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Foreword

Math Mammoth Grade 7 comprises a complete math curriculum for the seventh grade mathematics studies. It follows the Common Core Mathematics Standards (CCS) for 7th grade. Those standards are so constructed that students can continue to a traditional algebra 1 curriculum after studying this. However, you also have the option of following this course with Math Mammoth Grade 8, which provides a gentler and slower transition to high school math.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-B, covers ratios and proportions (chapter 6), percent (chapter 7), geometry (chapter 8), probability (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 7-A worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your student’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

In seventh grade, the first five chapters should be studied in order. Also, chapter 7 (Percent) should be studied before studying any of the chapters 8, 9, or 10 (Geometry, Probability, Statistics). You can be flexible with the rest of the scheduling.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your student!
- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for your student to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 4 pages long, intended for one day. Some 5 and 6-page lessons take two days. There are also a few optional lessons.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Worktext 7-A	
Chapter 1	8 days
Chapter 2	13 days
Chapter 3	9 days
Chapter 4	16 days
Chapter 5	16 days
TOTAL	**62 days**

Worktext 7-B	
Chapter 6	17 days
Chapter 7	12 days
Chapter 8	23 days
Chapter 9	10 days
Chapter 10	12 days
TOTAL	**74 days**

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page counts in the table below *include* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A	81	9	197	72	2.74	13.7
7-B	99	10	244	89	2.74	13.7
Grade 7 total	180	19	441	161	2.73	13.7

The table below is for you to use.

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A			197			
7-B			244			
Grade 7 total			441			

Let's say you determine that your student needs to study about 2.5 pages a day, or 12-13 pages a week on average in order to finish the curriculum in a year. As the student studies each lesson, keep in mind that sometimes most of the page might be reserved for workspace to solve equations or other problems. You might be able to cover more than the average number of pages on such a day. Some other day you might assign the student only one page of word problems.

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, seventh graders might spend 45-90 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 6: Ratios and Proportions
Introduction

Chapter 6 reviews the concept, which has already been presented in grade 6, of the **ratio** of two quantities. From this concept, we develop the related concepts of a **rate** (so much of one thing per so much of another thing) and a **proportion** (an equation of two ratios).

When two quantities are in proportion, we can consider the quantities as variables, write an equation to describe the relationship between them, and graph that equation. This study of proportional relationships takes the concept of *ratio* to a new level, and paves the way to the study of linear functions in 8th grade.

The first lessons focus on the concepts of ratio, rate, and unit rate. Students use tables of equivalent ratios and unit rates to solve a variety of problems involving rates. We especially focus on calculating unit rates when the quantities involve fractions.

Then we study proportional relationships, using the familiar tables of equivalent rates as a starting point. Students write and graph equations relating the two quantities (seen as variables now). They find the unit rate and plot it on the graph as a single point, and relate the different representations of proportional relationships (graph, table of values, wording, and equation) to each other. We also spend some time analyzing whether a given relationship between variables is proportional or not.

The next topic is proportions — equations where one ratio is equal to another. Students learn to solve proportions with cross-multiplying and to set them up in the correct way to solve a word problem. They also learn and compare different ways to solve problems with rates. It is not always necessary to set up a proportion!

Then we turn our attention to an application of all this in geometry: scaled figures, scale drawings, floor plans, and maps. Students encounter scales such as 1:90 or 2 cm = 30 m. They calculate dimensions in reality from the scale drawing and vice versa, and redraw scale drawings at a different scale. Floor plans use a scale also, and are hopefully an interesting topic to students.

The lesson on maps is optional. In today's world, most of us are using online map services which calculate the distances for us, so there is much less need to figure out distances using physical maps, but some students (and teachers) might find the topic interesting.

In 8th grade, students encounter proportional relationships again, as they learn the connection between the unit rate and the slope of the graph, and compare different proportional relationships represented in different ways.

I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

There are free videos matched to the curriculum at **https://www.mathmammoth.com/videos/** (choose 7th grade).

Good Mathematical Practices

- The chapter gives students many opportunities to persevere in problem solving through various multi-step word problems. Remind your student(s) that making a mistake is part of normal problem solving, and when you think about your mistake, your brain actually grows. There is no brain growth when doing problems that you already know how to solve.
- The study of proportional relationships lays a big foundational piece for mathematical modelling. In this chapter, students encounter many real-life situations (e.g. speed, price per kg). They learn to see the quantities involved as variables, to write an equation relating the variables, and to graph the equations. These are all essential skills in mathematical modelling.
- In the lesson Scale Drawings 2, students will explore how the area of a scaled figure relates to the scale factor, and they have the opportunity to find the general rule through repeated reasoning.

Pacing Suggestion for Chapter 6

This table does not include the chapter test because it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 6	page	span	suggested pacing	your pacing
Ratios and Rates	14	*3 pages*	1 day	
Solving Problems Using Equivalent Rates	17	*3 pages*	1 day	
Unit Rates	20	*4 pages*	1 day	
Proportional Relationships 1	24	*4 pages*	1 day	
Proportional Relationships 2	28	*4 pages*	1 day	
Proportional Relationship or Not?	32	*4 pages*	1 day	
Solving Proportions	36	*3 pages*	1 day	
Proportions and Problem Solving	39	*4 pages*	1 day	
More on Proportions	43	*4 pages*	1 day	
Scaling Figures	47	*4 pages*	1 day	
Scale Drawings 1	51	*3 pages*	1 day	
Floor Plans	54	*3 pages*	1 day	
Scale Drawings 2	57	*3 pages*	1 day	
Scale Drawings—More Practice (optional)	60	*2 pages*	1 day	
Maps (optional)	62	*6 pages*	2 days	
Chapter 6 Mixed Review	66	*3 pages*	1 day	
Chapter 6 Review	69	*5 pages*	2 days	
Chapter 6 Test (optional)				
TOTALS		*54 pages*	16 days	
with optional content		*(62 pages)*	(18 days)	

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr7ch6

SCAN ME

Ratios and Rates

A **ratio** is a comparison of two numbers, or quantities, using division.

For example, to compare the hearts to the stars in the picture, we say that the ratio of hearts to stars is 5:10 (read "five to ten").

The two numbers in the ratio are called the **first term** and the **second term** of the ratio. The order in which these terms are mentioned does matter! For example, the ratio of stars to hearts is *not* the same as the ratio of hearts to stars. The former is 10:5 and the latter is 5:10.

We can write this ratio in several different ways:

- The ratio of hearts to stars is 5:10.
- The ratio of hearts to stars is 5 to 10.
- The ratio of hearts to stars is $\frac{5}{10}$.
- For every five hearts, there are ten stars.

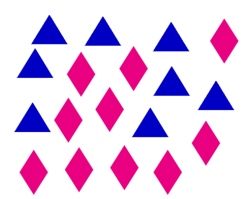
Note that we are not comparing two numbers to determine which one is greater (as in 5 < 10). The comparison is relative as in a multiplication problem. For example, the ratio 5:10 can be simplified to 1:2, and it indicates to us that there are twice as many stars as there are hearts.

We **simplify ratios** in exactly the same way we simplify fractions.

Example 1. In the picture at the right, the ratio of hearts to stars is 12:16. We can simplify that ratio to 6:8 and even further to 3:4. These three ratios (12:16, 6:8, and 3:4) are called **equivalent ratios**.

The ratio that is simplified to lowest terms, 3:4, tells us that for every three hearts, there are four stars.

1. Write the ratio and then simplify it to lowest terms.



The ratio of triangles to diamonds is _____ : _____ = _____ : _____ .

In this picture, there are ______ triangles to every _______ diamonds.

2. **a.** Draw a picture with pentagons and circles so that the ratio of pentagons to the total of all the shapes is 7:9.

b. What is the ratio of circles to pentagons?

3. **a.** Draw a picture in which (1) there are three diamonds for every five triangles, and (2) there is a total of 9 diamonds.

b. Write the ratio of all the diamonds to all the triangles, and simplify this ratio to lowest terms.

4. Write the equivalent ratios.

a. 5 to 45 = 1 to ________	**b.** 3 : _____ = 9 : 60	**c.** 280 : 420 = 2 : _____	**d.** $\frac{5}{13} = \frac{\square}{65}$

We can also form **ratios using quantities that have units**. If the units are the same, they cancel.

Example 2. Simplify the ratio 250 g : 1.5 kg.

First we convert 1.5 kg to grams and then simplify: $\frac{250\text{ g}}{1.5\text{ kg}} = \frac{250\text{ g}}{1500\text{ g}} = \frac{250}{1500} = \frac{1}{6}$.

5. Use a fraction line to write ratios of the given quantities as in the example. Then simplify the ratios.

a. 5 kg and 800 g $\frac{5\text{ kg}}{800\text{ g}} =$	**b.** 600 cm and 2.4 m
c. 1 litre and 750 ml	**d.** 325 cm and 1.25 m

We can generally **convert** ratios with decimals or fractions **into ratios of whole numbers**.

Example 3. Because we can multiply both terms of the ratio by 10, $\frac{1.5\text{ km}}{2\text{ km}} = \frac{15\text{ km}}{20\text{ km}}$.

Then: $\frac{15\text{ km}}{20\text{ km}} = \frac{15}{20} = \frac{3}{4}$. So the ratio 1.5 km : 2 km is equal to 3:4.

You can also see that the ratio is 3:4 by noticing that both 1.5 km and 2 km are evenly divisible by 500 m.

Example 4. Simplify the ratio ¼ mile to 5 miles.

First, the units cancel: ¼ mi : 5 mi = ¼ : 5. Multiplying both terms of the ratio by 4, we get ¼ : 5 = 1:20.

6. Use a fraction line to write ratios of the given quantities. Then simplify the ratios to whole numbers.

a. 5.6 km and 3.2 km	**b.** 0.02 m and 0.5 m
c. 1.25 m and 0.5 m	**d.** 1/2 L and 7 1/2 L
e. 1/4 cup and 3 1/2 cups	**f.** 2/3 km and 1 km

If the two terms in a ratio have *different* units, then the ratio is also called a **rate.**

Example 5. The ratio "8 km to 40 minutes" is a rate that compares the quantities "8 km" and "40 minutes," perhaps for the purpose of giving us the speed at which a person is running.

We can write this rate as 8 km : 40 minutes or $\frac{8\text{ km}}{40\text{ minutes}}$ or 8 km *per* 40 minutes.

The word "per" in a rate signifies the same thing as a colon or a fraction line.

This rate can be simplified: $\frac{8\text{ km}}{40\text{ minutes}} = \frac{1\text{ km}}{5\text{ minutes}}$. The person runs 1 km in 5 minutes.

Example 6. Simplify the rate "15 pencils per 100¢." Solution: $\frac{15\text{ pencils}}{100¢} = \frac{3\text{ pencils}}{20¢}$.

7. Write each rate using a colon, the word "per," or a fraction line. Then simplify it.

a. Jeff swims at a constant speed of 400 metres : 15 minutes.

b. The car can travel 45 km on 3 L of petrol.

8. Fill in the missing terms to form equivalent rates.

a. $\frac{1/2\text{ cm}}{30\text{ min}} = \frac{}{1\text{ h}} = \frac{}{15\text{ min}}$

b. $\frac{\$88.40}{8\text{ hr}} = \frac{}{2\text{ hr}} = \frac{}{10\text{ hr}}$

9. Simplify these rates. Don't forget to write the units.

a. 280 km per 7 hours

b. 10 cm : 1.5 minutes

10. A car is travelling at a constant speed of 72 km/hour. Fill in the table of equivalent rates: each pair of numbers in the table (distance/time) forms a rate that is equivalent to the rate 72 km/hour.

Distance (km)							
Time (min)	10	30	40	50	60	90	100

11. Eight pairs of socks cost $20. Fill in the table of equivalent rates.

Cost ($)								
Pairs of socks	1	2	4	6	7	8	9	10

Solving Problems Using Equivalent Rates

Example 1. It took Liam 1 ½ hours to paint 8 metres of fence. Painting at the same speed, how long will it take him to paint the rest of the fence, which is 28 metres long?

In this problem, we see a rate of 8 m per 1 ½ hours. There is another rate, too: 28 m per an unknown amount of time. These two are equivalent rates. We can use a table of equivalent rates to solve the problem.

Amount of fence (m)	8	4	28
Time (minutes)	90	45	315

(1) We figure that Liam can paint 4 m of fence in 45 minutes (by dividing the terms in the original rate by 2).

(2) Next we multiply both terms in the rate of 4 m/45 min by seven to get the rate 28 m/315 min.

It will take Liam 315 minutes, or 5 hours 15 minutes, to paint the rest of the fence.

Example 2. Sofia rides her bike 20 km in 45 minutes.
Riding at the same speed, how far will she go in 1 hour?

$$\frac{20 \text{ km}}{45 \text{ minutes}} \overset{\div 3}{\underset{\div 3}{=}} \frac{\square \text{ km}}{15 \text{ minutes}} \overset{\cdot 4}{\underset{\cdot 4}{=}} \frac{\square \text{ km}}{60 \text{ minutes}}$$

We can <u>multiply or divide both terms of a rate by the same number</u> to form another, equivalent rate. (You have used this same idea in the past with equivalent fractions.)

It's not easy to go directly from 45 minutes to 60, but we can use 15 as a "stepping stone" in between.

Recall that 20 ÷ 3 is easy to solve when you think of it as a fraction: 20/3 = 6 2/3. Sofia can ride 6 2/3 km in 15 minutes.

Then, we multiply both terms of that rate by 4. Again, don't be intimidated by the fraction:
4 · (6 2/3) = 4 · (20/3) = 80/3 = 26 2/3. So, Sofia can ride 26 2/3 km in 1 hour.

1. Fill in the tables of equivalent rates.

a.

Distance	15 km			
Time	3 hr	1 hr	15 min	45 min

b.

Pay	\$15			
Time	45 min	15 min	1 hr	1 hr 45 min

2. Fill in the missing terms in these equivalent rates.

a. $\frac{3 \text{ pies}}{8 \text{ boys}} = \frac{\_\_\_\_}{2 \text{ boys}} = \frac{\_\_\_\_}{12 \text{ boys}} = \frac{\_\_\_\_}{20 \text{ boys}}$

b. $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\_\_\_\_}{1 \text{ min}} = \frac{\_\_\_\_}{3 \text{ min}}$

3. Aiden can ride his bicycle 12 km in 28 minutes. At the same constant speed, how long will he take to go 54 km?

$$\frac{12 \text{ km}}{28 \text{ minutes}} = \frac{6 \text{ km}}{\square \text{ minutes}} = \frac{\square \text{ km}}{\square \text{ minutes}}$$

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Review: Percent

Percent (or **per cent**) means *per hundred* or “divided by a hundred.” (The word “cent” means one hundred.) So, percent means the rate per hundred, or a hundredth part.

To convert percentages into fractions, simply read the “per cent” as “per 100.” Thinking of hundredths, you can also easily write them as decimals.

Therefore, 8% = 8 per cent = 8 per 100 = 8/100 = 0.08.

Similarly, 167% = 167 per 100 = 167/100 = 1.67.

$\frac{5}{100}$ five per cent = 5%

1. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
	0.07	
52%		
		$\frac{59}{100}$

Percent	Decimal	Fraction
109%		
200%		
		$\frac{382}{100}$

A number with two decimal digits has hundredths, so it can easily be written as a percentage. For example, 0.56 = 56%. But we can write numbers with more decimal digits as percents, also.

Example 1. As a percentage, the number 0.5642 is 56.42%. Compare this to 0.56 = 56%. The digits “42” simply follow the digits “56”, and become the decimal digits for the percentage.

Decimal	Percent	Fraction
0.09	9%	$\frac{9}{100}$
0.091	9.1%	$\frac{91}{1000}$
0.09146	9.146%	$\frac{9146}{100\,000}$

2. Write as percentages, fractions, and decimals.

Percent	Decimal	Fraction
0.9%		
		$\frac{282}{1000}$
	0.8914	

Percent	Decimal	Fraction
		$\frac{91}{10\,000}$
2.391%		
	0.94284	

Writing fractions as percentages

Example 2. Sometimes you can convert a fraction into an equivalent fraction with a denominator of 100, 1000, or some other power of 10. After that it is easy to write it as a decimal and then as a percentage.

$$\frac{46}{25} \overset{\cdot 4}{=} \frac{184}{100} = 1.84 = 184\%$$

Example 3. For most fractions, we need to use *division* to convert the fraction to a decimal first, and then to a percentage.

Simply treat the fraction line as a division symbol and divide (using long division or a calculator), to get a decimal. Then write it as a percentage.

$$\frac{8}{9} = 0.888... \approx 0.889 = 88.9\%$$

```
    0.8888
9 ) 8.0000
   -7 2
      8 0
    - 7 2
        8 0
      - 7 2
          8 0
        - 7 2
            8
```

3. Fill in the table. First write each fraction as an equivalent fraction where the denominator is a power of ten.

Fraction	Fraction (denominator is a power of ten)	Decimal	Percent
$\frac{8}{25}$	$\frac{\boxed{}}{100}$		
$\frac{142}{200}$	$\frac{\boxed{}}{100}$		
$\frac{24}{20}$			
$\frac{31}{250}$			
$\frac{3}{8}$			

4. Write as percentages. Use long division. Round your answers to the nearest tenth of a percent.

a. 11/8	**b.** 11/24

5. Write the fractions as decimals and percentages. Round the decimals to four decimal digits. Use a calculator.

a. $\frac{2}{3} \approx$ ____________ = ____________%	**b.** $\frac{11}{6} \approx$ ____________ = ____________%
c. $\frac{17}{23} \approx$ ____________ = ____________%	**d.** $\frac{304}{57} \approx$ ____________ = ____________%

6. Match the fractions and percentages.

$\frac{3}{4}$	$\frac{17}{20}$	$\frac{4}{5}$	$\frac{18}{25}$	$\frac{5}{4}$	$\frac{7}{10}$	$\frac{9}{8}$	$\frac{23}{20}$
125%	80%	75%	115%	70%	72%	85%	112.5%

7. Remember mental math? Fill in the shortcuts for finding these easy percentages of a number.

To find 50% of a number, divide it by _______. To find 25% of a number, divide it by _______.	To find 10% of a number, divide it by ________. To find 1% of a number, divide it by ________.
To find 30% of a number, first find __________% of the number, then multiply that by _____. To find 75% of a number, first find __________% of the number, then multiply that by _____.	

8. Find various percentages of the number 360.

Percentage	Value
5%	
10%	
20%	
25%	
50%	
60%	
75%	
80%	
100%	360
125%	
150%	

9. Solve using mental math.

a. What is 25% of $84.00?

b. 17 is 25% of what number?

c. Find 60% of 300.

d. 8 is 1% of what number?

e. Find 3% of 2000 km.

f. 9 is 3% of what number?

g. What is 20% of $45?

h. 24 is 60% of what number?

i. Find 150% of $60.

j. 36 is 200% of what number?

Solving Basic Percentage Problems

All percentages are fractions. Recall that “percent” means “per 100”. For example, 34% is 34 per 100 or 34/100 — a fraction. Stated differently, a percentage is a “rate per 100”.

All percentage problems have to do with a **part** versus **total**. As a fraction, we write $\frac{part}{total}$.

As a percent, it is still a fraction, and has the same value, but we want the denominator to be 100.

For example, to find what percentage 2 is of 5, we can write: $\frac{2}{5} = \frac{40}{100}$, and then write 40/100 as 40%.

Example 1. What percentage is 14 km of 75 km?

We know the total (75 km) and we know the part (14 km). This is essentially asking what fraction 14 km is of 75 km, but we need to express the answer as a percentage, not as a fraction. Therefore:

1. We write the fraction $\frac{part}{total}$: it is $\frac{14\text{ km}}{75\text{ km}}$ but the units “km” cancel out so it becomes just $\frac{14}{75}$.

2. Then we use a calculator to divide 14/75 = $0.18\overline{6}$ and write that as a percentage: $0.18\overline{6}$ = $18.\overline{6}$%.

Normally, we round the result and say that 14 km is about 19% of 75 km.

Example 2. Find 59.2% of $2600.

Here we know the percentage — which means we know the fraction — and the total. We don’t know the *part* as a quantity. This is the same as asking for 592/1000 of $2600.

Recall that the word “of” translates into multiplication. Thus, we could use fraction multiplication (we could calculate (592/1000) · 2600) but often, the quickest way to do these types of calculations is to convert the percentage into a decimal first, and then use decimal multiplication.

So, instead of (592/1000) · $2600, we write 59.2% as 0.592, and calculate 0.592 · $2600 = $1539.20.

If the percentage is known and the total is known: (What is x% of y?)

This is the same as asking for a fraction of some total.

1. Write the percentage as a decimal.
2. Multiply that decimal by the total.

Or use mental math tricks for finding 1%, 10%, 20%, 30%, 25%, 50%, 75%, *etc*. of a number.

If you are asked the percentage:

Asking “what percentage” is essentially the same as asking “what part” or “what fraction.”

1. Write the fraction $\frac{part}{total}$.

2. Write this fraction as a percentage. Often, you will do this in two steps: first write that fraction as a decimal, and then that as a percentage.

1. Fill in the tables. Use mental math.

a.

Amount			75					
Percentage	10%	20%	30%	50%	100%	120%	150%	200%

b.

Amount		12	20	32			60	
Percentage	10%				100%	120%	150%	200%

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Angle Relationships 1

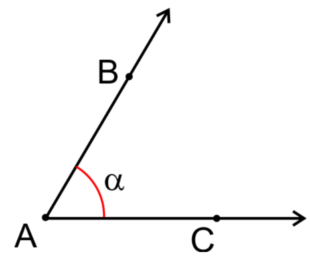
A **ray** has a starting point and continues indefinitely in one direction (indicated by one arrowhead).

In contrast to a ray, a **line** continues indefinitely in *two* directions (indicated by two arrowheads).

An **angle** consists of **two rays that start at the same point**, called the **vertex**. Each ray is called a **side** of the angle.

We can denote the angle on the right as angle BAC, or using the symbol "∠" for "angle," as ∠BAC.

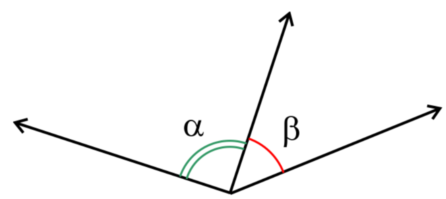
Note that we list the vertex point in the middle: it is ∠B**A**C, not ∠ABC. We could also name it ∠CAB.



In mathematics, we also often denote angles with the beginning letters of the Greek alphabet: α (alpha), β (beta), γ (gamma), and δ (delta). So ∠BAC can also be called "angle α."

Two angles are **adjacent** if they have a **common vertex and share one side.**

In the image on the right, ∠α and ∠β are adjacent (side-by-side) angles.



1. B is a point on line AD. Find the measures of the three angles, and also the angle sums. Do you notice any special numbers?

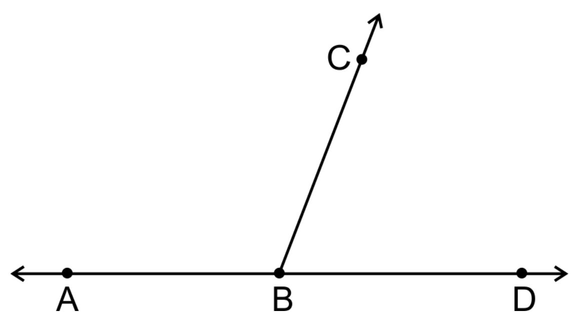
   ∠ABC = __________°

   ∠CBD = __________°

   ∠ABD = __________°

   sum of ∠ABC and ∠CBD : ____________°

   sum of all three angles: _____________ ° (This should be 360°.)

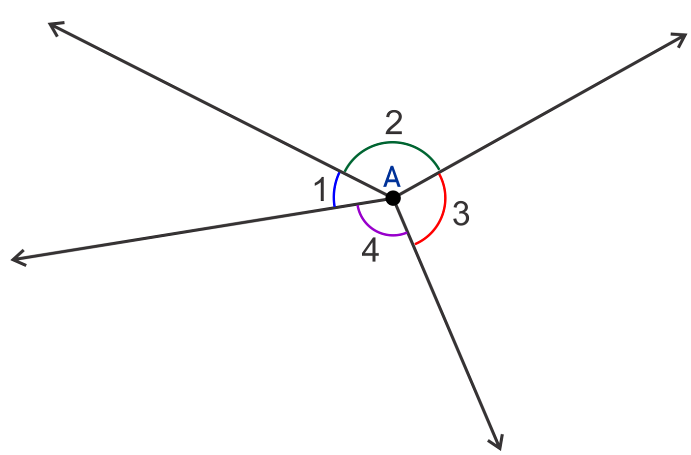


2. Several rays start at point A. Measure the angles. Calculate their sum.

   ∠1 = __________° ∠2 = __________°

   ∠5 = __________° ∠4 = __________°

   Sum of the angles = __________°

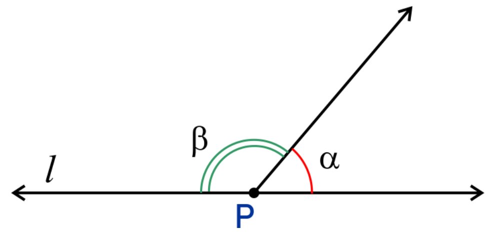


P is a point on line *l*. The angles ∠α and ∠β in this image are adjacent, and they form a straight angle (an angle of 180 degrees). They are called **supplementary angles**.

Two angles are supplementary if their **sum is 180 degrees:**

$$\angle\alpha + \angle\beta = 180°$$

We also say that α supplements β.

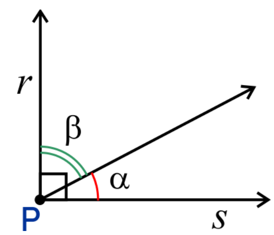


Rays *r* and *s* start at point P and form a right angle. The adjacent angles ∠α and ∠β form a right angle. They are called **complementary angles**.

Two angles are complementary if their **sum is 90 degrees:**

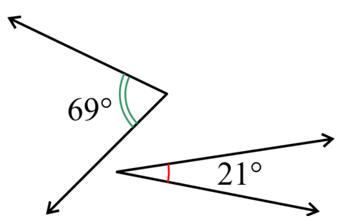
$$\angle\alpha + \angle\beta = 90°$$

We also say that α complements β.

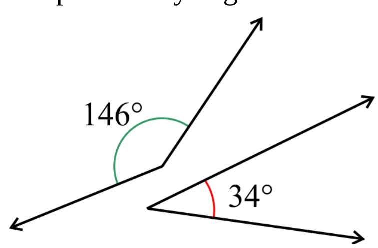


Here's a mnemonic to help you remember the difference: **<u>S</u>**upplementary angles form a **<u>S</u>**traight line, and **<u>C</u>**omplementary angles form a **<u>C</u>**orner (a right angle).

Supplementary angles don't have to be adjacent, and neither do complementary angles.



These are still complementary angles, because 21° + 69° = 90°.



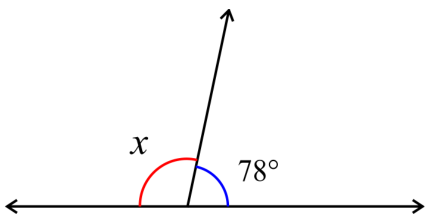
These are still supplementary angles, because 146° + 34° = 180°.

3. **a.** Draw a 38° angle. Then draw an adjacent angle that complements it.

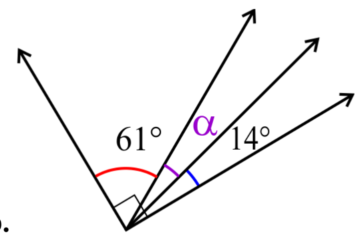
b. Draw an 82° angle. Then draw an adjacent angle that supplements it.

4. Write an equation for the unknown and solve it. Do not measure any angles.

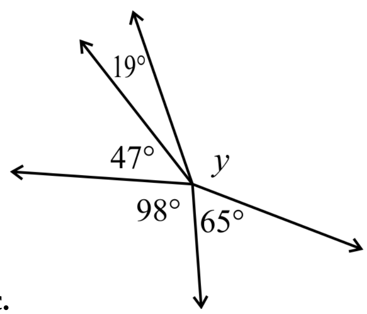
a.



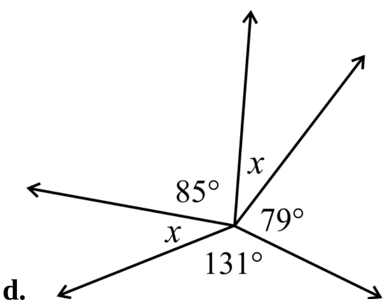
b.



c.

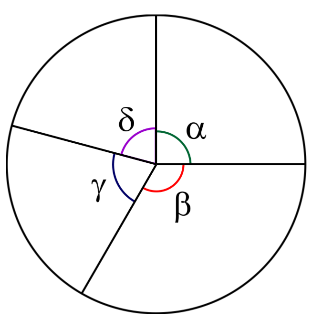


d.



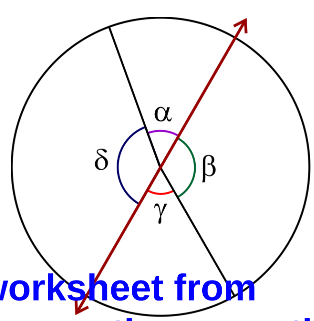
5. Figure out the missing entries in the tables without measuring any angles.

a.



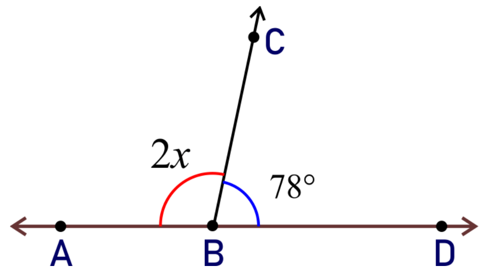
Angle	Degrees	Fraction	Percentage
α		1/4	
β	120°		
γ			
δ	75°		

b.



Angle	Degrees	Fraction	Percentage
α	50°		
β			
γ		1/6	
δ			

Example. Point B is on line AD. Write an equation to solve for the unknown. What is the measure of angle ABC?



Since angle ABD is a straight angle (180°), the equation is:

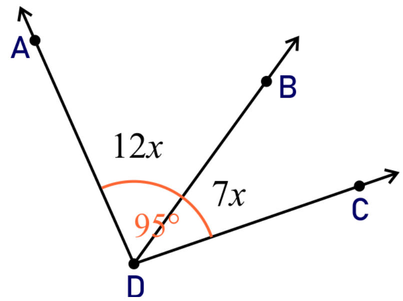
$$\begin{aligned} 2x + 78 &= 180 \\ 2x &= 102 \\ x &= 51 \end{aligned}$$

So, x is 51°. However, angle ABC does not measure 51° because its measure is $2x$, not x. So, we double the value of x to get that ∠ABC = 102°.

6. Angle ADC measures 95°.

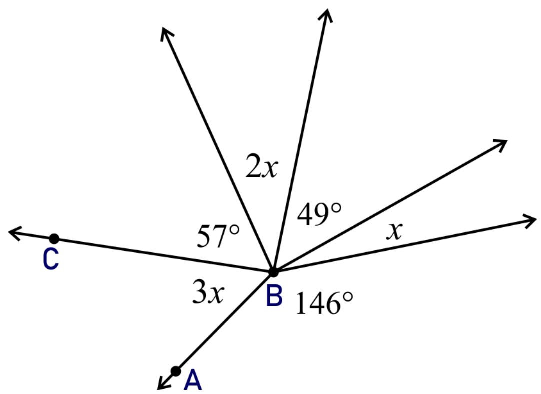
   a. Write an equation for the unknown and solve it.

   b. Find the measure of ∠BDC.



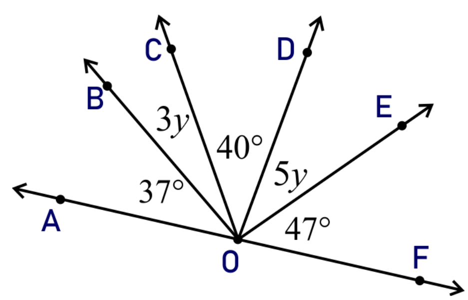
7. **a.** Write an equation for the unknown and solve it.

   b. Find the measure of ∠ABC.



8. **a.** Write an equation for the unknown and solve it.

   b. Find the measure of each angle in the image, excluding those whose angle measure is given.



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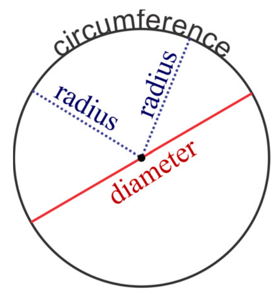
Circumference of a Circle

Circle terms

- The **circumference** of a circle is the perimeter, or outside curve, of the circle.
- The **radius** is any line segment from the centre point to the circumference.

In fact, what makes a circle, a circle is the fact that all the points on the circumference are **at the same distance from the centre point** of the circle. This distance is called the **radius** of the circle.

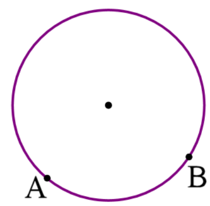
- The **diameter** is any line segment from circumference to circumference that goes through the centre point of the circle.



You may use a calculator for every problem in this lesson.

1. **a.** Draw a radius for this circle from the point A and a diameter from the point B.

   b. What simple relationship exists between the diameter and the radius of any circle?



2. A circle is drawn on the ground. Your 5-metre jump rope is just enough to go around it. Match the approximate measurements and the terms.

diameter	about 5 m
radius	about 0.8 m
circumference	about 1.6 m

3. There exists an amazing relationship between the circumference and the diameter of every circle! Let's study it now. Find at least **five circular objects**, such as a plate, a can, a glass, and so on. Measure the diameter (*d*) of each circle with a ruler. Measure the circumference (C) of each circle by placing a string around the object, and then measuring the length of the string. Record your results in the table.

   In the last column, divide the circumference by the diameter using a calculator (separately for each object). In other words, you will calculate the ***ratio* of the circumference to the diameter**.

Object	C	*d*	C ÷ *d*

What do you notice?

If you have measured accurately, for each object, the ratio of C to *d* should be a little over 3.

Even thousands of years ago, people knew that the circumference of a circle was *about* three times its diameter. Ancient Egyptians used the number **22/7** (which is about 3.14285) instead of 3 for this factor. For example, if the diameter of a circle was 2 units long, the Egyptians would have calculated the circumference to be (22/7) · 2 = 44/7 = 6 2/7 units.

4. Find the circumference of each circle, using 22/7 and a calculator. Round the answers to one decimal.

a. Diameter 5 cm	**b.** Radius 3 inches	**c.** Diameter 2 m
Circumference =	Circumference =	Circumference =

In reality, the ratio of a circle's circumference to its diameter (the ratio C/*d*) is neither exactly 3 nor 22/7. It is **pi**: a number that is about 3.1416 and denoted by the Greek letter π. The value of π has been calculated to many millions of decimal digits. Here are some of the first digits:

$$\frac{C}{d} = \pi \approx 3.1415926535897932384626433832795028841971693993751058209749445923078164...$$

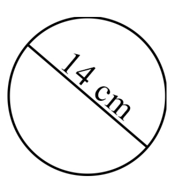
The decimal expansion of π goes on forever without ever having any pattern in the digits! This means it is an **irrational number**: you cannot write it as a fraction *a*/*b*, where *a* and *b* are integers.

In calculations, you can use π ≈ 3.14, π ≈ 22/7, or the π-button on your calculator (if you have it).

From the relationship (or equation) $\frac{C}{d} = \pi$ we can solve for C, and get the formula $C = \pi d$.

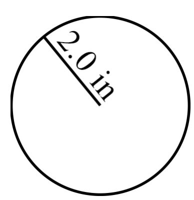
This formula allows us to calculate the circumference when the diameter is known.

5. Find the circumference of these circles. Use π ≈ 3.14. Give your answer to the same accuracy as the dimension given in the problem.



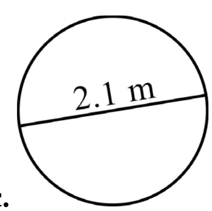
a.

C = ____________________



b.

C = ____________________



c.

C = ____________________

6. **a.** The diameter of a circle is 5.60 km. What is its circumference?
Its radius?

b. The circumference of a circle is 120 cm. What is its diameter?
Its radius?

7. Fill in the table. Use $\pi \approx 3.14$ and a calculator. Round to one decimal digit.

	Circle A	Circle B	Circle C	Circle D
Circumference	14 cm			7.5 km
Diameter		2.5 in		
Radius			8.3 m	

8. **a.** Learn to use a compass to draw circles (on blank paper). If you need help, check out this video: https://www.youtube.com/watch?v=02XRad7s1Io

b. Draw a circle with a radius of 3 cm. To do that, you need to set the radius on the compass to be 3 cm. One way to do that is by placing the compass next to a ruler, and adjusting the radius of the compass until it is 3 cm as measured by the ruler.

c. Find the circumference of the circle you drew in (b).

9. **a.** Draw a circle with a diameter of 5 cm. Calculate its circumference.

b. Sketch (freehand) a *square* that has the same perimeter as your circle. How long is the side of your square, to the nearest millimetre?

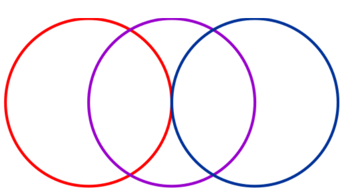
10. A Ferris wheel has a radius of 36.5 m. If you go around in it twice, what distance do you travel? Give your answer to the nearest metre.

11. A hula hoop has a diameter of 95 cm. Little Anna rolls it down the path for fun. How many complete turns of the hula hoop happen when the hoop rolls a distance of 6.5 metres?

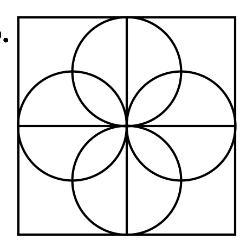
Puzzle Corner

Redraw these circle designs on blank paper. You can choose the size (radius) of the circles.

a.



b.



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Probability

You *probably* already have an intuitive idea of what *probability* is. In this lesson we look at some simple examples in order to study probability from a mathematical point of view.

If we flip a coin, the chance, or **probability**, of getting “heads” is 1/2. The chance of getting “tails” is also 1/2. “Heads” and “tails” are the two possible **outcomes** when tossing a coin, and they are equally likely.

When rolling a six-sided number cube (a die), you have six possible **outcomes**: you can roll either 1, 2, 3, 4, 5, or 6. These are all equally likely (assuming the die is fair).

Thus the probability of rolling a five is 1/6. The probability of rolling a three is also 1/6. In fact, the probability of each of the six outcomes is 1/6.

The probability of rolling an even number is 3/6, or 1/2, because three of the six possible outcomes are even numbers.

Simple probability has to do with situations where each possible outcome is equally likely.

Then the **probability** of an event is the fraction $\dfrac{\textbf{number of favourable outcomes}}{\textbf{number of possible outcomes}}$

“favourable outcomes” are those that make up the event you want. The examples will make this clear.

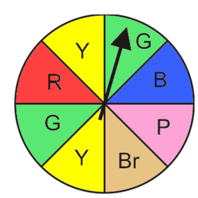
Example 1. What is the probability of getting a number that is less than 6 when tossing a fair die?

Count how many of the outcomes are “favourable” (less than 6). There are five: 1, 2, 3, 4, or 5.
And there are six possible outcomes in total.

Therefore, the probability is $\dfrac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \dfrac{5}{6}$.

In math notation we write “P” for probability and put the event in parentheses: **P(less than 6) = 5/6.**

Example 2. On this spinner the number of possible outcomes is eight, because the arrow is equally likely to land on any of the eight wedges. What is the probability of spinning yellow?



There are TWO favourable outcomes (yellow areas) out of EIGHT possible outcomes.

P(yellow) = 2/8 = 1/4.

(Because green and yellow each have two wedges, there are only six possible colours that can result. When we list the possible outcomes, we list the six colours. However, when we figure the probabilities, we must use the eight equal-sized wedges to find the probability.)

By convention, the probability of an event is always at least 0 and at most 1. In symbols: $0 \le P(\text{event}) \le 1$.

A probability of 0 means that the event does not occur; it is impossible. Probability of 1 means that the event is sure to occur; it is certain. A probability near 1 (such as 0.85) means that the event is likely to occur. A probability of 1/2 means that an event is neither likely nor unlikely.

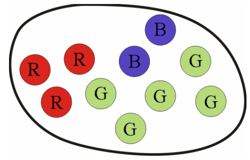
Example 3. What is the probability of rolling 8 on a standard six-sided die?

This is an impossible event, so its probability is zero: P(8) = 0.

Example 4. What is the probability of rolling a whole number on a die?

This is a sure event, so its probability is one. P(whole number) = 1.

1. There are three red marbles, two dark blue marbles, and five light green marbles in Michelle's bag. List all the possible outcomes if you choose one marble randomly from her bag.



2. Michelle chooses one marble at random from her bag. What is the probability that...

   a. the marble is blue?

   b. the marble is not red?

   c. the marble is neither blue nor green?

3. Make up an event with a probability of zero in this situation.

4. Suppose you choose one letter randomly from the word "PROBABILITY."

   a. List all the possible outcomes for this event.

   Now find the probabilities of these events:

   b. P(B)

   c. P(A or I)

   d. P(vowel)

   e. Make up an event for this situation that is likely to occur, yet not a sure event, and calculate its probability.

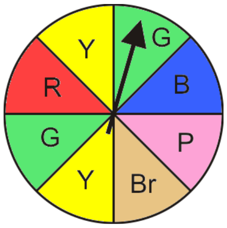
The complement of an event and the probability of "not"

The ***complement*** of any event A is the event that A does *not* occur.

If the probability of event A is a, then the probability of A not happening is simply $1 - a$.

5. The weatherman says that the chance of rain for tomorrow is 1/10. What is the probability of it not raining?

6. The spinner is spun once. Find the probabilities as simplified fractions.



   a. P(green) **b.** P(not green)

   c. P(not pink) **d.** P(not black)

   e. Make up an event for this situation that is not likely, yet not impossible either, and calculate its probability.

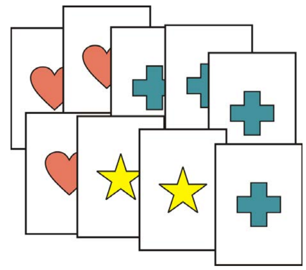
Probabilities are often given as percentages instead of fractions.

Example 5. Kimberly's sock bin contains 7 brown socks, 9 white socks, and 5 red socks. She picks one without looking. What is the probability that she gets a white sock?

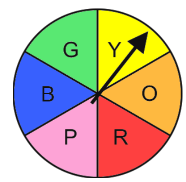
There are 9 white socks out of 21 socks in all. The probability is $9/21 = 3/7 \approx 0.42857 = 0.429 = 42.9\%$.

7. Suppose you were to draw one card from the set of cards on the right. Complete the table with the possible outcomes, and their probabilities both as fractions and as percentages (to the nearest tenth of a percent).

Possible outcomes	Probability (fraction)	Probability (percentage)



8. This "rainbow spinner" is spun once. Find the probabilities to the nearest tenth of a percent.



a. P(yellow)

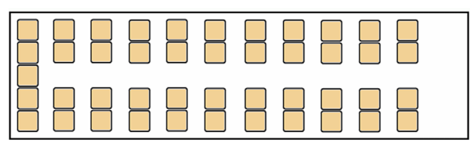
b. P(blue or green)

c. P(not orange)

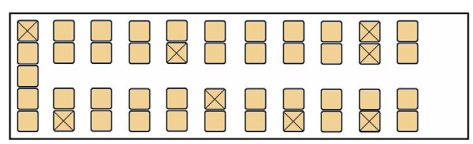
d. P(not red and not purple)

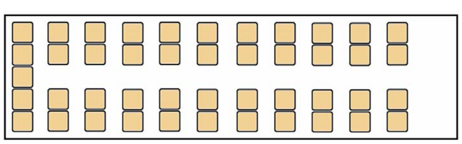
e. Make up an event for this situation with a probability of 1.

9. **a.** An empty bus has 45 seats, and 22 of them are window seats. If you are assigned a seat at random, what is the probability, to the nearest tenth of a percent, that you get a window seat?



b. Now each seat marked with an "x" is already occupied. If you choose a seat randomly, what is the probability, to the nearest tenth of a percent, that you get a window seat?





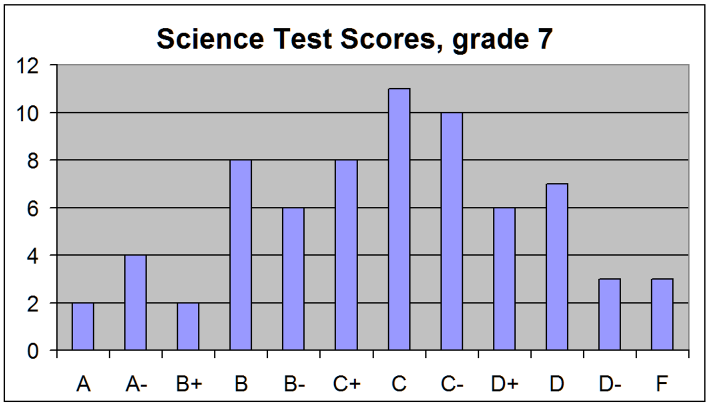
The chart shows you the seating arrangement of a bus. You enter the bus, and the driver informs you that fifteen seats are already occupied and that if you choose a seat randomly, the probability of getting a window seat is less than 25%.

How many window seats are occupied, at least?

Puzzle Corner

Probability Problems from Statistics

Example 1. The bar graph shows the science test scores of all seventy 7th graders in Westmont School. If you choose one of them at random, then what is the probability that the student's score was at least C− (in other words, C− or better)?



Sometimes when a probability question involves "at least," it is easier to look at the complement event — everything else — and find its probability first. The complement of "at least C−" is "at most D+" in other words, D+, D, D−, and F. From the graph, it is easier to sum the number of students who got the four low scores than to sum the number of students who got the eight high scores.

The number of students who got D+, D, D−, or F is 6 + 7 + 3 + 3 = 19 students. There are a total of 70 students, so P(at most D+) = 19/70. Now it's easy to calculate the original probability in question: P(at least C−) = 1 − 19/70 = 51/70.

1. You choose one student at random from the 7th graders in Westmont School.

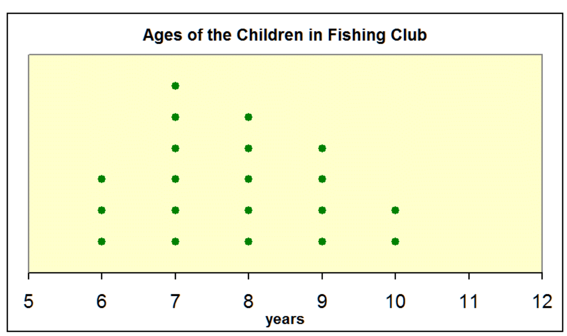
   a. What is the probability that the student's score was at least D?

   b. What is the probability that the student's score was at most B+?

2. The dotplot shows the age distribution of a children's fishing club. One child is chosen randomly from the group.

   a. What is the probability that the child is at most 9 years of age?

   b. What is the probability that the child is at least 7 years of age?



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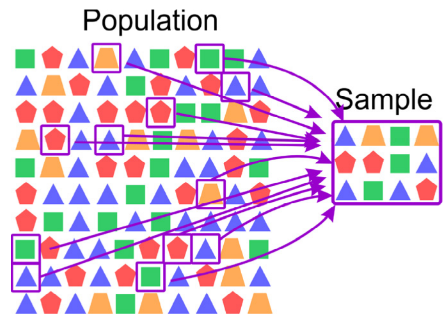
Random Sampling

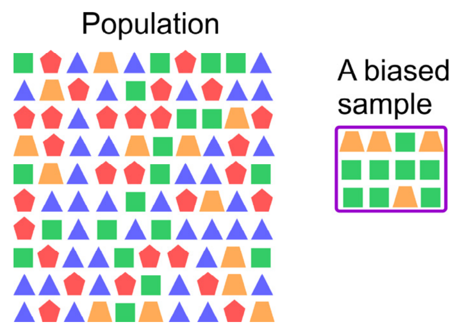
When researchers have a question concerning a large population, they obtain a **sample** (a part) of that population. That is because it is typically impossible to study the entire population.

For example, if you want to know how the citizens of France feel about climate change, you cannot just go and ask every person in France about it. You would choose for example 600 French citizens as your sample and ask them your question.



The way a sample is chosen is very important. Some methods of sampling may produce a sample that is *not* representative of the entire population. We call that a **biased sample**.

For example, if you are studying a student population of 630 in a school with close to an equal number of boys and girls, and you happen to choose a sample of 20 boys, then your sample is biased. It doesn't represent the entire population well.



We need to use **unbiased sampling methods** in order to get a sample that truly represents the population being studied. The best way to avoid biased samples is to select a **random sample.**

The main characteristics of a random sample are:

1. **Randomness:** each member of the population has an equal chance of being selected.

   Let's say a researcher is studying the types of cars Americans own. He decides to interview only people he finds at a local mall because that mall is close to where he lives, so it is convenient for him. His sample is biased because not every member of the US population even has a chance to be selected in his sample. Maybe the people at his local mall are predominantly rich people who own several cars per family, so in that respect those people would not be a good representation of the entire population of the US.

   We call this type of sample a **convenience sample** because it is convenient or easy to obtain.

2. **External selection:** respondents must be chosen by the researcher, not self-selected.

   If our researcher mails a questionnaire to various people across the US asking them to fill it out and return it, his sample is a **voluntary response sample**, which is a biased sample. Some people volunteer to return the questionnaire, but others don't. The people themselves decide whether or not to be a part of the sample.

   Why might this be a problem? Some of the people who would choose to take part may have an external reason to do so. They might want to show off how "good" they are in the particular aspect being studied, or they might just like to speak out about their opinions.

Our researcher could get a true random sample by choosing people randomly from a list of people living in the US and calling them. That way, each person has an equal chance of being selected in the sample (it is random), and the people cannot self-select to take part (the researcher chooses who takes part).

An unbiased sampling method is more likely to produce a representative sample.

1. You are studying whether students in a large college prefer to drink coffee black, with milk, with cream, or with sweetener, or whether they prefer not to drink coffee at all.

   a. Which of the six sampling methods listed below produce a voluntary response sample?

   b. Which methods don't give each member of the student population an equal chance to be selected for the sample?

   c. Which method is likely to produce a sample with only coffee drinkers, overlooking those who don't drink coffee?

   d. Which method will be the most likely to give you a representative (unbiased) sample?

   <u>Sampling Methods</u>

   (1) You interview 80 students in a cafe on the campus.

   (2) You interview 80 students who come in at the main door of the campus.

   (3) You interview the first 80 students you happen to meet on a certain day.

   (4) You choose 80 names randomly from a list of all the students. You call them to interview them.

   (5) You send an email to all the students in the college, asking them to fill in a form on a web page you have set up. You hope to get at least 80 responses.

   (6) You choose 80 names randomly from a list of all the students. You send them an email, asking them to fill in a form on a web page you have set up.

2. A recipe website posts a poll on their home page that any visitor to that website can take. In it, they ask if people are looking for a recipe for a dessert, a main dish, a side dish, bread, or salad. During the course of one Sunday, 4600 people visit the page, and 252 of them fill in the poll. Explain why the poll results will be based on a biased sample.

Some common random sampling methods are:

1. **Simple random sampling.** Each individual in the sample is chosen randomly and entirely by chance, perhaps by using dice, through pulling names out of a hat, or with a random number generator.

2. **Systematic random sampling.** The individuals of the population are placed in some order, and then each individual at a certain specified interval is selected for the sample.

   For example, a supermarket might study the shopping habits of its customers by choosing every 15th customer who enters the store for the sample.

3. **Stratified random sampling.** The population is first divided into categories (strata) and then a random sample is obtained from each category.

   For example, to study how much sleep students in a particular school get, you might first divide the students into groups by grade levels (the stratification), then select a random sample from each of the grade levels.

3\. A population to be studied doesn't have to be of people. A factory produces MP3 players. Out of the 500 units that the factory produces each day, a quality control inspector selects 25 for testing to study their quality and reliability. Which way should he choose those 25 so that his sample would best represent all the MP3 players that the factory produces?

a. Choose the first 25 produced on a given day.

b. First choose a number between 1 and 20 randomly. Select the player corresponding to that number, and after that, every 20th player, in the order they were produced that day.

c. Choose 25 players that have just been finished around 1 PM when the inspector is touring the factory.

d. Generate 25 random numbers between 1 and 500 and choose the corresponding 25 MP3 players in the order they were produced that day.

4\. Ryan has two large fields planted with green beans. He wants to compare the bean plants in one field with the plants in the other. Design a practical sampling method for him to produce an unbiased sample.

Sometimes it is not obvious how a particular sampling method might be biased.

If you are studying students' homework habits in a particular school, it might initially make sense to interview the first 25 students who come into the school in the morning. However, there could be an underlying factor that makes that method biased. What if students who are diligent with their homework also tend to come to school early? In that case, students who are not diligent don't have an equal chance of being selected for your sample. A better method is to use systematic random sampling and to choose, say, every 10th student entering the school for the sample.

5. Heather is studying the effect of how the method of feeding affects the health of a baby during its first year of life. She has already determined that babies who are fed with infant formula get sick more often than babies who are fed with human milk, but she especially wants to find out how often babies who are fed both formula and breast milk get sick. Explain why interviewing mothers in the places below will produce a biased sample:

   a. A pediatrician's office.

   b. A breastfeeding class for new moms.

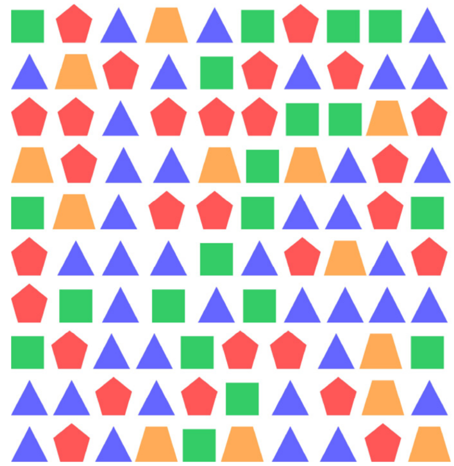
6. **a.** Devise a method that will produce a biased sample based on self-selection, and explain how that would happen, based on Heather's situation in Question 5.

   b. Design a sampling method for Heather that is most likely to produce a representative sample.

7. An organization that helps teenagers with drug problems has set up a telephone hot line for teens to call in to discuss their problems. After a few months of operations, the organization wants to evaluate the effectiveness of their service. Since they don't usually get as many calls on Tuesdays, they decide to choose a particular Tuesday to ask each teen at the end of the call to answer a few questions about how the service has helped. Is this a good method for selecting a sample? Explain.

Using Random Sampling

1. In this activity, you will make several samples of 10 from this population of shapes:



Since the shapes are in a 10 by 10 grid arrangement, you can easily assign a number from 1 to 100 for each shape. To obtain a random sample, you can use one of these ideas or come up with your own.

- Choose a random number between 1 and 10. Then, starting from that number, choose every 10th shape.
- Go to **https://www.random.org/integers/** and generate 10 random integers between 1 and 100. (If the set of numbers contains a duplicate, discard that set and make another.)

Here is an example sample (Sample 1) to help you get started:

It is based on generating these random numbers at the website above: `76 17 51 63 88 29 95 73 40 69`

Generate at least five more samples. Count the number of each kind of shape in each of your samples, and fill in the table. Lastly, calculate the average number of triangles, the average number of squares, and so on.

	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Sample 6	Averages
triangles	5						
squares	1						
pentagons	3						
trapezoids	1						
Total	10	10	10	10	10	10	—

2. Now imagine that you haven’t seen the entire population of shapes but you try to infer (conclude) something about the entire population of shapes based on these six samples.

   a. Which shape seems to be the most common?

   b. Which shape seems to be the least common?

   c. List the shapes in order, from the least common to the most common:

   d. Based on the average number of the shapes in the six samples, *estimate* how many of each shape there are in the entire population of 100 shapes:

   ______ triangles ______ squares ______ pentagons ______ trapezoids

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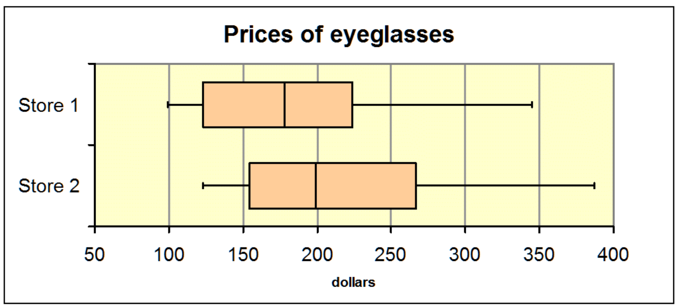
Chapter 10 Review

1. Jake practised shooting baskets on ten different days. In each practice he shot 50 times, and he recorded the number of baskets that he made. Sadly, he felt he didn't improve any.

Mon	Tue	Wed	Thu	Fri	Sun	Mon	Tue	Thu	Fri
26	33	30	34	29	27	33	27	31	35

Based on this data, estimate how many baskets Jake would make in 120 shots.

2. The boxplots below have to do with prices of eyeglasses in two different stores.



Compare the prices, including their variability, between these two stores.

3. Harry belongs to a club that helps abandoned animals. Harry would like for his club to purchase some new equipment, and he wants to find out whether the other members of the club support his idea. Harry plans to interview those club members who stay a little longer after their regular meeting to chat.

a. Explain why Harry's sampling method is biased.

b. Suggest an unbiased random sampling method for him.

4. The math teacher, Mrs. Riley, had almost 200 hundred test papers to grade and she knew she couldn't finish grading them in one evening. She decided to take a sample of 20 papers from 7th grade students and another sample of 20 papers from 8th grade students to get an idea of how well the students did.

These are the test results for the two samples:

GRADE 7: 56 59 61 64 66 68 68 73 75 75 76 77 79 79 83 84 88 90 96 97

GRADE 8: 52 54 55 58 60 62 62 63 65 68 69 70 72 72 74 77 82 85 92 99

a. Find the median, 1st and 3rd quartiles, and the interquartile range for each sample.

Grade 7: 1st quartile: __________ Median __________ 3rd quartile: __________ IQR: __________

Grade 8: 1st quartile: __________ Median __________ 3rd quartile: __________ IQR: __________

b. Make two boxplots of the results in the grid above.

c. Compare the two grade levels based on the overlap in the distributions, the measures of centre, and the measures of variability.

5. The table below shows the results of two separate surveys where students were asked who they would vote for in an upcoming school election.

	Hanley	**Johnson**	**Garcia**	**Wilson**	**Evans**	**Totals**
Survey 1	5	25	22	13	10	75
Survey 2	3	23	24	16	9	75

a. What can you infer based on these results?

b. Estimate how many of the school's 1230 students will vote for Wilson.
How far off do you expect that your estimate might be?