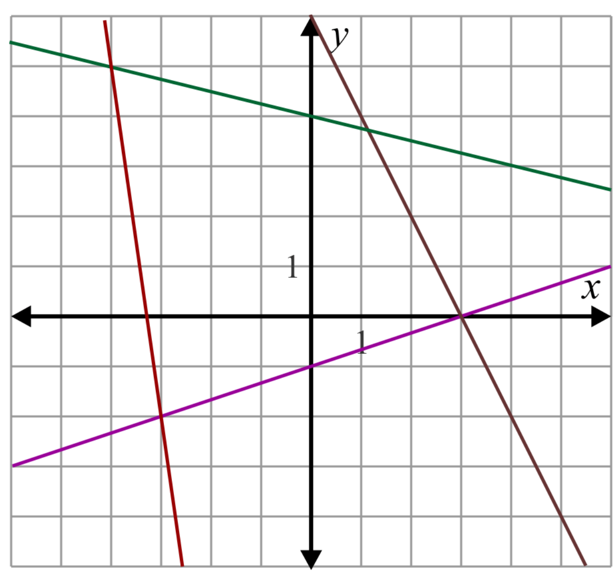
Solution: (______, ______)

b. $\begin{cases} y = -(1/4)x + 4 \\ y = -2x + 6 \end{cases}$

Solution: (______, ______)

c. $\begin{cases} -(1/3)x + y = -1 \\ 2x + y = 6 \end{cases}$

Solution: (______, ______)



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The Addition Method, Part 2

Example 1. In this system of equations, it will not work to add the equations. Neither the x nor the y terms have opposite coefficients, and will not cancel out.

$$\begin{cases} 5x - 6y = 12 \\ x - 2y = 4 \end{cases} \quad \Big| \cdot (-5)$$

But we can make it so! If we **multiply the bottom equation** by −5, the new set of equations will have $5x$ in the top equation and $-5x$ in the bottom. The $5x$ and $-5x$ are called **a zero pair**, and they will cancel out when we add the equations.

$$\begin{array}{rrcrl} & \{ \; 5x - 6y & = & 12 & \textbf{(1)} \\ + & \{ \; -5x + 10y & = & -20 & \textbf{(2)} \\ \hline & 4y & = & -8 & \\ & y & = & -2 & \end{array}$$

Doing that, we easily solve that $y = -2$. Then, substituting that value to the first equation, we find out that $x = 0$.

$$\begin{array}{rcl} 5x - 6(-2) & = & 12 \quad \textbf{(1)} \\ 5x & = & 0 \\ x & = & 0 \end{array}$$

So, the solution is the ordered pair (**0**, **−2**). Let's check that with the *original* equations:

$$\begin{cases} 5(\mathbf{0}) - 6(\mathbf{-2}) \stackrel{?}{=} 12 \\ \mathbf{0} - 2(\mathbf{-2}) \stackrel{?}{=} 4 \end{cases} \rightarrow \begin{cases} 12 = 12 \; ✓ \\ 4 = 4 \; ✓ \end{cases}$$

Why is this okay to do? If you multiply both sides of an equation by a non-zero number, the resulting equation is equivalent to the original: it has the same solution(s).

1. Solve each system of equations. In (a), first multiply the top equation by 2. In (b), you figure out which equation to multiply by what, at first. Then add the equations.

a. $\begin{cases} 3x + y = -7 \\ -6x - 4y = 8 \end{cases} \quad \Big| \cdot 2$

↓

$$\begin{array}{rl} & \{ \; \_\_\_x + \_\_\_y = \_\_\_\_\_ \\ + & \{ \; -6x - 4y = 8 \\ \hline \end{array}$$

b. $\begin{cases} -5x + 3y = -8 \\ 3x - y = 4 \end{cases}$

Example 2. In this system of equations, we will transform *both* the top and the bottom equations, in order to arrive at a zero pair.

When we multiply the top equation by −3 and the bottom equation by 2, the new set of equations will have $-6x$ in the top equation and $6x$ in the bottom (a zero pair).

Doing that, we easily solve that $y = -3$. Then, substituting that value to the second equation, we find out that $x = 4$.

So, the solution is the ordered pair (**4**, **−3**). Let's check that with the *original* equations:

$$\begin{cases} 2(4) - 4(-3) \stackrel{?}{=} 20 \\ 3(4) + 6(-3) \stackrel{?}{=} -6 \end{cases} \rightarrow \begin{cases} 8 + 12 = 20 \checkmark \\ 12 - 18 = -6 \checkmark \end{cases}$$

$$\begin{matrix} (1) \\ (2) \end{matrix} \begin{cases} 2x - 4y = 20 \\ 3x + 6y = -6 \end{cases} \left| \begin{matrix} \cdot (-3) \\ \cdot 2 \end{matrix} \right.$$

↓

$$\begin{array}{rrcl} & \begin{cases} -6x + 12y \\ 6x + 12y \end{cases} & = & \begin{matrix} -60 \\ -12 \end{matrix} \\ + & & & \\ \hline & 24y & = & -72 \\ & y & = & -3 \end{array}$$

↓

$$\begin{aligned} 6x + 12(-3) &= -12 \quad (2) \\ 6x &= 24 \\ x &= 4 \end{aligned}$$

2. By what numbers should you multiply these equations in order to create a zero pair? You do not have to solve the systems.

a. $\begin{matrix} (1) \\ (2) \end{matrix} \begin{cases} 4x - 7y = 5 \\ 5x + 3y = -11 \end{cases}$	**b.** $\begin{matrix} (1) \\ (2) \end{matrix} \begin{cases} 2x + 3y = 20 \\ 9x + 4y = -1 \end{cases}$

3. Solve each system of equations. In (a), first multiply the equations as indicated. In (b), you figure out which equation to multiply by what. Then add the equations. Lastly, check your solutions.

a. $\begin{cases} 2x + 7y = -3 \\ 3x - 2y = 3 \end{cases} \left| \begin{matrix} \cdot 3 \\ \cdot (-2) \end{matrix} \right.$

↓

$$\begin{array}{rl} & \begin{cases} \_\_\_x + \_\_\_y = \\ \_\_\_x + \_\_\_y = \end{cases} \\ + & \\ \hline \end{array}$$

b. $\begin{cases} 6x - 2y = -38 \\ -10x + 5y = 70 \end{cases}$

4. Solve each system of equations. Lastly (always!), check your solutions.

a. $\begin{cases} 6x - 2y = -20 \\ 12x + 6y = 30 \end{cases}$	**b.** $\begin{cases} 10x - 6y = 48 \\ (5/3)x - y = 8 \end{cases}$
c. $\begin{cases} 2x + 3y = -19 \\ -3x + 5y = 95 \end{cases}$	**d.** $\begin{cases} 2x - 5y = -11 \\ 7x + 3y = -100 \end{cases}$

5. Find the error in the solution of this system of equations. Then correct the error and solve the system.

$$\begin{cases} -7x - 9y = -5 \\ 6x + 8y = 4 \end{cases} \quad \begin{array}{|l} \cdot 8 \\ \cdot 9 \end{array} \quad \rightarrow \quad \begin{array}{r} \\ + \end{array}\begin{cases} -56x - 70y = -40 \\ 54x + 72y = 36 \end{cases}$$

$$x + 2y = -4$$

6. Find the error in the solution of this system of equations. Then correct the error and solve the system.

$$\begin{array}{l} (1) \\ (2) \end{array} \begin{cases} 10x + 3y = 5 \\ 2x + 7y = 3 \end{cases} \quad \begin{array}{|l} \\ \cdot 5 \end{array}$$

↓

$$\begin{cases} 10x + 3y = 5 \\ 10x + 35y = 15 \end{cases}$$

$$38y = 20$$

$$y = 10/19$$

↓

$$\begin{aligned} (1) \quad 10x + 3\mathbf{(10/19)} &= 5 \\ 10x + 30/19 &= 5 \\ 10x &= 5 - 30/19 \\ 10x &= 65/19 \\ x &= 65/190 = 13/38 \end{aligned}$$

But the solution (13/38, 10/19) does not check:

$$\begin{aligned} 2(13/38) + 7(10/19) &\overset{?}{=} 5 \\ 26/38 + 70/19 &\overset{?}{=} 5 \\ 26/38 + 140/38 &\overset{?}{=} 5 \\ 166/38 &\neq 5 \end{aligned}$$

7. Kayla DIVIDED instead of multiplying... is that OK?
Why or why not?

$$\begin{cases} -x + 5y = 11 \\ 3x - 12y = 18 \end{cases} \quad \begin{array}{|l} \\ \div 3 \end{array}$$

↓

$$\begin{array}{r} \\ + \end{array}\begin{cases} -x + 5y = 11 & (1) \\ x - 4y = 6 & (2) \end{cases}$$

$$y = 17$$

Finish solving the system.

8. Solve each system of equations and give the solutions rounded to two decimal digits. Note: in the intermediate steps, use at least five decimal digits. Round *only* the final answers to two decimals.

a. $\begin{cases} 3x + 8y = 10 \\ 15x + 20y = 4 \end{cases}$

b. $\begin{cases} -30x + 40y = 50 \\ 9x - 12y = -15 \end{cases}$

c. $\begin{cases} 0.4x - 0.5y = 0 \\ 2.5x + 0.3y = 1 \end{cases}$

d. $\begin{cases} 40x - 20y = 140 \\ -30x + 15y = 90 \end{cases}$

e. $\begin{cases} -0.3x + 0.7y = -0.5 \\ x + 0.9y = 0 \end{cases}$

f. $\begin{cases} 150x - 0.25y = 17.5 \\ 600x + y = 70 \end{cases}$

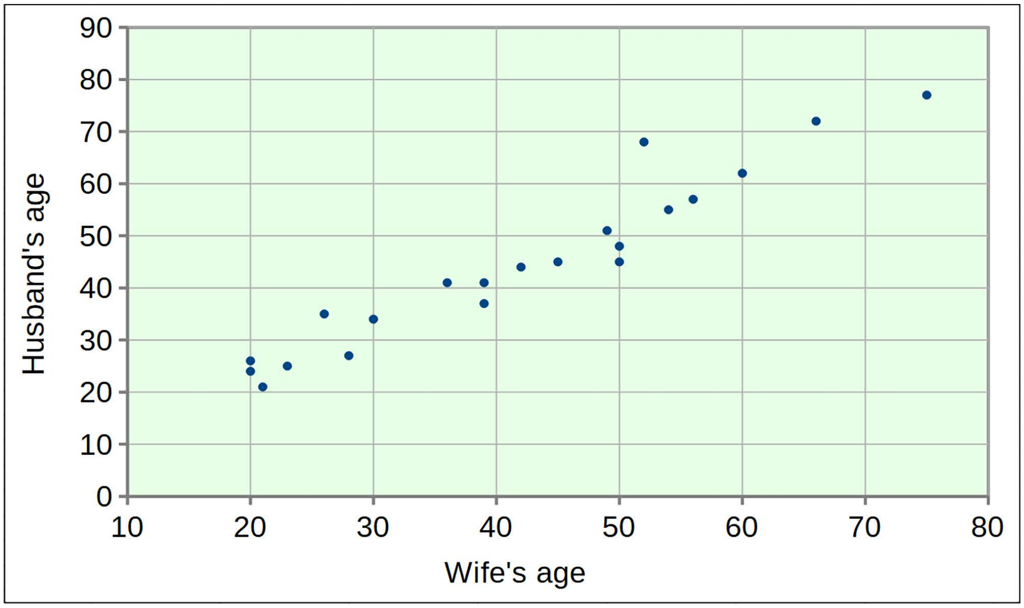
Puzzle Corner

Find the values of *a* and *b* if the system below has (1, −3) as its solution.

$\begin{cases} 6x - by = 2 \\ ax + by = 3 \end{cases}$

Scatter Plots

A **scatter plot** depicts **bivariate data**, meaning that the data involves **two variables**. In the scatter plot below, the variables are the husband's age and the wife's age. Each dot in this scatter plot represents a husband-wife couple. In other words, the coordinates of the dot give us the ages of the husband and the wife.



1. Refer to the scatter plot above.

a. Locate the dot with coordinates (36, 41). What does it signify?

b. Find two couples where the wife is the same age in both cases. Estimate the ages of their husbands.

c. Find the couple with the third oldest husband in this data set. How old is his wife?

d. Is it true that the youngest wife is married to the youngest husband? Explain.

e. Is it true that the oldest wife is married to the oldest husband? Explain.

f. Do you notice a relationship between the two variables? Explain what you see.