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Foreword

Math Mammoth Grade 6, Canadian Version, comprises a complete math curriculum for the sixth grade mathematics studies. This curriculum is essentially the same as the U.S. version of Math Mammoth Grade 6, only customised for Canadian audiences in a few aspects (listed below). The curriculum meets the Common Core Standards in the United States, but it may not perfectly align to the sixth grade standards in your province.

The Canadian version of Math Mammoth differs from the US version in these aspects:

- The curriculum uses metric measurement units, not both metric and customary (imperial) units.
- The spelling conforms to British international standards.
- The pages are formatted for Letter paper size.
- Large numbers are formatted with a space as the thousands separator (such as 12 394).
  (The decimals are formatted with a decimal point, as in the US version.)

This year starts out, in chapter 1 of part 6-A, with a revision of the four operations with whole numbers (including long division), place value and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practise writing expressions in different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple equations. We also briefly study inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we revise all of decimal arithmetic, just using more decimal digits than in 5th year. Students also convert measuring units in this chapter.

Ratios (chapter 4) is a new topic. Students are already familiar with finding fractional parts, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios and problem solving using bar models.

Percent (chapter 5) is an important topic, because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry and statistics.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.
- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don’t automatically assign all the exercises. Use your judgement, trying to assign just enough for your student’s needs. You can use the skipped exercises later for revision. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed revision lessons**, and the curriculum also provides you with additional **cumulative revision lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2.0	10
6-B	157	88	10	78	2.0	10
Grade 6 total	323	180	20	160	2.0	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional revision before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional revision. Then, to get a count of “pages/day”, divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and revisions	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let’s assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems (“drill”) or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional revision.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organise their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1 000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative revisions and the worksheet maker

The student books contain mixed revision lessons which revise concepts from earlier chapters. The curriculum also comes with additional cumulative revision lessons, which are just like the mixed revision lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Revisions book in the print version.

The cumulative revisions are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the revision come from. For example, "Cumulative Revision, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative revisions allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilise a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: Revision of the Basic Operations
Introduction

The goal of the first chapter in year 6 is to revise the four basic operations with whole numbers, place value and rounding, as well as to learn about exponents and problem solving.

A lot of this chapter is revision, and I hope this provides a gentle start for 6th year math. In the next chapter, we will then delve into some beginning algebra topics.

Special notes for this chapter: problem solving

This chapter doesn't have many new concepts – only the concept of exponents and powers. Besides revising how to perform the four basic operations with pencil and paper, students also get some practice for problem solving.

Solving (word) problems in math works much the same way as solving problems in real life. You may start out one way, come to a "dead end", and have to take an entirely different approach. Good problem solvers monitor their progress as they work, and change course if necessary.

Here is a list of general tips and strategies for solving mathematical problems that you can share with your student(s).

- If you cannot solve the original problem, try to **solve an easier, related problem first**. This may help you find a way to solve the original. For example, if the numbers in the problem seem intimidating, change them temporarily to really easy numbers and see if you can solve the problem then. Or reduce the details mentioned in the problem to make it simpler, solve the simpler problem, then go back to the original. You can also try special cases of the problem at hand at first.
- Drawing a sketch, a diagram (e.g. a bar model), or making a table can be very helpful.
- **Check your final answer** if at all possible, using a different method. For example, division problems can be checked by multiplication and subtractions by addition. Multi-step problems can often be solved in different ways or in a different order.

  At the very least, **check that your answer is reasonable** and actually makes sense. If the problem is asking how many days of vacation someone might get in a year, and you get an answer in the thousands, you can tell something went really wrong.

  And, once you find your answer is wrong – maybe it doesn't make sense – it is NOT time to cry and give up. Do you know how many times Thomas Alva Edison tried and failed, until he finally found a way to make a commercially viable electric light bulb? Thousands of times.

  Perseverance is something that is very necessary when you encounter problems in real life, and I don't mean math problems. Everyone fails, but it is those who keep trying who will ultimately succeed. Every successful entrepreneur can tell you that. And, failing is *not* a sign of being stupid. It is a sign of being a human. ALL of us make mistakes and fail. And ALL of us improve as we keep trying.
- Often, it is easier and neater to perform paper-and-pen calculations (long addition, subtraction, multiplication, division) on a grid paper.
- The space in the worktext may not be enough. Use as much scrap paper (extra paper) as necessary.
- Remember to include a unit (if applicable) in the answers to word problems.

General principles in using the curriculum

Please note that it is not recommended to assign all the exercises by default. Use your judgement, and strive to vary the number of assigned exercises according to the student's needs.

The specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about two pages daily or 10 pages a week in order to finish the curriculum in about 36 weeks.

See the user guide at in the beginning of this book or online at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

The Lessons in Chapter 1

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch1

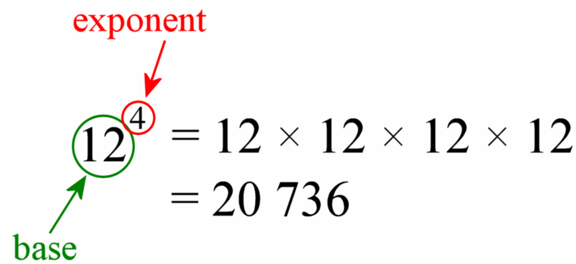
SCAN ME

Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{\ 3 \times 3 = 9\ }$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

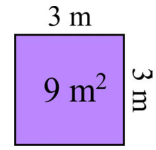
b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed



You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square metres and other units of area.

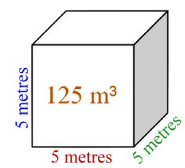
If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square metres.

Notice that the symbol for square metres is $\mathbf{m^2}$. This means "**metre × metre**." We are, in effect, squaring the unit *metre* (multiplying the unit of length *metre* by itself)!

The expression $(9\text{ cm})^2$ means 9 cm × 9 cm. We multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 $\mathbf{cm^2}$**. The square centimetre (cm^2) comes from multiplying "**centimetre × centimetre**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometres or square millimetres.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *metre* by itself three times (metre × metre × metre) to get the unit of volume "cubic metre" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometres: A = 12 km × 12 km = ______________	**b.** A square with sides 6 m long: A = ______________________________
c. A square with a side length of 6 centimetres: A = ______________________________	**d.** A square with a side with a length of 12 cm: A = ______________________________

4. Express the volume (V) as a multiplication, and solve.

a. A cube with an edge of 2 cm: V = 2 cm × 2 cm × 2 cm = ______________	**b.** A cube with edges 10 cm long each: V = ______________________________
c. A cube with edges 1 m in length: V = ______________________________	**d.** A cube with edges that are all 5 m long: V = ______________________________

5. **a.** The perimeter of a square is 40 centimetres. What is its area?

b. The volume of a cube is 64 cubic centimetres. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is the length of one edge?

The powers of 10 are very special —and very easy! Notice that the exponent tells us *how many zeros* there are in the answer.	$10^1 = 10$ $10^2 = 10 \times 10 = 100$ $10^3 = 10 \times 10 \times 10 = 1\,000$	$10^4 = 10\,000$ $10^5 = 100\,000$ $10^6 = 1\,000\,000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2\,187$, how can you use that result to find 3^8?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35\,184\,372\,088\,832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____________ of a _______________ with sides ______ units long.

b. 101^3 gives us the _____________ of a _______________ with edges ______ units long.

c. 2×6^2 gives us the ____________ of two _______________ with sides ______ units long.

d. 4×10^3 gives us the ____________ of ____ _______________ with edges ______ units long.

Puzzle Corner

Make a pattern, called a **sequence**, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, n. What value do you calculate for n^0?
Do you think it will come out this way for every base number?

Why or why not?

Place Value

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
2	**0**	**9**	**3**	**5**	**6**	**0**	**7**	**5**	**8**	**5**	**5**	**4**	**0**	**2**
trillions period			billions period			millions period			thousands period			ones period		

The letters "h t o" stand for hundreds, tens, ones.

Read this number as:

"Two hundred and nine trillion, three hundred and fifty-six billion, seventy-five million, eight hundred and fifty-five thousand, four hundred and two."

To write this number in its *expanded form*, take each digit's value, and write them all as a sum:

200 000 000 000 000 + 9 000 000 000 000 + 300 000 000 000 + 50 000 000 000 + 6 000 000 000
+ 70 000 000 + 5 000 000 + 800 000 + 50 000 + 5 000 + 400 + 2

This is easier to write using exponents:

$2 \times 10^{14} + 9 \times 10^{12} + 3 \times 10^{11} + 5 \times 10^{10} + 6 \times 10^{9}$

$+ 7 \times 10^{7} + 5 \times 10^{6} + 8 \times 10^{5} + 5 \times 10^{4} + 5 \times 10^{3} + 4 \times 10^{2} + 2 \times 10^{0}$

Remember that in powers of 10, the exponent tells you how many zeros are in the number.
For example, 10^{11} = 100 000 000 000 has eleven zeroes.
Notice especially: $\mathbf{10^0 = 1}$ (the number 1 has no zeros!).

The number system we use is based on *place value*. This means that a digit's *value* depends on its position or *place* within the number.

Our number system is called a *decimal*, or *base-ten*, system (from the Latin word `decima`, *a tenth part*). The value of each position or place is *one-tenth* of the value of the previous place.

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
0	**0**	**0**	**6**	**3**	**0**	**9**	**5**	**7**	**8**	**1**	**2**	**4**	**9**	**8**
trillions period			billions period			millions period			thousands period			ones period		

The digit "6" is in the hundred billions place.
Its value is 6 × a hundred billion, or 600 billion.

The digit "5" is in the ten millions place.
Its value is 5 × ten million, or 50 million.

1. Write the numbers in the place value chart. Answer the questions.

a. 89 million, 2 thousand, 4 hundred

What is the value of the digit "9"? ____________________

trillions period			billions period			millions period			thousands period			ones period		

b. 142 billion, 2 million, 139 thousand

What is the value of the digit "3"? ____________________

trillions period			billions period			millions period			thousands period			ones period		

c. 5 trillion, 47 million, 260

What is the value of the digit "4"? ____________________

trillions period			billions period			millions period			thousands period			ones period		

Chapter 2: Expressions and Equations
Introduction

In this chapter students start learning *algebra* – in a nutshell, the way to "do arithmetic with variables". Algebra enables us to solve real-life problems abstractly, in terms of variable(s) instead of numbers, and it is a very powerful tool.

Special notes for this chapter: algebra

The chapter focuses on two important basic concepts: **expressions** and **equations**. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). Integers are introduced in part 6-B, and then Math Mammoth grade 7 deals with algebraic concepts including with negative numbers.

We start out by revising the order of operations. Then the lessons focus on algebraic expressions. Students encounter the exact definition of an expression, a variable, and a formula, and practise writing expressions in many different ways. They study equivalent expressions and simplifying expressions. Length and area are two simple contexts I have used extensively for students to learn to write and simplify expressions.

In these lessons, students have opportunities to **write real-world scenarios in terms of variables**. In other words, they *decontextualise* – they abstract a given situation and represent it symbolically. Then, as they learn algebra, they learn to manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and to reason abstractly about those quantities represented by the variables.

The other major topic of the chapter is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few easy two-step equations.

Next, students solve and graph simple inequalities, again practising the usage of variables to represent quantities. Lastly, they are introduced to the usage of *two* variables in algebra, including how to graph that relationship on a coordinate plane. This is an important topic, as so many real-life situations involve a relationship between two quantities, and graphing that relationship is an important tool in mathematical modelling.

You will find free videos covering many topics of this chapter of the curriculum at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 2

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch2

SCAN ME

The Order of Operations

<u>The Order of Operations (BEMDAS)</u>

1) Solve what is within the **brackets (B)**.

2) Solve **exponents (E)**.

3) Solve **multiplication (M)** and **division (D)** from left to right.

4) Solve **addition (A)** and **subtraction (S)** from left to right.

<u>Note:</u> From now on, we will use the raised dot • for the multiplication symbol. This is because we will be studying algebra, and × can be confused with the letter *x*, often used in algebra.

So, for example, we will write $5 \cdot 2$ to signify five times two.

Example 1. Solve $200 - (10 - 4 + 5)^2$.

1. Solve what is within the brackets: $10 - 4 + 5$. Since subtractions and additions are on the same level, solve them from left to right: $10 - 4 + 5 = 11$. The expression is now $200 - 11^2$.
2. Next, solve the exponent: $11^2 = 121$. The expression is now $200 - 121$.
3. Lastly, subtract. $200 - 121 = 79$.

Example 2. $\dfrac{10 + 50}{12 - 6}$.

This expression is *not* the same as $10 + 50 \div 12 - 6$. Instead, the fraction line works as a grouping symbol, grouping together what is above and below the line, so that the division is to be done *last*. The expression is actually $(10 + 50) \div (12 - 6)$.

First, solve the expressions above and below the line (as if they were grouped using brackets), and lastly divide:

$$\frac{10 + 50}{12 - 6} = \frac{60}{6} = 10$$

Example 3a. Here is an expression that has only *multiplications* and *divisions*: $20 \cdot 2 \div 4 \cdot 10$.

Those operations are on the SAME level in the order of operations, but that does *not* mean that multiplications are solved before divisions. Instead, they are solved in order from left to right.

$$20 \cdot 2 \div 4 \cdot 10$$
$$= 40 \div 4 \cdot 10$$
$$= 10 \cdot 10 = 100$$

Example 3b. Let's rewrite the expression from 3a. using the fraction line for division—it will become easier!

Notice, there is a division by 4:

$$20 \cdot 2 \div 4 \cdot 10$$

This means that 4 needs to be in the denominator.

The expression can be written as $20 \cdot \dfrac{2}{4} \cdot 10$ or

as $\dfrac{20 \cdot 2}{4} \cdot 10$ (either is correct).

Comparing to the original expression $20 \cdot 4 \div 4 \cdot 10$, it looks quite different, but it is now easier to see what needs done. Verify that you get the same answer as in example 3a.

1. Put brackets into the equations to make them true.

a. $100 - 50 - 50 = 100$ **b.** $200 \div 10 + 10 + 5 = 15$ **c.** $50 + 50 \cdot 4 - 10 = 390$

2. Rewrite each expression using the fraction line, then solve. Compare each expression in the top row of boxes to the one below it. *Hint: Only whatever comes right after the ÷ sign needs to be in the denominator.*

a. $64 \div 8 \cdot 4$	**b.** $64 \div (8 \cdot 4) \cdot 2$	**c.** $4 \cdot 8 \div 4 \cdot 2$
d. $64 \div (8 \cdot 4)$	**e.** $64 \div 8 \cdot 4 \cdot 2$	**f.** $(4 \cdot 8) \div (4 \cdot 2)$

3. Find the value of these expressions.

a. $150 + 2 \cdot 10$	**b.** $5^2 \cdot 2^3$	**c.** $3^2 \cdot (150 + 900) \div 3$
d. $\dfrac{12 + 9}{4 + 1}$	**e.** $\dfrac{5^2}{3^2}$	**f.** $\dfrac{2^3}{8} + 10^3$
g. $(6 + 6)^2 \cdot (15 - 5)^2$	**h.** $40 + 80 \div 2 \cdot 4 - 15$	**i.** $\dfrac{7^2}{7} \cdot 7$

4. Write the expressions in a shorter way, using multiplication. Find their values.

a. $20\,000 - 500 - 500 - 500 - 500 - 500 - 500 - 500$

b. $70 + 70 + 70 + 70 + 70 + 70 + 120 + 120 + 120 + 120 + 120$

5. Write the expressions in a shorter way, using exponents. Find their values.

a. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 + 5 \cdot 5 \cdot 5$

b. $5 \cdot 100 \cdot 100 \cdot 100 + 2 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Expressions, Part 1

Expressions in mathematics consist of:	**Examples of expressions:**
• numbers; • mathematical operations (+ , − , · , ÷ , exponents); • letters, such as *x*, *y*, *a*, T and so on. These letters signify numbers whose value might *vary*. They are called *variables*.	$5 + 6$ $\frac{bh}{2}$ $12 \cdot 9 - 7 \cdot 5$ $2^4 - x$ $\frac{x + y}{2}$ $T - 5$

Note 1. Expressions do *not* have an "equals" sign (=)! (It is *equations* that do.)

Note 2. In algebra, the multiplication sign · is omitted between two variables and between a number and a variable. So, *bh* means *b* times *h*, and $9t^2$ means 9 times t^2.

What do we do with expressions?

One main thing is that we can find the *value* of an expression by calculating it. This is also called *evaluating the expression*. For example, the value of 5 + 6 is 11. The value of 12 · 9 − 7 · 5 is 73.

If the expression contains a variable, such as T − 5, then we cannot find its value unless we know the value of the variable. For example, if T is 12, then the expression T − 5 has the value 12 − 5 = __7__.

Example 1. Evaluate the expression $x^4 - y$ when *x* has the value 2, and *y* has the value 7.

Simply write "2" in place of *x* and "7" in place of *y*, and calculate:

$$2^4 - 7 = 16 - 7 = \underline{9}$$

Example 2. Evaluate 24 − 6*p* when *p* has the value 3.

Here, 6*p* signifies 6 times *p*. The multiplication sign is omitted between a number and a variable. We substitute 3 in place of *p*, and get:

$$24 - 6 \cdot 3 = 24 - 18 = \underline{6}$$

1. Evaluate the expressions when the value of the variable is given.

a. $7z$ when $z = 3$	**b.** $5x^2$ when $x = 2$
c. $2x + 18$ when $x = 5$	**d.** $\frac{35}{z} \cdot 13$ when $z = 5$
e. mn^2 when $m = 5$ and $n = 3$	**f.** $\frac{3}{5}s$ when $s = 25$
g. $\frac{x^4}{x^2}$, when $x = 10$	**h.** $\frac{1}{9}y - 4$ when $y = 81$

Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system and using decimals with measuring units. Most of these topics have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, consider assigning only one-fourth to one-half of the exercises initially. Monitor the student's progress and assign more if needed. The skipped problems can be used for revision later.

We start out by studying place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of revision, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

Since the chapter focuses on restudying the mechanics of decimal arithmetic, it is a good time to stress to your student(s) the need for **accurate calculations** and for **checking one's final answer**. Notice how the lessons often ask students to estimate the answer before calculating the exact answer. Estimation can be used as a type of check for the final answer: if the final answer is far from the estimation, there is probably an error in the calculation. It can also be used to check if an answer calculated with a calculator is likely correct.

In the lessons about multiplication and division of decimals, students work both with mental math and with standard algorithms. The lessons that focus on mental math point out various patterns and shortcuts for students, helping them to see the structure and logic in math. I have also explained why the common rules (or shortcuts) for decimal multiplication and decimal division actually work, essentially providing a mathematical proof on a level that 6th graders can hopefully understand.

The last lessons deal with measuring units and the metric system, rounding out our study of decimals.

Consider mixing the lessons here with lessons from some other chapter. For example, the student could study decimals and some other topic on alternate days, or study a little of each topic each day. Such somewhat spiral usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

As a reminder, check out this list of resources for challenging problems:
https://l.mathmammoth.com/challengingproblems

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 3

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch3

SCAN ME

Place Value with Decimals

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
4	**5**	**7**	**3.**	**9**	**1**	**6**	**0**	**7**	**2**

The various places on the place value chart are positioned symmetrically around the ONES place.

From the ones place, we have tens to the left, and tenths to the right. Two places away are hundreds to the left, and hundredths to the right. Three places away are thousands to the left, and thousandths to the right and so on.

In expanded form:

$$4 \cdot 1\,000 + 5 \cdot 100 + 7 \cdot 10 + 3 \cdot 1 + 9 \cdot \frac{1}{10} + 1 \cdot \frac{1}{100} + 6 \cdot \frac{1}{1000} + 7 \cdot \frac{1}{100000} + 2 \cdot \frac{1}{1000000}$$

Using decimals: 4 000 + 500 + 70 + 3 + 0.9 + 0.01 + 0.006 + 0.00007 + 0.000002

Example 1.

6 hundred-thousandths is $\frac{6}{100\,000}$ or 0.00006.

It has five decimal places, the same as one hundred thousand (100 000) has five zeros.

Example 2.

123 ten-thousandths is $\frac{123}{10\,000}$ or 0.0123.

There are four decimal places, the same as ten thousand (10 000) has four zeros.

Example 3. 7 millionths is 0.000007. It has six decimal places, the same as one million has six zeros.

0.000007 *also* happens to have six zeros, if you count the zero in the ones place. However, think of it as having six *decimal places*, instead, because that allows you to easily convert, for example, 453 millionths or 6 795 millionths into decimals: 0.000453 and 0.006795. They do not have six zeros, but they *do* have six decimal places.

Example 4. 465 hundredths is $\frac{465}{100}$.

As a decimal, it needs to have two decimal places because it is so many hundredths. (You can remember that because 100 has two zeros.) So it is 4.65.

Example 5.

2 180 964 ten-thousandths is $\frac{2\,180\,964}{10\,000}$.

As a decimal, it needs to have four decimal places because it is so many ten-thousandths (and 10 000 has four zeros). So it is 218.0964.

1. Draw lines to match the expressions that have the same value.

0.00006	6 parts per thousand	$\frac{6}{100\,000}$	$\frac{15}{100\,000}$
0.0015	15 hundred-thousandths	$\frac{6}{1000}$	$\frac{15}{1000}$
0.000006	15 ten-thousandths	$\frac{6}{100\,000}$	$\frac{15}{10\,000}$
0.00015	6 hundred-thousandths		$\frac{15}{1\,000\,000}$
0.006	15 parts per million		
0.000015	15 thousandths		
0.015	6 millionths		

2. Write as decimals.

a. three thousandths

b. 34 tenths

c. 1 and 1934 millionths

d. 34 ten-thousandths

e. 907 millionths

f. 837 hundred-thousandths

g. 52 hundredths

h. 8 hundred-thousandths

i. 3 and 17 thousandths

j. 91 millionths

k. 1 and 56 thousandths

l. 2 and 28 319 millionths

m. 291 ten-thousandths

n. 4 and 5 millionths

3. Write as pure fractions, *not* as mixed numbers—that is, the numerator (the number on the top) can be greater than the denominator (the number on the bottom).

a. 0.09

b. 0.005

c. 0.045

d. 0.00371

e. 0.02381

f. 3.0078

g. 2.9302

h. 2.003814

i. 5.3925012

j. 0.0000031

k. 3.294819

l. 45.00032

4. Write in expanded form, as a sum of fractions. Follow the example.

a. $2.67 = 2 \cdot 1 + 6 \cdot \frac{1}{10} + 7 \cdot \frac{1}{100}$

b. 0.594

c. 45.6

d. 0.004923

e. 0.00000506

5. Write as decimals.

a. $60 + 5 + \frac{2}{10} + \frac{8}{100} + \frac{6}{1000}$	**b.** $5 + \frac{5}{100} + \frac{5}{1000} + \frac{9}{1\,000\,000}$
c. $700 + \frac{1}{1000} + \frac{3}{100\,000} + \frac{7}{100}$	**d.** $\frac{1}{100} + \frac{3}{10\,000} + \frac{4}{1\,000\,000}$
e. $\frac{9}{100} + 6 + \frac{3}{10\,000} + \frac{5}{10}$	**f.** $\frac{2}{100} + 2 + \frac{1}{1000} + \frac{1}{100\,000}$

Comparing Decimals

Compare decimal numbers place by place (tenths with tenths, hundredths with hundredths, *etc*.), starting from the *biggest* place. A place value chart can help with this.

	0	.	0	0	3	8	0	5
	0	.	0	0	0	5	1	2
T	O		te	hu	th	t-th	h-th	mi

"T" means tens.
"O" means ones.
"te" means tenths.
"hu" means hundredths.
"th" means thousandths.
"t-th" means ten-thousandths.
"h-th" means hundred-thousandths.
"mi" means millionths.

This place-by-place comparison shows that 0.003805 > 0.000512. It has 3 thousandths as its largest place value, while the other number has no thousandths.

Here is a slick trick! If you make the decimals have the same amount of decimal digits by placing zeros on the end, you can simply look at the decimal parts and compare them the same as "apples to apples" (provided of course that the whole-number parts are equal).

Example. Which is more, 6.00198 or 6.003?

Place zeros on the end of 6.003 so that it has five decimal digits: 6.003 becomes 6.003**00**. Now we compare 6.00198 to 6.00300. Since the whole-number parts are equal, and since the decimal parts are both in hundred-thousandths, you can compare the decimal parts as "numbers" in themselves:

One number has 6 and 198 hundred-thousandths, and the other one has 6 and 300 hundred-thousandths. Clearly, 300 hundred-thousandths is more than 198 hundred-thousandths, so 6.00300 is more than 6.00198.

1. Compare the numbers and write < , = , or >. You can use the place value charts to help.

a. 0.067 ☐ 0.0098

	.						
	.						
O		te	hu	th	t-th	h-th	mi

b. 0.0005 ☐ 0.005

	.						
	.						
O		te	hu	th	t-th	h-th	mi

c. 1.828 ☐ 1.0828

	.						
	.						
O		te	hu	th	t-th	h-th	mi

d. 2.504040 ☐ 2.505404

	.						
	.						
O		te	hu	th	t-th	h-th	mi

e. 8.00014 ☐ 8.004

	.						
	.						
O		te	hu	th	t-th	h-th	mi

f. 0.91701 ☐ 0.917005

	.						
	.						
O		te	hu	th	t-th	h-th	mi

2. Underline the greatest number.
Use the place value charts to help.

	.						
	.						
O		te	hu	th	t-th	h-th	mi

	.						
	.						
O		te	hu	th	t-th	h-th	mi

a. 0.05 0.009 0.1	**b.** 1.04 1.2013 1.1	**c.** 0.905 0.86948 0.9
d. 0.0004 0.0000337	**e.** 9.082 9.1 9.09	**f.** 0.288391 0.284857

Chapter 4: Ratios
Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate and unit rate. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Lastly, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height or length, and they solve a variety of problems involving aspect ratio.

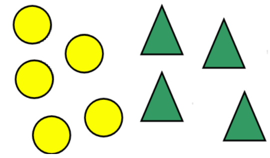
This chapter contains lots of opportunities for problem solving, once again. In the lessons that use bar models, encourage your student(s) to communicate their thinking and explain (justify) how they solved the problems. It doesn't have to be fancy. All we are looking for is some explanation of what the student did and why. The bar models provide an excellent way for the students to demonstrate their reasoning here. Essentially, they are practising constructing a **mathematical argument**.

Once again, there are some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 4

Ratios and Rates

A **ratio** is simply a *comparison* of two numbers or other quantities.

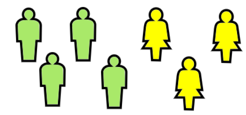


To compare the circles to the triangles in the picture, we say that the *ratio of circles to triangles* is 5:4 (read "five to four").

We can write this ratio (in text) in many different ways:

- The ratio of circles to triangles is 5:4 (read "5 to 4").
- The ratio of circles to triangles is 5 to 4.
- The ratio of circles to triangles is $\frac{5}{4}$.
- For each five circles, there are four triangles.

The two numbers in the ratio are called the **first term** and the **second term** of the ratio.



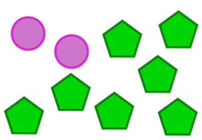
In this picture, the ratio of males to females is 4:3. However, the ratio of *females to males* is **3:4**. The order in which the terms are mentioned does matter!

We can also compare a part to the whole. The ratio of males to everyone is 4:7.

Also, we can use fractions to describe the same image: $\frac{4}{7}$ of the people are males, and $\frac{3}{7}$ are females.

1. Describe the images using ratios and fractions.

a.



The ratio of circles to pentagons is ____ : ____

The ratio of pentagons to all shapes is ____ : ____

$\frac{\quad}{\quad}$ of the shapes are pentagons.

b.

The ratio of hearts to stars is ____ : ____

The ratio of stars to all shapes is ____ : ____

$\frac{\quad}{\quad}$ of the shapes are stars.

2. **a.** Draw a picture: There are hearts and circles, and the ratio of hearts to all the shapes is 1:3.

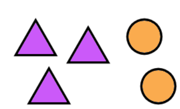
   b. What is the ratio of hearts to circles?

3. Look at the picture of the triangles and circles. If we drew more triangles and circles in the same ratio, how many circles would there be …

   a. … for 9 triangles?

   b. … for 15 triangles?

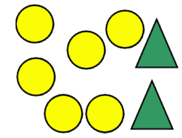
   c. … for 300 triangles?



We can **simplify** ratios in exactly the same way we simplify fractions (using division).

The ratio of circles to triangles is $\frac{6}{2} = \frac{3}{1}$. We say that 6:2 and 3:1 are **equivalent ratios**.

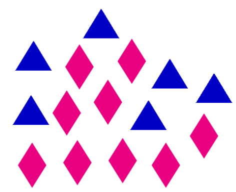
The simplified ratio 3:1 means that for each three circles, there is one triangle.



Example 1. When we simplify the ratio of hearts to stars to the *lowest terms*, we get $\frac{12}{16} = \frac{3}{4}$, or 12:16 = 3:4. This means that for each three hearts, there are four stars.

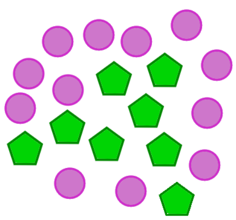
We could also simplify like this: 12:16 = 6:8. These two ratios are equivalent, but neither is simplified to the lowest terms.

4. Write the ratios, and then simplify them to lowest terms.



a. The ratio of diamonds to triangles is _____ : _____ or _____ : _____ .

There are ______ diamonds to every _______ triangles.



b. The ratio of pentagons to circles is _____ : _____

or _____ : _____ .

There are ______ circles to every _______ pentagons.

5. **a.** Draw a picture in which (1) there are five squares for each two hearts, and (2) there is a total of six hearts.

b. Write the ratio of all hearts to all squares, and simplify this ratio to lowest terms.

c. Write the ratio of all shapes to hearts, and simplify this to lowest terms.

6. Write the ratios using a colon. Simplify the ratios if possible.

a. 15 to 20

b. 16 to 4

c. 25 to 10

d. 13 to 30

7. Write the equivalent ratios. Think about equivalent fractions.

a. $\frac{5}{2} = \frac{20}{\square}$	**b.** 3 : 4 = 9 : _____	**c.** 16 : 18 = ____ : 9	**d.** $\frac{5}{1} = \frac{\square}{4}$

e. 2 to 100 = 1 to ________	**f.** _____ to 40 = 3 to 5	**g.** 5 : ____ = 1 to 20

We can also form ratios using quantities with units. For example, in the ratio 5 km : 8 km, both terms contain the unit "km". We can then simplify the ratio, cancelling the units "km": $\frac{5 \text{ km}}{8 \text{ km}} = \frac{5}{8}$.

8. Write ratios of the given quantities. Use the fraction line to write the ratios. Then, simplify the ratios. In most of the problems, you will need to *convert* one quantity so it has the same measuring unit as the other.

a. 2 kg and 400 g $\frac{2 \text{ kg}}{400 \text{ g}} = \frac{2\,000 \text{ g}}{400 \text{ g}} = \frac{2\,000}{400} = \frac{5}{1}$	**b.** 200 ml and 2 L
c. 400 ml and 5 L	**d.** 800 m and 1.4 km
e. 120 cm and 1.8 m	**f.** 3 cm 4 mm and 1 cm 4 mm

If the two terms in the ratio have *different* units, then the ratio is also called a **rate.**

Example 2. "5 kilometres to 40 minutes" is a rate. It is comparing the quantities "5 kilometres" and "40 minutes," perhaps for the purpose of giving us the speed at which a person is walking.

We can write this rate as 5 kilometres : 40 minutes or $\frac{5 \text{ kilometres}}{40 \text{ minutes}}$ or 5 kilometres *per* 40 minutes.

The word "per" in a rate signifies the same thing as a colon or a fraction line.

This rate can be simplified: $\frac{5 \text{ kilometres}}{40 \text{ minutes}} = \frac{1 \text{ kilometre}}{8 \text{ minutes}}$. The person walks 1 kilometre in 8 minutes.

Example 3. Simplify the rate "15 pencils per 100 c." Solution: $\frac{15 \text{ pencils}}{100\text{c}} = \frac{3 \text{ pencils}}{20\text{c}}$.

9. Write each rate using a colon, the word "per," or a fraction line. Then simplify it.

a. Annie walks at a constant speed of 3 kilometres in half an hour.

b. In this county, there are five teachers for every 60 pupils.

c. Each three kilograms of rice costs $4.50.

10. Fill in the missing numbers to form equivalent rates.

a. $\frac{2\text{ cm}}{30\text{ min}} = \frac{\quad}{15\text{ min}} = \frac{\quad}{45\text{ min}}$	**b.** $\frac{\$72}{8\text{ hr}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{10\text{ hr}}$
c. $\frac{1/4\text{ km}}{10\text{ min}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{5\text{ hr}}$	**d.** $\frac{\$84.40}{8\text{ hr}} = \frac{\quad}{2\text{ hr}} = \frac{\quad}{10\text{ hr}}$

11. Express these rates in lowest terms.

a. $44 : 4 hr	**b.** $30 : 8 kg
c. 420 km : 8 hr	**d.** 16 apples for $12

12. The rate of pencils to dollars is constant. Fill in the table.

Pencils	Dollars
1	
2	
3	0.75
6	
7	
8	

13. The rate of kilometres to litres remains constant. Fill in the table.

Kilometres							150	
Litres	1	2	3	4	5	10	15	50

14. An automobile travels at a constant speed of 80 km/hour. This means the *rate* of kilometres to hours remains the same. Fill in the table.

Km	10	20	80	100	150	200	500
Minutes							

15. You can use a table like in the previous problems to solve this problem. Six pairs of scissors cost $21. How much would five pairs cost?

16. You can use a table like in the previous problems to solve this problem. Mark can type at a constant rate 225 words in five minutes. How many words can he type in 12 minutes?

Unit Rates

In a **unit rate**, the second term of the rate is *one* of something, or a unit.

For example, 5 dollars per 1 kilogram is a unit rate. It is commonly said as “5 dollars per kilogram,” but the “per kilogram” means “per one kilogram.”

Examples of unit rates:

35 words per (one) minute	$3.70/kg	2/3 cup of sugar per 1 cup of flour
45 kilometres per hour	each student gets 3 pencils	$0.70 per marker

To change a rate that is not a unit rate into a unit rate, simply divide.

Example 1. To change the rate $16 for 6 cups into a unit rate, divide the numbers (16 divided by 6):

We get: $\frac{\$16}{6 \text{ cups}} = \frac{\$2.67}{1 \text{ cup}}$, which is more commonly written as $2.67 per cup.

Example 2. Two tablespoons of salt in 5 decilitres of water is the unit rate 2/5 tablespoons of salt per 1 decilitre of water.

To see that, write it using the fraction line: $\frac{2 \text{ tbsp}}{5 \text{ dl}} = 2/5$ tsp per dl.

1. Give two examples of unit rates (for example, a unit price and a speed).

2. Change to unit rates.

a. $15 for five cups	**b.** 180 kilometres in six hours	**c.** 10 000 people and five doctors

3. Change to unit rates. Give the rate using the word “per” or the slash /.

a. To paint 130 square metres, you need to use 15 litres of paint.

b. Joanne’s Internet speed is 100 megabits in 25 seconds.

c. Each group of five students gets two calculators.

d. 7 teaspoons of vanilla for each 4 cups of batter.

e. We paid $75 for fifteen lunches.

f. $111 for three mattresses.
Hint: use long division.

Chapter 5: Percent
Introduction

This chapter is all about the basics of the concept of percent—a very important topic in regards to real life. We focus on how to calculate percentages (e.g. what percentage is $20 of $50) and how to find a certain percentage of a given number or quantity (e.g. what is 20% of 80 km). In 7th grade, students learn about percent of change and how to make comparisons with percent.

The lessons emphasise the connection between percentages and fractions & decimals in various ways. After all, percentages *are* fractions: the word percent simply means "a hundredth part," and the concept of percent builds on the student's previous understanding of fractions and decimals.

Specifically, the student should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students using Math Mammoth have been practising that concept since 4th grade, and one reason why I have emphasized finding a fractional part of a whole in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered that, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

In this context of thinking of percentages as fractions, students learn how to find a percentage of a given number or quantity using **mental math techniques**. For example, students find 10% of $400 by thinking of it as 1/10 of $400, and thus dividing $400 by 10. They also learn to find a percentage of a quantity using *decimal* multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0.17 × 45 km.

In fact, in cases where mental math is not a good option, I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or fractions. That is because using decimals is simpler and quicker. Also, this method is often superior later on in algebra courses, when students need to write equations from verbal descriptions, and symbolically represent situations that involve percentages.

The last lesson of the chapter teaches students how to find the total when the percentage and the partial amount are known. For example: "Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?" Students solve these with the help of the visual bar models, which they are already familiar with.

As the lessons constantly refer back to fractions and decimals, students can relate calculations with percentages to their earlier knowledge, and thus see **the logical structure of mathematics**. It will also prevent students from memorising calculations with percentages without understanding what is going on.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Some students might only need half or even less of the available exercises, in order to understand the concepts.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

The Lessons in Chapter 5

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch5

SCAN ME

Percent

Percent (or **per cent**) means *per hundred* or "divided by a hundred."
That is because the word "cent" means one hundred.

The symbol for percent is **%**.

When you divide by 100, you get one hundredth (1/100).
Therefore, 8% means 8 per 100, which is 8/100.
Similarly, 67% means 67 divided by 100, or 67/100.

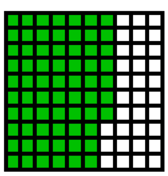
$\frac{5}{100}$ five per cent $= 5\%$

Since percentages are just hundredths, we can very easily write them as fractions and as decimals.

$63\% = \frac{63}{100} = 0.63$ $\qquad 9\% = \frac{9}{100} = 0.09$

1. Write the shaded part and the unshaded part as fractions, as decimals and as percentages.

a.



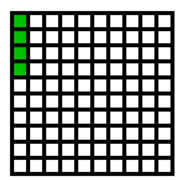
shaded

$\frac{\square}{\square}$ = ________ = _______ %

unshaded

$\frac{\square}{\square}$ = ________ = _______ %

b.



shaded

$\frac{\square}{\square}$ = ________ = _______ %

unshaded

$\frac{\square}{\square}$ = ________ = _______ %

2. Write as percentages, fractions and decimals.

a. $28\% = \frac{28}{100} = 0.28$	**b.** 17% = $\frac{\square}{\square}$ = ________	**c.** _______% = $\frac{\square}{\square}$ = 0.89
d. 60% = $\frac{\square}{\square}$ = ________	**e.** _______% = $\frac{5}{100}$ = ________	**f.** _______% = $\frac{\square}{\square}$ = 0.08

3. Typically, seven out of every 100 babies born in the River Creek Hospital have a birth defect, most of them minor defects.

   a. What typical percentage of the babies have birth defects?

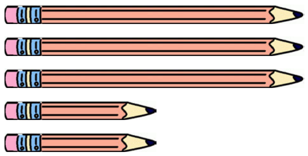
   b. What typical percentage of the babies do *not* have birth defects?

   c. About how many babies with birth defects would you expect to find in a group of 500 babies?

   d. About how many babies with birth defects would you expect to find in a group of 1 300 babies?

Other fractions as percentages

What part of the pencils are short?



Two out of five, or 2/5 of them are short.

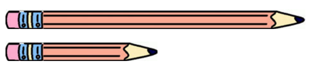
Let's rewrite 2/5 with a denominator of 100 using the method for equivalent fractions:

$$\frac{2}{5} \overset{\cdot 20}{=} \frac{40}{100}$$

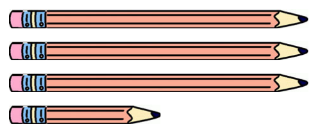
Now we can write 40/100 as 40%.

So, 40% of the pencils are short.

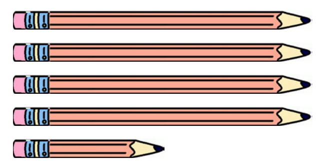
4. Write what part of the pencils are short, both as a fraction and as a percentage. Use equivalent fractions.



a. $\frac{\square}{\square} = \frac{\square}{100}$ = ______%



b. $\frac{\square}{\square} = \frac{\square}{100}$ = ______%

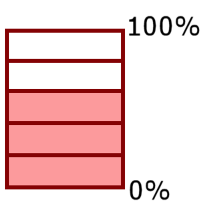


c. $\frac{\square}{\square} = \frac{\square}{100}$ = ______%

5. Convert the fractions into equivalent fractions with a denominator of 100, and then write them as percentages.

a. $\frac{4}{10} = \frac{\square}{100}$ = ______%	**b.** $\frac{11}{20} = \frac{\square}{100}$ = ______%	**c.** $\frac{8}{10} = \frac{\square}{100}$ = ______%
d. $\frac{3}{20} = \frac{\square}{100}$ = ______%	**e.** $\frac{6}{25} = \frac{\square}{100}$ = ______%	**f.** $\frac{4}{5} = \frac{\square}{100}$ = ______%

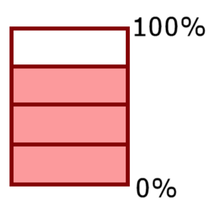
6. Write what part of the rectangle is shaded and what part is not shaded, both as fractions and percentages.



a.

Shaded: $\frac{\square}{\square}$ = _____%

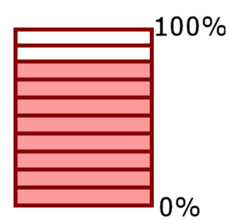
Not shaded: $\frac{\square}{\square}$ = _____%



b.

Shaded: $\frac{\square}{\square}$ = _____%

Not shaded: $\frac{\square}{\square}$ = _____%

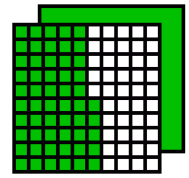


c.

Shaded: $\frac{\square}{\square}$ = _____%

Not shaded: $\frac{\square}{\square}$ = _____%

Percentages that are more than 100%



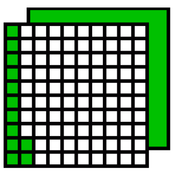
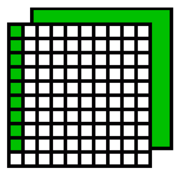
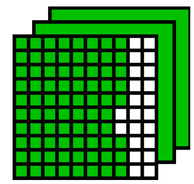
The image shows 1 whole and 55/100. As a mixed number, we write 1 55/100. As a decimal, we write 1.55.

Since 55/100 is 55%, and one whole is 100%, the image shows 155%.

We can use percentages that are more than 100%. Just remember that 100% is 1, and 1% is 0.01.

$$200\% = \frac{200}{100} = 2 \qquad\qquad 308\% = \frac{308}{100} = 3.08$$

7. Write as fractions, decimals and percentages.

		
a. $\frac{\square}{100}$ = ______ = _____%	**b.** $\frac{\square}{100}$ = ______ = _____%	**c.** $\frac{\square}{100}$ = ______ = _____%

8. Write as percentages, fractions and decimals.

a. 105% = $\frac{\square}{\square}$ = ______	**b.** 457% = $\frac{\square}{\square}$ = ______	**c.** _____% = $\frac{\square}{\square}$ = 2.09
d. _____% = $\frac{506}{100}$ = ______	**e.** _____% = $\frac{482}{100}$ = ______	**f.** _____% = $\frac{\square}{\square}$ = 3.11

9. Write the fractions as percentages.

a. About 4/5 (_________ %) of the population of the United States is 14 years old or older.

b. About 2/25 (_________ %) of the world's population lives in North America.

c. The continent of Africa covers about 1/5 (_________ %) of the Earth's total land mass.

10. There are two trees growing in Sandy's front yard. The taller one is 5/4 as tall as the shorter one.

a. Write the second sentence using a percentage instead of a fraction.

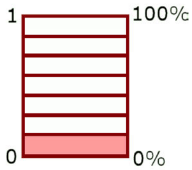
b. If the shorter tree is 160 cm tall, how tall is the taller tree?

Change any fraction to a percentage

To write 1/7 as a percentage, you can either:

- Divide 1 by 7 using long division or a calculator. You will get a decimal number. Express that as a percentage. OR,
- Find 1/7 of 100%; in other words divide 100 by 7. Then your answer is already a percentage.

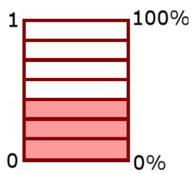
```
    0 1 4 . 2 8
7 ) 1 0 0 . 0 0
  -   7
      3 0
    - 2 8
        2 0
      - 1 4
          6 0
        - 5 6
            4
```



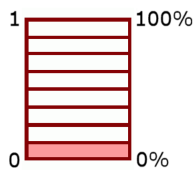
Dividing 100 by 7, we get 14.28...
Rounded to the nearest whole percent, that is 14%.

How many percent would 2/7 be?
What about 5/7?

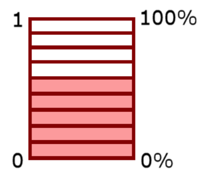
11. Write the fractions as percentages. Use long division. Round your answers to the nearest percent.



a. $\frac{}{}$ = _______%



b. $\frac{}{}$ = _______%



c. $\frac{}{}$ = _______%

12. Write the fractions as percentages. Round the answers to the nearest percent.

a. According to a 2020 estimate, about 1/20 (________ %) of the population of Guatemala is 65 years old or older.

b. In that same year, about 13/100 (_________ %) of the population of Australia was 65 years old or older.

c. The Indian Ocean covers approximately 7/50 (_________ %) of the Earth's surface.

d. About 3/5 (__________ %) of the world's population lives in Asia.

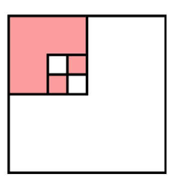
13. Write as a percentage. Round your answers to the nearest percent.

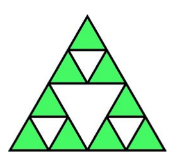
a. 8/7

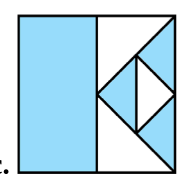
b. 1 3/8

Puzzle Corner

How many percent of each figure is coloured?

a. 

b. 

c. 

What Percentage . . . ?

What percentage of the height of a 4-m tree is a 1-m sapling?	A choir has 22 women and 18 men. **Find what percentage** of the choir's members are men.	One pair of jeans costs \$25 and another costs \$28. **How many percent** is the price of the cheaper jeans of the price of the more expensive jeans?

Look carefully at the questions above. Notice that the problems don't tell you the percentage; in other words, there is no number in the problem written as *x*%. Instead, they ask *you* to find it!

Questions with "What percentage . . . ?" or "How many percent . . . ?"

Asking "What percentage?" or "How many percent?" is the same as asking "How many hundredth parts?"

We can solve these questions in a two-part process:

1. First find out the part that is being asked for <u>as a fraction</u>. The denominator will probably not be 100.

2. Convert that fraction to a decimal. Then you can easily convert the decimal to a percentage!

Example 1. A choir has 22 women and 18 men. Find what percentage of the choir's members are men.

1. Find *what part* (fraction) of the choir's members are men. That is 18/40, or 9/20.

2. Write 9/20 as a percentage. You can use equivalent fractions: 9/20 = 45/100 = 45%.

Example 2. One pair of jeans costs \$25 and another costs \$28.
How many percent is the price of the cheaper jeans of the price of the more expensive jeans?

1. Write *what part* (fraction) the cheaper price is of the more expensive price. The answer is 25/28.

2. Write 25/28 as a percentage. A calculator gives 25/28 = 0.8928 . . .
Rounded to the nearest whole percent, that is 89%.

1. **a.** What percentage of the height of a 4-m tree is the height of a little 1-m sapling?

b. How many percent is \$12 of \$16?

2. Find how many percent the shorter object's height is of the taller object's height.


6 m

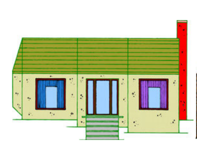

8 m

a.


300 cm


120 cm

b.


4 m


5 m

c.