Compound Probability

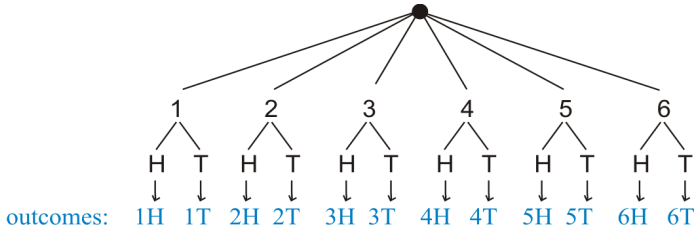
Compound probability means finding the probability where two events both occur. If the outcome of the one event does not affect the outcome of the other, they are said to be **independent.** In that case you can find the probability of two events occurring by multiplying the probabilities of the two events. Examples will make this clear.

Example 1. You toss a coin, and then you roll a die. What is the probability of getting 6 and heads?

P(6) is 1/6, and P(heads) is 1/2. Clearly, whether you get heads or tails on the coin does not affect what you get on the roll. The two events are *independent*. Therefore, we can multiply the two probabilities.

$$P(\text{6 and heads}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

You can also see this probability by looking at the tree diagram, because in only one outcome out of the twelve possible ones do we have 6 and heads.

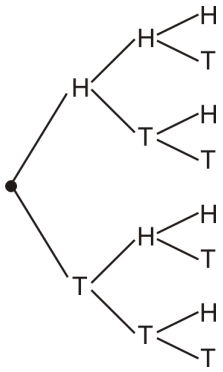


Example 2. You toss a coin three times. What is the probability of getting heads every time?

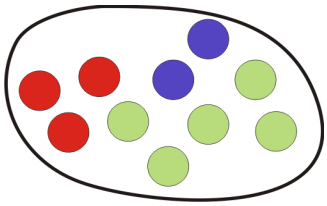
These three events—toss a coin, toss a coin, toss a coin— are independent. Getting heads on one toss doesn't affect whether you get heads or tails on the next.

P(heads) = 1/2. Therefore, P(heads and heads and heads) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

You can also see this from the tree diagram. There is only one outcome with "HHH", and a total of 8 possible outcomes.



Example 3. The bag has three red marbles, two dark blue marbles, and five light green marbles. You take one marble, and put it back. Then you take a marble again, and put it back. What is the probability of getting first a red marble, then a blue one?



Again, we simply multiply the individual probabilities:

$$P(\text{red, blue}) = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{3}{50}.$$

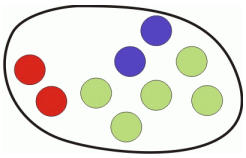
1. You toss a coin three times.

   a. What is the probability of getting tails, then heads, then tails?

   b. What is the probability that you get heads on your second toss?

   c. Use the tree diagram. What is the probability of getting two heads and one tails in three tosses? Note they can be in any order, such as THH or HTH.

2. You take a marble out of the bag and put it back. Then you take another marble. Find the probabilities.



a. P(red, then green)

b. P(green, then red)

c. P(not blue, not blue)

d. P(not red, not red)

3. You roll a six-sided die two times. Find the probabilities.

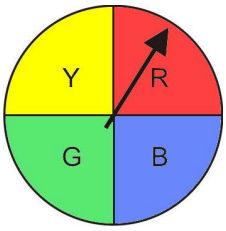
a. P(1; 5)

b. P(even; odd)

c. P(2; 5 or 6)

d. P(6; not 6)

4. The spinner is spun two times. Find the probabilities.



a. P(blue; blue)

b. P(green; not green)

c. P(not blue; yellow)

d. P(yellow or green; red or blue)

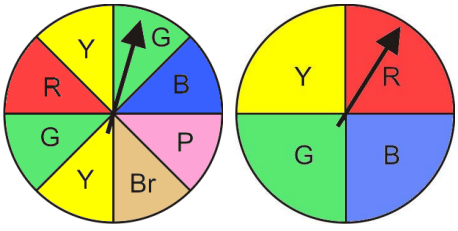
5. The weatherman says that the chance of rain is 20% for each of the next five days, and your birthday is in two days! You also know that the probability of your dad taking you to the amusement park on your birthday is 1/2.

a. What is the probability that you get to go the park, and it doesn't rain?

b. What is the probability that you get to go the park, and it rains?

Check: The sum of the probabilities in (a) and (b) should be 1/2.

6. The two spinners are spun. The first spinner has eight regions and the second spinner has four. Find the probabilities:



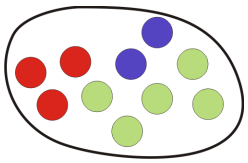
a. P(red, red)

b. P(blue, not blue)

c. P(yellow or green, yellow or green)

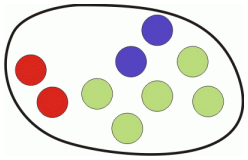
d. P(not red, red)

Taking an object without replacing it



Let's say you choose a marble and don't put it back. Then you choose another marble. This means you have in effect chosen two marbles. What is the probability that both are red?

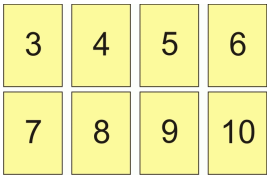
We first find the probability that the first marble is red. That is simply 3/10 since there are two red marbles and ten in all.



After you get a red marble, and don't put it back, the bag now has one red marble less. So, the probability of getting a red marble now is 2/9.

Thus, the probability of getting two red marbles is $\frac{\cancel{3}^{1}}{\cancel{10}^{5}} \times \frac{\cancel{2}^{1}}{\cancel{9}^{3}} = \frac{1}{15}$.

Example. You choose one card, without putting it back. Then you choose another. What is the probability that the first is an even number, and the second is 7?

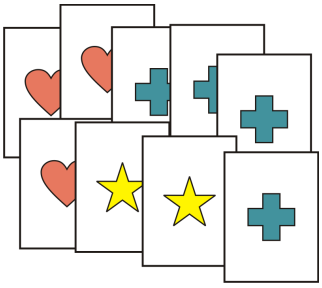


There are four cards with an even number, and eight total cards. So, P(even) = 4/8 = 1/2.

After one card with an even number has been drawn, there are seven cards left, and one of them has number 7. So, P(7) is 1/7.

P(even, 7) = $\frac{1}{2} \times \frac{1}{7} = \frac{1}{14}$.

7. You choose a card randomly from this group of cards. Then you choose another card, without replacing the first. Find the probabilities.



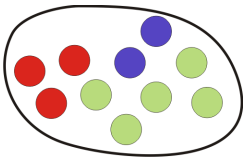
a. P(heart, heart)

b. P(star, cross)

c. P(not heart, not heart)

d. P(star, not star)

8. You choose two marbles randomly from the bag, without replacement.



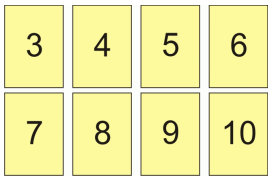
a. What is the probability that both are green?

b. What is the probability that the first is green, and the second is red?

c. What is the probability that the first is red, and the second is green?

d. Add the probabilities from (b) and (c) to get the probability that you get one red and one green marble when drawing two marbles.

9. You choose one card, without putting it back. Then you choose another. Find the probabilities. Give your answers as fractions and percents.



a. P(5; 6)

b. P(not 5; not 5)

c. P(9; even)

d. P(8 or 9; not 10)

10. A 6th grade classroom has 13 boys and 16 girls. The teacher randomly chooses two persons to be responsible for the cleanup after a bake sale. Give these probabilities to the tenth of a percent.

a. What is the probability that both are girls?

b. What is the probability that both are boys?

c. What is the probability that the first person chosen is a girl, and the second is a boy?

d. What is the probability that the first person chosen is a boy, and the second is a girl?

CHECK. The probabilities you get in (a), (b), (c), and (d) should total 100% because they are all the possible outcomes.

e. Add the probabilities in (c) and (d) to get the probability that one of the cleaners is a girl and one is a boy.

11. Michael has 10 white socks and 14 black socks mixed together in a drawer. He chooses one sock to wear randomly, and doesn't put it back. Then he chooses another sock. Find the probabilities:

a. P(white, white)

b. P(black, black)

c. P(black, white)

d. P(white, black)

CHECK. The four probabilities above should total 100%.

e. Add the probabilities in (a) and (b) to find the probability that Michael wears matching socks.

f. What is the probability Michael doesn't wear matching socks?

Puzzle Corner

Matthew has 8 white socks, 9 brown socks, and 10 black socks mixed together in a drawer. He chooses two socks randomly. Find the probability he gets to wear a matching pair. (That would be nice!)