Exponents and Powers

If you multiply the same number by itself repeatedly, such as 5 × 5 × 5 × 5 × 5 × 5, it is **repeated multiplication**. We have a shorthand notation for it: $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

Read 5^6 as "five to the sixth power." The number 5 is called the *base*. It tells us what number we are multiplying repeatedly. The little raised number is the *exponent*, and it tells how many times the number is repeatedly multiplied.

We can also solve that $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15{,}625$.

These repeated multiplications are called ***powers***. For example, 10 × 10 × 10 × 10 is "ten to the fourth power," and 10^7 is "ten to the seventh power." They are both **powers of ten**.

We have two other special ways to read powers when the exponent is 2 or 3:

- 10^2 is read "ten squared", because it gives us the area of a square with a side length of 10 units.
- 4^3 is read "four cubed", because it gives us the volume of a cube with an edge length of 4 units.

1. Read the powers aloud. Then write out the repeated multiplications, and solve.

a. $5^2 = 5 \times 5 = 25$	**e.** 10^3 = ______________________________
b. 2^3 = ____ × ____ × ____ = _______	**f.** 7^2 = ______________________________
c. 3^3 = ______________________________	**g.** 2^4 = ______________________________
d. 10^2 = ______________________________	**h.** 1^6 = ______________________________

2. Write using exponents, and solve.

a. 4 × 4 × 4 =	**e.** 1 × 1 × 1 × 1 × 1 =
b. 9 × 9 =	**f.** 2 × 2 × 2 × 2 × 2 =
c. 10 × 10 × 10 × 10 =	**g.** 3 × 3 × 3 × 3 =
d. five to the third power =	**h.** zero to the tenth power =

3. Multiplication is repeated addition, and a power is repeated multiplication. Compare.

a. 2 + 2 + 2 + 2 = 4 × 2 = _______	**b.** 5 + 5 + 5 = ___ × ___ = _____
2 × 2 × 2 × 2 = $\square^{\square}$ = __________	5 × 5 × 5 = $\square^{\square}$ = _______

4. Write these powers of ten as normal numbers. Notice there is a *shortcut* and a *pattern*!

a. 10^2 = ______________________________	**e.** 10^6 = ______________________________
b. 10^3 = ______________________________	**f.** 10^7 = ______________________________
c. 10^4 = ______________________________	**g.** 10^8 = ______________________________
d. 10^5 = ______________________________	**h.** 10^9 = ______________________________

SHORTCUT: In any power of ten, such as 10^8, the exponent tells us how many _________ the number has after the digit 1.

Remember? When you multiply numbers ending in zeros, multiply the "parts" without zeros and tag as many zeros onto the result as there are in the factors. Look at these examples:

6,000 × 500	2,300 × 20,000	200 × 5,000 × 70
Multiply 6 × 5 = 30, and tag 5 zeros to the result: 30←00000 = 3,000,000	Multiply 23 × 2 = 46, and tag 6 zeros to the result: 46←000000 = 46,000,000	Multiply 2 × 5 × 7 = 70, and tag 6 zeros to the result: 70←000000 = 70,000,000

5. Calculate the products mentally.

a. 200 × 30,000	**b.** 40 × 2 × 200,000	**c.** 500,000 × 3,000
d. 100 × 15,000	**e.** 30 × 900,000	**f.** 50,000 × 200 × 6
g. 120 × 20 × 200 × 50	**h.** 40 × 20 × 10 × 50 × 200	**i.** 50,000 × 20,000 × 8

6. Calculate.

a. 5×10^2 = _____________	**b.** 7×10^6 = _____________	**c.** 51×10^3 = _____________
5×10^3 = _____________	2×10^4 = _____________	161×10^6 = _____________
5×10^4 = _____________	6×10^7 = _____________	29×10^4 = _____________

Why does this work?

It is because we can break down such multiplications so that we multiply the single-digit numbers and the powers of ten separately.

For example, 300 × 9,000 is the same as 3 × 100 × 9 × 1,000. Since we can multiply in any order, we can multiply 3 × 9 and 100 × 1,000 separately, to get 27 × 100,000. And that equals 2,700,000.

7. Did you understand the above explanation? Fill in.

a. 200 × 3,000 is equal to	**b.** 6,000 × 200 × 50 is equal to
___ × 100 × ___ × 1,000, which is equal to	(___ × 1000) × (___ × 100) × (___ × 10)
___ × ___ × 100 × 1,000	= ___ × ___ × ___ × 1000 × 100 × 10
= _____ × _______________	= ______ × _______________
= _____________________	= _____________________

8. Find the missing exponent or power of ten.

a. $6 \times 10^{\square} = 6{,}000$	**b.** $3 \times 10^{\square} = 300{,}000$	**c.** $56 \times \square^{\square} = 560{,}000$
$71 \times 10^{\square} = 71{,}000{,}000$	$9 \times 10^{\square} = 90{,}000{,}000$	$295 \times \square^{\square} = 2{,}950{,}000{,}000$

9. Some challenges. Can you find a shortcut?

a. $10^3 \times 10^2$ = _____________	**b.** $5 \times 10^2 \times 10^4$ = _____________
c. $10^5 \times 10^3$ = _____________	**d.** $8 \times 10^4 \times 2 \times 10^3$ = _____________
e. $10^6 \times 10^2 \times 10^2 = 10^{\square}$	**f.** $10^3 \times 10^5 \times 10^2 \times 10^4 = 10^{\square}$

10. Astronomy involves some really big numbers. Write these numbers in the normal manner.

Pluto's surface area is about 17×10^6 km^2.

The Sun's average distance from Earth is 15×10^7 km.

Haumea is a dwarf planet located beyond Neptune's orbit.
The mass of Haumea is about 4×10^{21} kg.

Puzzle Corner

The Sun's mass is about 2×10^{30} kg and Jupiter's mass is about 2×10^{27} kg. About how many times heavier is the Sun than Jupiter?