Contents

Chapter 4: Rational Numbers

Chapter 5: Equations and Inequalities

Foreword

Math Mammoth Grade 7, Canadian Version, comprises a complete math curriculum for the seventh grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 7, only customized for Canadian audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the seventh grade standards in your province (it will more likely match with some 7th and some 8th grade Canadian standards).

The Canadian version of Math Mammoth has been customized for Canadian audiences in these aspects:

- The curriculum uses mostly metric measurement units. Since Canadians also commonly use certain customary units, there are a few exercises that use inches.
- The spelling conforms mostly to American English, taking into account a few key differences where Canadian English follows British English.
- Large numbers are formatted with a space as the thousands separator (such as 12 394). (Decimals are formatted with a decimal point, as in the US version.)
- The pages are formatted for Letter-size paper.

The main areas of study in Math Mammoth Grade 7 are:

- The basics of algebra (expressions, linear equations and inequalities);
- The four operations with integers and with rational numbers (including negative fractions and decimals);
- Ratios, proportions, and percent;
- Geometry;
- Probability and statistics.

This book, 7-A, covers the language of algebra (chapter 1), integers (chapter 2), one-step equations (chapter 3), rational numbers (chapter 4), and equations and inequalities (chapter 5). The rest of the topics are covered in the 7-B worktext.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your student’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously.

In seventh grade, the first five chapters should be studied in order. Also, chapter 7 (Percent) should be studied before studying any of the chapters 8, 9, or 10 (Geometry, Probability, Statistics). You can be flexible with the rest of the scheduling.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your student!
- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your student. Go through a few of the first exercises together, and then assign some problems for your student to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many students can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, students definitely will vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

Each chapter introduction contains a suggested pacing guide for that chapter. You will see a summary on the right. (This summary does not include time for optional tests.)

Most lessons are 3 or 5 pages long, intended for one day. Some 5-page lessons can take two days. There are also a few optional lessons.

It can also be helpful to calculate a general guideline as to how many pages per week the student should cover in order to go through the curriculum in one school year.

Worktext 7-A	
Chapter 1	8 days
Chapter 2	13 days
Chapter 3	9 days
Chapter 4	16 days
Chapter 5	16 days
TOTAL	**62 days**

Worktext 7-B	
Chapter 6	17 days
Chapter 7	12 days
Chapter 8	23 days
Chapter 9	10 days
Chapter 10	12 days
TOTAL	**74 days**

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day (36-week) school year. The page count in the table below *includes* the optional lessons.

Example:

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A	81	9	197	72	2.74	13.7
7-B	99	10	244	89	2.74	13.7
Grade 7 total	180	19	441	161	2.73	13.7

The table below is for you to use.

Grade level	School days	Days for tests and reviews	Lesson pages	Days for the student book	Pages to study per day	Pages to study per week
7-A			197			
7-B			244			
Grade 7 total			441			

Let’s say you determine that your student needs to study about 2.5 pages a day, or 12-13 pages a week on average in order to finish the curriculum in a year. As the student studies each lesson, keep in mind that sometimes most of the page might be reserved for workspace to solve equations or other problems. You might be able to cover more than the average number of pages on such a day. Some other day you might assign the student only one page of word problems.

Now, something important. Whenever the curriculum has lots of similar practice problems (a large set of problems), feel free to **only assign 1/2 or 2/3 of those problems**. If your student gets it with less amount of exercises, then that is perfect! If not, you can always assign the rest of the problems for some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, seventh graders might spend 45-90 minutes a day on math. If your student finds math enjoyable, they can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the student's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. By then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "outside the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the MathMammoth.com website.

Chapter 1: The Language of Algebra
Introduction

In the first chapter of *Math Mammoth Grade 7* we both review basic algebra topics from sixth grade, and also go deeper into them, plus study the basic properties of the four operations. Since a good part of this chapter is review, it serves as a gentle introduction to 7th grade math, laying a foundation for the rest of the year. For example, when we study integers in the next chapter, students will once again simplify expressions, just with negative numbers. When we study equations in chapters 3 and 5, and also in subsequent grade levels, students will use the skills from this chapter (such as simplifying expressions, using the distributive property) in solving equations.

The main topics are the order of operations, writing and simplifying expressions, and the properties of the four operations, including the distributive property. Students have studied most of these in 6th grade. The main principles are explained and practised both with visual models and in abstract form, and the lessons contain varying practice problems that approach the concepts from various angles.

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. See the user guide at the beginning of this book or at **https://www.mathmammoth.com/userguides/** for some further thoughts on using and pacing the curriculum.

You can find matching videos for topics in this chapter at **https://www.mathmammoth.com/videos/** (choose grade 7).

Good Mathematical Practices

- The student is embarking on a wonderful journey into algebra — learning to do arithmetic with letters. The familiar properties of the four operations still hold, just like they do with numbers. Algebra is such a wonderful tool because it allows us to abstract a given situation and represent it symbolically, and then manipulate the representing symbols as if they have a life of their own. It is the foundational tool that allows us to model real-world situations with mathematics.

Pacing Suggestion for Chapter 1

This table does not include the chapter test as it is found in a different book (or file).
Please add one day to the pacing for the test if you use it.

The Lessons in Chapter 1	page	span	suggested pacing	your pacing
Exponents and the Order of Operations	13	*4 pages*	1 day	
Expressions and Equations	17	*3 pages*	1 day	
Properties of the Four Operations	20	*4 pages*	1 day	
Simplifying Expressions	24	*4 pages*	1 day	
Growing Patterns 1	28	*3 pages*	1 day	
The Distributive Property	31	*5 pages*	2 days	
Chapter 1 Review	36	*2 pages*	1 day	
Chapter 1 Test (optional)				
TOTALS		*25 pages*	8 days	

Games at Math Mammoth Online Practice

Hexingo Game — Order of Operations
Practise the order of operations with the four basic operations, parentheses, and exponents.
https://www.mathmammoth.com/practice/order-operations#num=3&operations=add,sub,mult,div,exponents,parens

Expression Exchange
This online activity includes THREE separate work areas where you can explore how simple algebraic expressions work, and then one game. In the work areas, you can learn how to add and subtract simple algebraic terms in order to form an expression. In the game, you will go through practice exercises, forming the asked expressions from parts.
https://www.mathmammoth.com/practice/expression-exchange

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter, including pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look! Many of our customers love using these resources to supplement the bookwork. You can use these resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr7ch1

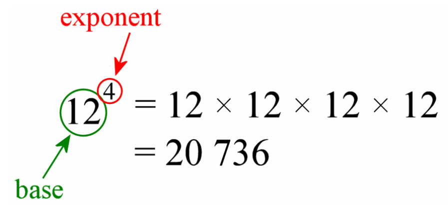
SCAN ME

Exponents and the Order of Operations

Let's review! Exponents are a shorthand for writing repeated multiplications by the same number.

For example, 0.9 · 0.9 · 0.9 · 0.9 · 0.9 is written 0.9^5.

The tiny raised number is called the **exponent**.
It tells us how many times the **base** number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 0.7^8 is read as "seven tenths to the eighth power" or "zero point seven to the eighth."

The "powers of 6" are simply expressions where 6 is raised to some power: for example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "eleven squared." That is because it gives us the <u>area of a square</u> with sides 11 units long.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 1.5^3 is read as "one point five cubed" because it is the <u>volume of a cube</u> with an edge 1.5 units long.

1. Evaluate.

a. 4^3 **b.** 10^5 **c.** 0.1^2

d. 0.2^3 **e.** 1^{100} **f.** 100 cubed

2. Write these expressions using exponents. Find their values.

a. 0 · 0 · 0 · 0 · 0 **b.** 0.9 · 0.9

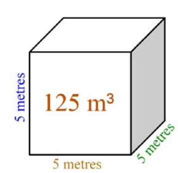
c. 5 · 5 · 5 + 2 · 2 · 2 · 2 · 2

d. 6 · 10 · 10 · 10 · 10 · 10 · 10 − 9 · 10 · 10 · 10 · 10

The expression $(5\text{ m})^3$ means that we multiply 5 metres by itself three times:

$(5\text{ m})^3 = 5\text{ m} \cdot 5\text{ m} \cdot 5\text{ m} = 125\text{ m}^3$

Notice that $(5\text{ m})^3$ is different from 5 m^3. The latter has no parentheses, so the exponent (the little 3) applies only to the unit "m" and not to the whole quantity 5 m.



3. Find the value of the expressions. Include the proper unit.

a. $(2\text{ cm})^3$ **b.** $(11\text{ m})^2$ **c.** $(1.2\text{ km})^2$ **d.** $(6\text{ cm})^2$

4. Match each of (a) and (b) with one expression on the right.

a. The volume of a cube with edges 2 cm long.

b. The volume of a cube with edges 8 cm long.

(i) 8 cm^3 **(ii)** $(8\text{ cm})^3$ **(iii)** 512 cm

The Order of Operations — PE[MD][AS]	
1) Solve what is within parentheses **(P)**. 2) Solve exponents **(E)**. 3) Solve the multiplicative operations — this includes both multiplications **(M)** and divisions **(D)** — from left to right. 4) Solve the additive operations — this includes both additions **(A)** and subtractions **(S)** — from left to right.	**Example 1**. In $15 - 2 + 3 \cdot 3$, we do $3 \cdot 3$ first, then the subtraction, and lastly the addition. You can remember PEMDAS with the silly mnemonic *Please Excuse My Dear Aunt Sally*. Or make up your own!

5. Find the value of each expression.

a. $120 - (9 - 4)^2$ **b.** $120 - 9 - 4^2$	**c.** $4 \cdot 5^2$ **d.** $(4 \cdot 5)^2$	**e.** $10 \cdot 2^3 \cdot 5^2$ **f.** $10 + 2^3 \cdot 5^2$
g. $(0.2 + 0.3)^2 \cdot (5 - 5)^4$	**h.** $0.7 \cdot (1 - 0.3)^2$	**i.** $20 + (2 \cdot 6 + 3)^2$

Example 2. Simplify $(10 - (5 - 2))^2$. Here we have double parentheses. First calculate what is within the *inner* parentheses: $5 - 2 = 3$. Then the expression becomes $(10 - 3)^2$. The rest is easy: $(10 - 3)^2 = 7^2 = 49$.	**Example 3.** Simplify $2 + \dfrac{1+5}{40-6^2}$. The fraction line works just like parentheses, as a grouping symbol, grouping both what is above the line and also what is below it. Therefore, first solve what is in the numerator and in the denominator (in either order). $2 + \dfrac{1+5}{40-6^2} = 2 + \dfrac{6}{4} = 2 + \dfrac{2}{3} = \dfrac{8}{3}$

6. Find the value of each expression.

a. $(12 - (9 - 4)) \cdot 5$	**b.** $12 - (9 - (4 + 2))$	**c.** $(10 - (8 - 5))^2$	**d.** $3 \cdot (2 - (1 - 0.4))$

7. Find the value of each expression.

a. $\dfrac{4 \cdot 5}{2} \cdot \dfrac{9}{3}$	**b.** $\dfrac{4 \cdot 5}{2} + \dfrac{9}{3}$	**c.** $\dfrac{4+5}{2} + \dfrac{9}{3-1}$

In algebra and beyond, the fraction line is preferred over the ÷ symbol, and it acts as a grouping symbol (just like parentheses).

Compare how each of these expressions looks when written either with the division symbol or with the fraction line. The latter usually makes the expressions easier to read.

$46 \div 2 + 50 \div 5$ vs. $\frac{46}{2} + \frac{50}{5}$

$48 \div (1 + 5) \cdot 3$ vs. $\frac{48}{1 + 5} \cdot 3$

Notice how only what comes directly after the ÷ symbol, whether a single number or an expression in parentheses, goes to the denominator.

Example 4. Rewrite the expression $(10 + 8) \div 4 + 3$ using the fraction line.

The denominator is just 4, not 4 + 3. The 10 + 8 will not need parentheses anymore because the fraction line in itself is a grouping symbol. So, this is written as $\frac{10 + 8}{4} + 3$.

The additions and subtractions that are done last (*not* additions and subtractions in parentheses or in the numerator/denominator) separate the expression into subsections that we call *terms*.

Example 5. This expression has four terms, separated by a +, then a −, and lastly a + sign.

$3^2 + \frac{2}{4} - \frac{30}{6 + 2} + 4 \cdot 8$

Example 6. Rewrite the expression $2 \div 4 + 3 \div (7 + 2)$ using the fraction line.

Now there are *two* divisions: the first by 4 and the second by (7 + 2), separated by an addition. This means we will use two fractions, or two terms, in the expression. It is written as $\frac{2}{4} + \frac{3}{7 + 2}$.

8. Match the expressions that are the same.

$2 \div 3 \cdot 4$ | $2 \div (3 \cdot 4)$ | $1 + 3 \div (4 + 2)$ | $1 + 3 \div 4 + 2$ | $(1 + 3) \div 4 + 2$ | $(1 + 3) \div (4 + 2)$

$1 + \frac{3}{4} + 2$ | $\frac{1 + 3}{4 + 2}$ | $\frac{2}{3} \cdot 4$ | $\frac{2}{3 \cdot 4}$ | $\frac{1 + 3}{4} + 2$ | $1 + \frac{3}{4 + 2}$

9. Rewrite each expression using the fraction line and then find its value.

a. $56 \div 7 + 6$	**b.** $7 \div (2 + 6)$	**c.** $16 \div (2 + 6) - 2$	**d.** $4 \div 5 - 1 \div 3$

To **evaluate an expression** means to find (calculate) its value.

Example 7. Evaluate the expression $x^2 - \frac{2+y}{y}$ when x is 10 and y is 3.

This means we substitute 10 for x and 3 for y in the expression and then calculate its value according to the order of operations:

$$x^2 - \frac{2+y}{y} = 10^2 - \frac{2+3}{3} = 100 - \frac{5}{3} = 98\ 1/3$$

However, in algebra and beyond, it is customary to *not* give answers as mixed numbers but as fractions, to avoid confusion. After all, 98 1/3 could easily be mistaken for 981/3. So let's go back to the expression 100 − (5/3) and simplify it so it becomes a fraction:

$$100 - \frac{5}{3} = \frac{300}{3} - \frac{5}{3} = \frac{295}{3} \quad \text{(This is the final value as a fraction.)}$$

10. Find the value of these expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $\frac{9^2}{9} \cdot 6$	**b.** $\frac{2^3}{3^2}$	**c.** $\frac{(5-3) \cdot 2}{8-1+2} + 3$

11. Evaluate the expressions. (When applicable, give your answer as a fraction, not as a decimal.)

a. $2x^2 - x$, when $x = 4$	**b.** $\frac{3s}{5} - \frac{2t}{5}$, when $s = 10$ and $t = 4$
c. $\frac{x^2}{x+1}$, when $x = 3$	**d.** $\frac{a+b}{b} + 2$, when $a = 1$ and $b = 3$

12. Why is it wrong to write the expression $2 + 5 \cdot 2 \div 4$ as $\frac{2 + 5 \cdot 2}{4}$?

Expressions and Equations

Expressions in mathematics consist of:

- numbers;
- mathematical operations (+ , − , · , ÷ , exponents);
- and letter variables, such as x, y, a, T, and so on.

Note: Expressions do *not* have an "equals" sign!

Examples of expressions: 5 $\frac{xy^4}{2}$ $T - 5 + \frac{x}{7}$

An **equation** has two expressions separated by an equals sign:

(expression 1) = (expression 2)

Examples: $0 = 0$ $2(a - 6) = b$

$9 = -8$ (a false equation) $\frac{x + 3}{2} = 1.5$

What do we do with expressions?

We can find the *value* of an expression (*evaluate* it). If the expression contains variables, we cannot find its value unless we know the value of the variables.

For example, to find the value of the expression $2x$ when x is 6/7, we simply substitute 6/7 in place of x.
We get $2x = 2 \cdot (6/7) = 12/7$.

Note: When we write $2x = 2 \cdot (6/7) = 12/7$, the equals sign is *not* signalling an equation to solve. (In fact, we already know the value of x!) It is simply used to show that the value of the expression $2x$ here is the same as the value of $2 \cdot (6/7)$, which is in turn the same as 12/7.

What do we do with equations?

If the equation has a variable (or several) in it, we can try to *solve* the equation. This means we find the values of the variable(s) that make the equation true.

For example, we can solve the equation $0.5 + x = 1.1$ for the unknown x.

The value 0.6 makes the equation true: $0.5 + 0.6 = 1.1$. We say $x = 0.6$ is the **solution** or the **root** of the equation.

1. This is a review. Write an expression.

a. $2x$ minus the sum of 40 and x.

b. The quantity 3 times x, cubed.

c. s decreased by 6

d. five times b to the fifth power

e. seven times the quantity x minus y

f. the difference of t squared and s squared

g. x less than 2 cubed

h. the quotient of 5 and y squared

i. 2 less than x to the fifth power

j. x cubed times y squared

k. the quantity $2x$ plus 1 to the fourth power

l. the quantity x minus y divided by the quantity x squared plus one

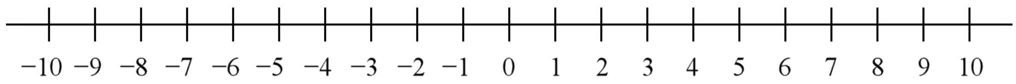
To read the expression $2(x + y)$, use the word ***quantity***: "two times the quantity x plus y."

There are other ways, as well, just not as common:

"two times the sum of x and y," or "the product of 2 and the sum x plus y."

(This page intentionally left blank.)

Integers



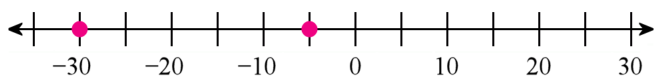
The **counting numbers** are 1, 2, 3, 4, 5, and so on. They work for addition. But counting numbers do not allow us to perform all possible subtractions; for example, the answer to the problem 2 − 7 is not any of them. That is when we come to the *negatives* of the counting numbers: −1, −2, −3, −4, −5, and so on.

Together with zero, all these form the set of **integers**: {..., −4, −3, −2, −1, 0, 1, 2, 3, 4, ...}.
Note: Zero is neither positive nor negative.

Read −1 as "negative one" and −5 as "negative five." Some people read −5 as "minus five." That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

Often, we need to put parentheses around negative numbers in order to avoid confusion with other symbols. Therefore, $^{-}5$, −5, and (−5) all mean "negative five."

Which is more, −30 or −5?



Which is *warmer*, −30°C or −5°C? Clearly −5°C is. Temperatures get colder and colder the more they move towards the negative numbers. We can write a comparison: −30°C < −5°C.

Similarly, we can write −\$500 < −\$200 to signify that to owe \$500 is a worse situation than to owe \$200.

Or, in elevation, −15 m > − 50 m means that 15 m below sea level is higher than 50 m below sea level.

1. Write comparisons using > , < , and integers. Don't forget to include the units!

   a. The temperature at 5 AM was 12°C below zero. Now, at 9 AM, it is 8°C below zero.

   b. I owe my mom \$12, and my sister owes her \$25.

   c. The bottom of the Challenger Deep trench is 11 033 m below sea level.
   Mt. Everest reaches to a height of 8848 m.

   d. The total electric charge of five electrons is −5*e*. The total electric charge of 5 protons is +5*e*.
   (The symbol *e* means elementary charge, or a charge of a single proton.)

   e. Dean has \$55, whereas Jack owes \$15.

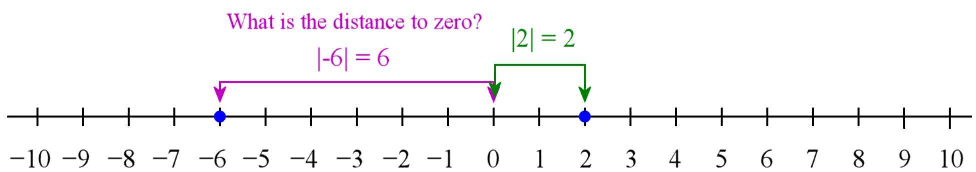
2. Which integer is...

   a. 3 more than −7 **b.** 8 more than −3 **c.** 7 less than 2 **d.** 5 less than −11

3. Write the numbers in order from the least to the greatest.

a. −5 −56 51 −15	**b.** 3 0 −31 −13

The **absolute value** of a number is its distance to zero.



We denote the absolute value of a number by putting vertical bars on either side of it.
So $|-4|$ means "the absolute value of 4," which is 4. Similarly, $|87| = 87$. In an expression we treat the absolute value bars like parentheses and solve them first.

Example 1. Simplify $|-4| - |1|$. First simplify the absolute values. We get $4 - 1 = 3$.

Let's say someone's account balance is −\$1000. That person is in debt. The absolute value of the debt is written as |−\$1000| and means that the *size* of the debt is \$1000.

If a diver is at a depth of −22 m, the absolute value |−22 m| tells us how far he is from the surface (22 m).

4. Simplify.

a. $\|-11\|$	**b.** $\|+7\|$	**c.** $\|0\|$	**d.** $\|-46\|$
e. $\|-5\| + \|-2\|$		**f.** $\|-5\| - \|2\|$	
g. $\|-5\| + \|-2\| + \|8\|$		**h.** $\|5\| + \|-2\| - \|-1\|$	

5. Answer, using the absolute value notation.

a. What is the distance between −153 and zero on a number line?

b. What is the distance between x and zero on a number line?

6. Interpret the absolute value in each situation.

a. A fishing net is at the depth of 5 metres. |−15 m| = _______ m

Here, the absolute value shows ____

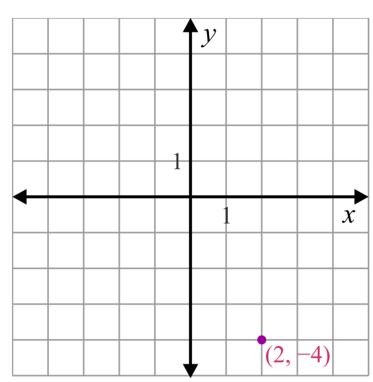
b. The temperature is −5°C. |−5°C | = _______°C

Here, the absolute value shows ____

c. Eric's balance is −\$7. |−\$7| = \$_______

The absolute value shows ______________________________________

____

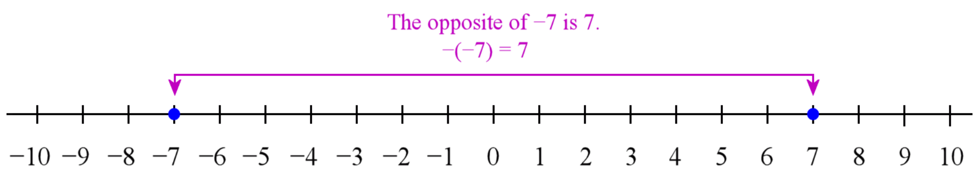


d. A point is drawn in the coordinate grid at (2, −4). |−4| = _______

Here, the absolute value shows ________________________________

____

The **opposite** of a number is the number that is on the opposite side of zero at the same distance from zero.



We denote the opposite of a number using the minus sign. For example, the opposite of 4 is written as −4. The opposite of −2 is written as −(−2), which is of course 2. So, −(−2) = 2.

The opposite of zero is zero itself. In symbols, −0 = 0.

"But wait," you might ask, "doesn't −4 mean 'negative four,' not 'the opposite of four'?"

It can mean either! Sometimes the context will help you tell which is which. Other times it isn't necessary to differentiate, because, after all, the opposite of four *is* negative four, or −4 = −4. ☺

In the expression −(4 + 5), the minus sign means the opposite of the sum 4 + 5, which equals negative nine.

Example 2. −| 7 | means the opposite of the absolute value of seven. It simplifies to −7.

Notice that there are *three* different meanings for the minus sign:

1. To indicate subtraction, as in 7 − 2.
2. To indicate negative numbers: "negative 7" is written −7.
3. To indicate the opposite of a number: −(*n* + 1) is the opposite of *n* + 1.

7. Write using symbols, and simplify if possible.

a. the opposite of 6

b. the opposite of −11

c. the opposite of the absolute value of 12

d. the absolute value of negative 12

e. the opposite of the sum 6 + 8

f. the opposite of the difference 9 − 7

g. the absolute value of the opposite of 8

h. the absolute value of the opposite of −2

8. Simplify.

a. −| 8 | **b.** −(−9) **c.** −| −7 | **d.** −0 **e.** −(−(−100))

9. Write with symbols. Use a variable for "*a number*" or "*a certain number*".

a. The absolute value of a number is equal to 6.

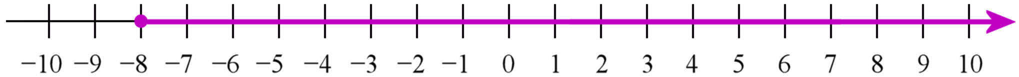
b. The opposite of a certain number is less than negative 2.

c. The absolute value of a certain number is greater than 15.

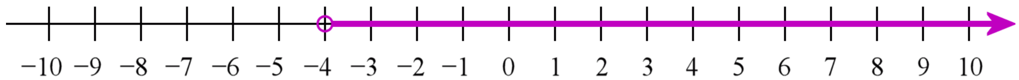
d. The opposite of *n* is equal to to the sum 56 + 5.

10. Daniel owed $5. Then he borrowed $10 more. Next, he paid off $7 of his debts. Lastly he made yet another debt of $4. Write one integer to express Daniel's money situation now.

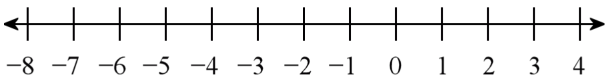
Remember **inequalities?** The number line below illustrates the inequality $x \geq -8$. Notice the arrow on the right, which shows that the ray continues to infinity. The closed circle denotes that −8 belongs to the solution set.



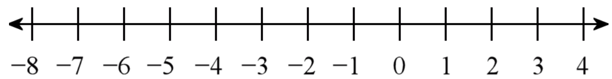
The inequality $x > -4$ is plotted on the number line below. The open circle indicates that −4 is not part of the solution set.



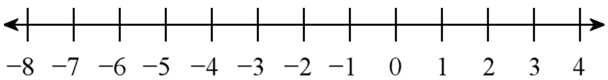
11. Plot these inequalities on the number line. Don't forget the arrow on the open end.



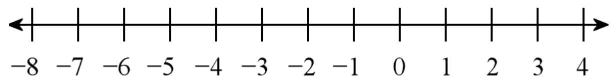
a. $x < -3$



b. $x > -1$



c. $x \geq -2$



d. $x \leq 2$

12. **a.** Solve the inequality $x < 2$ in the set {−3, −2, −1, 0, 1, 2, 3}.

b. Solve the inequality $x \geq -5$ in the set {−10, −8, −6, −2, −1, 0, 5}.

13. Write an inequality. Use negative integers where appropriate.

a. The pit is at most 10 m deep.

b. The pit is at least 12 m deep.

c. Tim's debt is no more than $500.

d. Nora owes at least $100.

e. For the skiing contest to take place, the temperature has to be warmer than 15 degrees below zero.

f. The freezer temperature should be colder than 10 degrees below zero.

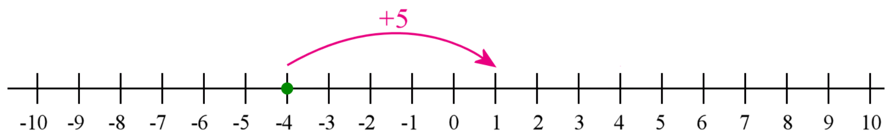
Puzzle Corner

Let a and b be two negative integers, with $b > a$.
What is the distance between them on the number line?
Write an expression.

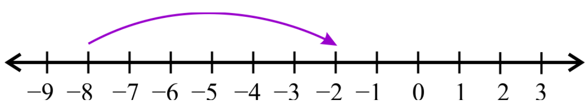
Addition and Subtraction on the Number Line 1

Addition can be modelled on the number line as a movement to the *right*.

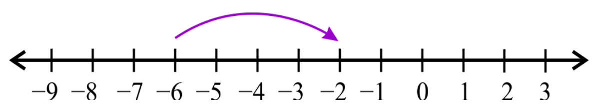
Suppose you are at −4, and you jump 5 steps to the right. You end up at 1. We write the addition −4 + 5 = 1.



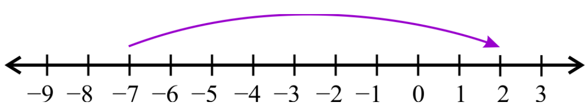
1. Write an addition equation to match each number line jump.



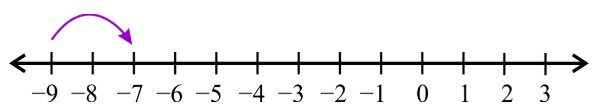
a. _______ + _______ = _______



b. _______ + _______ = _______

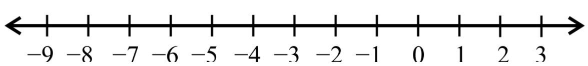


c. _______ + _______ = _______

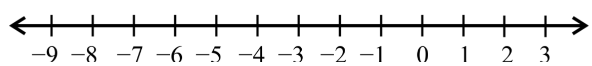


d. _______ + _______ = _______

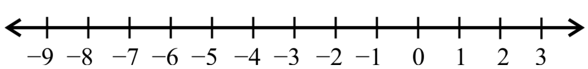
2. Draw a number line jump for each addition equation and solve.



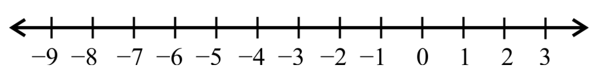
a. −8 + 3 = _______



b. −2 + 5 = _______



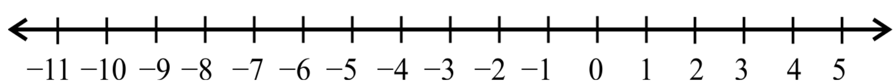
c. −4 + 4 = _______



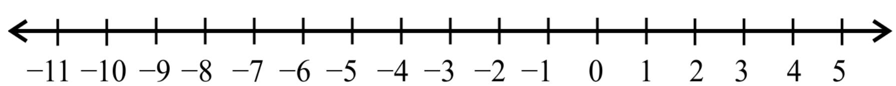
d. −10 + 12 = _______

3. What about adding more than one number? How could these additions be illustrated by number line jumps?

a. −4 + 2 + 3



b. −11 + 6 + 4



(This page intentionally left blank.)

Solving Equations

Do you remember? An **equation** has two expressions, separated by an equals sign:

(expression) = **(expression)**

To solve an equation, we can

- add the same quantity to both sides
- subtract the same quantity from both sides
- multiply both sides by the same number
- divide both sides by the same number

Notice that in any of these operations, the two expressions on the left and right sides of the equation will remain equal, even though the expressions themselves change!

Example 1. We will manipulate the simple equation 2 + 3 = 5 in these four ways. We will write in the margin the operation that is going to be done next to both sides.

Let's add six to both sides.	$2 + 3 = 5$	**+ 6**
Now, both sides equal 11. Next, we multiply both sides by 8.	$2 + 3 + 6 = 11$	**· 8**
Now, both sides equal 88. Next, we subtract 12 from both sides.	$16 + 24 + 48 = 88$	**− 12**
Now both sides equal 76. Next, we divide both sides by 2.	$16 + 24 + 48 - 12 = 76$	**÷ 2**
Now both sides equal 38.	$8 + 12 + 24 - 6 = 38$	

Of course, you do not usually work with equations like the one above, but with ones that have an unknown. Your goal is to **isolate** the unknown, or **leave it by itself,** on one side. Then the equation is solved.

We can model an equation with a **pan balance**. Both sides (pans) of the balance will have an *equal* weight in them, thus the sides are balanced (not tipped to either side).

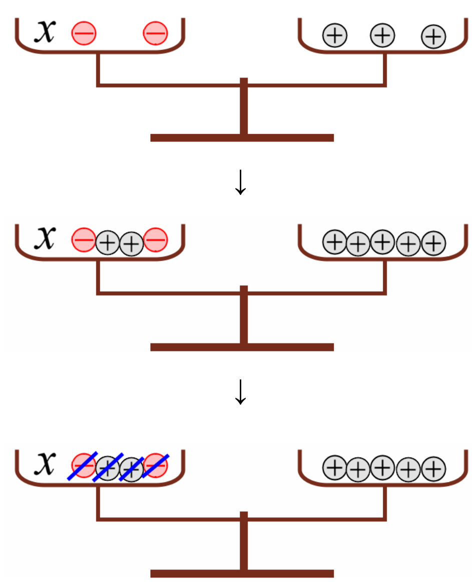
Example 2. Solve the equation $x - 2 = 3$.

We can write this equation as $x + (-2) = 3$ and model it using negative and positive counters in the balance.

Here x is accompanied by two negatives on the left side. Adding two positives *to both sides* will cancel those two negatives. We denote that by writing "**+2**" in the margin.

We write $x + (-2) + 2 = 3 + 2$ to show that 2 was added to both sides of the equation.

Now the two positives and two negatives on the left side cancel each other, and x is left by itself. On the right side we have 5, so x equals 5 positives.



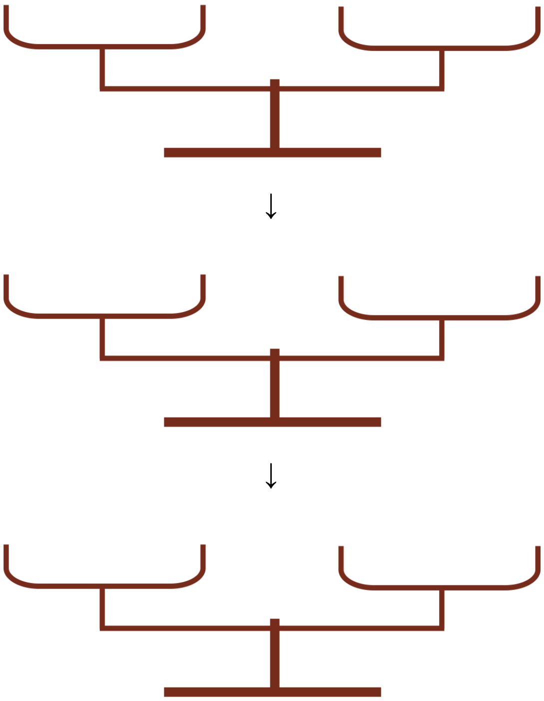
margin ↓

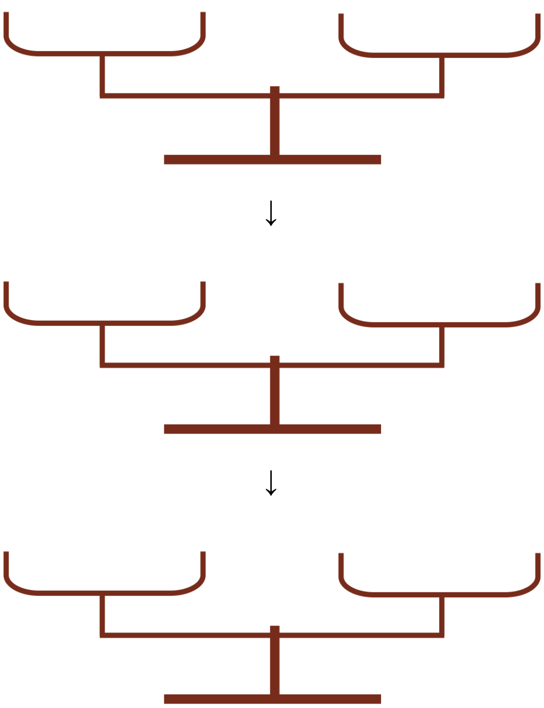
$x + (-2) = 3$ **| + 2**

$x + (-2) + 2 = 3 + 2$

$x = 5$

1. Solve the equations. Write in the margin what operation you do to both sides.

a. Balance	**Equation**	**Operation to do to both sides**
	$x + 1 = -4$	

b. Balance	**Equation**	**Operation to do to both sides**
	$x - 1 = -3$	

(This page intentionally left blank.)

4. Let's review a little! Which equation matches the situation?

a. A stuffed lion costs $8 less than a stuffed elephant.
Note: p_l signifies the price of the lion and p_e the price of the elephant.

$p_e = 8 - p_l$ | $p_e = p_l - 8$ | $p_l = p_e - 8$

b. A shirt is discounted by 1/5 of its price, and now it costs $16.

$p - 1/5 = \$16$ | $\frac{4p}{5} = \$16$ | $\frac{p}{5} = \$16$ | $p - p/5 = \$16$ | $\frac{5p}{4} = \$16$

5. Find the roots of the equation $\frac{6}{x + 1} = -3$ in the set {2, −2, 3, −3, 4, −4}.

6. Write an equation, then solve it using guess and check. Each root is between −20 and 20.

a. 7 less than x equals 5.

b. 5 minus 8 equals x plus 1

c. The quantity x minus 1 divided by 2 is equal to 4.

d. x cubed equals 8

e. −3 is equal to the quotient of 15 and y

f. Five times the quantity x plus 1 equals 10.

Addition and Subtraction Equations

You can **keep track of the operations you're using in a couple of different ways**. One way is to write the operation underneath the equation on both sides. Another is to write it in the right margin, like we did in the last lesson. But in either case, always check your solution: does it solve the original equation?	**One way:** $x + 9 = 4$ $\underline{-9} \quad \underline{-9}$ $x = -5$	**Another way:** $x + 9 = 4 \quad \vert -9$ $x + 9 - 9 = 4 - 9$ (This step is optional.) $x = -5$
	Check: $-5 + 9 \stackrel{?}{=} 4$. Yes, it checks.	

1. Solve these one-step equations. Keep track of the operations either under the equation or in the margin, whichever way your teacher prefers.

a. $x + 5 = 9$	**b.** $x + 5 = -9$
c. $x - 2 = 3$	**d.** $w - 2 = -3$
e. $z + 5 = 0$	**f.** $y - 8 = -7$

2. In these equations, first simplify, separately on the right and left sides. Look at (a) for an example.

a. $x - 4 - 3 = 2 + 8$ $x - 7 = 10$	**b.** $x - 5 - 5 = -9 + 5$
c. $2 + s + 3 = 3 + (-9)$	**d.** $t + 10 - 4 = -3 - 5$

(This page intentionally left blank.)

Rational Numbers

If you can write a number as a *ratio of two integers*, it is a **rational number.**

For example, 4.3 is a rational number because we can write it as the ratio $\frac{43}{10}$ or 43:10.

Note: To represent rational numbers, we usually indicate the ratio with a fraction line rather than a colon.

Examples of rational numbers

Since −10 can be written as $\frac{-10}{1}$, it is a rational number. It can also be written as $\frac{10}{-1}$.

Since 0.1 can be written as $\frac{1}{10}$, it is a rational number.

Since 3.24 can be written as $\frac{324}{100}$, it, too, is a rational number.

Negative fractions

The ratio of the integers 7 and −10 gives us the fraction $\frac{7}{-10}$. As we studied earlier, we usually write this as $-\frac{7}{10}$ and read it as “negative seven tenths.”

Obviously, all fractions, whether negative or positive, are rational numbers.

Negative fractions give us negative decimals.

For example, $-\frac{8}{10}$ is written as a decimal as −0.8, and $-5\frac{21}{100} = -5.21$.

You can write a rational number as a ratio of two integers in many ways.

For example, the decimal −1.4 can be written as a ratio of two integers in all these ways (and more!):

$$-1.4 = \frac{-14}{10} = \frac{-28}{20} = \frac{28}{-20} = \frac{42}{-30} = \frac{-42}{30} = \frac{-7}{5}$$

So −1.4 is *definitely* a rational number! ☺ But the same holds true for all rational numbers—you can always write them as a ratio of two integers in multitudes of ways.

1. Write these numbers as a ratio (fraction) of two integers.

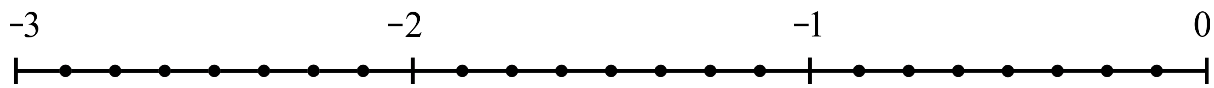
a. 6	**b.** −100	**c.** 0	**d.** 0.21
e. −1.9	**f.** −5.4	**g.** −0.56	**h.** 0.022

2. Are all percents, such as 34% or 5%, rational numbers? Justify your answer.

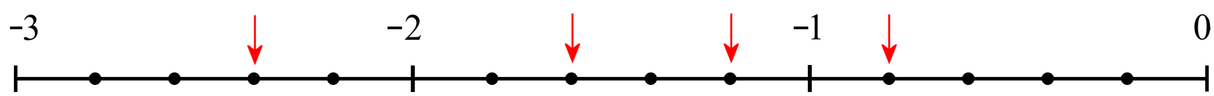
3. Form a fraction from the two given integers. Then convert it into a decimal.

a. 8 and 5	**b.** −4 and 10	**c.** 89 and −100
d. −5 and 2	**e.** 91 and −1000	**f.** −1 and −4

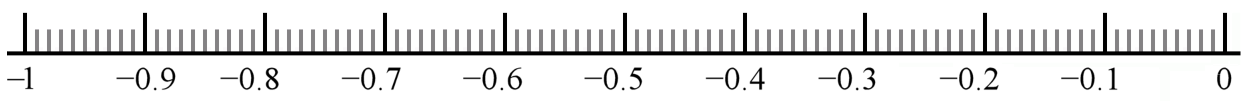
4. Mark the fractions and mixed numbers on the number line below: $-\frac{1}{2}$, $-\frac{7}{8}$, $-1\frac{5}{8}$, $-\frac{9}{4}$, $-2\frac{3}{4}$



5. Write the fractions marked by the arrows.



6. Mark the decimals on the number line: −0.11, −0.58, −0.72, −0.04



7. Sketch a number line from −3 to 0, with tick marks at every tenth. Then mark the following numbers on your number line: −0.2, −1.5, −2.8, −3/5, and −5/2.

8. Write these rational numbers as ratios of two integers (fractions) in a lot of different ways.

a. $-2 = -\frac{2}{1} =$
b. $0.6 = \frac{6}{10} =$

9. Compare, writing < or > in between the numbers.

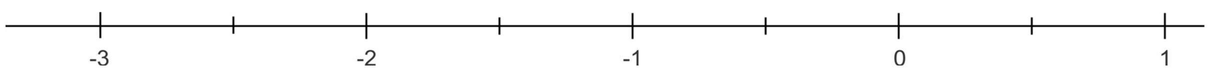
a. $-\frac{7}{8}$ $\quad$ -1	**b.** $-\frac{3}{4}$ $\quad$ $\frac{1}{2}$	**c.** $-\frac{15}{2}$ $\quad$ -7	**d.** -0.98 $\quad$ -1.4

10. Order these rational numbers in order, from the smallest to the greatest.

$2.1 \quad -\frac{1}{8} \quad -1 \quad -\frac{7}{3} \quad -2.01 \quad 1 \quad \frac{1}{3} \quad -0.5$

11. Mark the decimals *and* the fractions on the number line, approximately.

$0.3 \quad -\frac{2}{5} \quad -0.8 \quad -\frac{10}{4} \quad -2.1 \quad -1\frac{1}{2} \quad -\frac{17}{10} \quad 0.95$



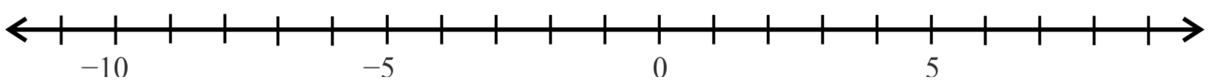
Recall that the absolute value of a number is its distance from zero.

Below, the thickened line shows the set of numbers that are greater than −3 and at the same time, less than 3. We can write it like this: the set of numbers x so that $-3 < x < 3$.

These are the numbers whose absolute value is less than 3, in other words the set of numbers for which $|x| < 3$. Their distance to zero is less than 3. For example, −2.8 and 0.492 and −6/5 belong to this set.

Note that 3 and −3 are not part of this set; that is why we use an open circle at 3 and −3.

12. **a.** Show on the number line the set of numbers x for which $|x| < 1.5$



b. List three rational numbers in that set that are not integers.

13. List three rational numbers r so that $|r| < 2$ and $r > -1$.

Repeating Decimals

As you already know, sometimes it is easy to write a fraction as a decimal. For example, 3/10 = 0.3 and 1/4 = 0.25. However, if you don't know of any other way to to find the decimal equivalent of a fraction, the technique that works all the time is to **treat the fraction as a division** and divide.

Example 1. Write $\frac{31}{40}$ as a decimal.

We will use long division. Note how we add many decimal zeros to the dividend (31) so that we can continue the division into the decimal digits.

This division **terminates** (comes out even) after just three decimal digits.

We get $\frac{31}{40}$ = 0.775. This is a **terminating decimal**.

```
      0 0.7 7 5
    ___________
40 ) 3 1.0 0 0 0
    -2 8 0
       3 0 0
    -  2 8 0
         2 0 0
       - 2 0 0
             0
```

1. Write as decimals, using long division. Continue the division until it terminates.

a. $\frac{3}{16}$

b. $\frac{51}{32}$

c. $\frac{17}{80}$

2. Use long division to write these fractions as decimals. Continue the division to at least 6 decimal digits. Notice what happens!

a. $\frac{2}{3}$

b. $\frac{7}{11}$

c. $\frac{8}{9}$

Example 2. Write $\frac{18}{11}$ as a decimal.

```
       0 1.6 3 6 3
11 ) 1 8.0 0 0 0
    -1 1
       7 0
     - 6 6
         4 0
       - 3 3
           7 0
         - 6 6
             4 0
           - 3 3
               7
```

We write 18 as 18.0000 in the long division "corner" and divide by 11. Notice how the digits "63" in the quotient and the remainders 40 and 70 start repeating.

So $\frac{18}{11}$ = 1.636363... . We can use an ellipsis (three dots, or "…") to indicate that the decimal is non-terminating. A better notation is to draw a **bar** (a line) over the digits that repeat: 1.636363… = $1.\overline{63}$.

This number is called a **repeating decimal** because the digits "63" repeat forever!

The decimal form of ANY rational number is either a terminating decimal or a repeating decimal.

This is an important fact. It says that when you write any fraction as a decimal, there are only two possibilities: either the decimal terminates or it repeats.

The converse is also true: if a decimal terminates or is a repeating decimal, it *can* be written as a fraction, thus is a rational number.

Example 3. The repeating decimal 1.9051050505… is written as $1.9051\overline{05}$. Notice that the bar marks only the digits that repeat ("05"). The digits "9051" that don't repeat are not included under the bar.
(If you're curious, as a fraction, this number is 1 886 054/990 000.)

Example 4. A calculator gives the decimal expansion of 5/13 as 0.38461538461538461538461538461538...

The repeating part is the digits "384615". So, 5/13 = $0.\overline{384615}$.

Example 5. The decimal 0.095 is a terminating decimal, but we *can* write it with an unending decimal expansion if we write zeros for all the decimal places after thousandths:

0.095 = 0.095000000000...

In other words, we can think of it as repeating the digit zero. In that sense, 0.095 = $0.095\overline{0}$.
However, as you know, we normally write terminating decimals without the extra zeros.

3. Write each decimal using a line over the repeating part.

a. 0.09090909… **b.** 5.6843434343… **c.** 0.19866666666…

4. Do it the other way around: write the repeating digits several times followed by an ellipsis (three dots).

a. $0.0\overline{887}$ **b.** $0.245\overline{6}$ **c.** $2.1\overline{7234}$

5. Which decimal is greater?

a. Which is more, $0.\overline{3}$ or 0.3? How much more?	**b.** Which is more, $0.\overline{55}$ or $0.\overline{5}$? How much more?
c. Which is more, $0.45\overline{0}$ or 0.45? How much more?	**d.** Which is more, $0.\overline{12}$ or 0.12? How much more?

(This page intentionally left blank.)

Two-Step Equations, Part 1

In two-step equations, we need to apply two different operations to both sides of the equation.

Example 1. On the side of the unknown (left), there is a multiplication by 2 and an addition of 3. To isolate the unknown, we need to undo those two operations, in two steps.

$$\begin{aligned} 3x + 2 &= 25 \quad &| -2 \\ 3x &= 23 \quad &| \div 3 \\ x &= 23/3 \end{aligned}$$

Check:

$$\begin{aligned} 3 \cdot (23/3) + 2 &\stackrel{?}{=} 25 \\ 23 + 2 &\stackrel{?}{=} 25 \\ 25 &= 25 \ \checkmark \end{aligned}$$

What if you divide first? That is possible:

$$\begin{aligned} 3x + 2 &= 25 \quad &| \div 3 \\ \frac{3x+2}{3} &= \frac{25}{3} \\ x + \frac{2}{3} &= \frac{25}{3} \quad &| -2/3 \\ x &= 23/3 \end{aligned}$$

Note that this leads to fractions in the middle of the solution process which is more error-prone. Then, the 2 on the left side also has to be divided by 3 (to become 2/3). This is something that is easy to forget and is therefore another reason why subtracting first is the "safer" way, in this case.

If this was a real-life application, we would probably give the answer as a decimal, rounded to a reasonable accuracy. Since it is a mathematical problem, we leave the answer as a fraction. (Why not as a mixed number? It is not wrong, but fractions are less likely to be misread. The mixed number 7 2/3 can easily be misread as 72/3.)

1. Solve. Check your solutions (as always!).

a. $5x + 2 = 67$	**b.** $3y - 2 = 70$	**c.** $3x + 11 = 74$
d. $8z - 2 = 98$	**e.** $75 = 12x + 3$	**f.** $55 = 4z - 11$

Example 2. This equation has decimals. The solution process works the same way. However, we give the final answer as a rounded decimal. Note that we first simplify $0.5n + 0.7n$ as $1.2n$.

$$\begin{aligned} 0.5n + 0.7n - 9.7 &= 0.45 \\ 1.2n - 9.7 &= 0.45 & \quad \mathbf{+\ 9.7} \\ 1.2n &= 10.15 & \quad \mathbf{\div\ 1.2} \\ n &\approx 8.5 \end{aligned}$$

When checking an equation with a rounded answer, we don't require precise equality. Near equality is taken as the equation checking.

$$\begin{aligned} 1.2 \cdot 8.5 - 9.7 &\stackrel{?}{=} 0.45 \\ 0.5 &\approx 0.45 \end{aligned}$$

In mathematics, the usage of a fraction typically implies that the value is precise, whereas a decimal implies that the number might be a rounded, approximate number and not precise. As you know, real-life applications often use decimals. So, in the case of an equation like this, we give the final answer rounded. In 8th grade, you will learn precise rounding rules governing these situations. For now, unless otherwise stated, round the final answer to the same accuracy as the least accurate decimal in the original equation.

2. Solve. Give the solutions to two decimal digits. Check your solutions (as always!).

a. $6.3y - 0.4 = 3$	**b.** $5.5 = 0.4y - 2.8$	**c.** $0.77s - 0.12 = 0.43$
d. $62.4 + 6x + 4x = 72.78$	**e.** $0.825 = 0.2y + 0.05y + 0.3$	**f.** $t + 1.27t - 3.12 = 3.098$

3. Check each solution on the right. If it is incorrect, find the error, and correct it.

a.

$$\begin{aligned} 10x - 14 &= 31 \\ 10x &= 17 \\ x &= 1.7 \end{aligned}$$

b.

$$\begin{aligned} 31 &= x + 15 + x \\ 31 &= 2x + 15 \\ 46 &= 2x \\ x &= 23 \end{aligned}$$