math MAMMOTH

Grade 6-A Worktext

Light Blue Series

- **R**eview of the basic operations
- **E**xpressions and equations
- **D**ecimals
- **R**atios
- **P**ercent

By Maria Miller

Contents

Chapter 4: Ratios

Chapter 5: Percent

Foreword

Math Mammoth Grade 6-A and Grade 6-B worktexts comprise a complete math curriculum for the sixth grade mathematics studies that meets and exceeds the Common Core Standards.

In sixth grade, students encounter the beginnings of algebra, algebraic expressions, one-variable equations and inequalities, integers, and ratios. We also review and deepen the students' understanding of rational numbers: both fractions and decimals are studied in depth, while percent is a new topic for 6th grade. In geometry, students learn to compute the area of various polygons, and also calculate volume and surface area of various solids. The last major area of study is statistics, where students learn to summarize and describe distributions using both measures of center and variability.

The year starts out with a review of the four operations with whole numbers (including long division), place value, and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practice writing expressions in many different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. We also study briefly inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we review all of decimal arithmetic, just using more decimal digits than in 5th grade. Students also convert measuring units in this chapter.

Ratios is a new topic (chapter 4). Students are already familiar with finding fractional parts from earlier grades, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios, and problem solving using bar models.

Percent (chapter 5) is an important topic to understand thoroughly, because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals, and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry, and statistics.

I wish you success in teaching math!

Maria Miller, the author

Chapter 1: Review of the Basic Operations
Introduction

The goal of the first chapter in grade 6 is to review the four basic operations with whole numbers, place value, and rounding, and to learn about exponents and problem solving.

A lot of this chapter is review, and I hope this provides a gentle start for 6th grade math. In the next chapter, we will delve into some beginning algebra topics.

The Lessons in Chapter 1

Helpful Resources on the Internet

LONG DIVISION

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Drag and Drop Math
An interactive tool to practice long division or long multiplication.
http://mrnussbaum.com/drag-and-drop-math/

Divide Decimals by Whole Numbers Quiz
Practice dividing decimals by whole numbers with this interactive online quiz.
http://www.ipracticemath.com/math-problem/decimal/dividing-decimals-by-whole-numbers

Short Division
A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.
http://www.themathpage.com/ARITH/divide-whole-numbers.htm

ALL FOUR OPERATIONS

Math Mahjong - Mixed Operations
Match all the tiles with the same value to win the game!
http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_3.html

Pop the Balloons
Pop the balloons in the order of their value. You need to use all four operations.
http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/

EXPONENTS

Exponent Pairs Game
Match each exponent with its value in this interactive memory game.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Pairs.asp?Topic=1

Exponents Quiz from ThatQuiz.org
Ten questions, fairly easy, and not timed. You can change the parameters as you like.
http://www.thatquiz.org/tq-2/?-j1-l4-p0

Pyramid Math
Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.
http://www.mathnook.com/math/pyramidmath.html

Picture Matching - Exponent Values
Match the powers with their values and reveal a pretty picture.
http://www.studystack.com/picmatch-275044

Exponent Game
A card game to practice exponents. I would limit the cards to small numbers, instead of using the whole deck.
http://www.learn-with-math-games.com/exponent-game.html

Otter Rush
Choose the correct value for "*x*" in exponent expressions (such as 2x = 16) in this racing game.
http://www.arcademics.com/games/otter-rush/otter-rush.html

Free Exponent Worksheets
Create a variety of customizable, printable worksheets to practice exponents.
http://www.homeschoolmath.net/worksheets/exponents.php

PLACE VALUE

Numbers
Practice place value, comparing numbers, and ordering numbers with this interactive online practice.
http://www.aaamath.com/B/grade6.htm#topic3

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Powers of Ten
A 9-minute movie that illustrates the dramatic changes of scale when zooming in or out by powers of ten (40 powers of ten), starting from a picnic blanket and ending in the universe, and then starting from a hand to the proton inside an atom.
http://www.youtube.com/watch?v=0fKBhvDjuy0

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Place Value Game
Create the largest possible number from the digits the computer gives you. Unfortunately, the computer will give you each digit one at a time and you will not know what the next number will be.
http://education.jlab.org/placevalue/

Large Numbers Quiz
Practice your large number skills with this multiple-choice quiz.
http://www.proprofs.com/quiz-school/story.php?title=NTczNDc3

Rounding to a Given Power of Ten
Practice your rounding skills with this self-check exercise.
http://www.transum.org/software/SW/Starter_of_the_day/Students/RoundingWN.asp?Level=7

Round Whole Numbers
Practice rounding whole numbers to the nearest hundred or thousand.
https://www.khanacademy.org/math/in-sixth-grade-math/knowing-numbers/new-topic-2015-11-17T22:50:37.680Z/e/rounding_whole_numbers

Estimate Quotients Quiz
Estimate the quotient. Choose "less than" or "greater than" to make the equation true.
http://www.mathgames.com/skill/6.35-estimate-quotients

PROBLEM SOLVING

Thinking Blocks - Fractions
Model and solve fraction word problems online.
http://www.mathplayground.com/tb_fractions/thinking_blocks_fractions.html

Speed, Time, and Distance Worksheets
Create customized worksheets to practice units of speed, time, and distance.
http://www.homeschoolmath.net/worksheets/speed_time_distance.php

Adding and Subtracting Decimals: Word Problems
Practice solving word problems by adding or subtracting decimal numbers.
https://www.khanacademy.org/math/arithmetic/decimals/adding-decimals/e/adding_and_subtracting_decimals_word_problems

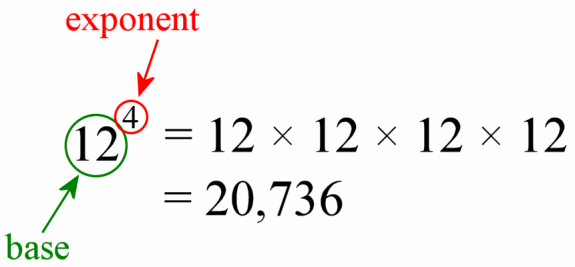
(This page intentionally left blank.)

Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the *exponent*. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45}, and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us the area of a square with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the volume of a cube with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them using mental math.

a. $3^2 = 3 \times 3 = 9$

b. 1^6

c. 4^3

d. 10^4

e. 5^3

f. 10^2

g. 2^3

h. 8^2

i. 0^5

j. 10^5

k. 50^2

l. 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. $8 \times 8 \times 8 \times 8 \times 8$

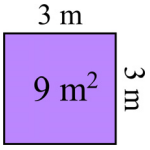
c. 40 squared

d. $10 \times 10 \times 10 \times 10$

e. nine to the eighth power

f. eleven cubed

You just learned that the expression 7^2 is read "seven *squared*" because it tells us the area of a *square* with a side length of 7 units. Let's compare that to square meters and other units of area.



If the sides of a square are 3 m long, then its area is 3 m × 3 m = 9 m^2 or nine square meters.

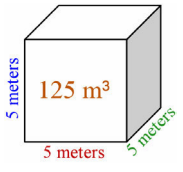
Notice that the symbol for square meters is **m^2**. This means "**meter × meter**." We are, in effect, squaring the unit *meter* (multiplying the unit of length *meter* by itself)!

Or, in the expression 9 cm × 9 cm, we multiply 9 by itself, but we also multiply the unit *cm* by itself. That is why the result is **81 cm^2**, and the square centimeter (cm^2) comes from multiplying "**centimeter × centimeter**."

We do the same thing with any other unit of length to form the corresponding unit for area, such as square kilometers or square millimeters.

With the customary units of area, such as square inches, square feet, and square miles, people often write "sq. in.", "sq. ft.", or "sq. mi." , instead of **in^2** , **ft^2**, and **mi^2**. Both ways are correct.

In a similar way, to calculate the volume of this cube, we multiply 5 m × 5 m × 5 m = 125 m^3. We not only multiply 5 by itself three times, but also multiply the unit *meter* by itself three times (meter × meter × meter) to get the unit of volume "cubic meter" or m^3.



3. Express the area (A) as a multiplication, and solve.

a. A square with a side of 12 kilometers: A = 12 km × 12 km = _____	**b.** A square with sides 6 m long: A = _____
c. A square with a side length of 6 inches: A = _____	**d.** A square with a side with a length of 12 ft: A = _____

4. Express the volume (V) as a multiplication, and solve.

a. A cube with a side of 2 cm: V = 2 cm × 2 cm × 2 cm = _____	**b.** A cube with sides each 10 inches long: V = _____
c. A cube with sides 1 ft in length: V = _____	**d.** A cube with edges that are all 5 m long: V = _____

5. **a.** The perimeter of a square is 40 cm. What is its area?

b. The volume of a cube is 64 cubic inches. How long is its edge?

c. The area of a square is 121 m^2. What is its perimeter?

d. The volume of a cube is 27 cm^3. What is the length of one edge?

The powers of 10 are very special —and very easy!	$10^1 = 10$	$10^4 = 10{,}000$
	$10^2 = 10 \times 10 = 100$	$10^5 = 100{,}000$
Notice that the exponent tells us *how many zeros* there are in the answer.	$10^3 = 10 \times 10 \times 10 = 1{,}000$	$10^6 = 1{,}000{,}000$

6. Fill in the patterns. In part (d), choose your own number to be the base.
Use a calculator in parts (c) and (d).

a.	**b.**	**c.**	**d.**
$2^1 =$	$3^1 =$	$5^1 =$	
$2^2 =$	$3^2 =$	$5^2 =$	
$2^3 =$	$3^3 =$	$5^3 =$	
$2^4 =$	$3^4 =$	$5^4 =$	
$2^5 =$	$3^5 =$	$5^5 =$	
$2^6 =$	$3^6 =$	$5^6 =$	

7. Look at the patterns above. Think carefully how each step comes from the previous one. Then answer.

a. If $3^7 = 2{,}187$, how can you use that result to find 3^8 ?

b. Now find 3^8 without a calculator.

c. If $2^{45} = 35{,}184{,}372{,}088{,}832$, use that to find 2^{46} without a calculator.

8. Fill in.

a. 17^2 gives us the _____ of a _____ with sides _____ units long.

b. 101^3 gives us the _____ of a _____ with edges _____ units long.

c. 2×6^2 gives us the _____ of two _____ with sides _____ units long.

d. 4×10^3 gives us the _____ of ____ _____ with edges _____ units long.

Puzzle Corner

Make a pattern, called a *sequence*, with the powers of 2, starting with 2^6 and going *backwards* to 2^0. At each step, *divide* by 2. What is the logical (though surprising) value for 2^0 from this method?

Make another, similar, sequence for the powers of 10. Start with 10^6 and divide by 10 until you reach 10^0. What value do you calculate for 10^0?

Try this same pattern for at least one other base number, n. What value do you calculate for n^0?

Do you think it will come out this way for every base number?

Why or why not?

(This page intentionally left blank.)

Chapter 2: Expressions and Equations
Introduction

In this chapter we concentrate on two important concepts: expressions and equations. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). The study of negative numbers is in part 6-B.

We start out by learning some basic vocabulary used to describe mathematical expressions verbally—terms such as the sum, the difference, the product, the quotient, and the quantity. Next, we study the order of operations once again. A lot of this lesson is review. The lesson *Multiplying and Dividing in Parts* is also partially review and leads up to the lesson on distributive property that follows later.

Then, we get into studying expressions in definite terms: students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways.

The concepts of equivalent expressions and simplifying expressions are important. If you can simplify an expression in some way, the new expression you get is equivalent to the first. We study these ideas first using lengths— it is a concrete example, and hopefully easy to grasp.

In the lesson *More On Writing and Simplifying Expressions* students encounter more terminology: term, coefficient, and constant. In exercise #3, they write an expression for the perimeter of some shapes in two ways. This exercise is once again preparing them to understand the distributive property.

Next, students write and simplify expressions for the area of rectangles and rectangular shapes. Then we study the distributive property, concentrating on the symbolic aspect and tying it in with area models.

The next topic is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few two-step equations as an optional topic.

Lastly, students get to solve and graph simple inequalities, and study the usage of two variables and graphing.

The Lessons in Chapter 2

Helpful Resources on the Internet

TERMINOLOGY

Coefficients, Like Terms, and Constants
This page contains a short description of coefficients, like terms, and constants in response to a student's question.
http://mathcentral.uregina.ca/QQ/database/QQ.09.07/h/maddie1.html

Algebra Definitions
This page illustrates and defines basic algebraic terms.
http://www.mathsisfun.com/algebra/definitions.html

Translating Words into Algebraic Expressions
Match the correct math expression with the corresponding English phrase, such as "7 less than a number". You can do this activity either as a matching game or as a concentration game.
https://www.quia.com/jg/1452190.html

Escape Planet
Choose the equation that matches the words.
http://www.harcourtschool.com/activity/escape_planet_6/

Writing Expressions with Variables
Practice writing algebraic expressions to match verbal descriptions of mathematical operations.
https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-variables-and-expressions/e/writing_expressions_1

ORDER OF OPERATIONS

Order of Ops
Save members of a Royal Family from prison by using order of operation skills to build stairways.
http://mrnussbaum.com/orderops/

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, −, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, *etc*., until it is solved. Lastly, the resource includes a game where you click on the falling blocks in the sequence that the order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.html

Order of Operations Quiz
Practice the order of operations with this multiple-choice self-check quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-860467-2&chapter=6&lesson=2&headerFile=4&state=fl

Math Operations Solitaire Game
Practice basic mental math with this math card game that you can play as one player or against the computer.
http://www.learn4good.com/games/card_and_dice/math-card-games.htm

Order of Operations Practice
A simple online quiz of 10 questions. Uses parentheses and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Choose A Math Operation
Choose the mathematical operation(s) so that the number sentence is true.
http://www.homeschoolmath.net/operation-game.php

MULTIPLY & DIVIDE IN PARTS

Multiply with Area Models
Use an area model to decompose factors and multiply in this interactive online activity.
https://www.khanacademy.org/math/5th-engage-ny/engage-5th-module-2/5th-module-2-topic-b/e/multiplying-2-digit-numbers-with-area-models

Divide Mentally to Get a Decimal Quotient
Practice dividing larger whole numbers to get a decimal quotient in this interactive exercise.
https://www.khanacademy.org/math/5th-engage-ny/engage-5th-module-2/5th-module-2-topic-g/e/dividing-decimals-without-the-standard-algorithm-2

EXPRESSIONS

BuzzMath Practice - Algebraic Expressions
Practice simplifying and writing algebraic expressions. Enter as a "guest" and then choose any of the items from the menu to practice that topic.
http://www.buzzmath.com/Menu#id=CC06E118

Evaluate Expressions
Practice entering values to evaluate real-world algebraic expressions. These are introductory problems, so the expressions aren't too complicated.
https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-variables-and-expressions/e/evaluating-expressions-3

Algebra Matching Game
Match the algebraic expressions to their simplified versions.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Matching.asp?Title=Matching&GoMathsID_Item=282&ImageFolder=/Software/SW/Starter_of_the_day/Students/Pairs_Algebra/img/images/&Topic=10

Writing Basic Expressions to Real-World Situations
Practice writing basic algebraic expressions to model real-world situations in this interactive online exercise.
https://www.khanacademy.org/math/in-sixth-grade-math/algebra-in/using-expressions-practically/e/writing-expressions-with-variables-word-problems

Algebraic Expressions Millionaire
For each question you have to identify the correct mathematical expression that models a given word expression.
http://www.math-play.com/Algebraic-Expressions-Millionaire/algebraic-expressions-millionaire.html

Expressions and Variables Quiz
Choose an equation to match the word problem or situation.
http://www.softschools.com/quizzes/math/expressions_and_variables/quiz815.html

Equivalent Expressions and the Distributive Property
Practice solving multi-step word problems with this interactive exercise. Click on “visitor” after going to this page (unless you wish to register with the site).
http://www.buzzmath.com/Docs#CC06E117

Equivalent Expressions
Practice determining whether or not two algebraic expressions are equivalent by manipulating the expressions. These problems require you to combine like terms and apply the distributive property.
https://www.khanacademy.org/math/algebra-basics/core-algebra-expressions/core-algebra-manipulating-expressions/e/equivalent-forms-of-expressions-1

EQUATIONS

One-step Equations Pong
Play the traditional pong game while solving one-step equations.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=105

Algebra Meltdown
Solve simple equations using function machines to guide atoms through the reactor. But don't keep the scientists waiting too long or they blow their tops. Again, includes negative numbers.
http://www.mangahigh.com/en/games/algebrameltdown

Balance when Adding and Subtracting Game
The interactive balance illustrates simple equations. Your task is to add or subtract x's and add or subtract 1's until you have isolated x on one side.
http://www.mathsisfun.com/algebra/add-subtract-balance.html

Equation Match
Match simple equations that have the same solution.
http://www.bbc.co.uk/schools/mathsfile/shockwave/games/equationmatch.html

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/algebra_puzzle.html

Solve Two-Step Equations
Practice solving equations that take two steps to solve in this interactive online exercise.
https://www.khanacademy.org/math/algebra/one-variable-linear-equations/alg1-two-steps-equations-intro/e/linear_equations_2

INEQUALITIES

Inequalities Word Problems
Practice writing inequalities with variables to describe real-world situations in this interactive online activity.
https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/greater-than-less-than/e/inequalities-in-one-variable-1

Plot Inequalities on a Number Line
Create number line graphs of inequalities in this interactive online exercise. Includes negative numbers.
https://www.khanacademy.org/math/pre-algebra/applying-math-reasoning-topic/greater-than-less-than/e/inequalities_on_a_number_line

Testing Solutions of Inequalities
Practice entering values of inequalities and checking to see if the inequalities hold true in this interactive online exercise.
https://www.khanacademy.org/math/algebra/one-variable-linear-inequalities/alg1-inequalities/e/testing-solutions-of-inequalities

TWO VARIABLES

Dependent and Independent Variables
Practice figuring out if a variable is dependent or independent in this interactive online activity.
https://www.khanacademy.org/math/algebra/introduction-to-algebra/alg1-dependent-independent/e/dependent-and-independent-variables

(This page intentionally left blank.)

The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum $(b + c)$ so that you multiply the numbers b and c separately by a, and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$. We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = 2x + 2 \cdot 1$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ____ $+ 3 \cdot$ ____ $=$	**b.** $7(50 + 6) = 7 \cdot$ ____ $+ 7 \cdot$ ____ $=$
c. $4(a + b) = 4 \cdot$ ____ $+ 4 \cdot$ ____ $=$	**d.** $2(x + 6) = 2 \cdot$ ____ $+ 2 \cdot$ ____ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply **each term** in the sum by the factor in front of the parentheses:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

Example 4. Now one of the terms in the sum has a coefficient (the 2 in $2x$):

$$6(2x + 3) = 6 \cdot 2x + 6 \cdot 3 = 12x + 18$$

3. Multiply using the distributive property.

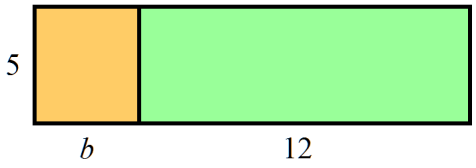
a. $2(3x + 5) =$	**b.** $7(7a + 6) =$
c. $5(4a + 8b) =$	**d.** $2(4x + 3y) =$
e. $3(9 + 10z) =$	**f.** $6(3x + 4 + 2y) =$
g. $11(2c + 7a) =$	**h.** $8(5 + 2a + 3b) =$

To understand even better why the the distributive property works, let's look at an area model (this, too, you have seen before!).

The area of the whole rectangle is 5 times $(b + 12)$.

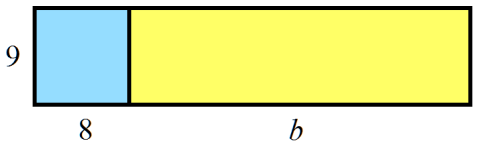
But if we think of it as *two* rectangles, the area of the first rectangle is $5b$, and of the second, $5 \cdot 12$.

Of course, these two expressions have to be equal:



$$5 \cdot (b + 12) = 5b + 5 \cdot 12 = 5b + 60$$

4. Write an expression for the area in two ways, thinking of one rectangle or two.

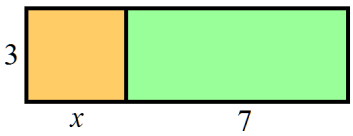


a. $9($ _____ $+$ _____ $)$ and

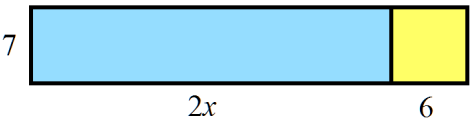
$9 \cdot$ _____ $+ 9 \cdot$ _____ $=$

b. $s($ _____ $+$ _____ $)$ and

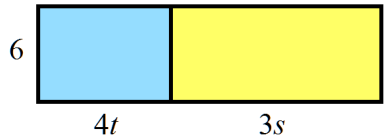
$s \cdot$ _____ $+ s \cdot$ _____ $=$



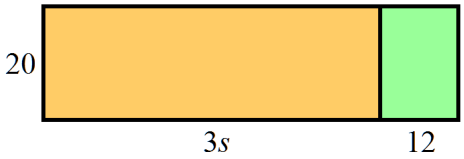
c. ____ (_____ + _____) and



d.



e.

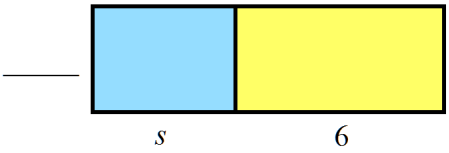


f.

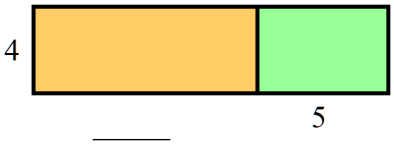
5. Find the missing number or variable in these area models.

a. ____ $(x + 2) = 3x + 6$

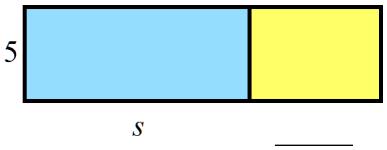
b. _____ $(t + 8) = 7t + 56$



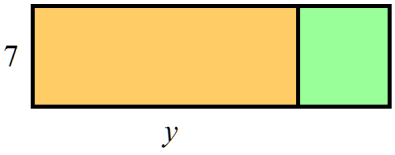
c. The total area is $9s + 54$.



d. $4($ ____ $+ 5) = 4z + 20$



e. $5(s +$ ____ $) = 5s + 30$



f. The total area is $7y + 42$.

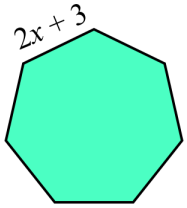
6. Find the missing number in the equations.

a. ____ $(x + 5) = 6x + 30$	**b.** $10(y +$ ____ $) = 10y + 30$
c. $6($ ____ $+ z) = 12 + 6z$	**d.** $8(r +$ ____ $) = 8r + 24$

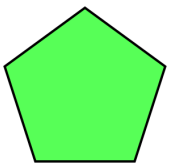
7. Find the missing number in the equations. These are just a little bit trickier!

a. ____ $(2x + 5) = 6x + 15$	**b.** ____ $(3w + 5) = 21w + 35$
c. ____ $(6y + 4) = 12y + 8$	**d.** ____ $(10s + 3) = 50s + 15$
e. $2($ _____ $+ 9) = 4x + 18$	**f.** $4($ _____ $+ 3) = 12x + 12$
g. $5($ _____ $+ 3) = 20y + 15$	**h.** $8($ ____ $+$ ____ $+ 7) = 40t + 8s + 56$

8. Write an expression for the perimeter of this regular heptagon as a *product*. Then multiply the expression using the distributive property.



9. The perimeter of a regular pentagon is $15x + 5$. How long is one of its sides?



When we use the distributive property "backwards," and write a sum as a product, it is called **factoring**.
Example 5. The sum $5x + 5$ can be written as $5(x + 1)$. We took the SUM $5x + 5$ and wrote it as a PRODUCT— something times something, in this case 5 times the quantity $(x + 1)$.
Example 6. The sum $24x + 16$ can be written as the product $8(3x + 2)$. *Notice* that the numbers 24 and 16 are *both divisible by 8*! That is why we write 8 as one of the factors.

10. Think of the distributive property "backwards," and factor these sums. Think of divisibility!

a. $6x + 6 =$ ____ $(x + 1)$	**b.** $8y + 16 = 8($ ____ + ____ $)$
c. $15x + 45 =$ ____ $(x +$ ____ $)$	**d.** $4w + 40 =$ ____ $(w +$ ____ $)$
e. $6x + 30 =$ ____ (____ + ____)	**f.** $8x + 16y + 48 =$ ____ (____ + ____ + ____)

11. Factor these sums (writing them as products). Think of divisibility!

a. $8x + 4 =$ ____ $(2x +$ ____ $)$	**b.** $15x + 10 =$ ____ $(3x +$ ____ $)$
c. $24y + 8 =$ ____ (____ + ____)	**d.** $6x + 3 =$ ____ (____ + ____)
e. $42y + 14 =$ ____ (____ + ____)	**f.** $32x + 24 =$ ____ (____ + ____)
g. $27y + 9 =$ ____ (____ + ____)	**h.** $55x + 22 =$ ____ (____ + ____)
i. $36y + 12 =$ ____ (____ + ____)	**j.** $36x + 9z + 27 =$ ____ (____ + ____ + ____)

12. The perimeter of a square is $48x + 16$. How long is its side?

Puzzle Corner

As a storekeeper, you need to purchase 1,000 items to get a wholesale (cheaper) price of \$8 per item, so you do. You figure you might sell 600 of them. You also want to advertise a \$3 discount to your customers. What should the non-discounted selling price be for you to actually earn a \$500 profit from the sale of these items?

Epilogue: It may be hard to see now where distributive property or factoring might be useful, but it IS extremely necessary later in algebra when solving equations.

To solve the problem above, you *can* figure it out without algebra, but it becomes fairly straightforward if we write an equation for it. Let p be the non-discounted price. Then $p - \$3$ is the price with the discount. We get:

$$\text{What we need to take in} = \text{pay to supplier + profit}$$

$$600(p - \$3) = 1{,}000 \cdot \$8 + \$500$$

To solve this equation, one needs to use the distributive property in the very first step:

$$600p - \$1800 = \$8{,}500$$

$$600p = \$10{,}300$$

(Can you solve this last step yourself?)

(This page intentionally left blank.)

Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system, and using decimals in measuring units. Most of the topics here have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, if the student has a good grasp of decimals already, consider assigning only 1/3 - 1/2 of the problems because the student should be able to go through this chapter quickly.

We study place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of review, just using longer decimals than in 5th grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

In the lessons about dividing decimals by decimals, I have tried to explain the principle behind the common shortcut ("Move the decimal point in both the divisor and the dividend enough steps that the divisor becomes a whole number"). This shortcut actually has to do with the principle that when you multiply the divisor and the dividend by the same number (*any* number), the value of the quotient does not change. This even ties in with equivalent fractions. Many school books never explain this principle in connection with decimal division.

The last lessons in this chapter deal with measuring units and the metric system, and they round out our study of decimals.

The Lessons in Chapter 3

Helpful Resources on the Internet

DECIMAL PLACE VALUE

Naming Decimals
Use this online quiz to improve your decimal skills.
http://www.themathpage.com/ARITH/Ar_Pr/dec_2.htm

Expanded Form
Practice decimal addition in this 10-question online quiz.
https://www.thatquiz.org/tq-c/?-j88-l7-p0

Write Decimals as Fractions
Practice rewriting decimals as fractions. These problems use decimals with up to four decimal places.
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-fracs-to-decimals/e/converting_decimals_to_fractions_2

Number Line Game
Slide the circles onto the number line from lowest to greatest. Complete all five rounds as quickly as you can. (Choose "fractions and decimals".)
http://www.abcya.com/number_line_fractions_percents_decimals.htm

Balloon Pop Decimals
Pop the balloons in order: from the smallest value to the largest value.
http://www.sheppardsoftware.com/mathgames/decimals/BalloonPopDecimals2.htm

Ordering Game
Get good at placing decimal numbers in order! This exercise includes negatives, decimals, fractions, and reverse order. Choose "Tricky decimals".
http://www.mathsisfun.com/numbers/ordering-game.php

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height. Requires Shockwave.
http://www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. After each guess you get feedback about whether your guess was too high or too low.
http://www.interactivestuff.org/sums4fun/decchall.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
http://www.mathwarehouse.com/games/our-games/decimal-games/decimal-place-value-math-game

Rounding Quiz
Practice rounding decimals in this 10-question online quiz.
https://www.thatquiz.org/tq-c/?-jg080-l5-p0

ADD AND SUBTRACT DECIMALS

Add and Subtract Decimals: Mixed Problems
Practice mental and written methods for adding and subtracting decimal numbers.
http://www.transum.org/Maths/Activity/Decimals/Add_Problems.asp?Level=3

Add and Subtract Decimals Quiz
Focus on adding and subtracting decimals with this interactive online quiz.
http://www.mrmaisonet.com/index.php?/Decimal-Quizzes/Adding-and-Subtracting-Decimals.html

MULTIPLY AND DIVIDE DECIMALS

Multiply Decimals by Whole Numbers - Mental Math
Practice mental and written methods for multiplying and dividing decimal numbers.
http://www.transum.org/Maths/Activity/Decimals/

Decimal Times - Multiplication
Practice multiplying 3-digit decimal numbers in this online exercise.
http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=3

Decimal Times - Division
Practice dividing decimals by integers in this interactive online activity.
http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=4

Long Division to Decimal Places
Read an illustrated lesson about long division to decimal places. Then, scroll down for interactive practice questions.
https://www.mathsisfun.com/long_division3.html

Decimal Times
Practice mental and written methods for multiplying and dividing decimal numbers in this interactive activity.
http://www.transum.org/Maths/Activity/Decimals/Default.asp?Level=5

Dividing Decimals Quiz
Practice decimal division with this interactive multiple-choice quiz.
http://www.eduplace.com/kids/hmm/practice/templates/rules.jsp?ID=hmm07_ep/gr5/1407&GRADE=5&UNIT=5&CHAPTER=14&LESSON=7

Divide Decimals
Reinforce your decimal division skills with this online activity.
http://www.buzzmath.com/Docs#CC06E67&page=1

Quiz: Multiply & Divide Decimals by Powers of Ten
Practice decimal arithmetic with this 10-question interactive quiz.
https://www.thatquiz.org/tq-3/?-j12c-l8-p0

Multiplying and Dividing by Powers of Ten
Practice multiplication and division by powers of ten with this interactive online exercise.
https://www.khanacademy.org/math/ab-sixth-grade-math/ab-number/ab-multiplication-division/e/multiplying-and-dividing-by-powers-of-10

WORD PROBLEMS

Rags to Riches: Decimal Word Problems
Practice decimal arithmetic with these multiple-choice word problems.
https://www.quia.com/rr/200480.html

More Decimal Word Problems
This page includes illustrated examples of how to solve decimal word problems, as well as problems to solve.
http://www.mathgoodies.com/lessons/decimals_part2/solve_more_problems.html

FRACTIONS AND DECIMALS

Fractions and Decimals Matching Game
Match each fraction to the equivalent decimal. Click the "read more" button to start the game.
http://www.mrmaisonet.com/index.php?/Decimal-Quizzes/Fraction-And-Decimal-Equivalents.html

Fraction Decimal Scale
Explore the relationship between fractions and decimals with this interactive activity.
http://www.mathplayground.com/Scale_Decimals.html

Fraction Snake - Ordering Game
Arrange the numbers on the snake in order from the largest on the head to the smallest at the tail.
http://www.transum.org/software/SW/fracorder/fraction_order.asp?Level=4

Recurring Decimals Guessing Game
Find pairs of whole numbers such that the first divided by the second results in the given recurring decimal number.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Recurring.asp

Order Fractions Quiz
Practice ordering fractions in this 10-question online quiz. Use your knowledge of decimals!
https://www.thatquiz.org/tq-6/?-j10-l7-p0

Fractions - Decimals calculator
Convert fractions to decimals, or decimals to fractions, including repeating (recurring) decimals to any number of decimal places, which normal calculators do not do.
http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/FractionsCalc.html

MEASUREMENT UNITS

Customary Units Activity
Practice converting between customary units of measurement with this interactive online exercise. Click on "visitor" after going to this page (unless you wish to register with the site).
www.buzzmath.com/Docs#CC06E973&page=3

Inequalities - Metric Units
Check that you know what inequality signs mean and how they are used to compare two quantities in this interactive exercise.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Inequalities.asp?Level=4

Metric and Customary Units - Approximations
Practice converting between metric and customary units of measurements in this online quiz.
http://www.mathgames.com/skill/6.85-convert-between-metric-and-customary-units

Unit Conversion in the Metric System
This video tutorial explains the metric system and how to make simple metric conversions. I show you the metric number line and how to use it to convert between units.
http://socratic.org/chemistry/measurement-in-chemistry/unit-conversions/unit-conversion-in-the-metric-system

(This page intentionally left blank.)

Fractions and Decimals

You already know how to change decimals to fractions. The number of decimal digits tells you the denominator—it is always a power of ten with as many zeros as you have decimal digits. For the numerator, just copy all the digits from the number. **Example:** $3.0928 = \frac{30{,}928}{10{,}000}$	You can write this as a mixed number, in which case you separate the whole number part from the decimal part, and the decimal digits form the numerator of the fraction: $15.30599 = \frac{1{,}530{,}599}{100{,}000} = 15\frac{30{,}599}{100{,}000}$

1. Write as fractions.

a. 0.09	**b.** 0.005	**c.** 0.045
d. 0.00371	**e.** 0.02381	**f.** 0.0000031

2. Write as fractions and also as mixed numbers.

a. 2.9302	**b.** 2.003814
c. 5.3925012	**d.** 3.0078
e. 3.294819	**f.** 45.00032

When changing a **fraction into a decimal**, we have several tools in our "toolbox." **Tool 1.** If the denominator of a fraction is already a power of ten, there is not much to do but to write it as a decimal. The number of zeros in the power of ten tells you the number of decimal digits you need.	$\frac{3}{10} = 0.3$	$\frac{451{,}593}{10{,}000} = 45.1593$

3. Write as decimals.

a. $\frac{36}{100}$	**b.** $\frac{5{,}009}{1000}$	**c.** $1\frac{45}{1000}$
d. $\frac{3908}{10{,}000}$	**e.** $2\frac{593}{100{,}000}$	**f.** $\frac{5903}{1{,}000{,}000}$
g. $\frac{45{,}039{,}034}{1{,}000{,}000}$	**h.** $\frac{435{,}112}{10{,}000}$	**i.** $\frac{450{,}683}{100{,}000}$

Tool 2. With some fractions, you can find an equivalent fraction with a denominator of 10, 100, 1000, *etc*. and then write the fraction as a decimal.	$\frac{27}{30} \overset{\div 3}{=} \frac{9}{10} = 0.9$	$\frac{66}{200} \overset{\div 2}{=} \frac{33}{100} = 0.33$	$\frac{3}{8} \overset{\times 125}{=} \frac{375}{1{,}000} = 0.375$

4. Write as decimals. Think of the equivalent fraction that has a denominator of 10, 100, or 1000.

a. $\frac{1}{5}$	**b.** $\frac{1}{8}$	**c.** $1\frac{1}{20}$
d. $3\frac{9}{25}$	**e.** $\frac{12}{200}$	**f.** $8\frac{3}{4}$
g. $4\frac{3}{5}$	**h.** $\frac{13}{20}$	**i.** $\frac{7}{8}$
j. $\frac{11}{125}$	**k.** $\frac{24}{400}$	**l.** $\frac{95}{500}$

5. In these problems, you see both fractions and decimals. Either change the decimal into a fraction, or vice versa. You decide which way is easier! Then, calculate mentally.

a. $0.2 + \frac{1}{4}$	**b.** $0.34 + 1\frac{1}{5}$	**c.** $2\frac{3}{5} + 1.3$	**d.** $\frac{5}{8} - 0.09$
e. $0.02 + \frac{3}{4}$	**f.** $1.9 + 3\frac{1}{8}$	**g.** $\frac{14}{20} - 0.23$	**h.** $\frac{18}{25} + 0.07$

Tool 3. Most of the time, in order to change a fraction to a decimal, you simply treat the fraction as a division problem and divide (with a calculator or long division).	$\frac{5}{6} = 5 \div 6 = 0.83333.... \approx 0.83$

6. Use long division in your notebook to write these fractions as decimals. Give your answers to three decimal digits.

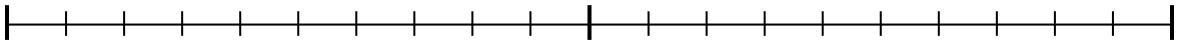
a. $\frac{2}{9} =$	**b.** $\frac{3}{7} =$	**c.** $\frac{7}{16} =$

7. Use a calculator to write these fractions as decimals. Give your answers to three decimal digits.

a. $\frac{1}{11} =$	**b.** $\frac{3}{23} =$	**c.** $\frac{47}{56} =$

8. Label the bold tick marks on the number line "0," "1," and "2." Then mark the following numbers on it where they belong.

$$0.2,\quad \frac{1}{4},\quad 0.65,\quad 1\frac{1}{3},\quad 0.04,\quad \frac{2}{5},\quad 1.22,\quad 1\frac{3}{4},\quad 1.95,\quad 1\frac{4}{5}$$



9. One bag of milk powder contains 900 g.
Another contains 3/4 kg.
What is the combined weight of the two?

10. A puzzle measures 14 3/8 inches by 20 3/8 inches.

a. Write these mixed numbers as decimals.

b. Calculate the area of the puzzle in square inches (as a decimal).

11. Flax seed costs $11.45 per kilogram. Sally bought 1 3/4 kg of it.
Calculate the total price of Sally's purchase (in dollars and cents).

12. Explain in two different ways how to calculate the price of 3/8 of a liter of oil, if one liter costs $12.95. (You do not have to calculate the price; just explain two ways *how* to do it.)

13. Give your answer to each of the following problems as both a fraction and as a decimal.

a. 0.3 × 5/8

b. 3/4 × 1.5

(This page intentionally left blank.)

Convert Metric Measuring Units

The metric system has one basic unit for each thing we might measure: For length, the unit is the **meter**. For weight, it is the **gram**. And for volume, it is the **liter**.

All of the other units for measuring length, weight, or volume are *derived* from the basic units using *prefixes*. The prefixes tell us what multiple of the basic unit the *derived unit* is.

For example, centiliter is 1/100 part of a liter (*centi* means 1/100).

Prefix	Abbreviated	Meaning
kilo-	k	1,000
hecto-	h	100
deka-	da	10
-	-	(the basic unit)
deci-	d	1/10
centi-	c	1/100
milli-	m	1/1000

Unit	Abbr	Meaning
kilometer	km	1,000 meters
hectometer	hm	100 meters
dekameter	dam	10 meters
meter	m	(the basic unit)
decimeter	dm	1/10 meter
centimeter	cm	1/100 meter
millimeter	mm	1/1000 meter

Unit	Abbr	Meaning
kilogram	kg	1,000 grams
hectogram	hg	100 grams
dekagram	dag	10 grams
gram	g	(the basic unit)
decigram	dg	1/10 gram
centigram	cg	1/100 gram
milligram	mg	1/1000 gram

Unit	Abbr	Meaning
kiloliter	kl	1,000 liters
hectoliter	hl	100 liters
dekaliter	dal	10 liters
liter	L	(the basic unit)
deciliter	dl	1/10 liter
centiliter	cl	1/100 liter
milliliter	ml	1/1000 liter

1. Write these amounts using the basic units (meters, grams, or liters) by "translating" the prefixes. Use both fractions and decimals, like this: 3 cm = 3/100 m = 0.03 m (since "centi" means "hundredth part").

a. 3 cm = *3/100 m* = *0.03 m*

5 mm = _____ m = _____ m

7 dl = _____ L = _____ L

b. 2 cg = _____ g = _____ g

6 ml = _____ L = _____ L

1 dg = _____ g = _____ g

2. Write the amounts in basic units (meters, grams, or liters) by "translating" the prefixes.

a. 3 kl = _____ L

8 dag = _____ g

6 hm = _____ m

b. 2 dam = _____ m

9 hl = _____ L

7 kg = _____ g

c. 70 km = _____ m

5 hg = _____ g

8 dal = _____ L

3. Write the amounts with derived units (units with prefixes) and a single-digit number.

a. 3,000 g = 3 kg

800 L = 8 _____

60 m = 6 _____

b. 0.01 m = _____ _____

0.2 L = _____ _____

0.005 g = _____ _____

c. 0.04 L = _____ _____

0.8 m = _____ _____

0.007 L = _____ _____

4. Write using prefixed units.

a. 0.04 meters = 4 cm

b. 0.005 grams = 5 _____

c. 0.037 meters = 37 _____

d. 400 liters = 4 _____

e. 0.6 meters = 6 _____

f. 2,000 meters = 2 _____

g. 0.206 liters = 206 _____

h. 20 meters = 2 _____

i. 0.9 grams = 9 _____

5. Change into the basic unit (either meter, liter, or gram). Think of the meaning of the prefix.

a. 45 cm = *0.45 m*

b. 65 mg =

c. 2 dm =

d. 81 km =

e. 6 ml =

f. 758 mg =

g. 2 kl =

h. 8 dl =

i. 9 dag =

Example 1. Convert 2.5 cg to grams.

					2.	5
kg	hg	dag	g	dg	cg	mg

→

			0.	0	2	5
kg	hg	dag	g	dg	cg	mg

Write 2.5 in the chart so that "2", which is in the ones place, is placed in the centigrams place.

Move the decimal point just after the grams place. Add necessary zeros. Answer: 0.025 g.

6. Write the measurements in the place value charts.

a. 12.3 m

km	hm	dam	m	dm	cm	mm

b. 78 mm

km	hm	dam	m	dm	cm	mm

c. 56 cl

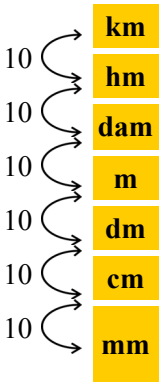
kl	hl	dal	l	dl	cl	ml

d. 9.83 hg

kg	hg	dag	g	dg	cg	mg

7. Convert the measurements to the given units, using the charts above.

	m	**dm**	**cm**	**mm**
a. 12.3 m	12.3			
b. 78 mm				78 mm
	L	**dl**	**cl**	**ml**
c. 56 cl				
	g	**dg**	**cg**	**mg**
d. 9.83 hg				



You can also convert measurements by thinking of how many steps apart the two units are in the chart and then multiplying or dividing by the corresponding power of ten.

Example 2. Convert 2.4 km into centimeters.

There are five steps from kilometers to centimeters. That means we would multiply 2.4 by 10, five times—or multiply 2.4 by 10^5.

$2.4 \times 100{,}000 = 240{,}000$, so 2.4 km = 240,000 cm.

Example 3. Convert 2,900 cg into hectograms.

"Centi" and "hecto" are four steps apart, so we will divide by $10^4 = 1000$.
$2{,}900 \div 10{,}000 = 0.29$, so 2,900 cg = 0.29 hg.

8. Convert the measurements. You can write the numbers in the place value charts or count the steps.

a. 560 cl = _____ L

b. 0.493 kg = _____ dag

c. 24.5 hm = _____ cm

d. 491 cm = _____ m

e. 35,200 mg = _____ g

f. 32 dal = _____ cl

g. 0.483 km = _____ dm

h. 0.0056 km = _____ cm

i. 1.98 hl = _____ dl

j. 9.5 dl = _____ L

kl	hl	dal	l	dl	cl	ml

kg	hg	dag	g	dg	cg	mg

km	hm	dam	m	dm	cm	mm

9. Each measurement has a flub, either in the unit or in the decimal point. Correct them.

a. The length of a pencil: 13 m

b. The length of an eraser: 45 cm

c. Circumference of Dad's waist: 9.2 m

d. The height of a room: 0.24 m

e. Jack's height: 1.70 mm

f. Jenny's height: 1.34 cm

10. Find the total …

a. … weight of books that weigh individually:
1.2 kg, 1.04 kg, 520 g, and 128 g.

b. … volume of containers whose individual volumes are:
1.4 L, 2.25 L, 550 ml, 240 ml, and 4 dl.

11. A dropper measures 4 ml. How many full droppers can you get from a 2-dl bottle?

12. Once a day, a nurse has to give a patient 3 mg of medicine for each kilogram of body weight. The patient weighs 70 kg. How many days will it take for the patient to take 2 g of medicine?

(This page intentionally left blank.)

Chapter 4: Ratios
Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate, and unit rate. The lesson *Equivalent Rates* allows students to solve a variety of word problems involving ratios and rates. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Then, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height, and solve a variety of problems involving aspect ratio.

Lastly, students learn how rates can be used to convert measurement units. This method is in addition to the methods for converting measurement units that were explained in the chapter on decimals. It does not mean that students should "change over" and forget what they learned earlier—it is simply a different method for doing the conversions. Some students may choose one method over another; some may be able to master all of the methods. Most will probably choose one method they prefer for doing these conversions.

The Lessons in Chapter 4

Helpful Resources on the Internet

Ratio Pairs Matching Game
Match cards representing equivalent ratios.
Easy: **http://nrich.maths.org/4824** Challenge: **http://nrich.maths.org/4821**

Ratio Stadium
A multi-player online racing game for matching equivalent ratios. The student with the fastest rate of correct answers will win the race.
http://www.arcademics.com/games/ratio-stadium/ratio-stadium.html

All About Ratios - Quizzes
Online quizzes about same and different ratios.
http://math.rice.edu/~lanius/proportions/

Equal Ratios Game
Stop the asteroids from crashing into the planet! Asteroids can be destroyed by neutralizing them with equal ratios. Blast the asteroid that matches the ratio at the bottom of the screen.
http://abcya.com/equal_ratios.htm

3-Term Ratios
Explore the amounts of red, blue, or green colors used in digital photography in this interactive mathematics resource by determining 3-term ratios in equivalent forms.
http://learnalberta.ca/content/mejhm/?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/object_interactives/3_term_ratio/use_it.html

Equivalent Ratios in Tables and Graphs
Choose the value that will make each statement true in this interactive exercise.
http://www.buzzmath.com/Docs#CC06E92&page=1

Ratio Tables
Practice filling tables of equivalent ratios in this interactive online exercise.
https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/solving-ratio-problems-with-tables

Quiz: Rates
Practice rates with these interactive multiple-choice word problems.
http://www.brainingcamp.com/lessons/rates/questions.php

Unit Rate Word Problems
Use rates to solve word problems in this interactive online activity.
https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/rate_problems_0.5

Comparing Rates
Practice comparing rates in this interactive activity from Khan Academy.
https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-1/6th-module-1-topic-c/e/comparing-rates

Comparing Two Speeds
Explore speeds and rates with this interactive activity.
http://www.brainingcamp.com/lessons/rates/manipulative.php

Free Ride
An interactive activity about bicycle gear ratios. Choose the front and back gears, which determines the gear ratio. Then choose a route, pedal forward, and make sure you land exactly on the five flags.
http://illuminations.nctm.org/Activity.aspx?id=3549

Rate lesson from BrainingCamp
A comprehensive interactive lesson on the concepts of ratio, rate, and constant speed (for 6th and 7th grades). Includes an animated lesson, a virtual manipulative, and questions and problems to solve.
http://www.brainingcamp.com/content/rates/

Thinking Blocks: Ratios
Model and solve word problems involving ratios and proportions with this interactive modeling tool.
http://www.mathplayground.com/tb_ratios/thinking_blocks_ratios.html

Practice with Ratios
An online quiz from Regents Exam Prep Center that includes both simple and challenging questions and word problems concerning ratios.
http://www.regentsprep.org/Regents/math/ALGEBRA/AO3/pracRatio.htm

USING RATIOS TO CONVERT MEASUREMENT UNITS

Unit Conversion Tool
Use this interactive tool for all types of unit conversion. Includes a slider that you can adjust to see various conversions.
http://www.mathsisfun.com/unit-conversion-tool.php

Converting Units with Dimensional Analysis
This page includes explanations, videos, and exercises to practice converting units.
https://www.texasgateway.org/resource/converting-between-measurement-systems

Ocean Math Worksheet
This ocean-themed worksheet contains a set of measurement conversion problems to solve.
http://www.mathgoodies.com/worksheets/ocean_wks.html

Dimensional Analysis Quiz
Use the conversions given in the table to help you answer the questions in this multiple-choice quiz.
http://ths.sps.lane.edu/chemweb/unit1/problems/dimensionalanalysis/

(This page intentionally left blank.)

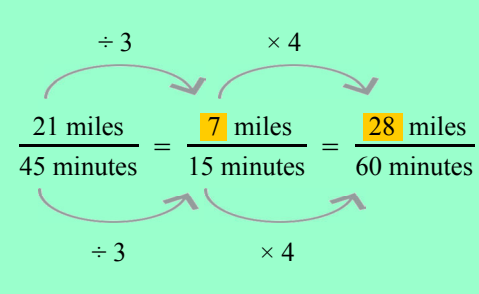
Using Equivalent Rates

Example 1. If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 21 miles per 45 minutes and hoping to arrive at so many miles per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, let's first figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.



Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 28 miles in one hour.

1. Write the equivalent rates.

a. $\frac{15\text{ km}}{3\text{ hr}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{15\text{ min}} = \frac{\quad}{45\text{ min}}$	**b.** $\frac{\$6}{45\text{ min}} = \frac{\quad}{15\text{ min}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{1\text{ hr }45\text{ min}}$
c. $\frac{3\text{ in}}{8\text{ ft}} = \frac{\quad}{2\text{ ft}} = \frac{\quad}{12\text{ ft}} = \frac{\quad}{20\text{ ft}}$	**d.** $\frac{115\text{ words}}{2\text{ min}} = \frac{\quad}{1\text{ min}} = \frac{\quad}{3\text{ min}}$

2. **a.** Jake can ride 8 miles in 14 minutes. How long will it take him to ride 36 miles? Use the equivalent rates.

$$\frac{8\text{ miles}}{14\text{ minutes}} = \frac{4\text{ miles}}{\quad\text{ minutes}} = \frac{36\text{ miles}}{\quad\text{ minutes}}$$

b. How many miles can Jake ride in 35 minutes?

3. A car can go 50 miles on 2 gallons of gasoline.

a. How many gallons of gasoline would the car need for a trip of 60 miles? Use the equivalent rates below.

$$\frac{50\text{ miles}}{2\text{ gallons}} = \frac{5\text{ miles}}{\quad\text{ gallons}} = \frac{60\text{ miles}}{\quad\text{ gallons}}$$

b. How far can the car travel on 15 gallons of gasoline?

Example 2. You get 20 erasers for $1.80. How much would 22 erasers cost?

Cost (C)			$0.90	$1.80	
Erasers (E)	1	2	10	20	22

You can solve this problem in several ways. Let's use a table of rates this time.

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of $1.80. Two erasers will cost one-fifth of that (divide by 5 to find it!). Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: In the table, each pair of numbers is a rate. For example, $1.80 for 20 erasers (or $1.80/20 erasers) is a rate, and so is $0.90 for 10 erasers.

Note 2: We can write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one): $C = 0.09E$. In other words, the cost is 0.09 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get with $1.35?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots? A table of rates can help you solve this.

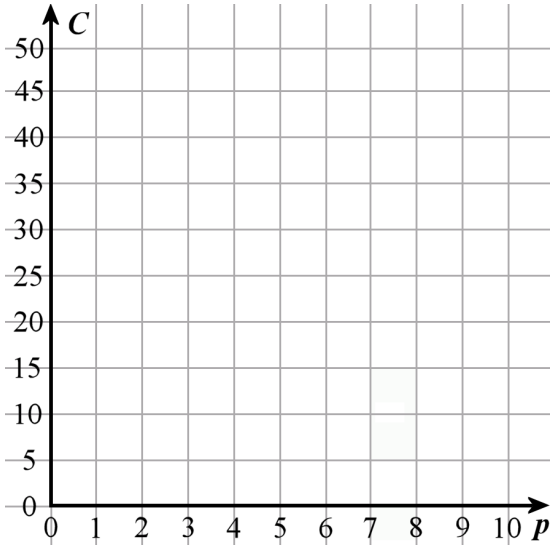
baskets						
shots						

7. **a.** Three pairs of socks cost $9. Fill in the table of rates. The variable *C* stands for cost, and *p* for pairs of socks.

C			9							
p	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates. Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (*p*).



8. **a.** You get 30 pencils for $4.50. How much would 52 pencils cost?

Cost						
Pencils						

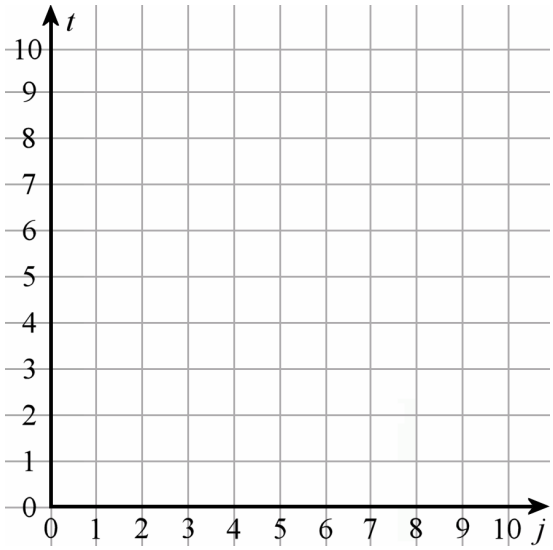
b. Write an equation relating the cost (C) and the number of pencils (P).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

a. Fill in the table. The variable *t* stands for the amount of tea, and *j* for the number of jars.

t					4					
j	1	2	3	4	5	6	7	8	9	10

b. Plot the number pairs from the table in this coordinate grid.



c. How many jars will Kate need for 20 liters of tea?

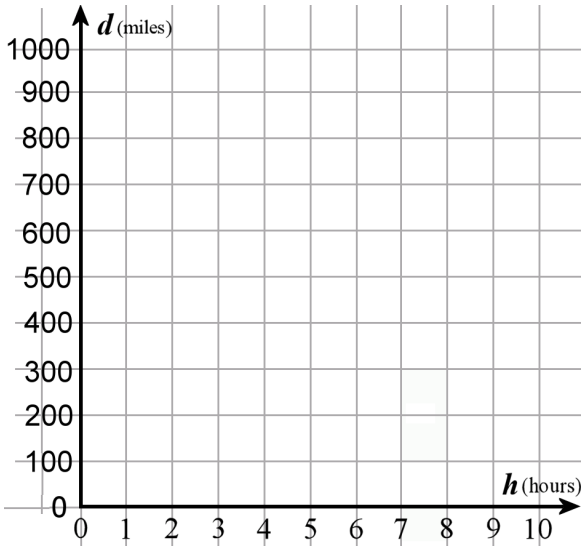
d. If Kate has 16 jars full of tea, how many liters of tea is in them?

10. **a.** A train travels at a constant speed of 80 miles per hour. Fill in the table of rates.

d										
h	1	2	3	4	5	6	7	8	9	10

b. Write an equation relating the distance (*d*) and the number of hours (*h*).

c. Plot the points in the grid on the right. The variable *h* stands for hours, and *d* for distance.



11. Another train travels at the constant speed of 60 miles per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. *How* can you see <u>from the graph</u> which train travels faster?

13. The plot shows the walking speeds for two people (t is in minutes, d is in miles). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates.
(Hint: For some of the points, you will need to use fractions and mixed numbers.)

Person 1 (red dot)

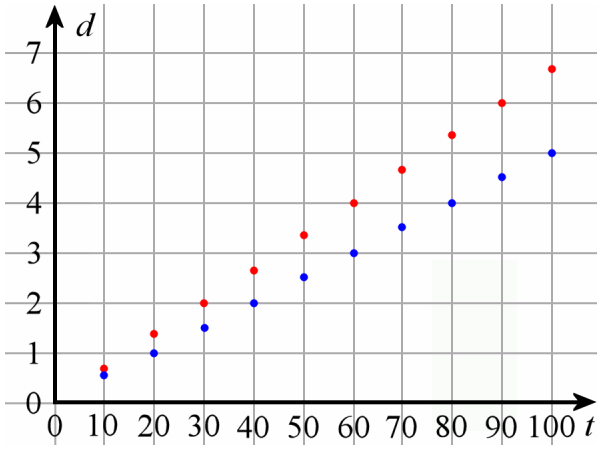
***d* (miles)**										
***t* (minutes)**										

Person 2 (blue dot)

***d* (miles)**										
***t* (minutes)**										

a. What is the speed of the first person in miles per hour?

b. What is the speed of the second person in miles per hour?



14. Train 1 travels at a constant speed of 240 miles in three hours. Train 2 travels 490 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: \$45 for eight bottles of shampoo, or \$34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to lowest terms:

640 people *out of* 1000 people = _____ people *out of* _____ people

b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

(This page intentionally left blank.)

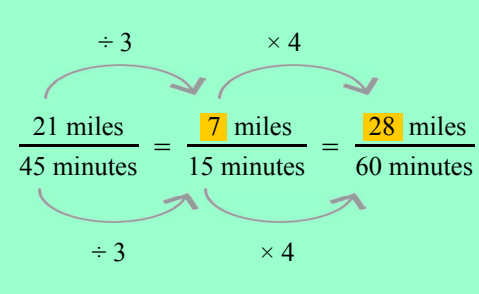
Using Equivalent Rates

Example 1. If Jake can ride his bike to a town that is 21 miles away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 21 miles per 45 minutes and hoping to arrive at so many miles per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, let's first figure the rate for 15 minutes, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.



Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 28 miles in one hour.

1. Write the equivalent rates.

a. $\frac{15\text{ km}}{3\text{ hr}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{15\text{ min}} = \frac{\quad}{45\text{ min}}$	**b.** $\frac{\$6}{45\text{ min}} = \frac{\quad}{15\text{ min}} = \frac{\quad}{1\text{ hr}} = \frac{\quad}{1\text{ hr }45\text{ min}}$
c. $\frac{3\text{ in}}{8\text{ ft}} = \frac{\quad}{2\text{ ft}} = \frac{\quad}{12\text{ ft}} = \frac{\quad}{20\text{ ft}}$	**d.** $\frac{115\text{ words}}{2\text{ min}} = \frac{\quad}{1\text{ min}} = \frac{\quad}{3\text{ min}}$

2. **a.** Jake can ride 8 miles in 14 minutes. How long will it take him to ride 36 miles? Use the equivalent rates.

$$\frac{8\text{ miles}}{14\text{ minutes}} = \frac{4\text{ miles}}{\quad\text{ minutes}} = \frac{36\text{ miles}}{\quad\text{ minutes}}$$

b. How many miles can Jake ride in 35 minutes?

3. A car can go 50 miles on 2 gallons of gasoline.

a. How many gallons of gasoline would the car need for a trip of 60 miles? Use the equivalent rates below.

$$\frac{50\text{ miles}}{2\text{ gallons}} = \frac{5\text{ miles}}{\quad\text{ gallons}} = \frac{60\text{ miles}}{\quad\text{ gallons}}$$

b. How far can the car travel on 15 gallons of gasoline?

Example 2. You get 20 erasers for $1.80. How much would 22 erasers cost?

Cost (C)			$0.90	$1.80	
Erasers (E)	1	2	10	20	22

You can solve this problem in several ways. Let's use a table of rates this time.

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of $1.80. Two erasers will cost one-fifth of that (divide by 5 to find it!). Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: In the table, each pair of numbers is a rate. For example, $1.80 for 20 erasers (or $1.80/20 erasers) is a rate, and so is $0.90 for 10 erasers.

Note 2: We can write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one): $C = 0.09E$. In other words, the cost is 0.09 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get with $1.35?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make when he tries 200 shots? A table of rates can help you solve this.

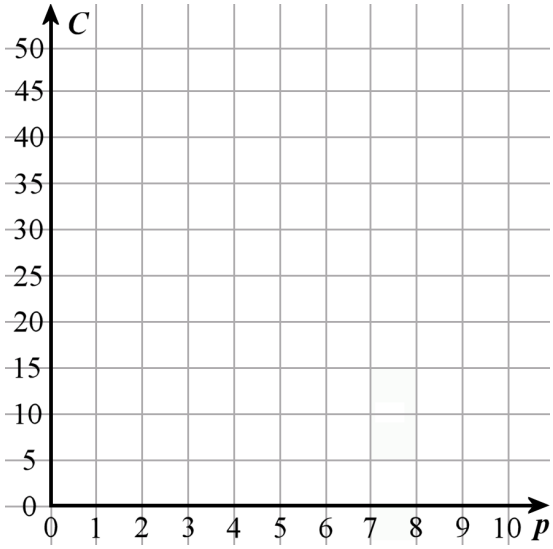
baskets						
shots						

7. **a.** Three pairs of socks cost $9. Fill in the table of rates. The variable *C* stands for cost, and *p* for pairs of socks.

C			9							
p	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates. Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (*p*).



8. **a.** You get 30 pencils for $4.50. How much would 52 pencils cost?

Cost						
Pencils						

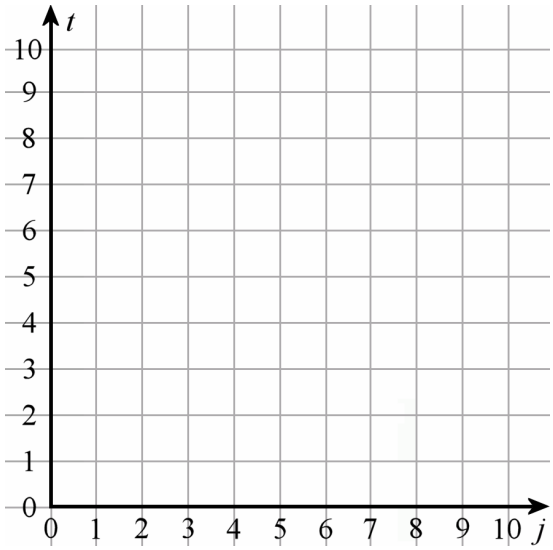
b. Write an equation relating the cost (C) and the number of pencils (P).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

a. Fill in the table. The variable *t* stands for the amount of tea, and *j* for the number of jars.

t					4					
j	1	2	3	4	5	6	7	8	9	10

b. Plot the number pairs from the table in this coordinate grid.



c. How many jars will Kate need for 20 liters of tea?

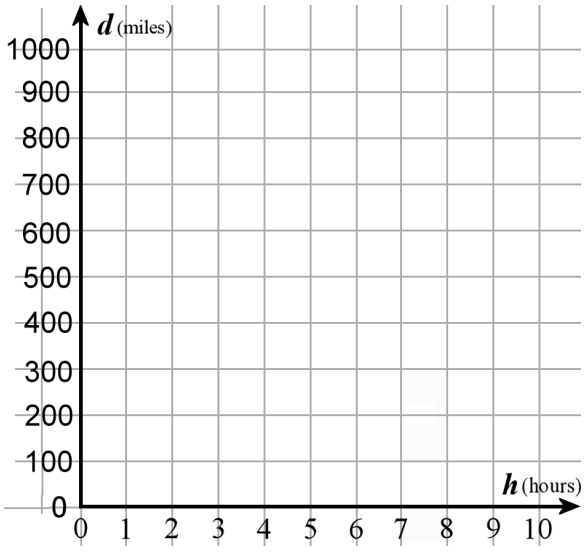
d. If Kate has 16 jars full of tea, how many liters of tea is in them?

10. **a.** A train travels at a constant speed of 80 miles per hour. Fill in the table of rates.

d										
h	1	2	3	4	5	6	7	8	9	10

b. Write an equation relating the distance (*d*) and the number of hours (*h*).

c. Plot the points in the grid on the right. The variable *h* stands for hours, and *d* for distance.



11. Another train travels at the constant speed of 60 miles per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. *How* can you see <u>from the graph</u> which train travels faster?

13. The plot shows the walking speeds for two people (t is in minutes, d is in miles). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates.
(Hint: For some of the points, you will need to use fractions and mixed numbers.)

Person 1 (red dot)

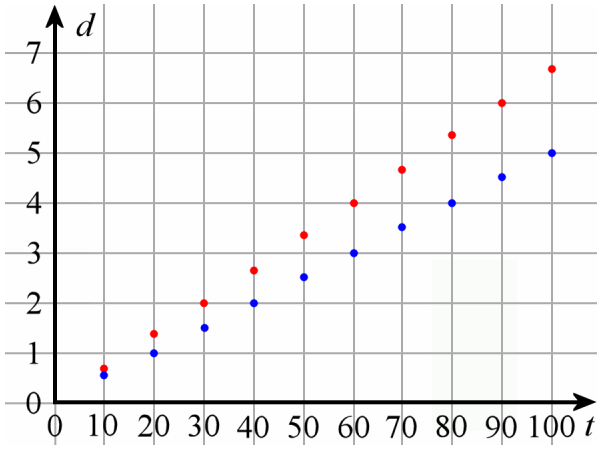
***d* (miles)**										
***t* (minutes)**										

Person 2 (blue dot)

***d* (miles)**										
***t* (minutes)**										

a. What is the speed of the first person in miles per hour?

b. What is the speed of the second person in miles per hour?



14. Train 1 travels at a constant speed of 240 miles in three hours. Train 2 travels 490 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: \$45 for eight bottles of shampoo, or \$34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to lowest terms:

640 people *out of* 1000 people = _____ people *out of* _____ people

b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

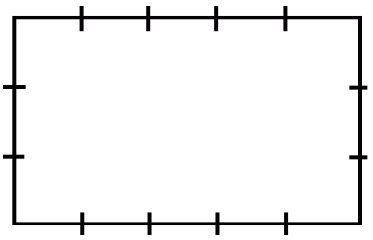
c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

(This page intentionally left blank.)

Aspect Ratio

You might have heard about the **aspect ratio** of the screens of televisions, computer monitors, and other monitors. The aspect ratio is simply **the ratio** of a **rectangle's width to its height**.

Example. A rectangle's width and height are in a ratio of 5:3. This means the aspect ratio is 5:3. If the rectangle's perimeter is 64 cm, what are its width and its height?



Let's draw the rectangle. Working from the 5:3 aspect ratio, let's divide the sides into "parts," or the same-sized segments, 5 for the width, and 3 for the height. We can see in the picture that perimeter is made up of 16 of these "parts." Since 64 ÷ 16 = 4, each part is 4 cm long.

Therefore, the rectangle's width is 5 × 4 cm = 20 cm, and its length is 3 × 4 cm = 12 cm.

1. The width and height of a rectangle are in a ratio of 9:2.

   a. Draw the rectangle, and divide its width and length into parts according to its aspect ratio.

   b. If the rectangle's perimeter is 220 cm, find its width and its height.

2. A rectangle's width is three times its height, and its perimeter is 120 mm. Find the rectangle's width and its height.

3. Find the aspect ratio of each rectangle:

   a. a rectangle whose height is 2/5 of its width

   b. a rectangle whose height is five times its width

   c. a square

4. The door of a fridge is 4/9 as wide as it is tall.

   a. What is the ratio of the door's width to its height?

   b. If the door is 54 cm wide, how tall is it?

5. Little Mary drew a picture on a rectangular piece of paper that was 6 inches wide and 9 inches high.

a. Write the aspect ratio, and simplify it to lowest terms.

b. If this picture were enlarged to be 20 inches *wide*, how high would it be? Use equivalent ratios.

6. Mr. Miller is ordering custom-made windows for his new house. He is considering windows of these sizes: 70 cm × 90 cm, 80 cm × 100 cm, 90 cm × 110 cm, and 100 cm × 120 cm.

a. Write the aspect ratios of all the windows and simplify them to lowest terms.

b. Do any of the windows share exactly the same aspect ratio when simplified? If so, then which ones? (That would mean they would have exactly the same shape.)

7. A sandbox is two times as wide as it is long.

a. What is its aspect ratio?

b. The perimeter of the sandbox is 15 ft. Find its length and width.

c. Find its area.

Don't confuse area with perimeter.

The aspect ratio pertains to the *length* and *width*, not to the area. However, once you know the length and the width, you can calculate the area.

8. Two television sets have the same perimeter, 150 cm. The aspect ratio of one is 16 : 9, and the aspect ratio of the other is 4 : 3.

a. Find the length and width of each television.

b. Which television has the larger area?

9. The area of a square is 49 in^2. If two of these squares are put side by side, we get a rectangle.

a. Find the aspect ratio of that rectangle.

b. Find the perimeter of the rectangle.

(This page intentionally left blank.)

Chapter 5: Percent
Introduction

The concept of percent builds on the student's understanding of fractions and decimals. Specifically, students should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students who have used Math Mammoth have been practicing that concept since 4th grade. One reason why I have emphasized finding a fractional part of a whole so much in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered how to find a fractional part a whole, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

The first lesson, *Percent*, practices the concept of percent as a hundredth part and how to write fractions and decimals as percentages. Next, we study how to find a percentage when the part and the whole are given (for example, if 15 out of 25 club members are girls, what percentage of them are girls?).

The following two lessons have to do with finding a certain percentage of a given number or quantity. First, we study how to do that using mental math techniques. For example, students find 10% of $400 by dividing $400 by 10. Next, students find a percentage of a quantity using decimal multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0.17 × 45 km.

I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or some other method (such as changing 17% into the fraction 17/100 for calculations). That is because using decimals is simpler: we simply change the percentage into a decimal and multiply, instead of having to build a proportion or use fractions. Also, decimals will be so much easier to use later on when solving word problems that require the usage of equations.

Next is a lesson about discounts, which is an important application from everyday life. Then we go on to the lesson *Practice with Percent*, which contrasts the two types of problems students have already studied: questions that ask to calculate a given percentage of a number and questions that ask to find the percentage. For example, the first type of question could be "*What is 70% of $380?*" and the second type could be "*What percentage is $70 of $380?*"

The last lesson lets students find the total when the percentage and the partial amount are known. For example: "Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?" We solve these with the help of bar models.

I have made several videos to match these lessons. You can watch them here:
http://www.mathmammoth.com/videos/percent.php

The Lessons in Chapter 5

Helpful Resources on the Internet

Percent videos by Maria
Videos on percent-related topics that match the lessons in this chapter!
http://www.mathmammoth.com/videos/percent.php

FRACTIONS, DECIMALS, AND PERCENTAGES

Fractions, Decimals, and Percentages Activity
This activity involves converting fractions to decimals, decimals to percentages, and percentages to fractions.
http://www.transum.org/Maths/Activity/FractionDecimalPercentage/

Fraction and Percentage Matching Game
Match each percentage to its equivalent fraction in this interactive activity.
http://transum.org/software/SW/Starter_of_the_day/Students/Matching.asp?ImageFolder=/Software/SW/Starter_of_the_day/Students/Pairs_Fractions_Percentages/img/&Topic=11

Fractions, Decimals, and Percentages Quiz
Practice converting between fractions, decimals, and percentages in this interactive 10-question quiz.
https://www.thatquiz.org/tq-e/?-jq8-l6-p0

Mission: Magnetite
Hacker tries to drop magnetite on Motherboard. To stop him, match up percentages, fractions, and images showing fractional parts.
http://pbskids.org/cyberchase/media/games/percent/

Fractions and Percent Matching Game
A simple matching game: match fractions and percentages.
http://www.mathplayground.com/matching_fraction_percent.html

Decention Game
Create teams consisting of three numbers. There must be one fraction, one decimal and one percent!
http://www.mathplayground.com/Decention/Decention.html

Fraction/Decimal/Percent Jeopardy
Answer the questions correctly, changing between fractions, decimals, and percentages.
http://www.quia.com/cb/34887.html

Flower Power
Grow and harvest flowers to make money in this addictive order-'em-up game. Practice ordering decimals, fractions, and percentages. The game starts with ordering decimals (daisies), and proceeds into fractions (tulips or roses).
http://www.mangahigh.com/en/games/flowerpower

Pie Charts Estimation Quiz
Estimate the size of the pie chart sectors in this self-check quiz.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Pie_Charts.asp

FINDING PERCENTAGES

Visual Percent
Enter any two values (part, whole, percent) and this manipulative will calculate the third value.
http://www.mathplayground.com/visualpercent.html

Penguin Waiter
Simple game where you calculate the correct tip to leave the penguin waiter.
http://www.funbrain.com/penguin/

Percent worksheets
Create an unlimited number of free customizable percent worksheets to print.
http://www.homeschoolmath.net/worksheets/percent-decimal.php
http://www.homeschoolmath.net/worksheets/percent-of-number.php
http://www.homeschoolmath.net/worksheets/percentages-words.php

Percentages of Something
See simple percentages illustrated in different ways.
http://www.bbc.co.uk/skillswise/game/ma16perc-game-percentages-of-something

A Conceptual Model for Solving Percent Problems
Explanation of how to use a 10 x 10 grid to explain the basic concept of percent, AND solve various types of percent problems.
http://illuminations.nctm.org/Lesson.aspx?id=960

Express as a Percentage Quiz
This self-check quiz requires you to work out what one quantity is as a percentage of a second quantity.
http://www.transum.org/Maths/Exercise/Express_As_A_Percentage.asp

Percentages Quiz - Mental Math
Test your understanding of simple percentages with this self-check quiz.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Percentages.asp?Level=5

Tipster Game
Learn to calculate tips at different percentages for different prices in this fun real-life game.
http://mrnussbaum.com/tipster-2/

Percentages Quiz
Practice finding the percentage of numbers with this interactive online quiz.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Percentages.asp?Level=6

Finding Percents
Find a percent of a quantity as a rate per 100, and solve problems involving finding the whole, given a part and the percent.
https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/finding_percents

Percent Shopping
Choose toys to purchase. In level 1, you find the sale price when the original price and percent discount are known. In level 2, you find the percent discount when the original price and the sale price are known.
http://www.mathplayground.com/percent_shopping.html

Percent: Finding the Whole
Practice finding the whole when the percentage is given.
http://www.mathplayground.com/percent02.html

Percent Word Problems
Practice solving word problems involving percentages in this interactive online exercise.
https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-ratios-rates-percentages/e/percentage_word_problems_1

(This page intentionally left blank.)

Percentage of a Number (Mental Math)

100% of something means *all* of it. 1% of something means 1/100 of it.
Since one percent means "a hundredth part," calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions**!
How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg. **To find 1% of something (1/100 of something), divide by 100.** Do you remember how to divide by 100 mentally? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.
To find 2% of some quantity, first find 1% of it, and double that. For example, let's find 2% of $6. Since 1% of $6 is $0.06, then 2% of $6 is $0.12.
To find 10% of some quantity, divide by 10. Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity! For example, 10% of $780 is $78. And 10% of $6.50 is $0.65. (To divide by 10 mentally, just move the decimal point one place to the left.)
Can you think of a way to find 20% of a number? (Hint: Start with finding 10% of the number.) _____

1. Find 10% of these numbers.

a. 700 _____ **b.** 321 _____ **c.** 60 _____ **d.** 7 _____

2. Find 1% of these numbers.

a. 700 _____ **b.** 321 _____ **c.** 60 _____ **d.** 7 _____

3. One percent of Mom's paycheck is $22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage ↓ number →	**1,200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _____.	50% of 244 is _____.
10% is $\frac{1}{\square}$. To find 10% of a number, divide by _____.	10% of 47 is _____.
1% is $\frac{1}{\square}$. To find 1% of a number, divide by _____.	1% of 530 is _____ .
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • **First find _____% of the number, and** • **then multiply by 2, 3, 4, 6, 7, 8, or 9.**	10% of 120 is _____. 30 % of 120 is _____. 60 % of 120 is _____.

6. Find the percentages. Use mental math.

a. 10% of 60 kg _____ 20% of 60 kg _____	**b.** 10% of $14 _____ 30% of $14 _____	**c.** 10% of 5 mi _____ 40% of 5 mi _____
d. 1% of $60 _____ 4% of $60 _____	**e.** 10% of 110 cm _____ 70% of 110 cm _____	**f.** 1% of $1,330 _____ 3% of $1,330 _____

7. David pays a 20% income tax on his $2,100 salary.

a. How many dollars is the tax?

b. How much money does he have left after paying the tax?

c. What percentage of his salary does he have left?

8. Nancy pays 30% of her $3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of $55. Peter's solution: 10% of $55 is $5.50 So, I subtract 100% − $5.50 = $94.50	**b.** Find 6% of $1,400. Patricia's solution: 1% of $1,400 is $1.40. So, 6% is six times that, or $8.40.

Some more mental math "tricks"	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	**25% of a quantity** 25% is the same as 1/4. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	**75% of a quantity** 75% is 3/4. First find 1/4 of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 in _____	**b.** 25% of 40 ft _____	**c.** 80% of 45 m _____
d. 75% of $4.40 _____	**e.** 90% of 1.2 m _____	**f.** 25% of 120 lb _____

11. Fill in the mental math method for finding 12% of $65.

10% of $65 is $_____. 1% of $65 is $_____. 2% of $65 is $_____.

Now, add to get 12% of $65: $_____ + $_____ = $_____

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is _____ kg. 1% of 44 kg is _____ kg.

Subtract _____ kg − _____ kg = _____ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut, and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

(This page intentionally left blank.)

Discounts

Other than figuring sales tax, the area of life in which you will probably most often need to use percentages is in calculating discounts.

A laptop that costs $600 is 20% off. What is the sale price?

Method 1. We calculate 20% of $600. That is the discounted amount in *dollars*.
Then we subtract that from the original price, $600.

20% of $600 is $120. And $600 − $120 = $480. So the sale price is $480.

Method 2. Since 20% of the price has been removed, 80% of the price is *left*.
By calculating 80% of the original price, you will get the new discounted price: 0.8 × $600 = $480

Two methods for calculating the discounted price:

1. Calculate the discount amount as a percentage of the original price. Then subtract.
2. Find what percentage of the price is left. Then calculate that percentage of the normal price.

1. All of these items are on sale. Calculate the discount in dollars and the resulting sale price.

a.  Price: $90
20% off

Discount amount: $ 18

Sale price: $_____

b.  Price: $5
40% off

Discount amount: $_____

Sale price: $_____

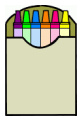
c.  Price: $15
30% off

Discount amount: $_____

Sale price: $_____

2. A $25 swimsuit was on sale for 20% off.
Monica tried to calculate the discounted price this way: $25 − $20 = $5.
What did she do wrong? Find the correct discounted price.

3. All these items are on sale. Find the discounted price.

a. Price: $1.20
25% off 

Discount amount: $_____

Discounted price: $_____

b. Price: $18
25% off 

Discount amount: $_____

Discounted price: $_____

c. Price: $150
30% off 

Discount amount: $_____

Discounted price: $_____

d. Price: $20
40% off 

Discount amount: $_____

Discounted price: $_____

e. Price: $2.20
10% off 

Discount amount: $_____

Discounted price: $_____

f. Price: $1.30
50% off 

Discount amount: $_____

Discounted price: $_____

You can often use **estimation** when calculating the discounted price.

Example 1. A $198.95 bicycle is discounted by 25%. What is the discounted price?

To estimate, round the original price of the bicycle to $200. Then, 25% of $200 is $50 (it is 1/4 of it). So the discounted price is about $150.

Example 2. A $425.90 laptop is discounted by 28%. What is the discounted price?

Round the discount percentage to 30%, and the price of the laptop to $430. 10% of $430 is $43. 30% of $430 is three times that much, or $129. Subtract using rounded numbers: $430 − $130 = $300.

4. *Estimate* the discounted price.

   a. 30% off of a $39.90 book

   b. 17% off of a $12.50 block of cheese

   c. 75% off of a $75.50 pair of shoes

5. Which is a better deal? Estimate using rounded numbers and mental math.

   a. 75% off of a $199 brand-name mp3 player.
   OR an equivalent off-brand mp3 player for $44.99

   b. 40% off of a new, $89 textbook
   OR a used copy, like new, of the same textbook for $39.90.

6. A company sells a computer program for $39.99. They estimate they would sell 50 copies of it in a week, with that price. If they discount the price by 25%, they think they could sell 100 copies. *Estimate* which way they would earn the most money.

Example 3. A pair of shoes costing $50 is discounted and now costs only $35.
What is the discount percentage?
Think about what *fraction* of the price "disappeared." Then, write that fraction as a percentage.
We see that $15 of the price "went away." The fraction of the price that was taken off is 15/50.
Now we simply rewrite 15/50 as 30/100, which is, as a percentage, 30%. So it was discounted by 30%.

7. Find the discounted percentage.

   a. Some jeans: original price, $50; discounted price, $45.

   b. A phone: original price, $40; discounted price, $30.

   c. A haircut: original price, $25; discounted price, $20.

8. Which of these methods work for calculating a discounted price of 25% off of $46?

$0.25 \times \$46$	$0.75 \times \$46$	$\$46 - \frac{\$46}{25}$	$\$46 - \frac{\$46}{4}$	$\frac{\$46}{4}$	$\frac{\$46}{4} \times 3$