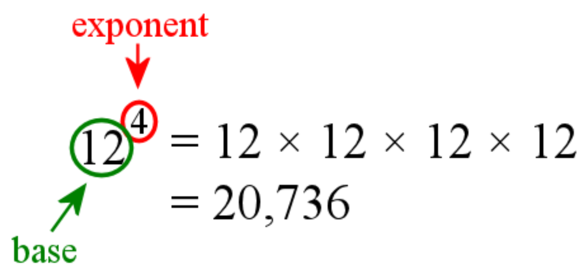
Powers and Exponents

Exponents are a kind of “shorthand” for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read “two raised to the fifth power,” “two to the fifth power,” or even just “two to the fifth.” Similarly, 7^9 is read “seven raised to the ninth power,” “seven to the ninth power,” or “seven to the ninth.” The “powers of 6” are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6.

However, expressions with powers of 2 and 3 are almost always read differently:
The expression 11^2 is usually read as “eleven squared” because it describes the area of a square with sides 11 units long. Similarly, 31^3 is generally read as “thirty-one cubed” because it gives the volume of a cube with sides 31 units long.

1. Write out these expressions as multiplications, then solve them.

a. $3^2 = \underline{\ 3 \times 3 = 9\ }$

b. 1^6

c. 4^3

d. 10^6

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^3

j. 10^5

2. Rewrite these expressions as multiplication. Then use a calculator to solve them.

a. 6^4

b. 11^3

c. 13^3

d. 27^5

3. Rewrite each expression using an exponent, then solve it. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

The expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with sides 7 units long.

For example, if the sides of a square are 5 cm long, then its area is 5 cm × 5 cm = 25 cm^2.

Notice that the symbol for "square centimeters" is $\mathbf{cm^2}$. This means "centimeter × centimeter." We are, in effect, squaring the measuring unit! In fact, we do the same thing when we use the units "square meters" and "square kilometers."

We could also write that expression as $(5\text{ cm})^2$ or "the quantity, five centimeters, squared." This means that both the 5 and the unit "cm" are squared, which makes 25 cm^2. Without the parenthesis it would be 5 cm^2 and mean "five square centimeters," which is something very different.

We can do the same thing with the traditional units of inches, feet, and miles. People often write "sq. in." for square inches, or "sq. ft." for square feet, instead of $\mathbf{in^2}$ and $\mathbf{ft^2}$, but both ways are correct.

Similarly, 7^3 is read "seven *cubed*" because it gives the volume of a *cube* with sides 7 units long.

For example, if the sides of a cube are 10 cm long, then its volume is $(10\text{ cm})^3$ = 1,000 cm^3, or "one thousand cubic centimeters.".

4. Express the area using exponents and solve.

a. A square with a side of 12 kilometers: The area is $(12\text{ km})^2$ = ___________	**b.** A square with sides 6 m long: Its area is
c. A square with sides each 6 inches long: Its area is	**d.** A square with a side with a length of 12 ft: The area is

5. Express the volume using exponents and solve.

a. A cube with a side of 2 cm: The volume is	**b.** A cube with sides each 10 inches long:
c. A cube with sides 1 ft in length:	**d.** A cube with edges that are all 5 m long:

6. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its side?

c. The area of a square is 121 m^2. What is its perimeter?

d. The *area* of one face of a cube is 64 in^2. What is its volume?

Notice something special about powers of 0 and powers of 1.

$0^5 = 0 \times 0 \times 0 \times 0 \times 0$ is simply 0! You can easily see that 0^3, 0^7, 0^{21}, and all of the other powers of 0 (0 raised to any whole-number power) are equal to 0.

$1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1$ is simply 1! It's easy to see that 1^3, 1^9, 1^{65}, and all of the other powers of 1 (1 raised to any whole-number power) are equal to 1.

The powers of 10 are also very special—and very easy!

Notice that the exponent tells us *how many zeroes* there are in the answer.

$10^1 = 10$	$10^4 = 10{,}000$
$10^2 = 10 \times 10 = 100$	$10^5 = 100{,}000$
$10^3 = 10 \times 10 \times 10 = 1{,}000$	$10^6 = 1{,}000{,}000$

7. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

8. Look at the patterns above. Think carefully about how each step comes from the previous one. Then answer the questions.

a. If you are given that $3^7 = 2{,}187$,
how can you use that result to find 3^8 ?

b. Find 3^8 without a calculator.

c. If you are given that $2^{45} = 35{,}184{,}372{,}088{,}832$,
how can you use that result to find 2^{46} ?

d. Find 2^{46} without a calculator.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method? Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?
Try this same pattern for at least one other base number, n. What value do you calculate for n^0? Do you think it will come out this way for every base number? Why or why not?