Factoring and Primes 2

Some numbers *only have two divisors*: 1 and the number itself. Such numbers are called **prime numbers.** 11 is one of them.

factor	factor	product
1 ×	**11** =	**11**

In the last lesson, we found that the prime numbers between 1 and 20 are **2, 3, 5, 7, 11, 13, 17 and 19**. 1 is usually not counted as a prime number (see the last lesson for an explanation why it is not).

Prime factorization using a factor tree

A factor tree is a handy way to factor numbers to their prime factors. The factor tree starts at the root and grows upside down!

We want to factor 24 so we write 24 on top. First, 24 is factored into 4 × 6. However, 4 and 6 are not primes, so we can *continue* factoring. Four is factored into 2 × 2 and six is factored into 2 × 3.

```
(root)          24
               /  \
              4  ×  6
             / \   / \
(leaves)    2 × 2 × 2 × 3
```

We will not factor 2 or 3 any further because they are prime numbers. Once you get to the primes in your “tree”, they are the “leaves”, and you stop factoring in that “branch”. So 24 = 2 × 2 × 2 × 3. This is the ***prime factorization of 24.***

Examples:

```
  30
 /  \
5 × 6
   / \
  2 × 3
```

5 is a prime number—it is a “leaf”. Once done, “pick the leaves”—you can even circle them to see them better! So, 30 = 2 × 3 × 5.

```
  21
 /  \
3 × 7
```

Both 3 and 7 are prime numbers, so we cannot factor them any further.

So 21 = 3 × 7.

```
   66                  66
  /  \                /  \
11 × 6       OR      2 × 33
    / \                  / \
   2 × 3               11 × 3
```

You can start the factoring process any way you want. The end result is the same: 66 = 2 × 3 × 11.

```
     72
    /   \
  12  ×  6
  / \    / \
 3 × 4 × 2 × 3
    / \
   2 × 2
```

72 has lots of factors so factoring takes many steps.

72 = 2 × 2 × 2 × 3 × 3

We could have also started by writing 72 = 2 × 36 or 72 = 4 × 18.

```
 57
/  \
```

How can you get started? Check:
- is 57 in any of the times tables?
- is it divisible by 2? By 3? By 5?

```
 65
/  \
```

How can you get started? Check:
- is 65 in any of the times tables?
- is it divisible by 2? By 3? By 5?

1. Factor the following numbers to their prime factors.

a. 18 / \	**b.** 6 / \	**c.** 14 / \
d. 8 / \	**e.** 12 / \	**f.** 20 / \
g. 16 / \	**h.** 24 / \	**i.** 27 / \
j. 25 / \	**k.** 33 / \	**l.** 15 / \

Prime numbers are like building blocks of all numbers. They are the first and foremost, and other numbers are “built” from them. “Building numbers” is like factoring backwards. We start with the building blocks—the primes—and see what number we get:

```
2 × 5 × 2 × 2
 \ /     \ /
 10   ×   4
   \     /
     40
```

```
2 × 3 × 2 × 3 × 2
 \ /     \ /    |
  6   ×   6   × 2
  |        \   /
  6   ×     12
   \       /
       72
```

```
5 × 2 × 7
 \ /    |
 10   × 7
   \   /
    70
```

```
2 × 7 × 2×3
 \ /     \ /
 14   ×   6
   \     /
     84
```

By using the process above (building numbers starting from primes) you can build ANY whole number there is! Can you believe that?

We can say this in another way: **ALL numbers** can be factored so the factors are prime numbers. That is sort of amazing! This fact is known as the *fundamental theorem of arithmetic*. And indeed, it is fundamental.

So, no matter what the number is—992 or 83,283 or 150,282—it can be written as a product of primes.

992 is factored on the right. $992 = 2^5 \times 31$. For 83,283 we get $3 \times 17 \times 23 \times 71$ and $151{,}282 = 2 \times 3^3 \times 11^2 \times 23$.
To find these factorizations, you need to test-divide the numbers by various primes so it is a bit tedious. Of course, today's computer programs can do the division very quickly.

```
        992
       /   \
     4   ×  248
    / \     /  \
   2 × 2 × 4 × 62
          / \   / \
         2 × 2 × 2 × 31
```

2. Build numbers from primes.

a. 2 × 5 × 11 \ / |	**b.** 3 × 2 × 2 × 2 \ / \ /	**c.** 2 × 3 × 7 \ / |
d. 11 × 3 × 2 | \ . /	**e.** 3 × 3 × 2 × 5 \ / \ /	**f.** 2 × 3 × 17

3. Build more numbers from primes.

a. $2 \times 5 \times 13$	b. $13 \times 13 \times 2$	c. $19 \times 3 \times 3$

4. Try it on your own! Pick 3-4 primes as you wish (you can use the same prime several times), and see what number is built from them.

Puzzle Corner

Ready for a challenge? Use your knowledge of divisibility tests and the calculator, and factor to prime factors:

a. 2,145 **b.** 3,680 **c.** 10,164