math MAMMOTH

Grade 5-A Worktext

Light Blue Series

- The four operations
- Large numbers and the calculator
- Problem solving
- Decimals

By Maria Miller

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Foreword

Math Mammoth Grade 5-A Worktext comprises a complete math curriculum for the first half of fifth grade mathematics. The fifth grade is time for fractions and decimals, in particular. In part 5-A, we study decimals in depth and with substantial detail. Students also deepen their understanding of whole numbers, learn much more problem solving, and get introduced to the calculator.

The year starts out with a study of whole numbers, their operations, and problem solving. Students get to review multi-digit multiplication and long division. Then we study equations with the help of a balance and bar models (simple diagrams). The main idea is to get students used to the idea of an equation and what it means to solve an equation.

In the second chapter, the focus is on large numbers and using a calculator. This is the first time a calculator is introduced in Math Mammoth complete curriculum—thus far, all calculations have been done mentally, or with paper and pencil. I want students to learn to be critical in their use of the calculator—use it with good judgment. Every exercise where calculator use is to be allowed is marked with a little calculator symbol.

The third chapter is about decimals and their operations. It is a long chapter because now is the time to learn decimal operations well. It is assumed that the student already has a solid foundation for decimal place value, as taught in Math Mammoth 4th grade curriculum. That is the true means of preventing common misconceptions, or students resorting to rote memorization of the decimal operations.

In part 5-B, students study graphing, fractions, and geometry.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 1: The Four Operations
Introduction

We start fifth grade by studying the four basic operations. This includes studying the order of operations, simple equations and expressions, long multiplication, long division, divisibility, primes, and factoring.

The main line of thought throughout this chapter is that of a mathematical *expression*. In mathematics, an expression consists of numbers, letters, and operation symbols, but does not contain an equal sign (an equation does). Students write simple expressions for problems they solve. They study the correct order of operations in an expression.

An *equation* in mathematics consists of an expression that equals another expression (expression = expression). We study simple equations, both with the help of visual bar models and also without. Bar models are also used for simple multiplication and division equations.

Next, we review multi-digit multiplication (multiplying in columns), starting with multiplying in parts (partial products) and how that can be visualized geometrically. Then it is time for long division, especially practicing long division with two-digit divisors. We also study why long division works, in the lesson *Long Division and Repeated Subtraction*. All along there are also word problems to solve.

Lastly, we study the topics of divisibility, primes, and factoring. Students learn the common divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10. In prime factorization, we use factor trees.

Although the chapter is named "The Four Operations," please notice that the idea is not to practice each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are trying to develop students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, "undo" operations, and so on. Many of the ideas in this chapter are preparing them for algebra in advance.

The Lessons in Chapter 1

	page	span
Warm Up: Mental Math	11	*2 pages*
The Order of Operations and Equations	13	*3 pages*
Review: Addition and Subtraction	16	*3 pages*
Review: Multiplication and Division	19	*4 pages*
Multiplying in Parts	23	*6 pages*
The Multiplication Algorithm	29	*5 pages*
More Multiplication	34	*5 pages*
Long Division	39	*4 pages*
A Two-Digit Divisor 1	43	*4 pages*
A Two-Digit Divisor 2	47	*3 pages*
Long Division and Repeated Subtraction	50	*5 pages*
Divisibility Rules	55	*5 pages*
Review: Factors and Primes	60	*4 pages*
Prime Factorization	64	*5 pages*
Chapter 1 Review	69	*3 pages*

Helpful Resources on the Internet

Long division & multiplication

Rectangle Multiplication
An interactive tool that illustrates multiplying in parts using the area model. Choose the "common" option for this grade level, to show multiplying in parts.
http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html

Snork's Long Division Game
Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems. In the beginning, choose the highest number you want to work with (the divisor) to be a two-digit number, in order to practice with two-digit divisors.
http://www.kidsnumbers.com/long-division.php

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Short Division
A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.
http://www.themathpage.com/ARITH/divide-whole-numbers.htm

All four operations

Math Mahjong
A Mahjong game where you need to match tiles with the same value. It uses all four operations and has three levels.
http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

Pop the Balloons
Pop the balloons in the order of their value. You need to use all four operations.
http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/index.html

Order of operations

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

Connect-the-Four
Solve very simple math problems about the order of operations and get to play connect-the-four game. Requires Java.
http://www.shodor.org/interactivate/activities/OrderOfOperationsFou/

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parentheses in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, —, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.htm

Order of Operations Practice
A simple online quiz of 10 questions. Uses parentheses and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate
Practice your arithmetic of all four operations plus the order of operations.
http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Factors and primes

Factor Game
Interactive game to practice divisibility among numbers 1-100. Play against the computer or a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

Factor Feeder
Eat factors of the given number, and avoid numbers that are not factors of the given number in this Pacman-style game. Use Arrow Keys to move.
http://hoodamath.com/games/factorfeeder.php

Primes, Factors and Divisibility - Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes/

Sliding Tile Factorization Game
Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.
http://www.visualmathlearning.com/Games/sliding_factors.html

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

Octopus Factors
Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.
http://www.counton.org/games/map-numbers/octopus/

Factors Millionaire Game
A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.
http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Not a Factor
Choose a number that is NOT a factor of the given number.
http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

MathGoodies Interactive Factor Tree Game
Type in a missing number to the factor tree, and the program will find the other factor, and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.pompuzzle.com/Snake

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Primes, Factors and Divisibility—Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes

Prime Number Calculator
This calculator tests if a number is a prime, and tells you its smallest divisor if it is not prime.
http://www.basic-mathematics.com/prime-number-calculator.html

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many are there, and more.
http://primes.utm.edu/

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips, and historical information how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.
http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20

Primality of 1 from Wikipedia
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and Against the Primality of 1
http://primefan.tripod.com/Prime1ProCon.html

Unique Prime Factorization
A video explaining the fundamental theorem of arithmetic: that each composite number has a unique prime factorization.
http://www.youtube.com/watch?v=5kl28hmhin0

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Order of Operations and Equations

- ***First*** **solve whatever is inside parentheses.**
- ***Next,*** **solve multiplications and divisions "on the same level," from left to right.**
- ***Last,*** **solve additions and subtractions "on the same level," from left to right.**

1. Solve what is within parenthesis first. You can enclose the operation to be done first in a "bubble."

$(36 + 4) \div (5 + 5)$ $= 40 \div 10$ $= 4$	**a.** $(50 - 2) \div (3 + 5)$	**b.** $20 \times (1 + 7 + 5)$
	c. $2 \times (600 \div 60) + (19 - 8)$	**d.** $180 \div (13 - 7 + 3)$

2. Solve. When there are many multiplications and divisions, do them from left to right.

$24 \div 3 \times 2 \div 4$ $= 8 \times 2 \div 4$ $= 16 \div 4 = 4$	**a.** $36 \div 4 \div 3$	**b.** $1{,}200 \div 4 \times 5 \div 3$
	c. $7 \times 90 \div 2 \times 2 \div 10$	**d.** $5 \times 6 \div 3 \div 2 \times 20$

3. Solve in the right order. You can enclose the operation to be done first in a "bubble" or a "cloud."

a. $12 \times 5 + 8 =$ _____	**b.** $10 + 2 \times 9 + 8 =$ _____
c. $45 + 5 \times 7 =$ _____	**d.** $10 + 2 \times (9 + 8) =$ _____
e. $(8 + 16) \div 3 \div 2 =$ _____	**f.** $2 \times (100 - 80 + 20) =$ _____
g. $120 - 2 \times (11 - 5) =$ _____	**h.** $25 + 8 \times 5 \div 2 =$ _____

4. Division can also be written with a fraction line. Solve in the right order.

a. $6 + \frac{24}{2} =$ _____	**b.** $\frac{32}{2} - 6 =$ _____	**c.** $\frac{54}{6} - 6 - 2 =$ _____

An **equation** has numbers, letters, operation symbols, and one equal sign, "=".
It's called an *equa*tion because it contains an *equal* sign. For example, "5 = 1 + 4" is an equation.

An **expression** only has numbers, letters, and operation symbols—but no equal sign.
For example, "40 × 2 + 6 × 5" is an expression.

5. Equation or expression? (Do not solve these.)

a. $4t = 180$ **b.** $2 + 60 \times 345 \div 9$ **c.** $15 = x + y$

d. $\frac{5.4 - 2.12}{0.4} = 8.2$ **e.** $1{,}000 = 1{,}000$ **f.** $12 - \frac{24 \div 0.8}{189}$

6. Which expression matches each problem? Also, solve the problems.

a. Mark bought three light bulbs for $8 each, and paid with $50. What was his change?

(1) $3 \times \$8 - \$50$ **(2)** $\$50 - \$8 + \$8 + \$8$
(3) $\$50 - 3 \times \$8$ **(4)** $\$50 - (\$8 - \$8 - \$8)$

b. Shirts costing $16 each are discounted by $5, so mom buys six of them. What is her total cost?

(1) $\$16 - \$5 \times 6$ **(2)** $6 \times (\$16 - \$5)$
(3) $\$16 \times 6 - \$5$ **(4)** $(\$16 - 6) \times 5$

c. Andy buys a salad for $8 and a pizza for $13, and shares the cost evenly with his friend. How many dollars is Andy's share of the cost?

(1) $\$8 + \$13 \div 2$ **(2)** $\$2 \div (\$8 + \$13)$
(3) $2 \times \$8 + 2 \times \$13$ **(4)** $(\$8 + \$13) \div 2$

d. Melissa shares equally the cost of a meal with three other people and the cost of a taxi with two other people. The meal costs $48 and the taxi costs $30. How much does Melissa pay?

(1) $\$48 \div 4 + \$30 \div 3$ **(2)** $(\$48 + \$30) \div 3 \div 2$
(3) $\$48 \div 3 + \$30 \div 2$ **(4)** $(\$48 + \$30) \div 5$

$120 - 75 = 3 \times 15$ This is the left side of the equation. This is the right side of the equation. Do the left and right sides have the same value? Just calculate $120 - 75$, then calculate 3×15, and check. If yes, it's a **true equation**. If not, it's a **false equation**.	$2 = 5$ left side right side This is a very simple equation —but it is false! $4 + 5 = 21 - 3$ left side right side This is also a false equation!	$18 = x - 3$ left side right side Solving the equation means finding the value of x (the **unknown**) that makes it true. The value $x = 21$ makes this equation true, so we say $x = 21$ is the solution.

7. If the equation is false, change one number in it to make it true.

a. $6 + \frac{32}{8} = 5$	**b.** $(6 - 2) \times 3 = 5 + 5$	**c.** $5 \times 2 = 16 \div 2 + 2$

8. Place parenthesis into these equations to make them true.

a. $10 + 40 + 40 \times 2 = 180$	**b.** $144 = 3 \times 2 + 4 \times 8$	**c.** $40 \times 3 = 80 - 50 \times 4$

9. Find a number to fit in the box so the equation is true.

a. $40 = (\square + 9) \times 2$	**b.** $4 \times 8 = 5 \times 6 + \square$	**c.** $4 + 5 = (20 - \square) \div 2$
d. $81 = 9 \times (2 + \square)$	**e.** $\square \times 11 = 12 + 20 \times 6$	**f.** $(4 + 5) \times 3 = \square \div 2$

10. Solve these simple equations.

a. $s \times 2 = 660$ $s =$ _____	**b.** $\frac{x}{2} = 5$ $x =$ _____	**c.** $200 - y = 60$ $y =$ _____

11. Build at least three true equations using (only) the symbols and numbers given. You may use the same number or symbol many times.

11, 3, 1, −, +, ×, ( ), =

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A Two-Digit Divisor 1

Often, it is helpful to write the multiplication table of the divisor before you divide.

Example 1. The division is by 16. Here is the multiplication table of 16:

3 × 16 = 48
4 × 16 = 64
5 × 16 = 80
6 × 16 = 96
7 × 16 = 112
8 × 16 = 128
9 × 16 = 144

```
  0 3
16)5 5 6 8
```

16 goes into 5 zero times, so we look at 55.

How many times does 16 go into 55?

Check in the table on the left. We see it goes into 55 three times.

```
  0 3 4
16)5 5 6 8
  -4 8
     7 6
```

Now, how many times does 16 go into 76?

From the table we can see that it is four times.

```
  0 3 4 8
16)5 5 6 8
  -4 8
     7 6
   - 6 4
     1 2 8
   - 1 2 8
         0
```

Lastly, 16 goes into 128 exactly 8 times, and the division is over.

Example 2. We are dividing by 32. Here is the multiplication table of 32:

3 × 32 = 96
4 × 32 = 128
5 × 32 = 160
6 × 32 = 192
7 × 32 = 224
8 × 32 = 256
9 × 32 = 288

```
  0 1
32)4 7 0 7
  -3 2
   1 5
```

32 goes into 47 once.

```
  0 1 4
32)4 7 0 7
  -3 2
   1 5 0
  -1 2 8
     2 2
```

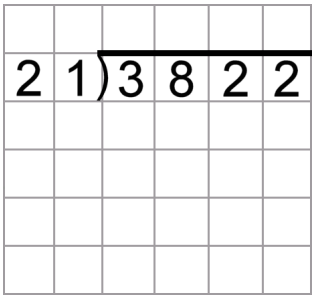
32 goes into 150 four times.

```
  0 1 4 7
32)4 7 0 7
  -3 2
   1 5 0
  -1 2 8
     2 2 7
   - 2 2 4
         3
```

32 goes into 224 seven times. Notice there is a remainder.

1. Divide. First write a multiplication table for the divisor. Check each answer by multiplying.

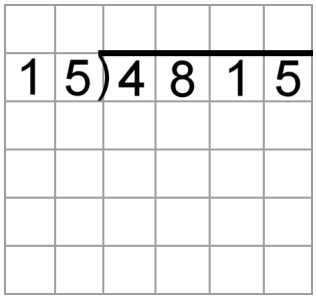
Table of 21:
2 × 21 =
3 × 21 =
4 × 21 =
5 × 21 =
6 × 21 =
7 × 21 =
8 × 21 =
9 × 21 =



2. Divide. First write a multiplication table for the divisor. Check each answer by multiplying.

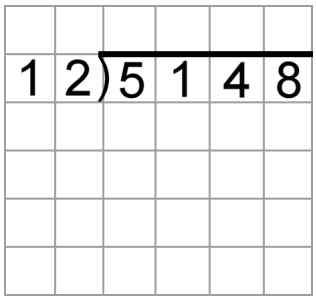
a.

Table of 15:
2 × 15 =
3 × 15 =
4 × 15 =
5 × 15 =
6 × 15 =
7 × 15 =
8 × 15 =
9 × 15 =



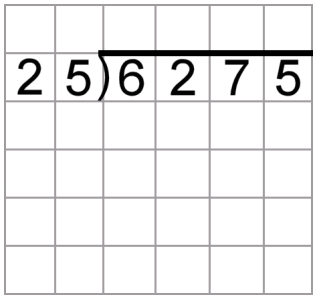
b.

Table of 12:
2 × 12 =
3 × 12 =
4 × 12 =
5 × 12 =
6 × 12 =
7 × 12 =
8 × 12 =
9 × 12 =



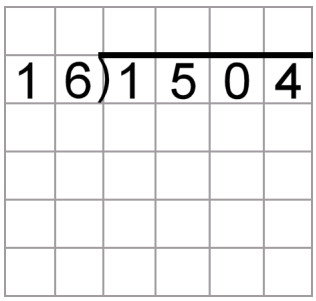
c.

Table of 25:
2 × 25 =
3 × 25 =
4 × 25 =
5 × 25 =
6 × 25 =
7 × 25 =
8 × 25 =
9 × 25 =



d.

Table of 16:
2 × 16 =
3 × 16 =
4 × 16 =
5 × 16 =
6 × 16 =
7 × 16 =
8 × 16 =
9 × 16 =



3. Divide. Check each answer by multiplying.

a.

Table of 12:
2 × 12 =
3 × 12 =
4 × 12 =
5 × 12 =
6 × 12 =
7 × 12 =
8 × 12 =
9 × 12 =

$12\overline{)888}$

b.

Table of 22:
2 × 22 =
3 × 22 =
4 × 22 =
5 × 22 =
6 × 22 =
7 × 22 =
8 × 22 =
9 × 22 =

$22\overline{)6710}$

c.

Table of 14:
2 × 14 =
3 × 14 =
4 × 14 =
5 × 14 =
6 × 14 =
7 × 14 =
8 × 14 =
9 × 14 =

$14\overline{)1736}$

d.

Table of 51:
2 × 51 =
3 × 51 =
4 × 51 =
5 × 51 =
6 × 51 =
7 × 51 =
8 × 51 =
9 × 51 =

$51\overline{)7548}$

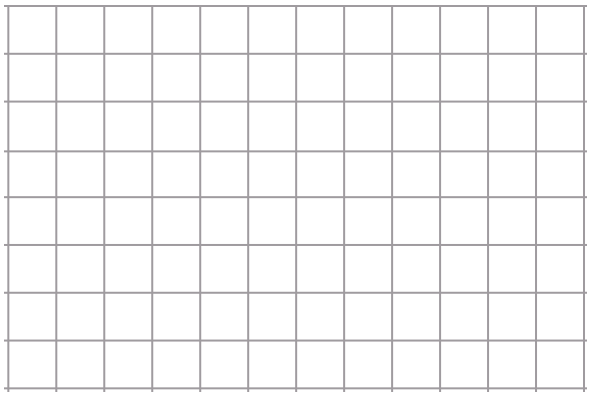
4. Mental math! If 20 goes into 800 forty times, then 20 goes into 820 one time more, or 41 times. In each box, use the top problem to help you solve the bottom problem.

a. 800 ÷ 20 = 820 ÷ 20 =	**b.** 700 ÷ 50 = 750 ÷ 50 =	**c.** 150 ÷ 15 = 300 ÷ 15 =
d. 480 ÷ 40 = 520 ÷ 40 =	**e.** 600 ÷ 30 = 690 ÷ 30 =	**f.** 1,200 ÷ 60 = 1,320 ÷ 60 =

5. **a.** How many inches are in one foot?

   b. Convert 245 inches into feet and inches.

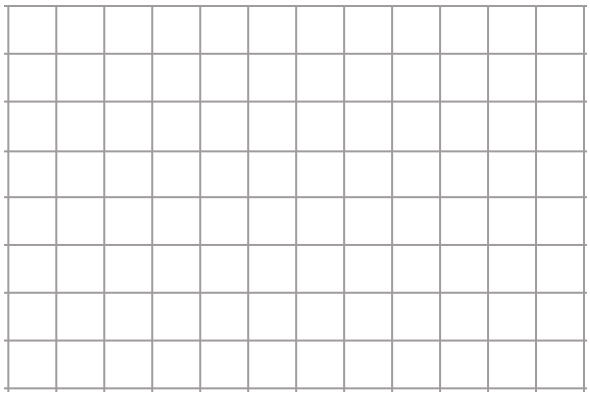
   c. Convert 387 inches into feet and inches.



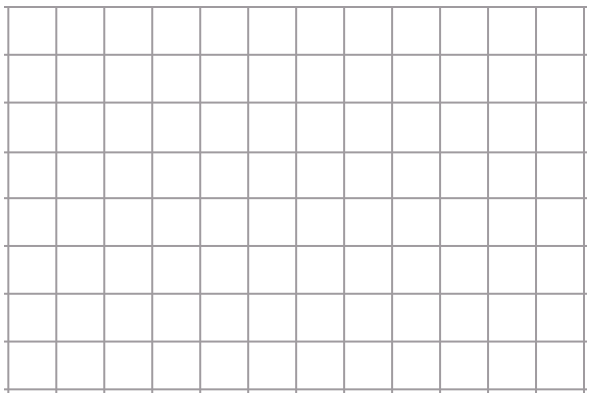
6. **a.** How many ounces are in one pound?

   b. Convert 163 ounces into pounds and ounces.

   c. Convert 473 ounces into pounds and ounces.



7. A newborn baby gains weight at approximately one ounce per day. Suppose that the baby gained weight at that rate for a FULL YEAR. (In reality, babies don't; their growth rate slows down.) How many pounds and ounces would the baby gain in a year?



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Chapter 2: Large Numbers and the Calculator

Introduction

In this chapter, we study large numbers and place value up to billions—that is, up to 12-digit numbers. We study adding, subtracting, rounding, exponents, and using a calculator.

This is the first time the calculator is introduced in Math Mammoth complete curriculum. I have delayed introducing the use of a calculator (as compared to many math curricula) for good reasons. I have received numerous comments on the harm that indiscriminate calculator usage can cause. In a nutshell, if children are allowed to use calculators freely, their minds get "lazy," and they will start relying on calculators even for simple things such as 6 × 7 or 320 + 50. It is just human nature!

As a result, students enter college without even knowing their multiplication tables by heart. Then they have tremendous trouble if they are required to use mental math to solve simple problems.

Therefore, we educators need to *limit* calculator usage until the students are much older. Children can *not* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is extremely useful, and students do need to learn to use it. In this curriculum, I strive to show the students not only *how* to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practice estimating the result before calculating it with a calculator. In the last lesson, students need to choose whether mental math or a calculator is the best "tool" for the calculation.

The Lessons in Chapter 2

	page	span
A Little Bit of Millions	75	*3 pages*
Place Value Up to Billions	78	*3 pages*
Exponents and Powers	81	*3 pages*
Adding and Subtracting Large Numbers	84	*3 pages*
Rounding	87	*3 pages*
The Calculator and Estimating	90	*3 pages*
When to Use the Calculator	93	*2 pages*
Mixed Review	95	*2 pages*
Review	97	*3 pages*

Helpful Resources on the Internet

Naming Numbers
These pages teach number naming skills covered in K8 math courses. Each page has an explanation, interactive practice and challenge games about naming numbers.
http://www.aaamath.com/B/nam.htm

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Powers of Ten
A 9-minute movie that illustrates the dramatic changes of scale when zooming in or out by powers of ten (40 powers of ten), starting from a picnic blanket and ending in the universe, and then starting from a hand to the proton inside an atom.
http://www.youtube.com/watch?v=0fKBhvDjuy0

Cookie Dough
Practices naming big numbers.
www.funbrain.com/numwords/index.html

Keep My Place
Fill in the big numbers to this cross-number puzzle.
http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html

Estimation
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences.
http://www.aaamath.com/B/est.htm

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Place Value Game
Create the largest possible number from the digits the computer gives you.
Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be.
http://education.jlab.org/placevalue/index.html

Free Exponent Worksheets
Create a variety of customizable, printable worksheets to practice exponents.
http://www.homeschoolmath.net/worksheets/exponents.php

Baseball Exponents
Choose the right answer from three possibilities before the pitched ball comes.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=95

Exponents Quiz from ThatQuiz.org
Ten questions, fairly easy, and not timed. You can change the parameters as you like to include negative bases, square roots, and even logarithms.
http://www.thatquiz.org/tq-2/?-j1-l4-p0

Exponents Jeopardy
The question categories include evaluating exponents, equations with exponents, and exponents with fractional bases.
http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html

Pyramid Math
Simple practice of either exponents, roots, LCM, or GCF. Drag the triangle with the right answer to the vase.
http://www.mathnook.com/math/pyramidmath.html

Exponents Battleship
A regular battleship game against the computer. Each time you "hit", you need to answer a math problem involving exponents (and multiplication).
http://www.quia.com/ba/1000.html

Exponent Battle
A card game to practice exponents. I would limit the cards to small numbers, instead of using the whole deck.
http://www.learn-with-math-games.com/exponent-game.html

Pirates Board Game
Steer your boat in pirate waters in this online board game, and evaluate powers.
http://mathgames4children.com/fun-board-games/6th-grade/pirate/exponents-pirate-waters-grade-6-game.html

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Adding and Subtracting Large Numbers

Just like 25 marbles + 54 marbles = 79 marbles, so will 25 million + 54 million = 79 million.

Just keep in mind: **a thousand thousands** makes a **million**, and **a thousand millions** makes a **billion**.

800,000 + 200,000	**Half a million**
Think of it as 800 thousand + 200 thousand. The answer is 1,000 thousand or 1,000,000.	Think of it as half of a thousand thousands, or 500 thousands = 500,000.
34,999,000 + 1,000	**2 billion − 300 million**
This is 34 million 999 thousand + 1 thousand, making 34 million 1000 thousand, or 35 million.	Think of it as 2,000 million − 300 million, which makes 1,700 million, or 1,700,000,000.

1. Add.

	a. 90,000	**b.** 99,000,000	**c.** 999,000
+ 1,000			
+ 10,000			
+ 100,000			
+ 1,000,000			

2. Match.

a.		**b.**	
1/2 million	750,000	1 million − 50,000	100,000,000
a hundred hundreds	100,000	1 million − 500,000	500,000
1/10 million	10^6	10^8	950,000,000
1/4 million	500,000	1 billion − 500 million	1/2 billion
3/4 million	10^4	1 billion − 50 million	950,000
a thousand thousands	200,000	1 million − 5,000	995,000
2/10 million	250,000	1 billion − 5 million	995,000,000

3. Add and subtract. Simply write the numbers under each other, lining up the place values. Use the usual addition or subtraction algorithm, regrouping the same way as you have learned before.

a. 329,145,000 + 2,809,125,093

b. 5,049 + 45,390,000 + 5,483,700

c. 45,700 + 90,567,000 + 2,560 + 2,300,560

d. 290,800 + 254,000,230 + 56,391 + 2,381

e. 480,560,000 − 23,980,000

f. 1,000,000 − 156,990

g. 22,300,000 − 4,431,190

h. 7,014,289,000 − 3,103,559,391

4. Subtract and compare.

a. 1 million − 100 thousand =

1 million − 10 thousand =

1 million − 1 thousand =

b. 7 million − 500 thousand =

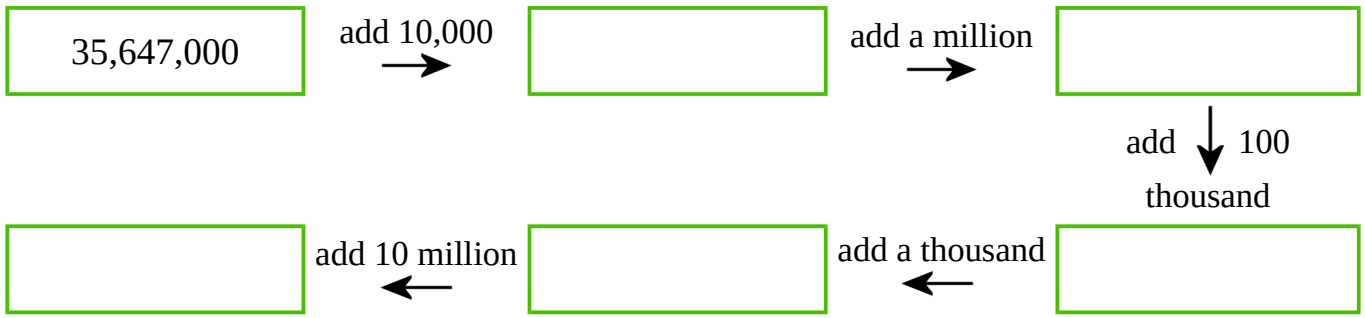
7 million − 50 thousand =

7 million − 5 thousand =

5. Continue counting for seven more numbers in each set:

a.	**b.**	**c.**
458,000,000 468,000,000 478,000,000	79,650,000 79,800,000 79,950,000	450,996,000 450,997,000 450,998,000
Each difference is _____	Each difference is _____	Each difference is _____

6. Complete the addition path.



7. Solve for x.

a. $x + 400{,}000 = 4{,}000{,}000$ $x =$ _____	**b.** $x - 350{,}000 = 2{,}000{,}000$ $x =$ _____
c. $200{,}000 + x + 600{,}000 = 7{,}000{,}000$ $x =$ _____	**d.** $2x = 3{,}000{,}000$ $x =$ _____

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Chapter 3: Problem Solving
Introduction

First in this chapter, students solve some equations, presented as pan balance puzzles. Then we study mixture equations, such as $4x + 38 = 128$, once again using the bar model as a visual model.

The bulk of this chapter is spent on problem solving. We use the bar model a lot. The problems include a fractional part of a whole, a fractional part more, the total is known, one part is more than the other, and so on.

Encourage the student to draw the bar model for the problems, as it is such a helpful tool. Some of the problems here could even be found in regular Algebra 1 textbooks where they would be solved with algebra. However, the bar model enables us to solve them without algebra; yet, it helps students' algebraic thinking! Essentially, one block in the bar model corresponds to the unknown x in an equation.

The Lessons in Chapter 3

	page	span
Balance Problems and Equations	102	*5 pages*
More Equations	107	*4 pages*
Problem Solving with Bar Models 1	111	*3 pages*
Problem Solving with Bar Models 2	114	*2 pages*
Problem Solving with Bar Models 3	116	*2 pages*
Problem Solving with Bar Models 4	118	*4 pages*
Mixed Review	122	*2 pages*
Chapter 3 Review	124	*3 pages*

Helpful Resources on the Internet

Pan Balance - Numbers
Enter a numerical expression in one pan and then in the other. The pans will move up and down depending on which expression is greater. When the expressions are equivalent, the pans will balance and the full equation will be entered into the *Balanced Equations* table. This tool strengthens understanding and computation of numerical expressions and equality. In understanding equality, one of the first things students must realize is that equality is a relationship, not an operation. Many students view "=" as "find the answer." For these students, it is difficult to understand equations such as $11 = 4 + 7$ or $3 \times 5 = 17 - 2$.

http://illuminations.nctm.org/ActivityDetail.aspx?ID=26

Pan Balance - Shapes
An online balance that builds your algebraic thinking. Find the unknown weight of each shape by placing shapes on the two pans, and trying to find situations where the weights are equal. One square always weighs 1 unit.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=33

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Thinking Blocks
An interactive math tool developed to help students learn how to solve multi-step word problems. Using brightly colored blocks, students model the relationships among the components of each word problem. The website has addition/subtraction problems, multiplication/division problems, and ratio problems. This block model corresponds to the bar model used in this book.
http://www.thinkingblocks.com/

Algebraic Reasoning
Find the value of an object based on two scales.
http://www.mathplayground.com/algebraic_reasoning.html

Algebra Puzzle
Find the value of each of the three objects presented in the puzzle. The numbers given represent the sum of the objects in each row or column.
http://www.mathplayground.com/algebra_puzzle.html

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

SpeedMath Deluxe
Create an equation from the four given digits using addition, subtraction, multiplication and division. Make certain that you remember the order of operations. Includes negative numbers sometimes.
http://education.jlab.org/smdeluxe/index.html

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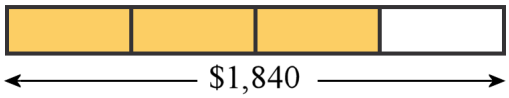
Problem Solving with Bar Models, Part 1

A fractional part of the whole

Jackie earns $1,840 monthly and Jessie earns 3/4 as much.
How much does Jessie earn?



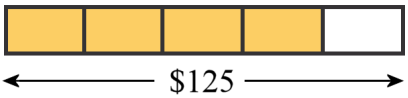
In the model, Jackie's salary is divided into four equal parts (blocks). To find 3/4 of it, *first find 1/4 of it*, which is **one block** in the model.

$$\$1{,}840 \div 4 = \$460$$

Then multiply that result by three: $3 \times \$460 = \$1{,}380$.
So, Jessie earns $1,380.

Solve. Draw a bar model. Write an expression (number sentence) for *each* calculation you do.

1. A $125 camera was discounted by 1/5 of its price.
   What is its new price?



_____ ÷ _____ = _____

_____

2. A pizza that weighs 680 g is divided into five equal pieces.
   How much do two pieces weigh?

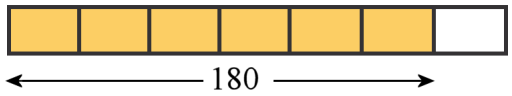
_____ ÷ _____ = _____

_____ × _____ = _____

3. A bottle of water costs 2/3 as much as a $1.50 juice.
   How much do *two* bottles of water and *two* juices cost?

A Fractional Part More

The school year in country A is 180 days long. In country B it is 1/6 part longer than that. How long is the school year in country B?



First, we divide the 180-day school year into 6 parts, to find how much one "block" is in the model:

180 ÷ 6 = 30. So, one block is 30 days.

Then we *add* one-sixth more to the whole bar model, and that is how long the school year is in country B.

180 + 30 = 210

So, the school year in country B is 210 days long.

Solve. Draw a bar model. Write an expression (number sentence) for each calculation you do.

4. The price of a $12 train ride went up by 1/6.
   What is the new price?

   _____ ÷ _____ = _____

   _____ + _____ = _____

5. A cafeteria lunch used to cost $4.50 but the price was increased by 1/5. What is the price now?

6. A one-way bus ride from Helen's home to town costs $1.
   The bus company will raise the price by 1/10 in June.

   a. How much will a one-way ride cost in June?

   b. How much more will a two-way ride (home-town-home) cost Helen in June than in May?

7. A T-shirt cost $10.50, but now it is discounted by 2/5 of its price.
Annie buys *ten* shirts with the discounted price. What is her total bill?

8. Duckville has 3,687 inhabitants, which is 3/5
of the number of inhabitants in Eagleby.
How many people *in total* live in Eagleby *and* Duckville?

9. A package of 10 small envelopes costs $2.50,
and a package of 10 large ones costs 2/5 more.
Find the total cost of buying 50 envelopes of each kind.

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Chapter 4: Decimals
Introduction

In this chapter, we study place value with decimals and learn to perform the four basic operations with decimal numbers.

The chapter starts with a short review of tenths and hundredths, after which, we study numbers with three decimal digits (thousandths). Students also compare and round numbers with up to three decimal digits.

The rest of the chapter is spent studying the four basic operations with decimals. We start with addition and subtraction, which we are familiar with from fourth grade, and then spend a considerable amount of time with multiplication and division of decimals.

I have tried to emphasize mental calculations based on the conceptual understanding of decimals. For that reason, the text often includes little "tricks" that can help with mental calculations. Along with that, the chapter has lessons on long multiplication and long division with decimals.

Problems accompanied by a small picture of a calculator are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

We also study using decimal numbers in measuring units, the metric system, and conversions between the customary units of measurement. I have tried to emphasize sensible and intuitive methods for converting measuring units within the metric system, instead of relying on mechanical formulas.

You might wonder why *Math Mammoth Grade 5* presents decimals before fractions. The traditional way is to teach fractions first because fractions are more general, and then, to show that decimals are simply a specific type of fractions with denominators that are powers of ten.

There are several reasons I present decimals before fractions. First, students have studied some about both decimals and fractions in earlier grades, so they should have the necessary background to comprehend that decimals are fractions. Therefore, I see no need to study all fraction arithmetic in 5th grade before decimal arithmetic.

Secondly, I feel that decimal arithmetic is somewhat easier than fraction arithmetic and students already know more about it than they know about all the fraction arithmetic that is studied in 5th grade (in 5-B). Thus, studying decimal arithmetic first may be easier for some students.

The Lessons in Chapter 4

	page	span
Review: Tenths and Hundredths	130	*3 pages*
More Decimals: Thousandths	133	*3 pages*
Comparing Decimals	138	*2 pages*
Rounding	140	*2 pages*
Add and Subtract Decimals	142	*4 pages*
Multiplying Decimals by Whole Numbers	146	*4 pages*
Multiplying Decimals in Columns	150	*2 pages*
Multiplying Decimals by Decimals	152	*4 pages*
More Decimal Multiplication	156	*3 pages*
Long Multiplication	159	*1 page*

Dividing Decimals—Mental Math	160	*5 pages*
Long Division with Decimals	165	*4 pages*
More Long Division with Decimals	169	*5 pages*
Multiply and Divide by Powers of Ten	172	*5 pages*
Divide Decimals by Decimals 1	177	*3 pages*
Divide Decimals by Decimals 2	180	*4 pages*
Decimals in Measuring Units and More	184	*4 pages*
Rounding and Estimating	188	*2 pages*
The Metric System	190	*3 pages*
Converting Between Customary Units of Measurement	193	*4 pages*
Number Rule Puzzles	197	*1 page*
Problem Solving	198	*4 pages*
Mixed Review	202	*2 pages*
Review	204	*5 pages*

Helpful Resources on the Internet

Decimal Arithmetic
These are my videos that go through all of the important decimal arithmetic: adding, subtracting, multiplying, dividing, comparing and rounding decimals, plus some problem solving. Great for grades 5, 6, and 7.
http://www.youtube.com/user/MathMammoth#grid/user/CCFD68119A0DA3E8

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers in ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

Smaller and Smaller Maze
Practice ordering decimal numbers to find your way through the maze.
http://www.counton.org/magnet/kaleidoscope/smaller/index.html

Decimal and Whole Number Jeopardy
Review place value and comparing and rounding numbers. Also, practice number patterns.
www.quia.com/cb/8142.html

Decimals in Space
An Asteroids-style game where you first answer a question about the smallest decimal and then get to shoot asteroids, earning points based on the numbers on them.
http://themathgames.com/arithmetic-games/place-value/decimal-place-value-math-game.php

Sock
Push the green blocks into the holes to make the target number.
www.interactivestuff.org/sums4fun/sock.html

Decimal Squares Blackjack
Play cards with decimals, trying to get as close to 2 as possible without going over.
www.decimalsquares.com/dsGames/games/blackjack.html

A Decimal Puzzle
Make every circle add up to 3.
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.htmlsopen=instructions&from=category_g_2_t_1.html

FunBrain Decimal Power Football
Simple games for addition, subtraction, multiplication, and division of decimals, including some with a missing factor or divisor. Solve a problem, and the football player moves down the field.
http://www.funbrain.com

Exploring Division of Decimals
Use a square to explore the products of two numbers with one decimal digit. The product is shown as an area.
www.hbschool.com/activity/elab2004/gr6/1.html

Decimal Speedway
Practice decimal multiplication in this fun car-racing game.
www.decimalsquares.com/dsGames/games/speedway.html

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More Long Division with Decimals

Fractions and division

Remember? The fraction line is *also* a division symbol. So $\frac{1}{8}$ can mean both one-eighth (a fraction) and a division problem $1 \div 8$. This gives us a means of writing fractions as decimals!

Example. Write $\frac{8}{9}$ as a decimal, to three decimal digits.

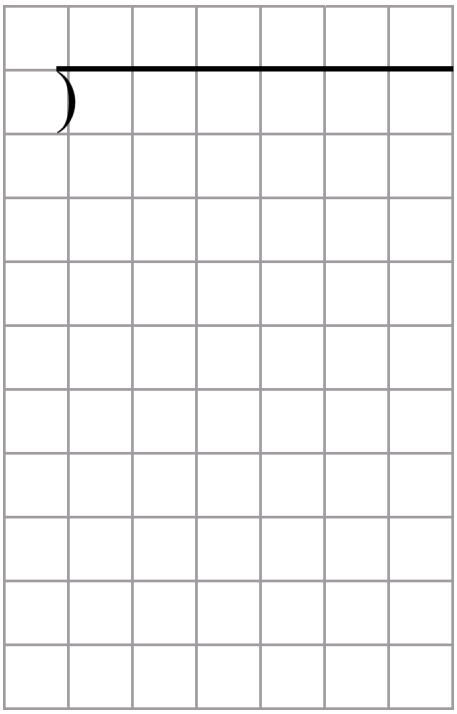
We simply divide 8 by 9, but writing 8 as 8.0000—with lots of decimal zeros. Look at the division on the right. We need to find *four* decimal digits for the quotient before we can round it to *three* decimal digits:

$\frac{8}{9} = 8 \div 9 \approx 0.889.$

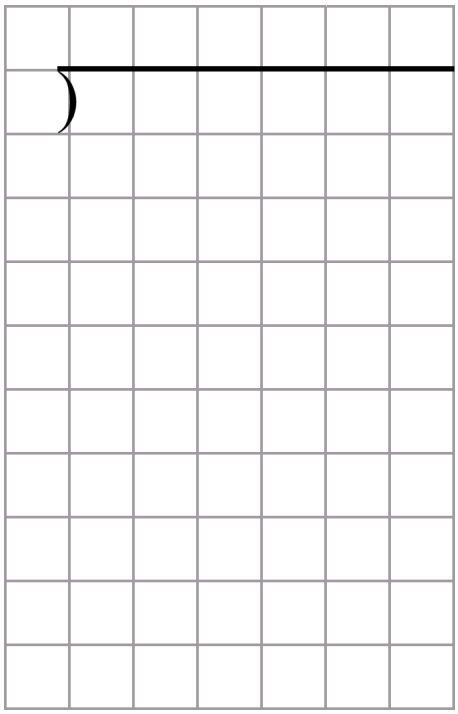
$$
\begin{array}{r}
0.8\,8\,8\,8 \\
9\overline{)8.0\,0\,0\,0} \\
\underline{7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8\,0 \\
\underline{-\,7\,2} \\
8
\end{array}
$$

1. Write the fractions as decimals, to three decimal digits.

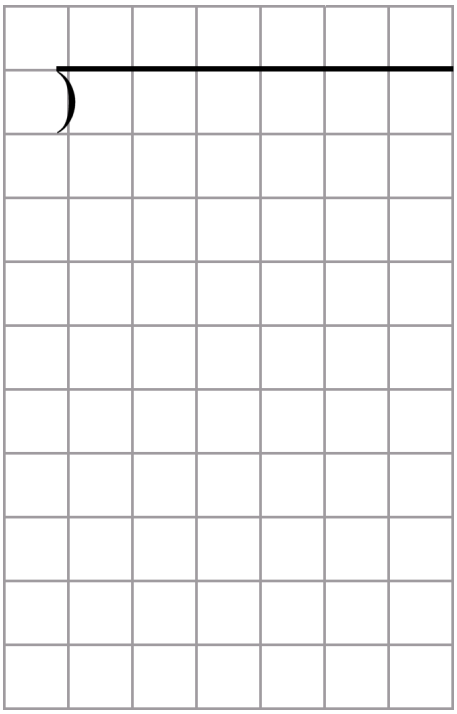
a. $\frac{5}{8} =$



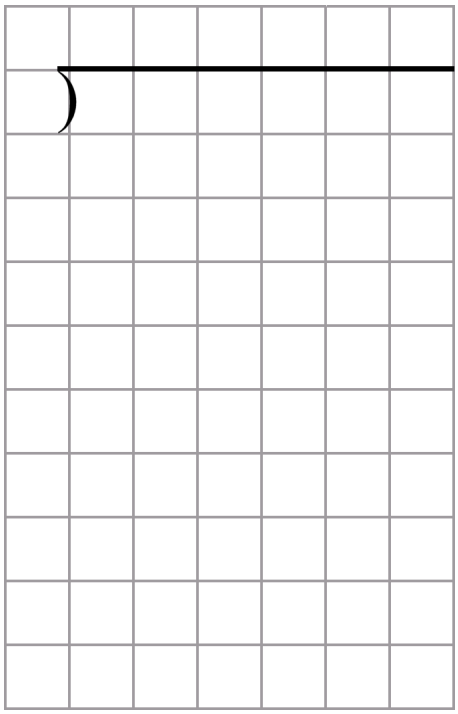
b. $\frac{6}{7} =$



c. $\frac{1}{6}$ =

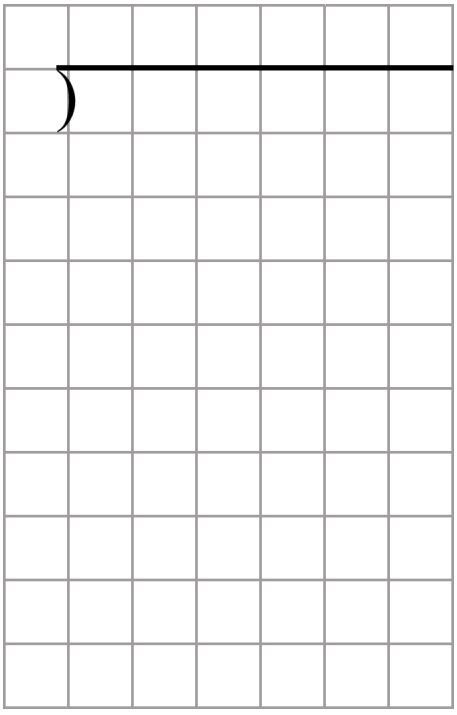


d. $\frac{7}{20}$ =

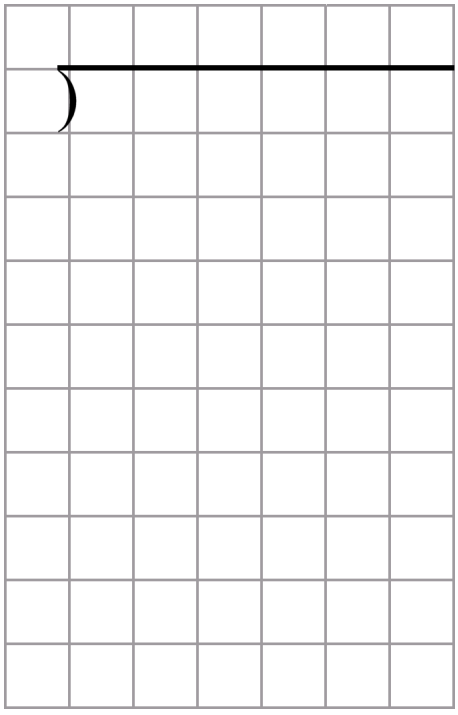


2. Calculate. You will need to add decimal zeros to the dividend.

a. 250 ÷ 6 to two decimal digits



b. 37.5 ÷ 11 to three decimal digits



3. **a.** Fill in the explanation as to how to solve the problem.

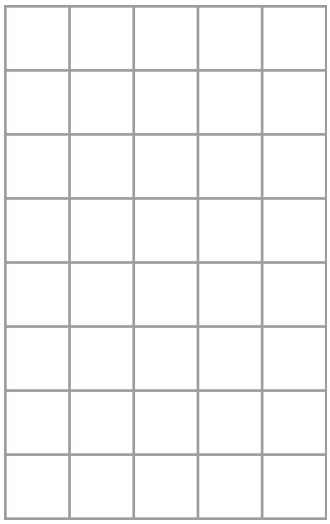
Three packs of transistors and seven packs of capacitors cost a total of $8.70. One capacitor pack costs $0.60. Find the cost of one transistor pack.

First _____ the cost of seven capacitor packs

from _____. Then divide that result by _____.

b. Write a *single* expression to match the explanation above.

c. Solve the problem.



4. Three friends equally shared the cost of a taxi fare, $35.40, and the cost of a meal, $128.95. How much did each person pay?

5. Write a word problem that matches each calculation below. *You do not have to calculate anything.*

a. ($50 − $26) ÷ 3 = $8

b. 25 × $1.40 ÷ 2 = $17.50

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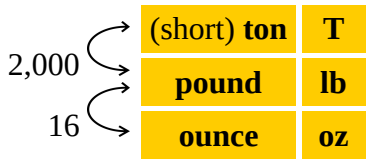
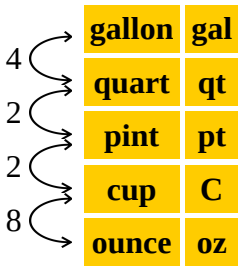
Converting Between Customary Units of Measurement

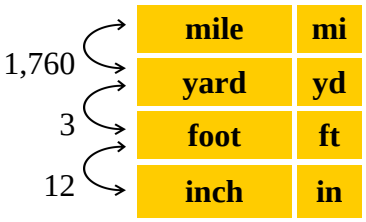
Units of weight



Units of volume



Units of length



To convert from one neighboring unit to another, either **multiply** or **divide** by the conversion factor.

If you do not know which, THINK if the result needs to be a smaller or bigger number.

Example. Convert 53 ounces into cups.

Ounces are smaller units than cups, so 53 ounces as cups will make *fewer* cups (you need fewer cups since they are the bigger units). So, we need to divide by the factor 8 (*since 8 ounces makes a cup*).

53 ÷ 8 = 6 R5. The results means 54 ounces is 6 cups and 5 (leftover) ounces.

You can also think of it this way: since 8 ounces makes a cup, we need to figure how many cups or how many "8 ounces" there are in 53 ounces... or how many 8s are in 53? The answer to that is solved by division.

1. Convert.

a. 6 ft = _____ in. 7 ft 5 in. = _____ in.	**b.** 25 in = _____ ft _____ in 45 in = _____ ft _____ in	**c.** 13 ft 7 in = _____ in 71 in. = _____ ft _____ in

2. Convert.

a. 2 lb 8 oz = _____ oz 45 oz = _____ lb _____ oz	**b.** 8 lb = _____ oz 56 oz = _____ lb _____ oz	**c.** 43 oz = _____ lb _____ oz 90 oz = _____ lb _____ oz

3. Convert.

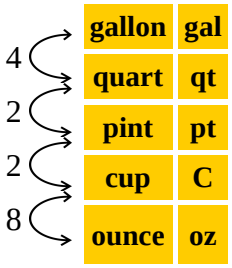
a. 3 C = _____ oz 55 oz = _____ C_____ oz	**b.** 4 C = _____ pt 3 pt = _____ C	**c.** 7 gal = _____ qt 45 qt = _____ gal _____ qt

Example. Convert 3 quarts into ounces.

We are going from bigger units (quarts) to smaller units (ounces), so there will be LOTS more of them. We need to multiply.

This time, quarts and ounces are not neighboring units in the chart. We need to **multiply 3 quarts by all of the factors** between them:

by 2, by 2, and by 8. So, we multiply 3 qt × 2 × 2 × 8 = 96 oz.



Example. Convert 742 inches into yards.

Between inches and yards we have the conversion factors 12 and 3. Also, yards are bigger units than inches... so we need *fewer* of them. That means we divide 742 by both 12 and 3—or we can divide it by 36.

742 ÷ 36 = 20 R 22. This means 742 in. = 20 yd 22 in. The remainder 22 is <u>in the same unit</u> as the dividend: in inches.

But, 22 inches is *also* 1 ft 10 in! So, in the end 742 in. = 20 yd 1 ft 10 in.

```
    0 2 0
36 )7 4 2
   7 2
     2 2
   -   0
     2 2
```

4. Convert. Use long division or multiplication.

a. 11 yd = _____ in.

b. 711 in. = _____ ft _____ in

c. 982 in. = _____ yd _____ ft _____ in

d. 254 oz. = _____ C _____ oz

Now, convert the cup-amount of your answer above into quarts and cups.

254 oz. = _____ qt _____ C _____ oz

Lastly, convert the quart-amount into gallons and quarts.

254 oz. = ____ gal _____ qt _____ C _____ oz

Example. Convert 4.52 lb into ounces.

We are going from bigger units (pounds) to smaller units (ounces), so there will be LOTS more of them. We need to multiply.

Using a calculator, multiply 4.52 × 16 = 72.32 oz.

2,000 →	(short) **ton**	**T**
	pound	**lb**
16 →	**ounce**	**oz**

Example. How many miles is 8,400 feet?

Since one mile is 5,280 feet, then 8,400 feet would be somewhere between 1 and 2 miles.
To find out exactly, use division, and round the answer: **8,400 ÷ 5,280 = 1.59090909... ≈ 1.59 miles.**

5. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 7.4 mi = _____ ft 16,000 ft = _____ mi	**b.** 1,500 ft = _____ yd 7,500 yd = _____ mi	1,760 → **mile** mi 3 → **yard** yd 12 → **foot** ft **inch** in 1 mile = 5,280 feet
c. 900 ft = _____ mi 2.56 mi = _____ yd	**d.** 12.54 mi = _____ ft 82,000 ft = _____ mi	

6. Convert. Use a calculator. Round your answer to two decimal digits, if necessary.

a. 15.2 lb = _____ oz 655 oz = _____ lb	**b.** 4.78 T = _____ lb 7,550 lb = _____ T	**c.** 78 oz = _____ lb 0.702 T = _____ lb

7. Solve the riddle. Use the calculator for the problems that you feel cannot be solved mentally.

F 0.6 mi = _____ ft

A 5,632 yd = _____ mi

O 10 qt = _____ C

H 2 lb 11 oz = _____ oz

L 1.3 mi = _____ yd

G 7 C = _____ oz

R 6,200 lb = _____ T

S 3 lb 5 oz = _____ oz

E 5 ft 2 in = _____ in

O 40 oz = _____ lb

I 14,256 ft = _____ mi

W 6 ft 7 in = _____ in

L 732 in = _____ ft

D 42 in = _____ ft

P 3 gal = _____ pt

A 0.75 mi = _____ ft

What did one potato chip say to the other?

53	43	3960	61	2288		57	62		56	40

3168	2.5	3.1		3.2		3.5	2.7	24	
									?

8. Solve.

a. If you serve 1-cup servings of juice to 30 people, how many *whole* gallons of juice will you need?

b. Mom was making applesauce in 2-gallon batches and bottling it in 1-quart jars. After 9 batches, how many jars of applesauce had she made?

c. How many 8-inch pieces can you cut out of 9 3/4 ft of ribbon?

d. A 4-ounce serving of coffee costs $1.20. What would a 5-ounce serving cost?

e. A bottle of shampoo weighs 13 oz, and there are 20 of them in a box. The box itself weighs 8 oz. How much does the box with the bottles of shampoo weigh in total (in pounds and ounces)?

f. Mark drinks three 5-ounce servings of coffee a day. Find how much coffee he drinks in a month (30 days). Give your answer in units other than ounces.

g. Erica lost 5 lb of weight over 4 weeks of time. How much weight did she lose daily, on average?

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math MAMMOTH

Grade 5-B Worktext

Light Blue Series

- Graphing and statistics
- Fractions: add and subtract
- Fractions: multiply and divide
- Geometry

By Maria Miller

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Foreword

Math Mammoth Grade 5-B Worktext comprises a complete math curriculum for the second half of fifth grade mathematics. Fifth grade focuses on fractions and decimals, in particular. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, and decimal arithmetic. In this part, 5-B, we study graphing, fraction arithmetic, and geometry.

This book starts with chapter 5, where we study graphing in a coordinate grid, line and bar graphs, and average and mode. Today's world has become increasingly complex, with lots of data in the media, so our children need a good grasp of graphs to be able to make sense of all that information.

Chapter 6 is about the addition and subtraction of fractions—another focus topic for 5th grade, besides decimals. Students learn to add and subtract unlike fractions, using the technique of first converting them to equivalent like fractions. In chapter 7, we study the multiplication and division of fractions (division only in special cases), relying first on visual models, and then proceeding to the abstract shortcuts.

Chapter 8 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success in your math teaching!

Maria Miller, the author

Chapter 5: Statistics and Graphing
Introduction

This chapter starts out with a study of the coordinate grid, but only in the first quadrant. Besides learning how to plot points, students also plot ordered pairs (points) from number patterns or rules. This is actually the beginning of the study of *functions*.

Practicing the use of the coordinate grid is a natural "prelude" to the study of line graphs, which follows next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories. The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study average (also called the *mean*) and mode, and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is plenty of time to learn that concept in subsequent grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all that many students are able to grasp from that is only the calculation procedures. I feel it is better to introduce and contrast initially only the two concepts, the mean and the mode, in order to give the student a solid foundation. We can introduce the median later, and then compare and contrast it with the other two.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 5

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Coordinate Grid	10	*3 pages*
Number Patterns in the Coordinate Grid	13	*4 pages*
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Line Graphs	21	*4 pages*
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Helpful Resources on the Internet

Coordinate Grid

Billy Bug Returns at Primary Games
Move Billy Bug to the feeding place with given coordinates.
http://www.primarygames.co.uk/pg2/bug2/bug2.html

Co-ordinate Game
You will see a red circle on the grid. Enter the co-ordinates and click "check."
http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/maths/coordinate_game/game1.htm

Graphit
A graphing tool that plots both functions and ordered pairs.
http://www.shodor.org/interactivate/activities/graphit/index.html

Graph Mole
A fun game about plotting points on a coordinate plane. Plot points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Coordinate Grid Quiz from ThatQuiz.org
This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0

Graphing and Graphs

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool:
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness, and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Data Grapher
This basic data grapher can be used to create bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=204

Histogram Tool
Create a histogram from your data, or analyze histograms from pre-made data.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Create a Graph
Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Graphs Quiz from That Quiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, box plot, scatter graph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
http://www.thatquiz.org/tq-5/math/graphs

Math Goodies Interactive Data and Graphs Lessons
Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs, and exercises.
http://www.mathgoodies.com/lessons/toc_vol11.html

Data Analysis Gizmos from Explorelearning.com
Interactive online simulations or activities, with lesson plans. Topics include creating a bar graph or aline graph, pictographs, mean and median, and a reaction time gizmo. This is an excellent resource. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=383

Statistics Interactive Activities
(scroll down to Statistics and Probability concepts)
A set of interactive tools for exploring histograms, pie charts, box plots, stem-leaf plots, and mean, median, variance, and standard deviation of data. You can enter your own data or explore the examples.
http://www.shodor.org/interactivate/activities/tools.html

Mean, Median, Mode, Range

Using and Handling Data
Simple explanations for finding mean, median, or mode.
http://www.mathsisfun.com/data/index.html#stats

Math Goodies Interactive Statistics Lessons
Clear lessons with examples and interactive quiz questions, including range, arithmetic mean, non-routine mean, median, and mode, practice exercises, and challenge exercises.
http://www.mathgoodies.com/lessons/toc_vol8.htm

Mean, Median, and Mode
How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.
http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median Lessons
Explanations with simple examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/data/measuresofaveragerev1.shtml

Measures Activity
Enter you own data and the program will calculate mean, median, mode, range and some other statistical measures.
http://www.shodor.org/interactivate/activities/Measures

Mean/Mode Quiz
A 10-question quiz about calculating the mode and mean.
http://www.thatquiz.org/tq-p-z1/?-j6g00-l5-p0

Landmark Shark Game
You are dealt five number cards, and using that as your data set you need to choose which of the range, median, or mode is the largest number.
http://media.emgames.com/emgames/demosite/playdemo.html?activity=M5A006&activitytype=dcr&level=3

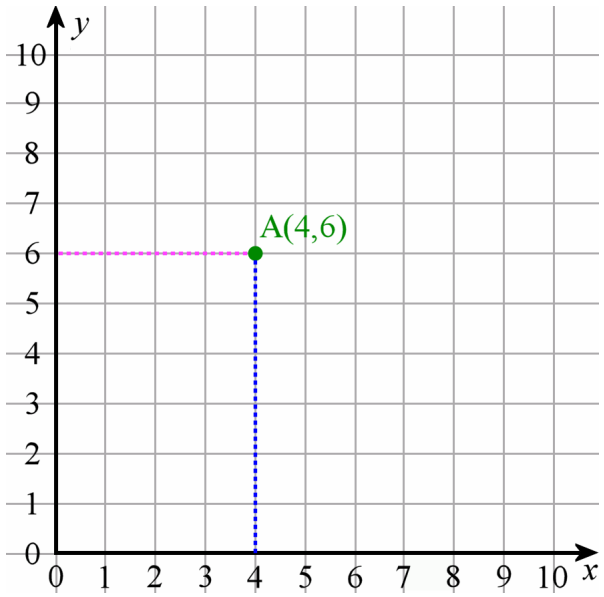
Coordinate Grid

This is a **coordinate grid**.

The long black line across the bottom, with the "*x*" near its end, is called the ***x*-axis**.

The vertical line that has "*y*" near the top is called the ***y*-axis**.

You can see one point, called "A," that is drawn or *plotted* on the grid.



It has two numbers *associated*, or matched, with it. Those two numbers are called the **coordinates** of the point A.

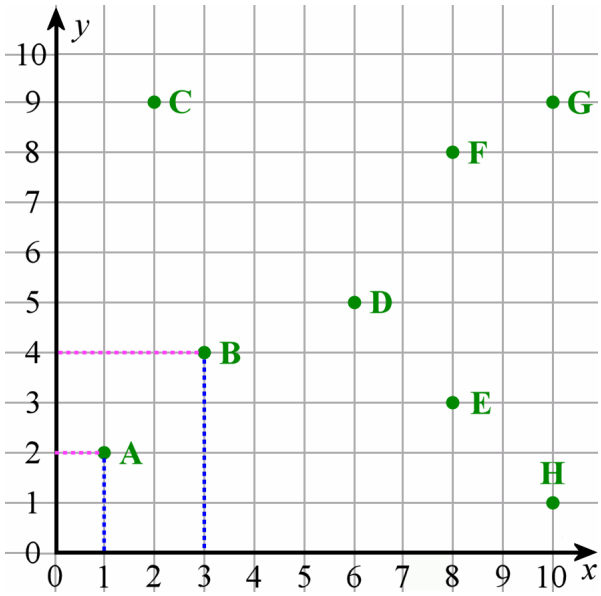
The first number is called the **x-coordinate** of the point A, and the second number is called the **y-coordinate** of the point A.

The x-coordinate of the point A is 4 because if you drew a line straight down from A, it would *intersect*, or "hit," the *x*-axis at 4. The y-coordinate of the point A is 6 because if you drew a line straight left from A, it would intersect the *y*-axis at 6.

We write the two coordinates of a point inside parentheses, separated by a comma.

Note: The order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is ALWAYS the *y*-coordinate, not the other way around.
So (5, 8) means the *x*-coordinate is 5 and the *y*-coordinate is 8.

1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in.



A (___, ___) B (___, ___)

C (___, ___) D (___, ___)

E (___, ___) F (___, ___)

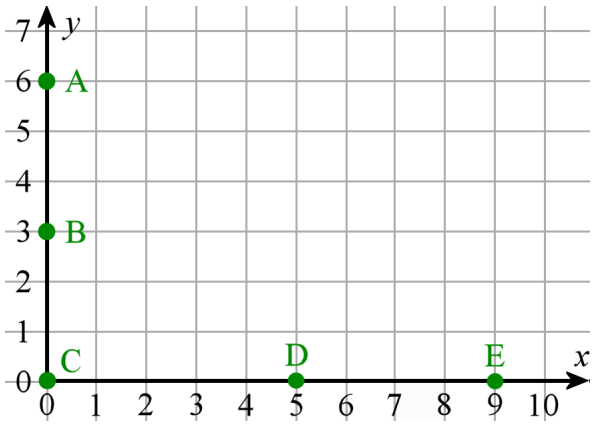
G (___, ___) H (___, ___)

Notice especially the points that are located on the two axes.

If a point lies on the *y*-axis, its *x*-coordinate is zero. A is (0, 6), and B is (0, 3) .

If the point lies on the *x*-axis, its *y*-coordinate is zero. D is (5, 0) and E is (9, 0).

The point C has the coordinates (0, 0). This point (0, 0) is called the *origin*.

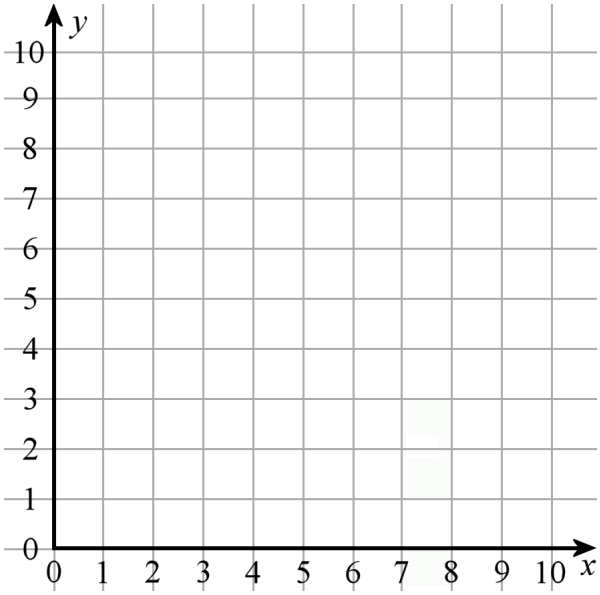


2. Plot and label the following points on the coordinate grid.

A (2, 8) B (0, 5) C (4, 0)

D (9, 10) E (8, 5) F (1, 4)

G (1, 0) H (0, 8) I (3, 7)



3. The coordinate grid is very useful for many things. For example, computer drawing programs use it frequently. Let's say "LINE (5,6) - (2,7)" means a straight line segment that is drawn from the point (5, 6) to the point (2, 7).

Draw the following line segments.
What figure is formed?

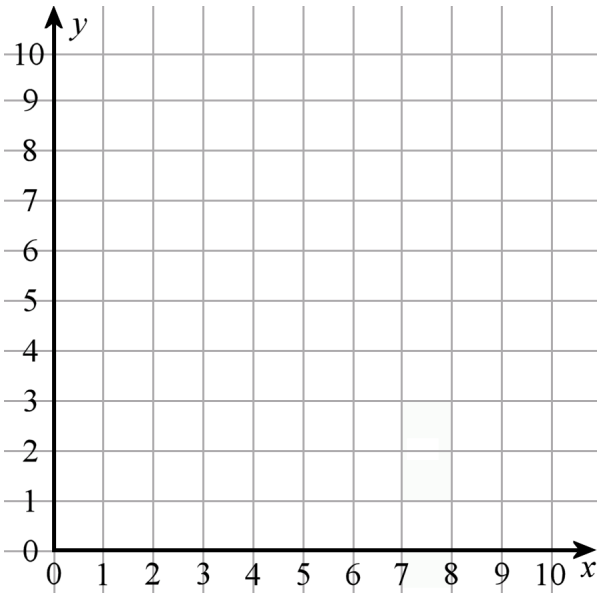
LINE (1, 0) - (7, 0) LINE (7, 0) - (7, 5)

LINE (1, 0) - (1, 5) LINE (1, 5) - (0, 5)

LINE (0, 5) - (4, 7) LINE (4, 7) - (8, 5)

LINE (8, 5) - (7, 5) LINE (3, 0) - (3, 3)

LINE (5, 0) - (5, 3) LINE (3, 3) - (5, 3)

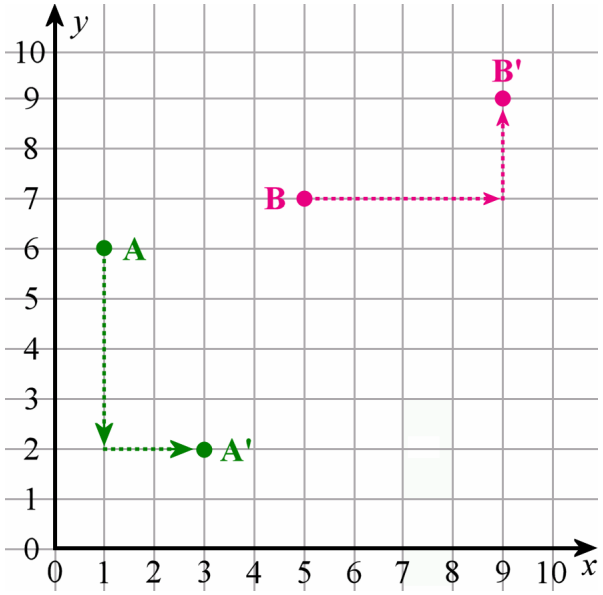


This example shows point A moving four units down and then two units to the right. The new location is called point A′ (read "A prime").

Originally A's coordinates were (1, 6).
After the movement, the coordinates are (3, 2)

Notice how you can just subtract four units from the *y*-coordinate (the movement four units straight down) and add two units to the *x*-coordinate (movement two units to the right).

Point B is originally at (5, 7). It moves four units to the right and two up. You add four to the *x*-coordinate, and two to the *y*-coordinate. Its new coordinates are (9, 9).

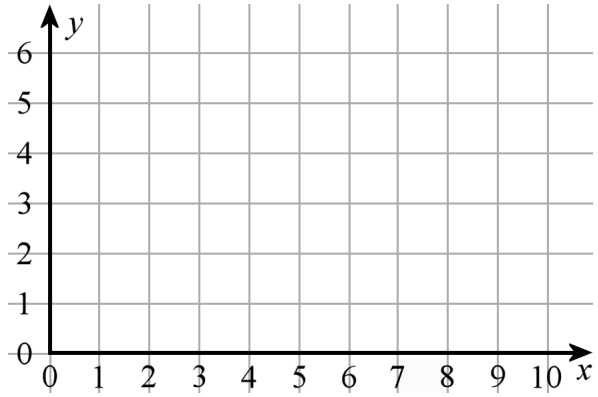


Movement up or down affects the *y*-coordinate.
Movement right or left affects the *x*-coordinate.
In other words, movement *parallel* to an axis affects that same coordinate.

4. The three vertices of a triangle are (2, 0), (5, 1) and (3, 4). The triangle is moved three units to the right and two up.

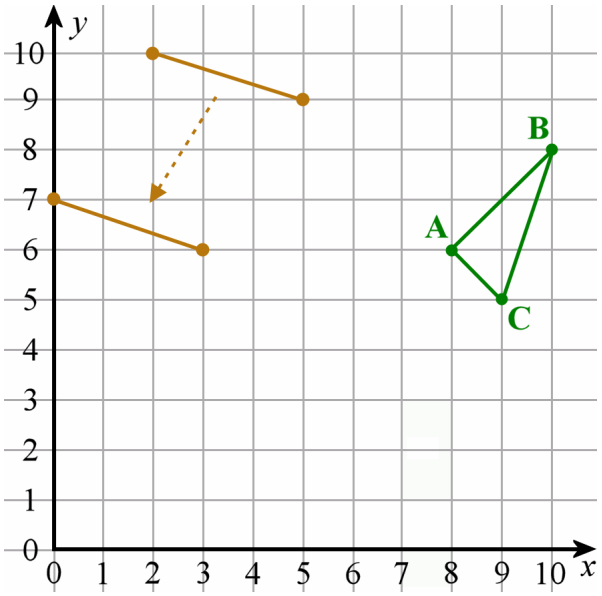
   a. Plot the vertices of the triangle before and after the movement.

   b. Write the coordinates of the vertices after the movement.



5. **a.** Determine how the line segment has been moved, and move the triangle ABC the same way. Let's call the new triangle A′B′C′. Write down the coordinates of the vertices of the triangle A′B′C′ after the movement.

   b. Let's say the point (3, 5) moves to (2, 7). Move the triangle ABC in a similar way. Write down the coordinates of the triangle's vertices after the movement.



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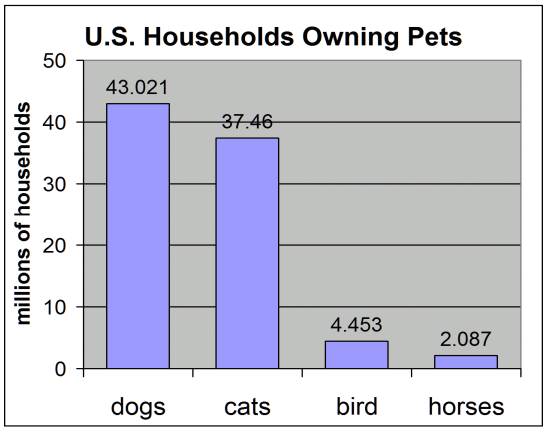
Mean, Mode, and Bar Graphs



Do you think you could calculate the average from the data shown in the bar graph? After all, there are numbers involved.

Actually, we cannot. To see why, you need to think *what kind of original data* produced this graph. What was asked of the people in the study? What did they respond?

The people were asked something like, "What pets do you have?" The people would have answered, "cat," or "dog," and so on.

The original data set consists simply of the words "cat," "dog," "bird," and "horse"—each one listed many times, because each mention of a "cat" would mean the answer of one particular household.

cat, cat, dog, dog, dog, bird, dog, dog, bird, cat, dog, horse, dog, cat, dog, ...

We cannot calculate anything from this kind of data set because it is not numerical data! However, we CAN find the most commonly occurring item, and that is called the ***mode***.

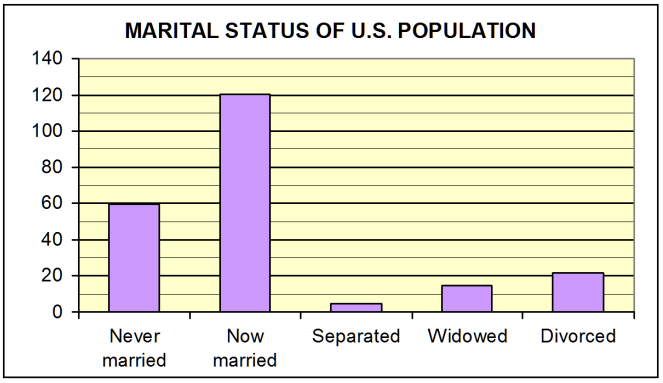
In this case, the mode is *dog.* It made the highest bar on the graph.

The *mode* is the most commonly occurring item in a data set.

- Sometimes a set of data has two or more modes. For example, the data set *green, green, blue, blue, black, brown, hazel* has two modes: both green and blue are equally common.

- If none of the items occurs twice or more, there is no mode. For example, this data: *green, blue, pink, red, black, brown, purple* has no mode.

1. Find the mode of the data set shown in the bar graph on the right.



2. **a.** Find the mode of this data:

   water, pop, juice, pop, juice, water, milk, water, pop, pop, juice, pop

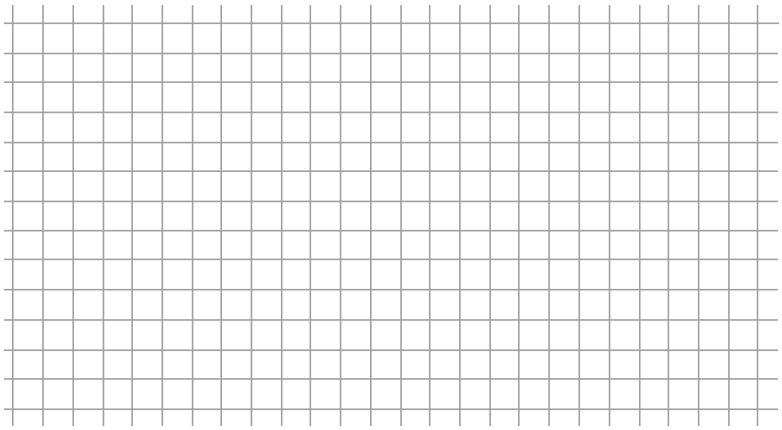
   b. If the above are the answers of 12 people to some question, what could have been the question?

3. Nineteen children were asked about their favorite ice cream flavor. Here are their responses:
strawberry, vanilla, chocolate, vanilla, chocolate chip, chocolate, pecan, pecan, vanilla, vanilla, strawberry, chocolate chip, vanilla, chocolate, chocolate, vanilla, strawberry, chocolate chip, vanilla.

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.



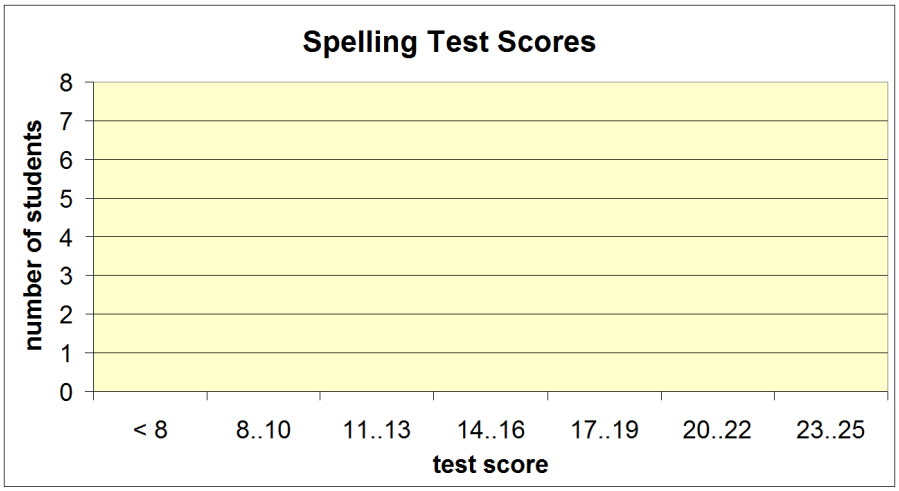
4. These are the spelling test scores of a fifth grade class:
4 5 7 9 9 10 10 11 11 12 12 12 13 14 17 18 18 18 19 19 19 20 24 25

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.

Test Score	Frequency
< 8	
8..10	
11..13	
14..16	
17..19	
20..22	
23..25	



5. **a.** Find the mode.

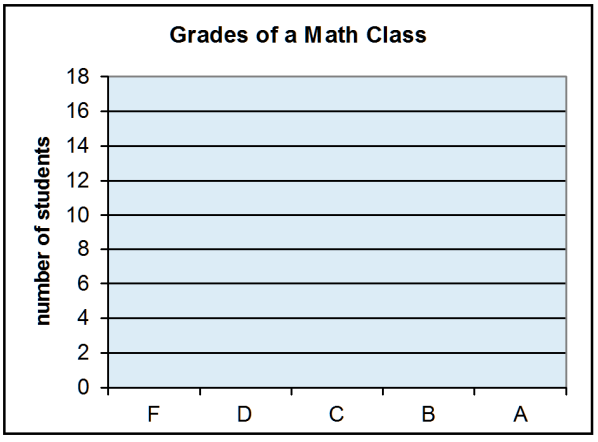
b. Draw a bar graph.

c. If possible, calculate the average.

d. There were ____ students in all. What *fraction* of the students got grade B?

Grades of a math class

Grade	Frequency
F	3
D	8
C	12
B	17
A	10



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Chapter 6: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade. I hope that students have already built a solid conceptual understanding in their minds in previous years, so we can build on that foundation.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a "stumbling block." Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions.

First, we review how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions. This chapter ends with a lesson on measuring in inches, using units as small as 1/16 of an inch.

The Lessons in Chapter 6

	page	*span*
Fraction Terminology	50	*1 page*
Review: Mixed Numbers	51	*4 pages*
Adding Mixed Numbers	55	*3 pages*
Subtracting Mixed Numbers 1	58	*4 pages*
Subtracting Mixed Numbers 2	62	*2 pages*
Equivalent Fractions 1	64	*3 pages*
Equivalent Fractions 2	67	*2 pages*
Adding and Subtracting Unlike Fractions	69	*3 pages*
Finding the (Least) Common Denominator	72	*3 pages*
Add and Subtract: More Practice	75	*3 pages*
Adding and Subtracting Mixed Numbers	78	*5 pages*
Comparing Fractions	83	*4 pages*
Measuring in Inches	87	*5 pages*
Mixed Review	92	*3 pages*
Review	95	*2 pages*

Helpful Resources on the Internet

General

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages. The activity includes several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the representations of the fractions and the models.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software
Games for addition and subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.
http://www.sheppardsoftware.com/math.htm#fractions

Who Wants Pizza?
This tutorial explains fraction addition and multiplication using a pizza, and then includes some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Fractioncity
Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Equivalent Fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Equivalent Fractions with Visual Models
Create custom-made worksheets for equivalent fractions. Choose to include pie images or not.
http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

Addition and Subtraction

Fraction Videos 1: Addition and Subtraction
My own videos that cover equivalent fractions, addition and subtraction of like and unlike fractions, and of mixed numbers.
http://www.mathmammoth.com/videos/fractions_1.php

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding Fractions
Illustrates how to find the common denominator when adding two unlike fractions using interactive pie models.
http://nlvm.usu.edu/en/nav/frames_asid_106_g_3_t_1.html

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure of adding two unlike fractions. A free registration required.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
The computer gives you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction.php

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Comparing Fractions Tool at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/app/tool/comparing-fractions

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Comparing/Ordering Fractions Worksheets
Create free worksheets for comparing two fractions or ordering 3-8 fractions. Compare fractions with the same denominator, fractions with the same numerator, or you compare a fraction to 1/2, or to 1, and so on. You can also include images (fraction pies).
http://www.homeschoolmath.net/worksheets/comparing_fractions.php

Measure It!
Practice measuring lines in inches. Multiple choice questions.
http://onlineintervention.funbrain.com/measure/index.html

Fraction Terminology

As we study fractions and their operations, it is important that you understand the terms, or words, that we use. This page is a reference. You can even post it on your wall or make your own fraction poster based on it.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.
The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

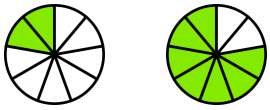
A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\frac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

Like fractions have the same denominator. They have the same kind of parts.

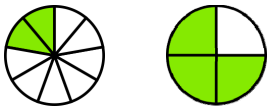
It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts.

It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

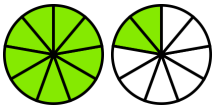


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

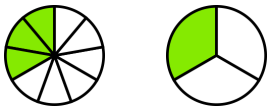
$\frac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). Since it is called a *fraction*, it is written as a fraction and *not* as a mixed number.



$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

Simplifying a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

$\frac{9}{12}$ simplifies to $\frac{3}{4}$.

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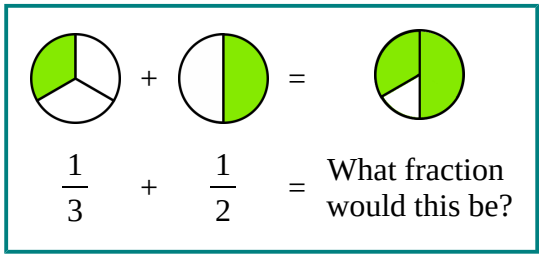
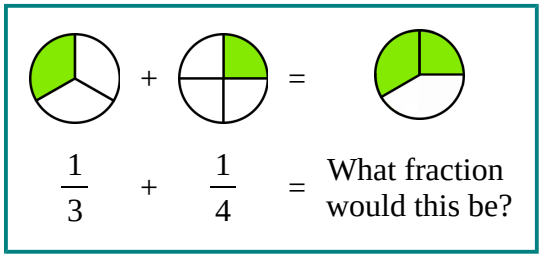
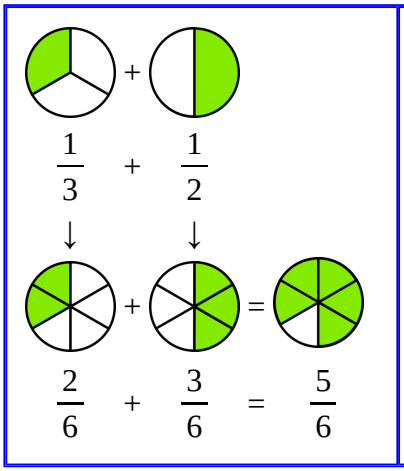
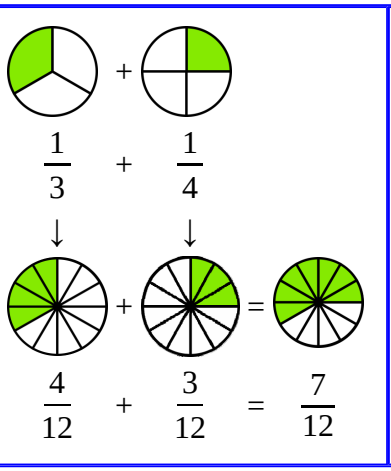
Adding and Subtracting Unlike Fractions

Cover the page below the black line. Then try to figure out the addition problems below.









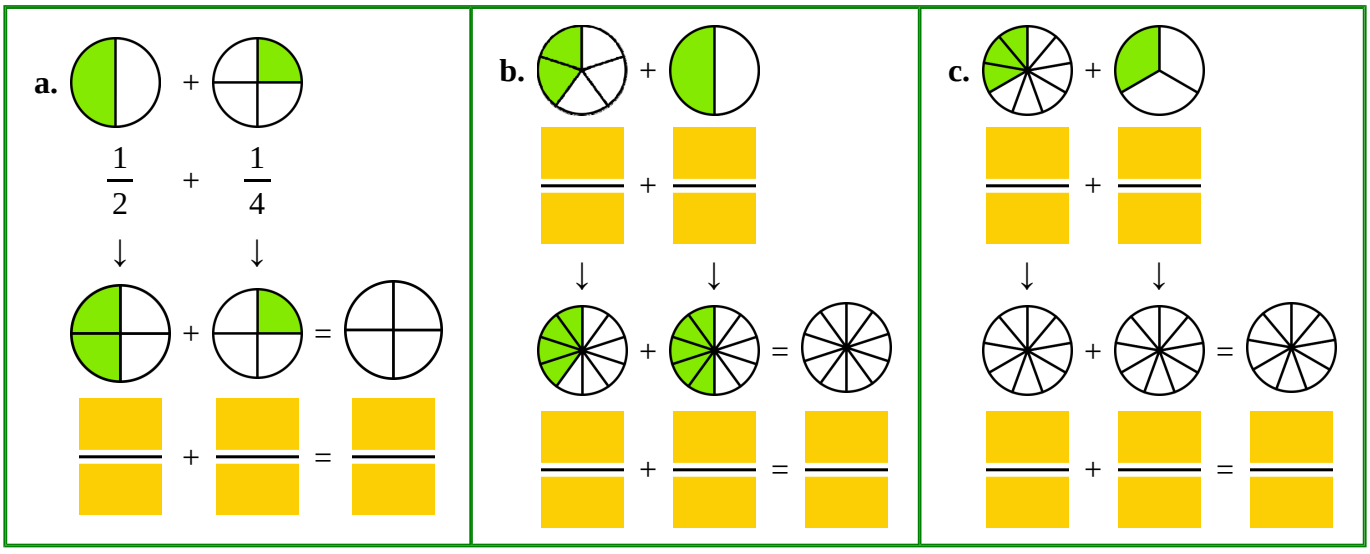
Did you solve the problems above?

The solution is this:

We convert the fractions so that they become *like* fractions (the *same* denominator), using equivalent fractions.

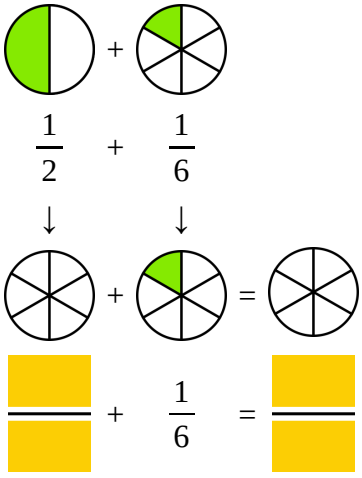
Then we can add or subtract.

1. Write the fractions shown by the pie images. Convert them into *equivalent fractions with the same denominator* (like fractions), and then add them. Color the missing parts.



2. Convert the fractions to like fractions first, and then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!

a.

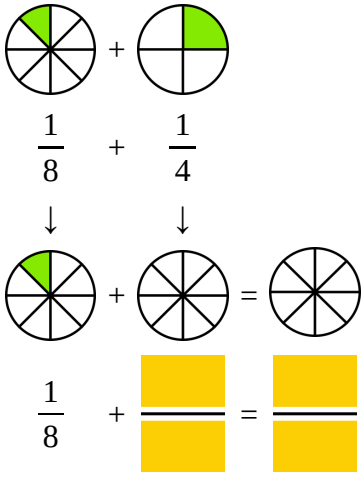


$\frac{1}{2} + \frac{1}{6}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{1}{6} = \frac{\square}{\square}$

b.

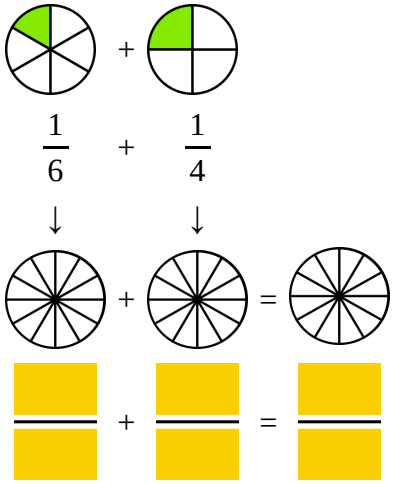


$\frac{1}{8} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{1}{8} + \frac{\square}{\square} = \frac{\square}{\square}$

c.



$\frac{1}{6} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

d. $\frac{5}{6} - \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{5}{6} - \frac{\square}{\square} = \frac{\square}{\square}$

e. $\frac{5}{8} - \frac{1}{4}$

$\downarrow \quad \downarrow$

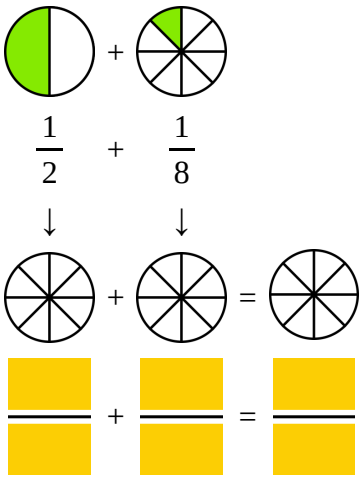
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

f. $\frac{5}{6} - \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

g.

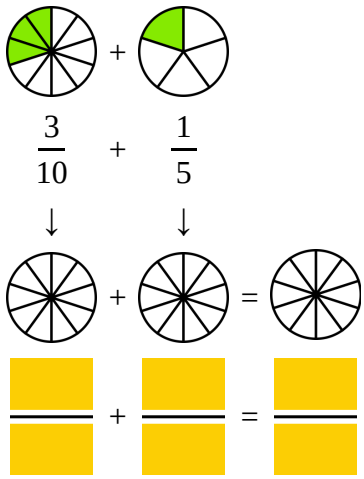


$\frac{1}{2} + \frac{1}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

h.

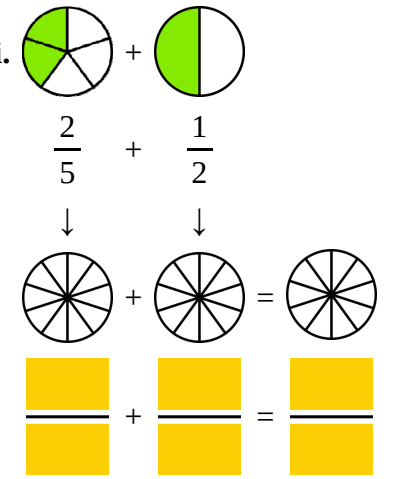


$\frac{3}{10} + \frac{1}{5}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

i.



$\frac{2}{5} + \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

j. $\frac{1}{2} + \frac{3}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

k. $\frac{9}{10} - \frac{2}{5}$

$\downarrow \quad \downarrow$

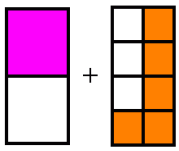
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

l. $\frac{4}{5} - \frac{1}{2}$

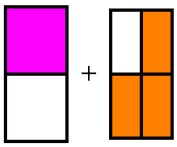
$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

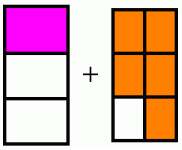
3. Split the parts only in the *first* fraction so that both fractions will have the same kind of parts. Add.



a. ___ + $\frac{5}{8}$ =

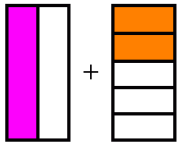


b. ___ + $\frac{3}{4}$ =

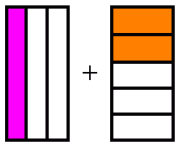


c. ___ + $\frac{5}{6}$ =

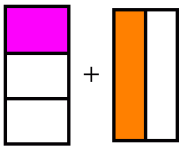
Now split the parts in *both* fractions so that they will have the same kind of parts. Add.



d. $\frac{\quad}{10}$ + $\frac{\quad}{10}$ =



e. $\frac{\quad}{15}$ + ___ =



f. ___ + ___ =

4. Fill in the table based on the problems above. What kind of parts did the two fractions have at first? What kind of parts did you use in the final addition?

	Types of parts:	Converted to:
a.	2nd parts and 8th parts	*8th* parts
b.	2nd parts and 4th parts	_____ parts
c.	3rd parts and 6th parts	_____ parts

	Types of parts:	Converted to:
d.	2nd parts and 5th parts	_____ parts
e.	3rd parts and 5th parts	_____ parts
f.	3rd parts and 2nd parts	_____ parts

5. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table above?

6. **Challenge:** If you think you know what kind of parts to convert these fractions into, then try these problems. Do not worry if you do not know how to do them—we will study this in the next lesson.

a. $\frac{1}{2}$ + $\frac{2}{3}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

b. $\frac{2}{3}$ − $\frac{2}{5}$

↓ ↓

$\frac{\quad}{\quad}$ − $\frac{\quad}{\quad}$ =

c. $\frac{1}{3}$ + $\frac{3}{4}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

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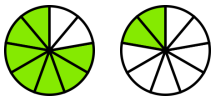
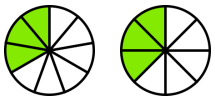
Comparing Fractions

Sometimes it is easy to know which fraction is the greater of the two. Study the examples below!



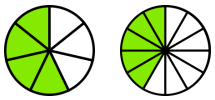
$\frac{7}{9} > \frac{2}{9}$

With **like fractions**, all you need to do is to check which fraction has more "slices," and that fraction is greater.



$\frac{3}{9} < \frac{3}{8}$

If both fractions have the **same number of pieces**, then the one with bigger pieces is greater.

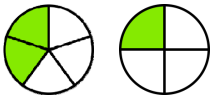


$\frac{4}{7} > \frac{5}{12}$

Sometimes you can **compare to 1/2**. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.

$\frac{6}{5} > \frac{9}{10}$

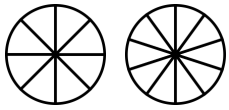
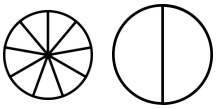
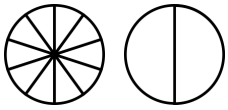
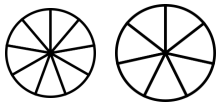
Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here, 6/5 is more than 1, and 9/10 is less than 1.



$\frac{2}{5} > \frac{1}{4}$

If you can imagine the pie pictures in your mind, you can sometimes "see" which fraction is bigger. For example, it is easy to see that 2/5 is more than 1/4.

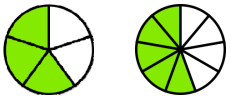
1. Compare the fractions, and write > , < , or = .

 **a.** $\frac{1}{8}$ $\frac{1}{10}$	 **b.** $\frac{4}{9}$ $\frac{1}{2}$	 **c.** $\frac{6}{10}$ $\frac{1}{2}$	 **d.** $\frac{3}{9}$ $\frac{3}{7}$
e. $\frac{8}{11}$ $\frac{4}{11}$	**f.** $\frac{7}{4}$ $\frac{7}{6}$	**g.** $\frac{5}{14}$ $\frac{5}{9}$	**h.** $\frac{4}{20}$ $\frac{2}{20}$
i. $\frac{2}{11}$ $\frac{2}{5}$	**j.** $\frac{1}{2}$ $\frac{5}{8}$	**k.** $\frac{3}{6}$ $\frac{1}{2}$	**l.** $\frac{1}{20}$ $\frac{1}{8}$
m. $\frac{1}{2}$ $\frac{3}{4}$	**n.** $\frac{8}{7}$ $\frac{3}{3}$	**o.** $\frac{49}{100}$ $\frac{61}{100}$	**p.** $\frac{7}{8}$ $\frac{8}{7}$
q. $\frac{9}{10}$ $\frac{3}{4}$	**r.** $\frac{6}{5}$ $\frac{3}{4}$	**s.** $\frac{4}{4}$ $\frac{9}{11}$	**t.** $\frac{1}{3}$ $\frac{3}{9}$

Sometimes none of the "tricks" explained in the previous page work, but we do have one more up our sleeve!

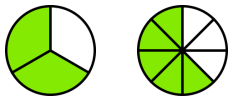
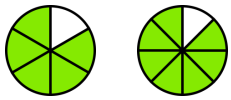
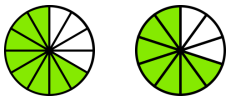
Convert both fractions into like fractions. Then compare.

In the picture on the right, it is hard to be sure if 3/5 is really more than 5/9. Convert both into 45th parts, and then it is easy to see that 27/45 is more than 25/45. Not by much, though!



$\frac{3}{5}$ ↓ $\frac{27}{45}$ > $\frac{5}{9}$ ↓ $\frac{25}{45}$

2. Convert the fractions into like fractions, and then compare them.

a.  $\frac{2}{3}$ ↓ $\frac{5}{8}$ ↓	**b.**  $\frac{5}{6}$ ↓ $\frac{7}{8}$ ↓	**c.**  $\frac{1}{3}$ ↓ $\frac{3}{10}$ ↓	**d.**  $\frac{8}{12}$ ↓ $\frac{7}{10}$ ↓
e. $\frac{5}{8}$ ↓ $\frac{7}{12}$ ↓	**f.** $\frac{11}{8}$ ↓ $\frac{14}{10}$ ↓	**g.** $\frac{6}{10}$ ↓ $\frac{58}{100}$ ↓	**h.** $\frac{6}{5}$ ↓ $\frac{11}{9}$ ↓
i. $\frac{7}{10}$ ↓ $\frac{5}{7}$ ↓	**j.** $\frac{43}{100}$ ↓ $\frac{3}{10}$ ↓	**k.** $\frac{9}{8}$ ↓ $\frac{8}{7}$ ↓	**l.** $\frac{7}{10}$ ↓ $\frac{2}{3}$ ↓

3. One cookie recipe calls for 1/2 cup of sugar. Another one calls for 2/3 cup of sugar. Which uses more sugar, a triple batch of the first recipe, or a double batch of the second?

How much more?

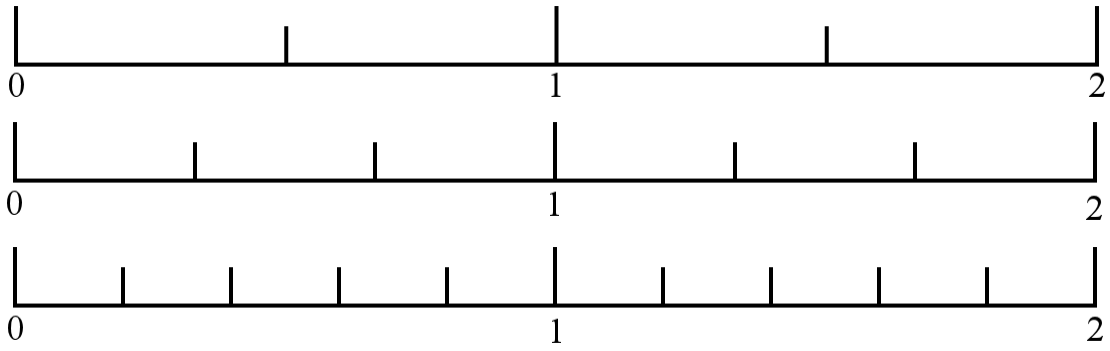
4. Compare the fractions using any method.

a. $\frac{5}{12}$ $\frac{3}{8}$	**b.** $\frac{5}{12}$ $\frac{4}{11}$	**c.** $\frac{3}{10}$ $\frac{1}{5}$	**d.** $\frac{3}{8}$ $\frac{4}{7}$
e. $\frac{4}{15}$ $\frac{1}{3}$	**f.** $\frac{5}{6}$ $\frac{11}{16}$	**g.** $\frac{7}{6}$ $\frac{10}{8}$	**h.** $\frac{5}{12}$ $\frac{5}{8}$
i. $\frac{3}{4}$ $\frac{4}{11}$	**j.** $\frac{13}{10}$ $\frac{9}{8}$	**k.** $\frac{2}{13}$ $\frac{1}{5}$	**l.** $\frac{1}{10}$ $\frac{1}{11}$

5. A coat costs $40. Which is a bigger discount:
1/4 off the normal price, or 3/10 off the normal price?

Does your answer change if the original price of the coat was $60 instead? Why or why not?

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



a. $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

___ < ___ < ___ < ___ < ___

b. $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

___ < ___ < ___ < ___ < ___

7. Write the three fractions in order.

a. $\frac{7}{8}, \frac{9}{10}, \frac{7}{9}$ ____ < ____ < ____	**b.** $\frac{1}{3}, \frac{4}{10}, \frac{2}{9}$ ____ < ____ < ____

8. Rebecca made a survey of a group of 600 women. She found that 1/3 of them never exercised, that 22/100 of them swam regularly, 1/5 of them jogged regularly, and the rest were involved in other sports.

a. Which was a bigger group, the women who jogged or the women who swam?

b. What fraction of this group of women exercise?

c. *How many women* in this group exercise?

d. How many women in this group swim?

Puzzle Corner

The seven dwarfs could not divide a pizza into seven equal slices. The oldest suggested, "Let's cut it into eight slices, let each dwarf have one piece, and give the last piece to the dog."

Then another dwarf said, "No! Let's cut it into 12 slices instead, and give each of us 1 ½ of those pieces, and the dog gets the 1 ½ pieces left over."

Which suggestion would give more pizza to the dog?

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Chapter 7: Fractions: Multiply and Divide

Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 8 on geometry in between chapters 6 and 7.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 6, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication (2/3) × 18 km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as 5 ÷ 3. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, 20 ÷ 6 = 3 2/6).

Then students encounter sharing divisions with fractions. For example, if two people share equally 4/5 of a pizza, how much will each person get? This is represented by the division (4/5) ÷ 2 = 2/5. Another case we study is dividing unit fractions by whole numbers (such as (1/2) ÷ 4).

We also divide whole numbers by unit fractions, such as 6 ÷ (1/3). Students will solve these thinking how many times the divisor "fits into" the dividend.

The last lesson is an introduction to ratios, and is optional. Ratios will be studied a lot in 6th and 7th grades, especially in connection with proportions. We are laying the groundwork for that here.

The Lessons in Chapter 7

	page	*span*
Simplifying Fractions 1	100	4 pages
Simplifying Fractions 2	104	4 pages
Multiply Fractions by Whole Numbers	108	4 pages
Multiplying Fractions by Fractions	112	5 pages
Fraction Multiplication and Area	117	6 pages
Simplifying Before Multiplying	123	4 pages
Multiplying Mixed Numbers	127	5 pages
Multiplication as Scaling/Resizing	132	4 pages
Fractions Are Divisions	136	4 pages
Dividing Fractions 1: Sharing Divisions	140	4 pages
Dividing Fractions 2: Fitting the Divisor	145	4 pages
Introduction to Ratios	149	4 pages
Mixed Review	153	3 pages
Review	156	4 pages

Helpful Resources on the Internet

General

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
http://www.mathmammoth.com/videos/fractions_2.php

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games; for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Simplifying & Equivalent Fractions

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
http://www.homeschoolmath.net/worksheets/fraction.php

Multiplication and Division

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiply and Reduce Fractions Battleship Game
When you hit the enemy's battleship, you need to solve a fraction multiplication problem.
http://www.quia.com/ba/57713.htm

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Number line bars
Fraction bars that illustrate visually how many times a fraction "fits into" another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

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Multiplying Fractions by Fractions

We have studied how to find a fractional part of a whole number using multiplication.

For example, $\frac{3}{5}$ of 80 is written as a multiplication: $\frac{3}{5} \times 80 = \frac{240}{5} = 48$.

NOTE: The word *OF* translates here into **MULTIPLICATION**.

We can use the same idea to find *a fractional part* ***of*** *a fraction*!

One-half of  is  .

As a multiplication, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

One-fourth of  is  .

As a multiplication, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

a. Find $\frac{1}{2}$ of 

$\frac{1}{2} \times \frac{1}{4} =$

b. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

c. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

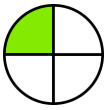
d. Find $\frac{1}{3}$ of 

e. Find $\frac{1}{3}$ of 

f. Find $\frac{1}{3}$ of 

g. Find $\frac{1}{4}$ of 

h. Find $\frac{1}{4}$ of 

i. Find $\frac{1}{4}$ of 

Did you notice a shortcut? If so, calculate $\frac{1}{5} \times \frac{1}{6} = \frac{\square}{\square}$

Shortcut: multiplying fractions of the type 1/*n*

To multiply fractions of the form 1/*n* where *n* is a whole number, simply multiply the denominators to get the new denominator →

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ or $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

2. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$	**b.** $\frac{1}{13} \times \frac{1}{3}$	**c.** $\frac{1}{5} \times \frac{1}{20}$

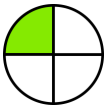
We have now studied how to find 1/2 or 1/3 or 1/5 of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

Review: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part *of* another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First, we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$. Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.

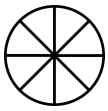
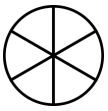
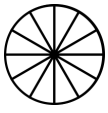
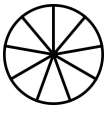
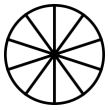
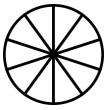
$\frac{2}{3}$ of  =

Example. Find $\frac{4}{5}$ of $\frac{1}{7}$.

First, we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction. It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Color in a picture to show the answer.

a. $\frac{3}{4} \times$  = 	**b.** $\frac{2}{3} \times$  = 
c. $\frac{3}{4} \times$  = 	**d.** $\frac{2}{3} \times$  = 
e. $\frac{2}{5} \times$  = 	**f.** $\frac{4}{5} \times$  = 

4. Solve the multiplications by using two helping multiplications. Lastly, simplify if possible.

a. $\frac{2}{3} \times \frac{1}{8} =$

First find 1/3 of 1/8, then multiply the result by 2.

$\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$ and $\frac{1}{24} \times 2 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

b. $\frac{3}{4} \times \frac{1}{10} =$

First find 1/4 of 1/10, then multiply the result by 3.

$\frac{1}{4} \times \frac{1}{10} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad}$

c. $\frac{3}{5} \times \frac{1}{6} =$

First find 1/5 of 1/6, then multiply the result by 3.

$\frac{1}{5} \times \frac{1}{6} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

d. $\frac{5}{6} \times \frac{1}{9} =$

First find 1/6 of 1/9, then multiply the result by 5.

$\frac{1}{6} \times \frac{1}{9} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 5 = \frac{\quad}{\quad}$

e. $\frac{2}{3} \times \frac{1}{7} =$

f. $\frac{3}{8} \times \frac{1}{4} =$

A shortcut for multiplying fractions

Multiply the numerators to get the numerator for the answer.
Multiply the denominators to get the denominator for the answer.

Study the examples on the right.

Remember always to give your final answer as a mixed number and in lowest terms (simplified).

$\frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$

$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1\frac{1}{10}$

5. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$

b. $\frac{11}{12} \times \frac{1}{6}$

c. $\frac{1}{3} \times \frac{3}{13}$

d. $9 \times \frac{2}{3}$

e. $\frac{2}{9} \times \frac{6}{7}$

f. $10 \times \frac{5}{7}$

COMPARE	
The roundabout way	**The shortcut**
$\frac{5}{6} \times \frac{1}{2} = ?$ First find 1/6 of 1/2, then multiply the result by 5. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ Find 1/8 of 3/5, then multiply that result by 2. And to find 1/8 of 3/5, first find 1/8 of 1/5, and then multiply that by 3. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$. That multiplied by 3 is $\frac{1}{40} \times 3 = \frac{3}{40}$. Then, that multiplied by 2 is $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$.	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.	

6. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8} =$	**b.** $\frac{7}{10} \times \frac{8}{5} =$
c. $\frac{9}{20} \times \frac{4}{5} =$	**d.** $\frac{2}{5} \times \frac{1}{3} =$
e. $\frac{1}{4} \times \frac{2}{7} =$	**f.** $\frac{9}{4} \times \frac{1}{3} =$
g. $\frac{2}{3} \times \frac{11}{8} =$	**h.** $\frac{2}{9} \times \frac{3}{10} =$

7. There was 1/4 of the pizza left. Marie ate 2/3 of that.

a. What part of the *original* pizza did she eat?

b. What part of the *original* pizza is left now?

8. Theresa has painted 5/8 of the room.

a. What part is still left to paint?

b. Now, Theresa has painted half of what was still left.
Draw a bar model of the situation.
What part of the room is still left to paint?

9. Ted has completed 2/3 of a job that his boss gave him.

a. What part is still left to do?

b. Now Ted has completed a third of what was still left to do.
Draw a bar model of the situation.
What (fractional) part of the original job is still left undone?

What part is completed?

10. Sally wants to make 1/3 of the recipe on the right.
How much does she need of each ingredient?

Carob Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
1/2 cup honey
1 teaspoon vanilla
3/4 cup whole wheat flour
3/4 teaspoon baking powder
1 cup walnuts or other nuts

11. For an upcoming get-together, Alison needs to multiply the coffee recipe. Assume that half of the guests drink one serving, and the other half drink two servings. Find how much coffee she will need, if she has:

a. 30 guests

b. 50 guests

c. 80 guests.

Coffee (5 servings)

3 1/2 cups water
1/4 cup coffee

Puzzle Corner Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$	**b.** $\times \frac{1}{4} = \frac{5}{16}$	**c.** $\times \frac{3}{8} = \frac{1}{16}$	**d.** $\times \frac{2}{5} = \frac{3}{10}$

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Chapter 8: Geometry
Introduction

The problems in this chapter involve lots of drawing. Geometry is a hands-on subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol “  ” are meant to be done in a notebook or on blank paper.

This chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and how to draw a perpendicular line through a given point on a line. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

The Lessons in Chapter 8

	page	*span*
Review: Angles	164	2 pages
Review: Drawing Polygons	166	4 pages
Circles	170	3 pages
Quadrilaterals	173	4 pages
Equilateral, Isosceles, and Scalene Triangles	177	5 pages
Area and Perimeter Problems	182	3 pages
Volume	185	5 pages
Volume of Rectangular Prisms (Cuboids)	190	4 pages
A Little Bit of Problem Solving	194	2 pages
Mixed Review	196	3 pages
Review	199	3 pages

Helpful Resources on the Internet

General/Review/Fun things

Turtle Pond
Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Shape Explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Interactive Tangram Puzzle
Place the tangram pieces so that they form a given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation! Requires Java.
http://www.shodor.org/interactivate/activities/Tessellate

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry
A collection of interactive activities: fractals, geoboard activities, golden rectangle, ladybug leaf, ladybug mazes, tangrams, tessellations, transformations, and more.
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Quadrilaterals

Interactive Quadrilaterals
See all the different kinds of quadrilateral "in action". You can drag the corners, see how the angles change, and observe what properties do not change.
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Complete the Quadrilateral
Join the dots to complete these quadrilaterals. Where there are options, try to find the one on the grid with the largest possible area. The author, Fawn Nguyen, wishes, "If appropriate for your class, I hope you will consider doing this lesson — it will make the hours I spent recreating the handout and key worth it! :)" Great for 5th grader or whenever you study classifying quadrilaterals.
http://fawnnguyen.com/2013/02/08/don-stewards-complete-the-quadrilateral.aspx

Polygon Matching Game
Many of the polygons included are quadrilaterals.
http://www.mathplayground.com/matching_shapes.html

Classify Quadrilaterals Worksheets
Make free printable worksheets for classifying (idenfitying, naming) quadrilaterals. There are seven special types of quadrilaterals: square, rectangle, rhombus, parallelogram, trapezoid, kite, scalene, and these worksheets ask students to name the quadrilaterals among these seven types.
http://www.homeschoolmath.net/worksheets/classify_quadrilaterals.php

Quadrilaterals Quest
First, the quest asks you to choose all the quadrilaterals with the given properties. After several of those types of activities follows a quiz.
http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html

Quadrilateral Properties
Investigate the properties of square, rectangle, rhombus, an isosceles trapezoid, and a non-isosceles trapezoid in this dynamic, online activity.
http://www.glencoe.com/sites/texas/student/mathematics/assets/interactive_lab/geometry/G_08/G_08

Quadrilateral Classification Game
A virtual manipulative that challenges students to "draw" quadrilaterals with specific characteristics by moving vertices on a coordinate grid. Includes some challenging vocabulary, which is explained below the activity, such as orthodiagonal quadrilateral, cyclic, or convex quadrilateral.
http://www.uff.br/cdme/jcq/jcq-html/jcq-en.html

Triangles

Classify Triangles Worksheets
Make free printable worksheets for classifying triangles by their sides, angles, or both.
http://www.homeschoolmath.net/worksheets/classify_triangles.php

Triangle Classification at Cut The Knot
A tutorial and an applet about classifying triangles by their sides and angles. In the applet, you can drag any of the vertices of the triangle, and the applet tells you whether your triangle is acute, obtuse, or right, or equilateral, isosceles, or scalene.
http://www.cut-the-knot.org/triangle/Triangles.shtml

Rags to Riches: Classify Triangles by Sides and Angles
Answer multiple-choice questions about the angles of a triangle and classification of triangles in a quest for fame and fortune.
http://www.quia.com/rr/457498.html

Identify Triangles Quiz
A simple multiple-choice quiz about identifying (classifying) triangles either by their sides or angles. You can modify some of the quiz parameters, such as the number of problems in it.
http://www.thatquiz.org/tq-A/?-j1-l34-p0

Triangles & Quadrilaterals Classification Game
Look at the shapes as they go past, and drag them into the right groups (equilateral, isosceles, or scalene triangles, and quadrilaterals with 4 congruent sides, 2 congruent sides, or no congruent sides).
http://www.bbc.co.uk/bitesize/ks2/maths/shape_space/shapes/play/

Classifying Triangles Game
A fast-paced game where you drag triangles into the correct basket as fast as you can (acute, obtuse, right).
http://www.math-play.com/classifying-triangles/classifying-triangles.html

Triangle Classification Exploration Tool
Line segment AB is drawn in the plane. Where should point C be placed so that ABC is a right triangle? …so that it is an isosceles triangle? …so that it is obtuse? This activity will allow you to explore these questions.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=142

Volume

Geometric Solids
Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

Cuboid Exploder and **Isometric Shape Exploder**
These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes, and then "explode" them to the unit cubes, illustrating volume.
www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html and
www.teacherled.com/resources/isoexplode/isoexplodeload.html

Geometry Volume/Surface Area Quiz from ThatQuiz.org
An online quiz, asking either the volume or surface area of cubes, prisms, spheres, cylinders, or cones. You can modify the quiz parameters to your liking, for example to omit some shapes, solve only for volume or surface area, or instead of solving for volume/surface area, you solve for an unknown dimension (side or radius) when the volume or surface area is given.
www.thatquiz.org/tq-4/?-j3vu0-lc-m2kc0-na-p0

Cubes
An online tool where you can explore filling a rectangular prism (a box) with unit cubes, rows of cubes, or layers of cubes. You can use this to let the student find the rule for finding the volume of a box if you know its width, depth, and height. Requires Java.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

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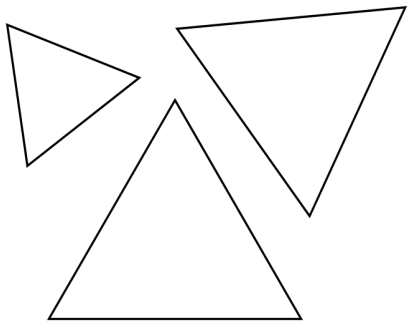
Equilateral, Isosceles, and Scalene Triangles

If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

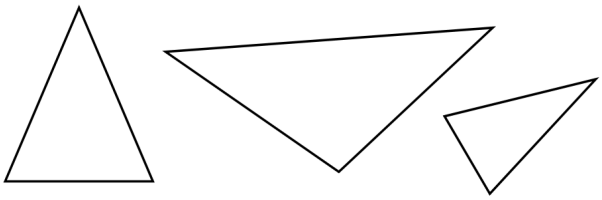
Equi- refers to things that are the "same" or "equal", and *lateral* means "sided." Think of it as a "same-sided" triangle.



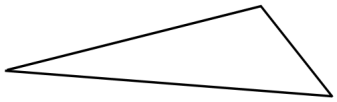
If just *two* of a triangle's sides are congruent, then it is called an **isosceles triangle**.

Think of it as a "same-legged" triangle, the "legs" being the two sides that are the same length.

Mark the two congruent sides of each isosceles triangle:

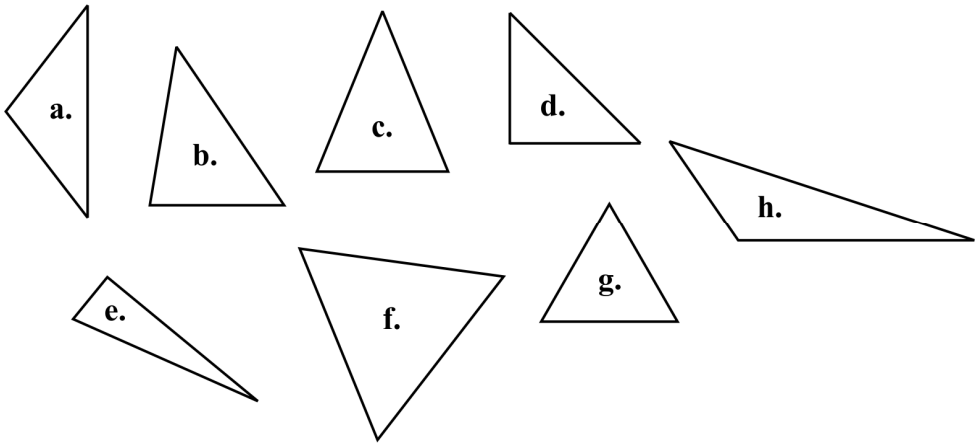


Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.



1. Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

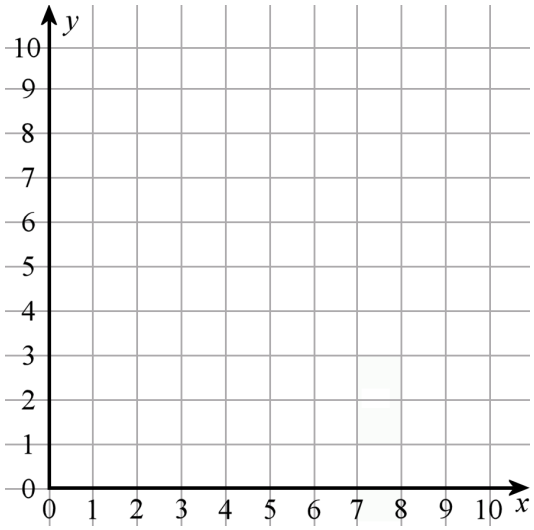
   You can mark each triangle with an "*e*," "*i*," or "*s*" correspondingly.



2. Fill in the table by classifying the triangles labeled as (a), (d), (e), and (g) above as "acute," "right," or "obtuse" (by their angles), and also as "equilateral," "isosceles," or "scalene" (by their sides).

Triangle	Classification by the sides	Classification by the angles
a		
d		
e		
g		

3. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles and sides.



Triangle 1: (0, 0), (4, 0), (0, 4)

_____ and

_____

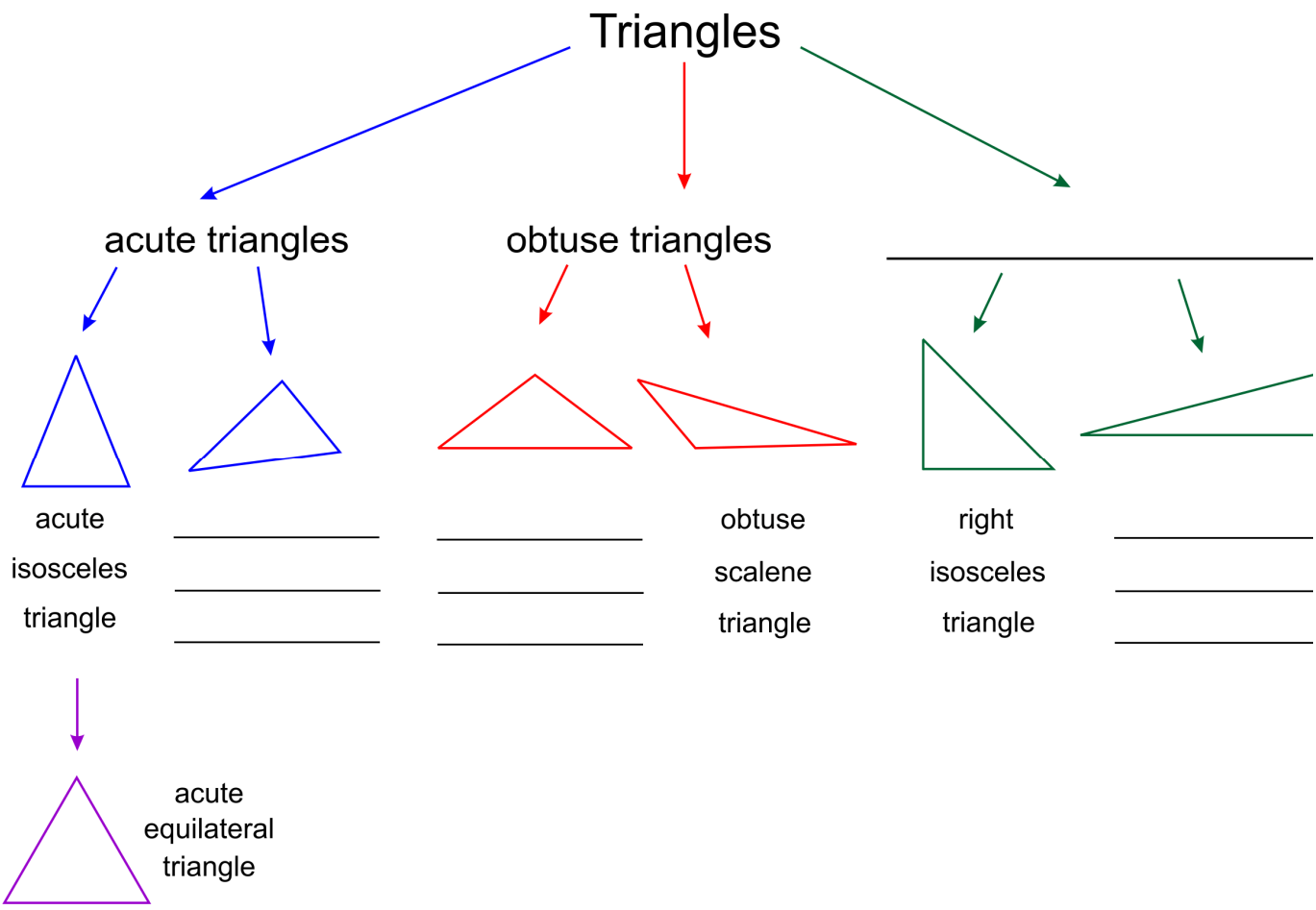
Triangle 2: (5, 5), (1, 8), (9, 4)

_____ and

_____

4. Plot in the coordinate grid an acute scalene triangle.

5. Fill in the missing parts in this tree diagram classification for triangles.



6. **a.** Draw a scalene obtuse triangle where one side is 3 cm and another is 7 cm.
Hint: Draw the 7-cm side first, then the 3-cm side forming any obtuse angle with the first side.

b. Measure the third side.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

7. **a.** Draw an isosceles right triangle whose two sides measure 5 cm.
Hint: Draw a right angle first. Then, measure off the 5-cm sides. Then draw in the last side.

b. Measure the third side. It is _____ cm.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

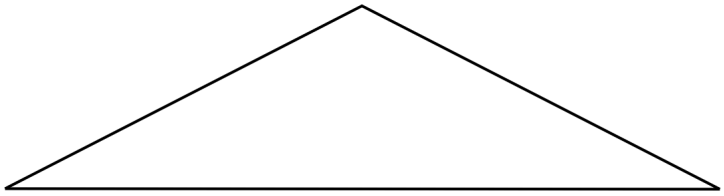
8. **a.** Draw any isosceles triangle.
Hint: Draw any angle. Then, measure off the two congruent sides, making sure they have the same length. Then draw the last side.

b. Measure the angles of your triangle. They measure _____ °, _____ °, and _____ ° .

The angle sum is _____ ° .

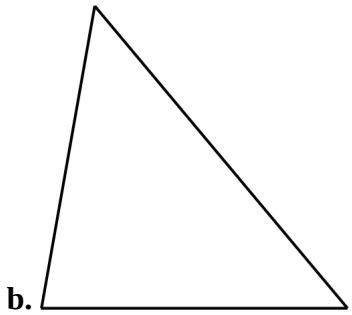
9. Measure all the angles in the isosceles triangles (a) and (b).
Continue their sides, if necessary.

a.



_____ °, _____ °, and _____ ° .

The angle sum is _____ ° .



b.

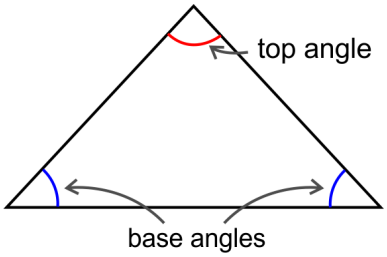
_____ °, _____ °, and _____ °.

The angle sum is _____ ° .

What do you notice?

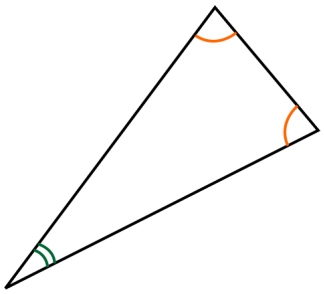
_____

_____



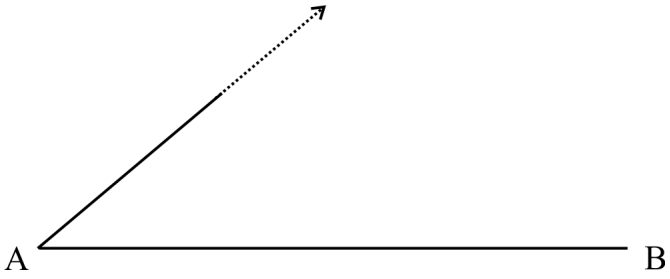
There are two angles in an isosceles triangle that have the same angle measure. They are called the **base angles.**

The remaining angle is called the **top angle**.



Can you find the top angle and the base angles in this isosceles triangle?

10. The angle at A measures 40°. Draw another angle of 40° at B, and then continue its side so that you get an isosceles triangle with 40° base angles.



Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

11. **a.** Draw an isosceles triangle with 75° base angles. (The length of the sides can be anything.)
Hint: start by drawing the base side (of any length). Then, draw the 75° angles.

b. Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

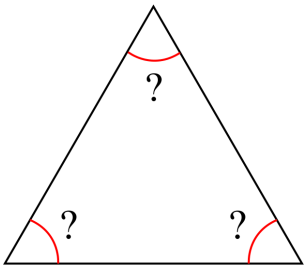
c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

12. **a.** Draw an isosceles triangle with a 50° top angle.
Hint: start by drawing a 50° angle. The two sides of the angle you drew are the two congruent sides of the triangle, so choose how long those sides should be, measure, and mark them. Then draw in the third side.

b. The base angles are _____ ° each. The three angle measures add up to _____ ° .

c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

13. Make a guess about the angle measures in an equilateral triangle: _____°
Measure to check.



14. **a.** Could an equilateral triangle be a right triangle?
If yes, sketch an example. If not, explain why not.

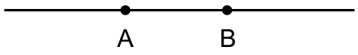
b. Could a scalene triangle be obtuse?
If yes, sketch an example. If not, explain why not.

c. Could an acute triangle be scalene?
If yes, sketch an example. If not, explain why not.

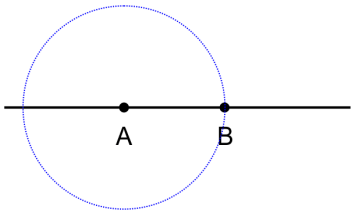
d. Could a right triangle be scalene?
If yes, sketch an example. If not, explain why not.

e. Could an obtuse triangle be equilateral?
If yes, sketch an example. If not, explain why not.

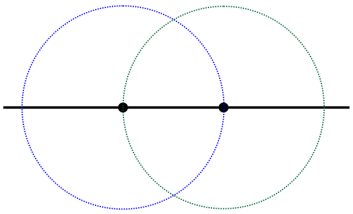
Draw an equilateral triangle with a compass and a ruler (optional)



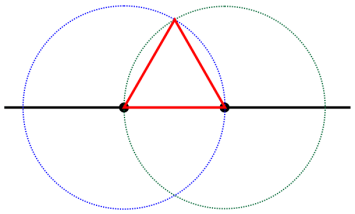
Draw a line segment and mark two points on it. These points mark one of the sides of the triangle.



Draw a circle using point A as the center point and the length AB as the radius. The third vertex of the triangle MUST lie on this circle. Can you see why?



Can you see what was done in this picture?



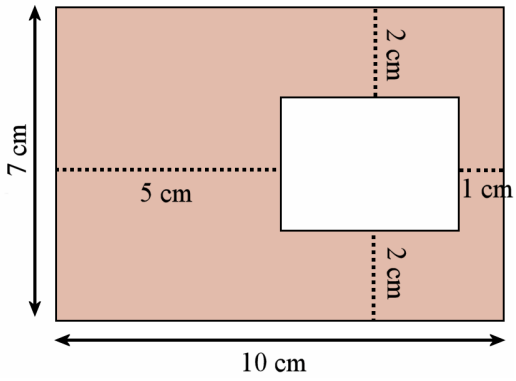
The triangle is finished!

15. Draw at least three different equilateral triangles in your notebook using the method above.
Can you explain why this method works?

Important Terms
• *equilateral triangle* • *isosceles triangle* • *scalene triangle*

Area and Perimeter Problems

Find the area of the shaded figure.



The easiest way to do this is:
(1) Find the area of the larger outer rectangle,
(2) find the area of the white inner rectangle, and
(3) subtract.

1. The area of the large rectangle is 7 cm × 10 cm = 70 cm^2.

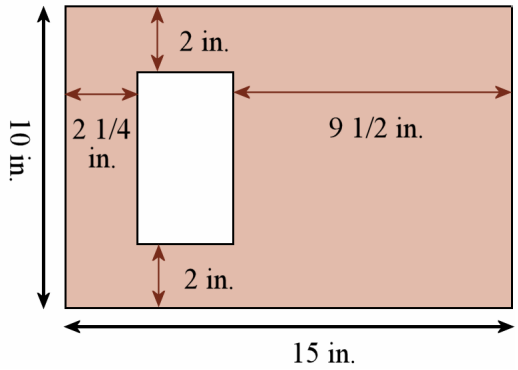
2. We find the *sides* of the white rectangle by subtracting.

   The longer side of the white rectangle is
   10 cm − 5 cm − 1 cm = 4 cm.
   The shorter side is 7 cm − 2 cm − 2 cm = 3 cm.

   So, the area of the white rectangle is 4 cm × 3 cm = 12 cm^2.

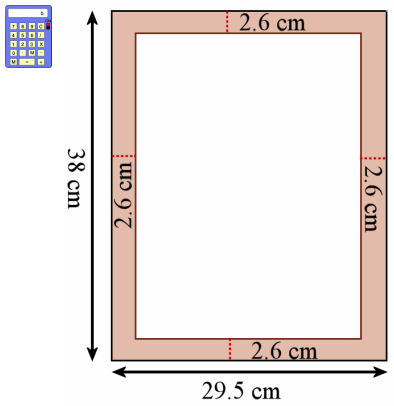
3. Now we subtract to find the shaded area: 70 cm^2 − 12 cm^2 = 58 cm^2.

1. **a.** Find the area of the white rectangle.
   All lines meet at right angles.



b. Find the area of the shaded figure.

2. The image on the right shows a picture frame.
   Find the area of the actual frame (that is, of the shaded part).
   All lines meet at right angles.



math MAMMOTH

Grade 5-B Worktext

Light Blue Series

- Graphing and statistics
- Fractions: add and subtract
- Fractions: multiply and divide
- Geometry

By Maria Miller

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Foreword

Math Mammoth Grade 5-B Worktext comprises a complete math curriculum for the second half of fifth grade mathematics. Fifth grade focuses on fractions and decimals, in particular. In part 5-A, students have studied the four operations with whole numbers, large numbers, problem solving, and decimal arithmetic. In this part, 5-B, we study graphing, fraction arithmetic, and geometry.

This book starts with chapter 5, where we study graphing in a coordinate grid, line and bar graphs, and average and mode. Today's world has become increasingly complex, with lots of data in the media, so our children need a good grasp of graphs to be able to make sense of all that information.

Chapter 6 is about the addition and subtraction of fractions—another focus topic for 5th grade, besides decimals. Students learn to add and subtract unlike fractions, using the technique of first converting them to equivalent like fractions. In chapter 7, we study the multiplication and division of fractions (division only in special cases), relying first on visual models, and then proceeding to the abstract shortcuts.

Chapter 8 takes us to geometry, starting with a review of angles and polygons. From there, students will learn to draw circles, to classify triangles and quadrilaterals, and the concept of volume in the context of right rectangular prisms (boxes).

I wish you success in your math teaching!

Maria Miller, the author

Chapter 5: Statistics and Graphing
Introduction

This chapter starts out with a study of the coordinate grid, but only in the first quadrant. Besides learning how to plot points, students also plot ordered pairs (points) from number patterns or rules. This is actually the beginning of the study of *functions*.

Practicing the use of the coordinate grid is a natural "prelude" to the study of line graphs, which follows next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs and histograms from a given set of data.

In order to make histograms, it is necessary to understand how to group the data into categories. The lesson *Making Histograms* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study average (also called the *mean*) and mode, and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is plenty of time to learn that concept in subsequent grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all that many students are able to grasp from that is only the calculation procedures. I feel it is better to introduce and contrast initially only the two concepts, the mean and the mode, in order to give the student a solid foundation. We can introduce the median later, and then compare and contrast it with the other two.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 5

	page	span
Coordinate Grid	10	*3 pages*
Number Patterns in the Coordinate Grid	13	*4 pages*
More Number Patterns in the Coordinate Grid	17	*4 pages*
Line Graphs	21	*4 pages*
Reading Line Graphs	25	*2 pages*
Double and Triple Line Graphs	27	*2 pages*
Making Bar Graphs	29	*2 pages*
Making Histograms	31	*2 pages*
Double Bar Graphs	33	*2 pages*

Average (Mean)	35	*3 pages*
Mean, Mode, and Bar Graphs	38	*2 pages*
Statistics Project (optional)	40	*1 page*
Mixed Review	41	*3 pages*
Review	44	*2 pages*

Helpful Resources on the Internet

Coordinate Grid

Billy Bug Returns at Primary Games
Move Billy Bug to the feeding place with given coordinates.
http://www.primarygames.co.uk/pg2/bug2/bug2.html

Co-ordinate Game
You will see a red circle on the grid. Enter the co-ordinates and click "check."
http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/maths/coordinate_game/game1.htm

Graphit
A graphing tool that plots both functions and ordered pairs.
http://www.shodor.org/interactivate/activities/graphit/index.html

Graph Mole
A fun game about plotting points on a coordinate plane. Plot points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Coordinate Grid Quiz from ThatQuiz.org
This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0

Graphing and Graphs

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool:
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness, and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Data Grapher
This basic data grapher can be used to create bar graphs, line graphs, pie charts, and pictographs. You can enter multiple rows and columns of data, select which set(s) to display in a graph, and choose the type of representation.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=204

Histogram Tool
Create a histogram from your data, or analyze histograms from pre-made data.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Create a Graph
Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Graphs Quiz from That Quiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, box plot, scatter graph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
http://www.thatquiz.org/tq-5/math/graphs

Math Goodies Interactive Data and Graphs Lessons
Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs, and exercises.
http://www.mathgoodies.com/lessons/toc_vol11.html

Data Analysis Gizmos from Explorelearning.com
Interactive online simulations or activities, with lesson plans. Topics include creating a bar graph or aline graph, pictographs, mean and median, and a reaction time gizmo. This is an excellent resource. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=383

Statistics Interactive Activities
(scroll down to Statistics and Probability concepts)
A set of interactive tools for exploring histograms, pie charts, box plots, stem-leaf plots, and mean, median, variance, and standard deviation of data. You can enter your own data or explore the examples.
http://www.shodor.org/interactivate/activities/tools.html

Mean, Median, Mode, Range

Using and Handling Data
Simple explanations for finding mean, median, or mode.
http://www.mathsisfun.com/data/index.html#stats

Math Goodies Interactive Statistics Lessons
Clear lessons with examples and interactive quiz questions, including range, arithmetic mean, non-routine mean, median, and mode, practice exercises, and challenge exercises.
http://www.mathgoodies.com/lessons/toc_vol8.htm

Mean, Median, and Mode
How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.
http://www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median Lessons
Explanations with simple examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/data/measuresofaveragerev1.shtml

Measures Activity
Enter you own data and the program will calculate mean, median, mode, range and some other statistical measures.
http://www.shodor.org/interactivate/activities/Measures

Mean/Mode Quiz
A 10-question quiz about calculating the mode and mean.
http://www.thatquiz.org/tq-p-z1/?-j6g00-l5-p0

Landmark Shark Game
You are dealt five number cards, and using that as your data set you need to choose which of the range, median, or mode is the largest number.
http://media.emgames.com/emgames/demosite/playdemo.html?activity=M5A006&activitytype=dcr&level=3

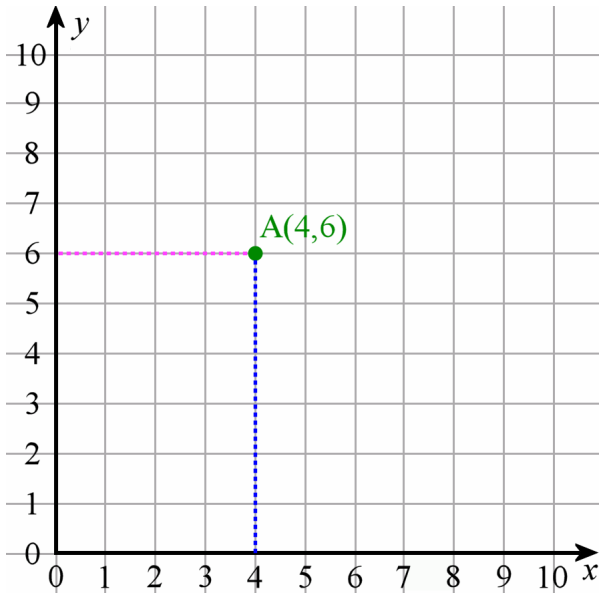
Coordinate Grid

This is a **coordinate grid**.

The long black line across the bottom, with the "*x*" near its end, is called the ***x*-axis**.

The vertical line that has "*y*" near the top is called the ***y*-axis**.

You can see one point, called "A," that is drawn or *plotted* on the grid.



It has two numbers *associated*, or matched, with it. Those two numbers are called the **coordinates** of the point A.

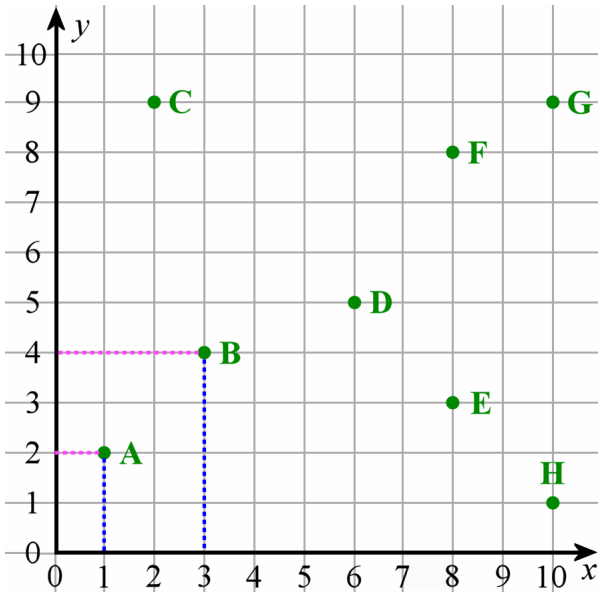
The first number is called the **x-coordinate** of the point A, and the second number is called the **y-coordinate** of the point A.

The x-coordinate of the point A is 4 because if you drew a line straight down from A, it would *intersect*, or "hit," the *x*-axis at 4. The y-coordinate of the point A is 6 because if you drew a line straight left from A, it would intersect the *y*-axis at 6.

We write the two coordinates of a point inside parentheses, separated by a comma.

Note: The order of the two coordinates matters. The *first* number is ALWAYS the *x*-coordinate, and the *second* number is ALWAYS the *y*-coordinate, not the other way around.
So (5, 8) means the *x*-coordinate is 5 and the *y*-coordinate is 8.

1. Write the two coordinates of the points plotted on the coordinate grid. For points A and B, the helping lines are drawn in.



A (___, ___) B (___, ___)

C (___, ___) D (___, ___)

E (___, ___) F (___, ___)

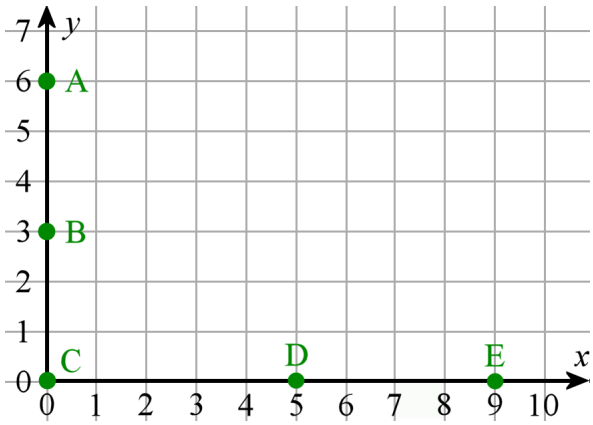
G (___, ___) H (___, ___)

Notice especially the points that are located on the two axes.

If a point lies on the *y*-axis, its *x*-coordinate is zero. A is (0, 6), and B is (0, 3) .

If the point lies on the *x*-axis, its *y*-coordinate is zero. D is (5, 0) and E is (9, 0).

The point C has the coordinates (0, 0). This point (0, 0) is called the *origin*.

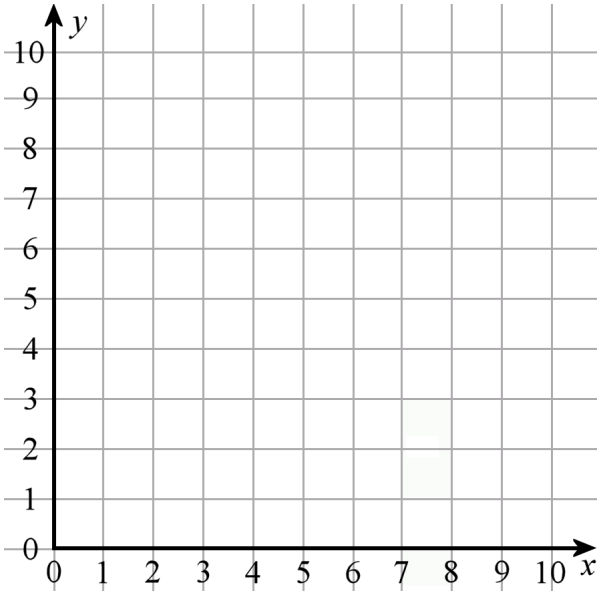


2. Plot and label the following points on the coordinate grid.

A (2, 8) B (0, 5) C (4, 0)

D (9, 10) E (8, 5) F (1, 4)

G (1, 0) H (0, 8) I (3, 7)



3. The coordinate grid is very useful for many things. For example, computer drawing programs use it frequently. Let's say "LINE (5,6) - (2,7)" means a straight line segment that is drawn from the point (5, 6) to the point (2, 7).

Draw the following line segments.
What figure is formed?

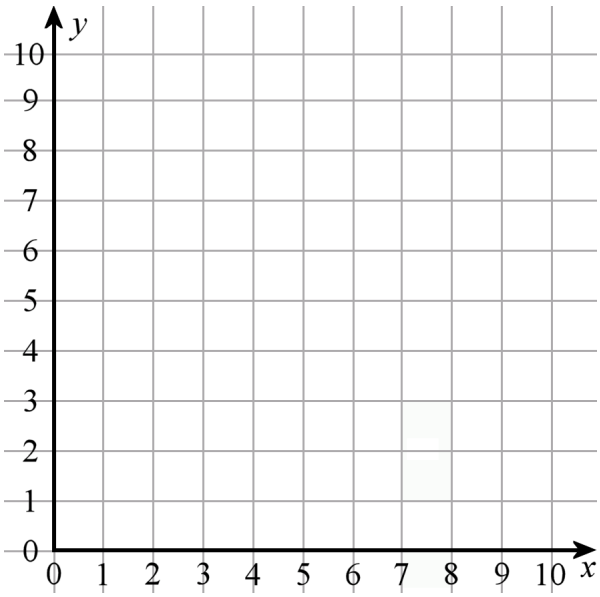
LINE (1, 0) - (7, 0) LINE (7, 0) - (7, 5)

LINE (1, 0) - (1, 5) LINE (1, 5) - (0, 5)

LINE (0, 5) - (4, 7) LINE (4, 7) - (8, 5)

LINE (8, 5) - (7, 5) LINE (3, 0) - (3, 3)

LINE (5, 0) - (5, 3) LINE (3, 3) - (5, 3)

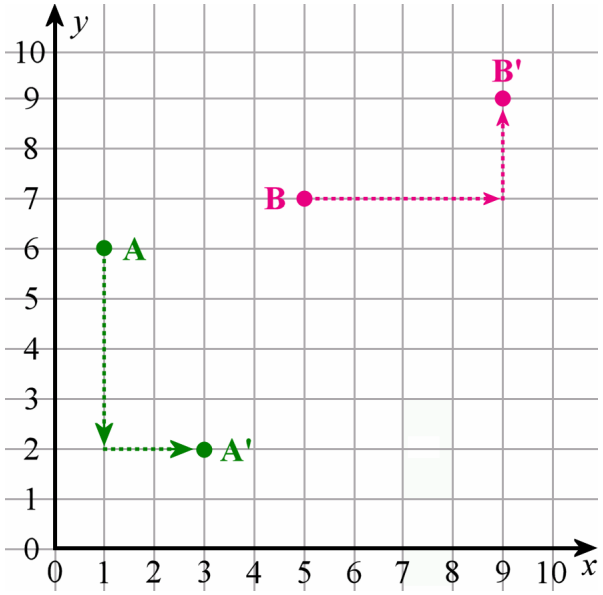


This example shows point A moving four units down and then two units to the right. The new location is called point A′ (read "A prime").

Originally A's coordinates were (1, 6).
After the movement, the coordinates are (3, 2)

Notice how you can just subtract four units from the *y*-coordinate (the movement four units straight down) and add two units to the *x*-coordinate (movement two units to the right).

Point B is originally at (5, 7). It moves four units to the right and two up. You add four to the *x*-coordinate, and two to the *y*-coordinate. Its new coordinates are (9, 9).

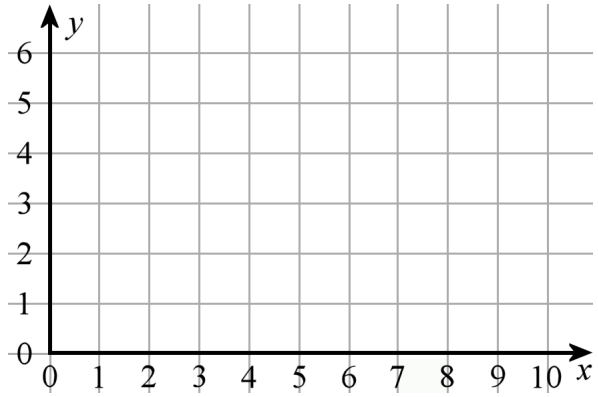


Movement up or down affects the *y*-coordinate.
Movement right or left affects the *x*-coordinate.
In other words, movement *parallel* to an axis affects that same coordinate.

4. The three vertices of a triangle are (2, 0), (5, 1) and (3, 4). The triangle is moved three units to the right and two up.

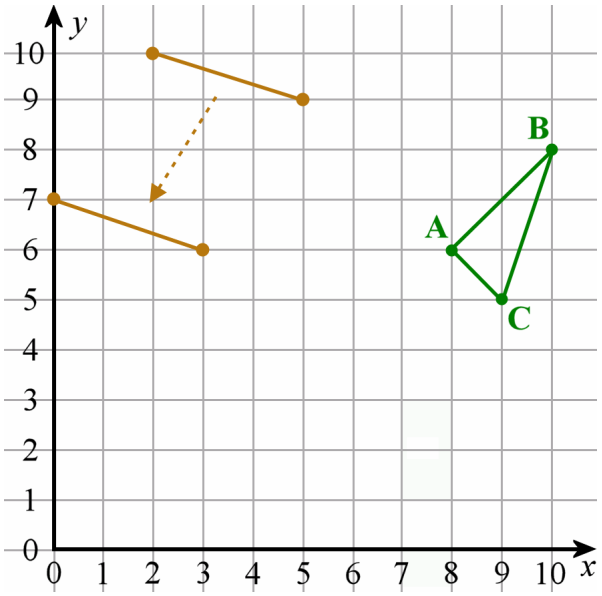
   a. Plot the vertices of the triangle before and after the movement.

   b. Write the coordinates of the vertices after the movement.



5. **a.** Determine how the line segment has been moved, and move the triangle ABC the same way. Let's call the new triangle A′B′C′. Write down the coordinates of the vertices of the triangle A′B′C′ after the movement.

   b. Let's say the point (3, 5) moves to (2, 7). Move the triangle ABC in a similar way. Write down the coordinates of the triangle's vertices after the movement.



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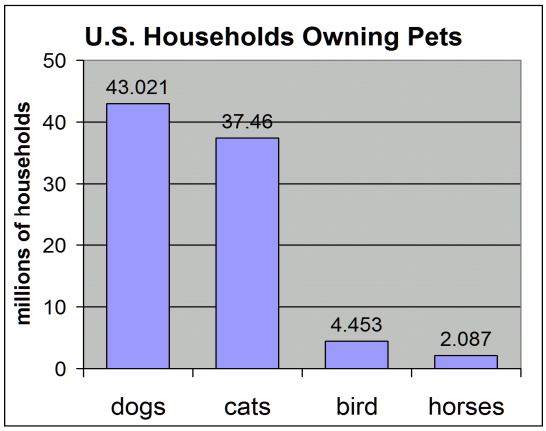
Mean, Mode, and Bar Graphs



Do you think you could calculate the average from the data shown in the bar graph? After all, there are numbers involved.

Actually, we cannot. To see why, you need to think *what kind of original data* produced this graph. What was asked of the people in the study? What did they respond?

The people were asked something like, "What pets do you have?" The people would have answered, "cat," or "dog," and so on.

The original data set consists simply of the words "cat," "dog," "bird," and "horse"—each one listed many times, because each mention of a "cat" would mean the answer of one particular household.

cat, cat, dog, dog, dog, bird, dog, dog, bird, cat, dog, horse, dog, cat, dog, ...

We cannot calculate anything from this kind of data set because it is not numerical data! However, we CAN find the most commonly occurring item, and that is called the ***mode***.

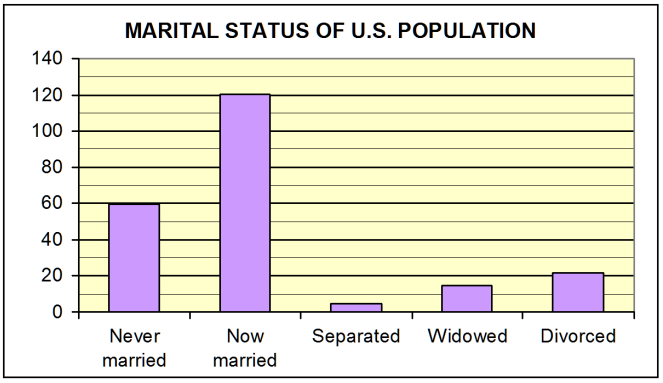
In this case, the mode is *dog.* It made the highest bar on the graph.

The *mode* is the most commonly occurring item in a data set.

- Sometimes a set of data has two or more modes. For example, the data set *green, green, blue, blue, black, brown, hazel* has two modes: both green and blue are equally common.

- If none of the items occurs twice or more, there is no mode. For example, this data: *green, blue, pink, red, black, brown, purple* has no mode.

1. Find the mode of the data set shown in the bar graph on the right.



2. **a.** Find the mode of this data:

   water, pop, juice, pop, juice, water, milk, water, pop, pop, juice, pop

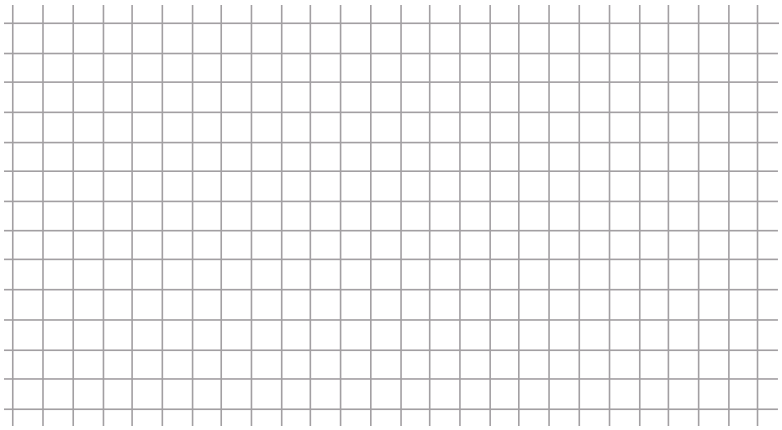
   b. If the above are the answers of 12 people to some question, what could have been the question?

3. Nineteen children were asked about their favorite ice cream flavor. Here are their responses:
strawberry, vanilla, chocolate, vanilla, chocolate chip, chocolate, pecan, pecan, vanilla, vanilla, strawberry, chocolate chip, vanilla, chocolate, chocolate, vanilla, strawberry, chocolate chip, vanilla.

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.



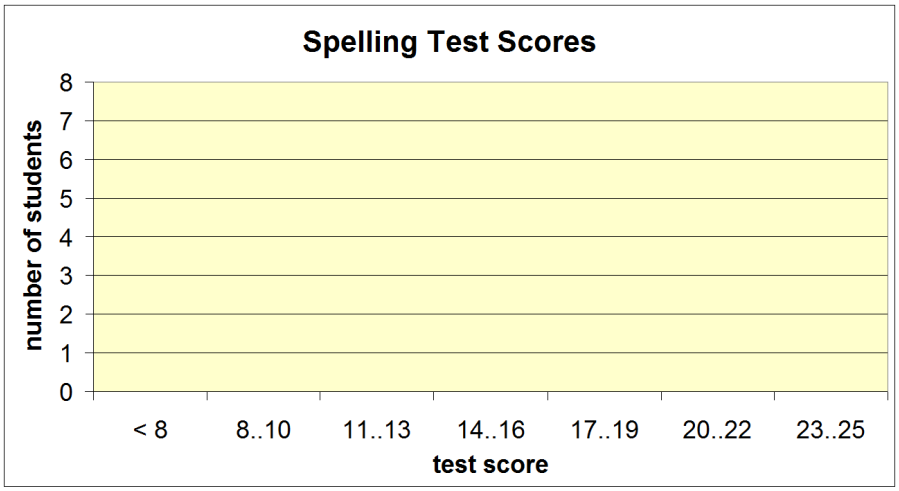
4. These are the spelling test scores of a fifth grade class:
4 5 7 9 9 10 10 11 11 12 12 12 13 14 17 18 18 18 19 19 19 20 24 25

a. Find the mode.

b. Draw a bar graph.

c. If possible, calculate the mean.

Test Score	Frequency
< 8	
8..10	
11..13	
14..16	
17..19	
20..22	
23..25	



5. **a.** Find the mode.

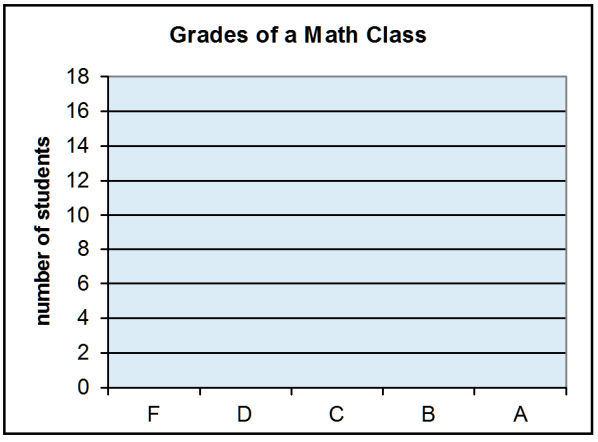
b. Draw a bar graph.

c. If possible, calculate the average.

d. There were ____ students in all. What *fraction* of the students got grade B?

Grades of a math class

Grade	Frequency
F	3
D	8
C	12
B	17
A	10



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Chapter 6: Fractions: Add and Subtract
Introduction

In 5th grade, students study most aspects of fraction arithmetic: addition, subtraction, multiplication, and then in some special cases, division. Division of fractions is studied in more detail in 6th grade. I hope that students have already built a solid conceptual understanding in their minds in previous years, so we can build on that foundation.

The chapter starts out with lessons on various ways to add and subtract mixed numbers. These are meant partially to review and partially to develop speed in fraction calculations. The lesson *Subtracting Mixed Numbers 2* presents an optional way to subtract, where we use a negative fraction. This is only meant for students who can easily grasp subtractions such as $(1/5) - (4/5) = -3/5$, and is not intended to become a "stumbling block." Simply skip the method if your student does not understand it easily.

Students have already added and subtracted *like* fractions in fourth grade. Now it is time to "tackle" the more complex situation of *unlike* fractions.

First, we review how to convert fractions into other equivalent fractions. We begin with a visual model of splitting pieces of pie, and from that, we develop the common procedure for equivalent fractions.

This skill is used immediately in the next lessons about adding and subtracting unlike fractions. We begin this topic by using visual models. From the visual and concrete we gradually advance toward the abstract. Several lessons are devoted to understanding and practicing the basic concept, and also to applying this new skill to mixed numbers.

The lesson *Comparing Fractions* reviews some mental math methods for comparing fractions. Students also learn a "brute force" method based on converting fractions to equivalent fractions. This chapter ends with a lesson on measuring in inches, using units as small as 1/16 of an inch.

The Lessons in Chapter 6

	page	*span*
Fraction Terminology	50	*1 page*
Review: Mixed Numbers	51	*4 pages*
Adding Mixed Numbers	55	*3 pages*
Subtracting Mixed Numbers 1	58	*4 pages*
Subtracting Mixed Numbers 2	62	*2 pages*
Equivalent Fractions 1	64	*3 pages*
Equivalent Fractions 2	67	*2 pages*
Adding and Subtracting Unlike Fractions	69	*3 pages*
Finding the (Least) Common Denominator	72	*3 pages*
Add and Subtract: More Practice	75	*3 pages*
Adding and Subtracting Mixed Numbers	78	*5 pages*
Comparing Fractions	83	*4 pages*
Measuring in Inches	87	*5 pages*
Mixed Review	92	*3 pages*
Review	95	*2 pages*

Helpful Resources on the Internet

General

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages. The activity includes several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the representations of the fractions and the models.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software
Games for addition and subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.
http://www.sheppardsoftware.com/math.htm#fractions

Who Wants Pizza?
This tutorial explains fraction addition and multiplication using a pizza, and then includes some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Fractioncity
Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Equivalent Fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Equivalent Fractions with Visual Models
Create custom-made worksheets for equivalent fractions. Choose to include pie images or not.
http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

Addition and Subtraction

Fraction Videos 1: Addition and Subtraction
My own videos that cover equivalent fractions, addition and subtraction of like and unlike fractions, and of mixed numbers.
http://www.mathmammoth.com/videos/fractions_1.php

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding Fractions
Illustrates how to find the common denominator when adding two unlike fractions using interactive pie models.
http://nlvm.usu.edu/en/nav/frames_asid_106_g_3_t_1.html

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure of adding two unlike fractions. A free registration required.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
The computer gives you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction.php

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Comparing Fractions Tool at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/app/tool/comparing-fractions

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Comparing/Ordering Fractions Worksheets
Create free worksheets for comparing two fractions or ordering 3-8 fractions. Compare fractions with the same denominator, fractions with the same numerator, or you compare a fraction to 1/2, or to 1, and so on. You can also include images (fraction pies).
http://www.homeschoolmath.net/worksheets/comparing_fractions.php

Measure It!
Practice measuring lines in inches. Multiple choice questions.
http://onlineintervention.funbrain.com/measure/index.html

Fraction Terminology

As we study fractions and their operations, it is important that you understand the terms, or words, that we use. This page is a reference. You can even post it on your wall or make your own fraction poster based on it.

$\frac{3}{11}$ The top number is the **numerator**. It *enumerates*, or numbers (counts), *how many* pieces there are.
The bottom number is the **denominator**. It *denominates*, or names, *what kind* of parts they are.

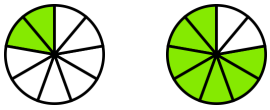
A mixed number has two parts: a whole-number part and a fractional part.

For example, $2\frac{3}{7}$ is a mixed number. Its whole-number part is 2, and its fractional part is $\frac{3}{7}$.

The mixed number $2\frac{3}{7}$ actually means $2 + \frac{3}{7}$.

Like fractions have the same denominator. They have the same kind of parts.

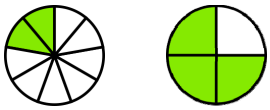
It is easy to add and subtract like fractions, because all you have to do is look at *how many* of that kind of part there are.



$\frac{2}{9}$ and $\frac{7}{9}$ are like fractions.

Unlike fractions have a different denominator. They have different kinds of parts.

It is a little more complicated to add and subtract unlike fractions. You need to first change them into like fractions. Then you can add or subtract them.

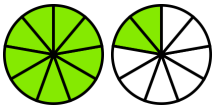


$\frac{2}{9}$ and $\frac{3}{4}$ are unlike fractions.

A proper fraction is a fraction that is less than 1 (less than a whole pie). 2/9 is a proper fraction.

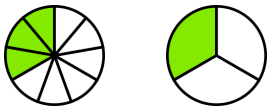
$\frac{2}{9}$ is a proper fraction.

An improper fraction is more than 1 (more than a whole pie). Since it is called a *fraction*, it is written as a fraction and *not* as a mixed number.



$\frac{11}{9}$ is an improper fraction.

Equivalent fractions are equal in value. If you think in terms of pies, they have the same amount of "pie to eat," but they are written using different denominators, or are "cut into different kinds of slices."



$\frac{3}{9}$ and $\frac{1}{3}$ are equivalent fractions.

Simplifying a fraction means that, for a given fraction, you find an equivalent fraction that has a "simpler," or smaller, numerator and denominator. (It has fewer but bigger slices.)

$\frac{9}{12}$ simplifies to $\frac{3}{4}$.

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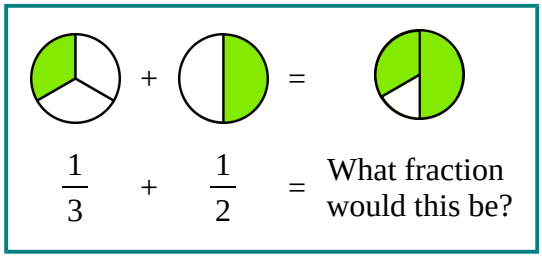
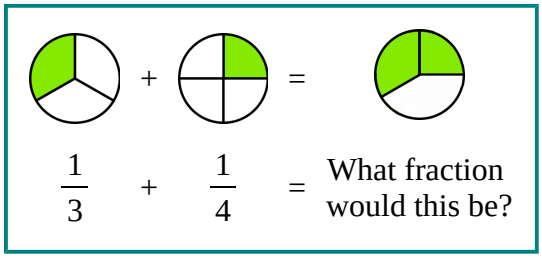
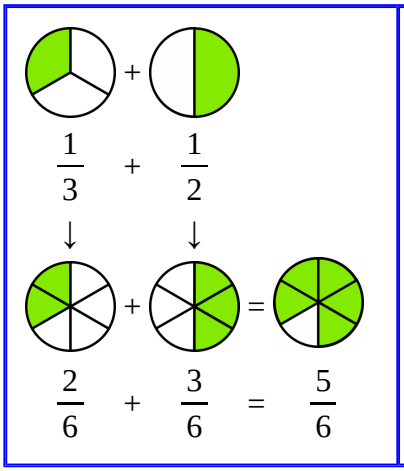
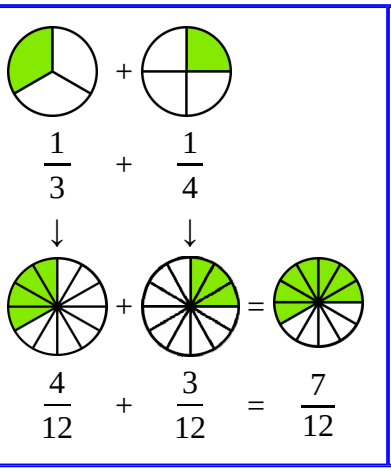
Adding and Subtracting Unlike Fractions

Cover the page below the black line. Then try to figure out the addition problems below.









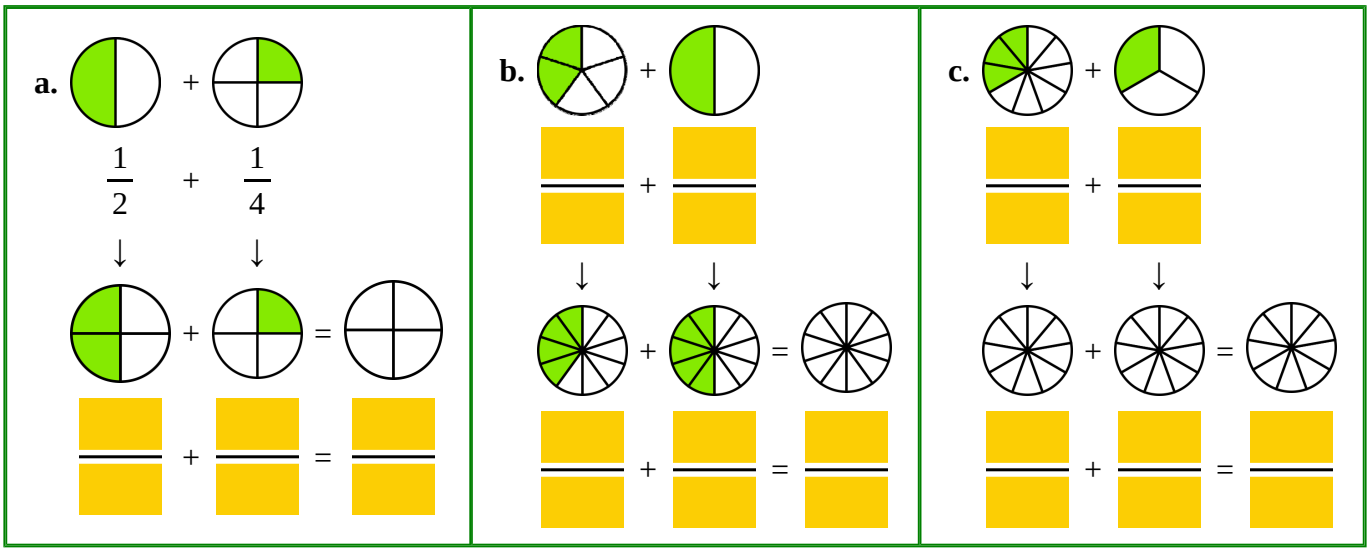
Did you solve the problems above?

The solution is this:

We convert the fractions so that they become *like* fractions (the *same* denominator), using equivalent fractions.

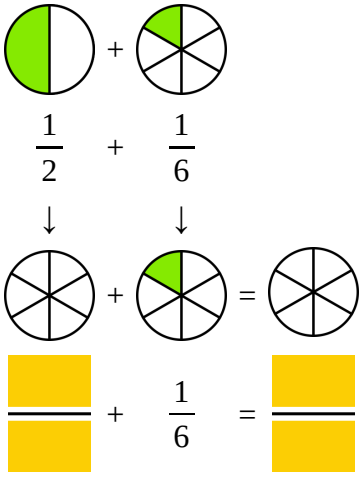
Then we can add or subtract.

1. Write the fractions shown by the pie images. Convert them into *equivalent fractions with the same denominator* (like fractions), and then add them. Color the missing parts.



2. Convert the fractions to like fractions first, and then add or subtract. In the bottom problems (d-f), you need to figure out what kind of pieces to use, but the *top* problems (a-c) will help you do that!

a.

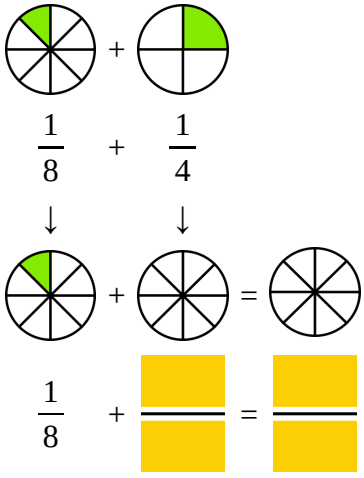


$\frac{1}{2} + \frac{1}{6}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{1}{6} = \frac{\square}{\square}$

b.

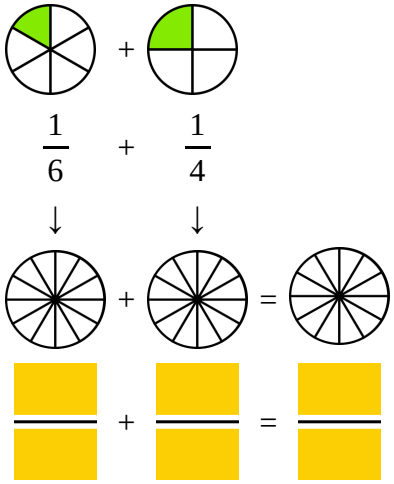


$\frac{1}{8} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{1}{8} + \frac{\square}{\square} = \frac{\square}{\square}$

c.



$\frac{1}{6} + \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

d. $\frac{5}{6} - \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{5}{6} - \frac{\square}{\square} = \frac{\square}{\square}$

e. $\frac{5}{8} - \frac{1}{4}$

$\downarrow \quad \downarrow$

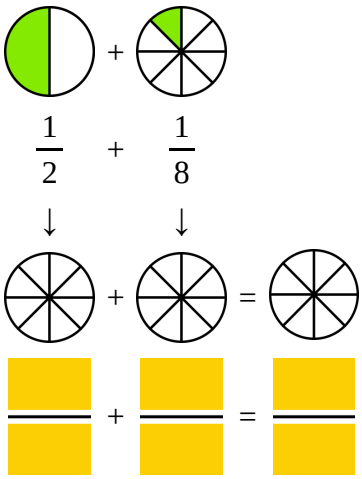
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

f. $\frac{5}{6} - \frac{1}{4}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

g.

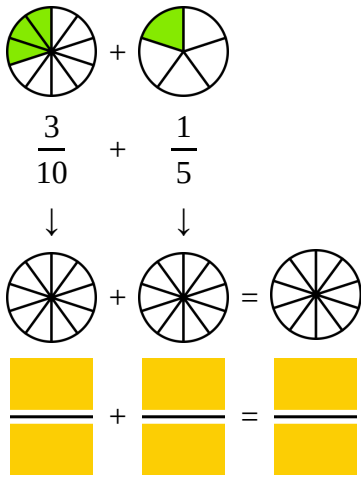


$\frac{1}{2} + \frac{1}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

h.

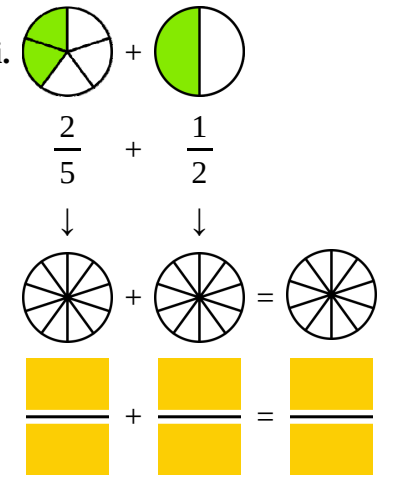


$\frac{3}{10} + \frac{1}{5}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

i.



$\frac{2}{5} + \frac{1}{2}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

j. $\frac{1}{2} + \frac{3}{8}$

$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

k. $\frac{9}{10} - \frac{2}{5}$

$\downarrow \quad \downarrow$

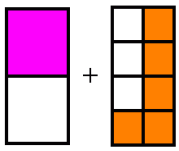
$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

l. $\frac{4}{5} - \frac{1}{2}$

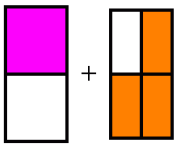
$\downarrow \quad \downarrow$

$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

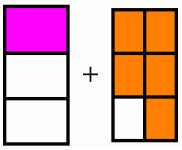
3. Split the parts only in the *first* fraction so that both fractions will have the same kind of parts. Add.



a. ___ + $\frac{5}{8}$ =

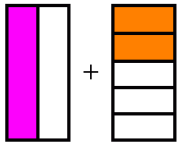


b. ___ + $\frac{3}{4}$ =

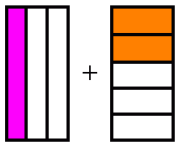


c. ___ + $\frac{5}{6}$ =

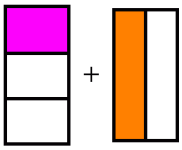
Now split the parts in *both* fractions so that they will have the same kind of parts. Add.



d. $\frac{\quad}{10}$ + $\frac{\quad}{10}$ =



e. $\frac{\quad}{15}$ + ___ =



f. ___ + ___ =

4. Fill in the table based on the problems above. What kind of parts did the two fractions have at first? What kind of parts did you use in the final addition?

	Types of parts:	Converted to:
a.	2nd parts and 8th parts	*8th* parts
b.	2nd parts and 4th parts	_____ parts
c.	3rd parts and 6th parts	_____ parts

	Types of parts:	Converted to:
d.	2nd parts and 5th parts	_____ parts
e.	3rd parts and 5th parts	_____ parts
f.	3rd parts and 2nd parts	_____ parts

5. Now think: How can you know into what kind of parts to convert the fractions that you are adding? Can you see any patterns or rules in the table above?

6. **Challenge:** If you think you know what kind of parts to convert these fractions into, then try these problems. Do not worry if you do not know how to do them—we will study this in the next lesson.

a. $\frac{1}{2}$ + $\frac{2}{3}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

b. $\frac{2}{3}$ − $\frac{2}{5}$

↓ ↓

$\frac{\quad}{\quad}$ − $\frac{\quad}{\quad}$ =

c. $\frac{1}{3}$ + $\frac{3}{4}$

↓ ↓

$\frac{\quad}{\quad}$ + $\frac{\quad}{\quad}$ =

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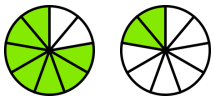
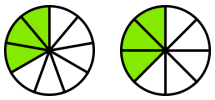
Comparing Fractions

Sometimes it is easy to know which fraction is the greater of the two. Study the examples below!



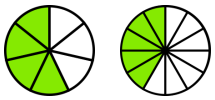
$\frac{7}{9} > \frac{2}{9}$

With **like fractions**, all you need to do is to check which fraction has more "slices," and that fraction is greater.



$\frac{3}{9} < \frac{3}{8}$

If both fractions have the **same number of pieces**, then the one with bigger pieces is greater.

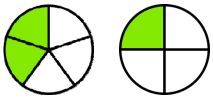


$\frac{4}{7} > \frac{5}{12}$

Sometimes you can **compare to 1/2**. Here, 4/7 is clearly more than 1/2, and 5/12 is clearly less than 1/2.

$\frac{6}{5} > \frac{9}{10}$

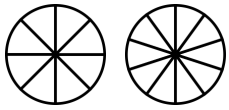
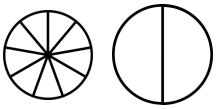
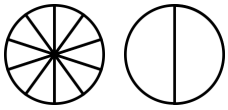
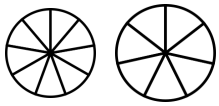
Any fraction that is bigger than one must also be bigger than any fraction that is less than one. Here, 6/5 is more than 1, and 9/10 is less than 1.



$\frac{2}{5} > \frac{1}{4}$

If you can imagine the pie pictures in your mind, you can sometimes "see" which fraction is bigger. For example, it is easy to see that 2/5 is more than 1/4.

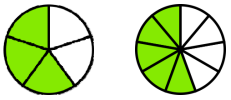
1. Compare the fractions, and write > , < , or = .

 **a.** $\frac{1}{8}$ $\frac{1}{10}$	 **b.** $\frac{4}{9}$ $\frac{1}{2}$	 **c.** $\frac{6}{10}$ $\frac{1}{2}$	 **d.** $\frac{3}{9}$ $\frac{3}{7}$
e. $\frac{8}{11}$ $\frac{4}{11}$	**f.** $\frac{7}{4}$ $\frac{7}{6}$	**g.** $\frac{5}{14}$ $\frac{5}{9}$	**h.** $\frac{4}{20}$ $\frac{2}{20}$
i. $\frac{2}{11}$ $\frac{2}{5}$	**j.** $\frac{1}{2}$ $\frac{5}{8}$	**k.** $\frac{3}{6}$ $\frac{1}{2}$	**l.** $\frac{1}{20}$ $\frac{1}{8}$
m. $\frac{1}{2}$ $\frac{3}{4}$	**n.** $\frac{8}{7}$ $\frac{3}{3}$	**o.** $\frac{49}{100}$ $\frac{61}{100}$	**p.** $\frac{7}{8}$ $\frac{8}{7}$
q. $\frac{9}{10}$ $\frac{3}{4}$	**r.** $\frac{6}{5}$ $\frac{3}{4}$	**s.** $\frac{4}{4}$ $\frac{9}{11}$	**t.** $\frac{1}{3}$ $\frac{3}{9}$

Sometimes none of the "tricks" explained in the previous page work, but we do have one more up our sleeve!

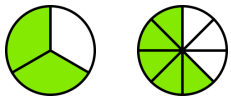
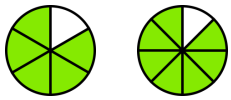
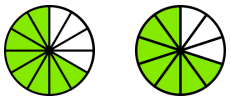
Convert both fractions into like fractions. Then compare.

In the picture on the right, it is hard to be sure if 3/5 is really more than 5/9. Convert both into 45th parts, and then it is easy to see that 27/45 is more than 25/45. Not by much, though!



$\frac{3}{5}$ ↓ $\frac{27}{45}$ > $\frac{5}{9}$ ↓ $\frac{25}{45}$

2. Convert the fractions into like fractions, and then compare them.

a.  $\frac{2}{3}$ ↓ $\frac{5}{8}$ ↓	**b.**  $\frac{5}{6}$ ↓ $\frac{7}{8}$ ↓	**c.**  $\frac{1}{3}$ ↓ $\frac{3}{10}$ ↓	**d.**  $\frac{8}{12}$ ↓ $\frac{7}{10}$ ↓
e. $\frac{5}{8}$ ↓ $\frac{7}{12}$ ↓	**f.** $\frac{11}{8}$ ↓ $\frac{14}{10}$ ↓	**g.** $\frac{6}{10}$ ↓ $\frac{58}{100}$ ↓	**h.** $\frac{6}{5}$ ↓ $\frac{11}{9}$ ↓
i. $\frac{7}{10}$ ↓ $\frac{5}{7}$ ↓	**j.** $\frac{43}{100}$ ↓ $\frac{3}{10}$ ↓	**k.** $\frac{9}{8}$ ↓ $\frac{8}{7}$ ↓	**l.** $\frac{7}{10}$ ↓ $\frac{2}{3}$ ↓

3. One cookie recipe calls for 1/2 cup of sugar. Another one calls for 2/3 cup of sugar. Which uses more sugar, a triple batch of the first recipe, or a double batch of the second?

How much more?

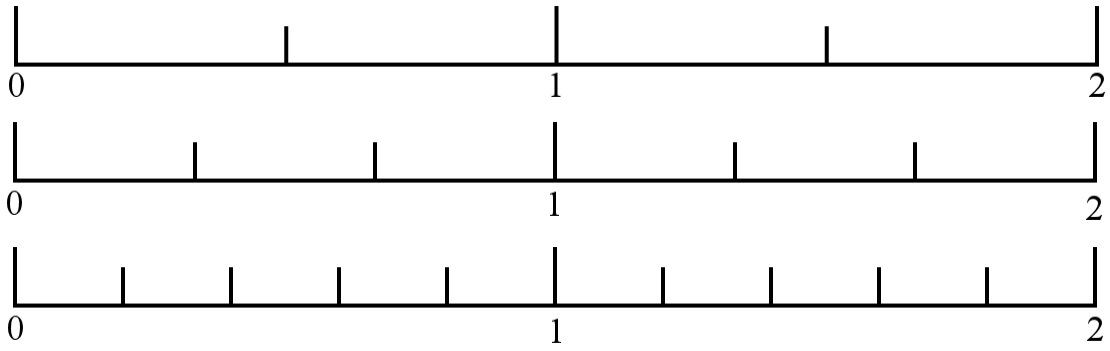
4. Compare the fractions using any method.

a. $\frac{5}{12}$ $\frac{3}{8}$	**b.** $\frac{5}{12}$ $\frac{4}{11}$	**c.** $\frac{3}{10}$ $\frac{1}{5}$	**d.** $\frac{3}{8}$ $\frac{4}{7}$
e. $\frac{4}{15}$ $\frac{1}{3}$	**f.** $\frac{5}{6}$ $\frac{11}{16}$	**g.** $\frac{7}{6}$ $\frac{10}{8}$	**h.** $\frac{5}{12}$ $\frac{5}{8}$
i. $\frac{3}{4}$ $\frac{4}{11}$	**j.** $\frac{13}{10}$ $\frac{9}{8}$	**k.** $\frac{2}{13}$ $\frac{1}{5}$	**l.** $\frac{1}{10}$ $\frac{1}{11}$

5. A coat costs $40. Which is a bigger discount:
1/4 off the normal price, or 3/10 off the normal price?

Does your answer change if the original price of the coat was $60 instead? Why or why not?

6. Here are three number lines that are divided respectively into halves, thirds, and fifths. Use them to help you put the given fractions in order, from the least to the greatest.



a. $\frac{1}{3}, \frac{2}{5}, \frac{2}{3}, \frac{1}{5}, \frac{1}{2}$

___ < ___ < ___ < ___ < ___

b. $\frac{7}{5}, \frac{3}{2}, \frac{4}{3}, \frac{6}{5}, \frac{2}{2}$

___ < ___ < ___ < ___ < ___

7. Write the three fractions in order.

a. $\frac{7}{8}, \frac{9}{10}, \frac{7}{9}$ ____ < ____ < ____	**b.** $\frac{1}{3}, \frac{4}{10}, \frac{2}{9}$ ____ < ____ < ____

8. Rebecca made a survey of a group of 600 women. She found that 1/3 of them never exercised, that 22/100 of them swam regularly, 1/5 of them jogged regularly, and the rest were involved in other sports.

a. Which was a bigger group, the women who jogged or the women who swam?

b. What fraction of this group of women exercise?

c. *How many women* in this group exercise?

d. How many women in this group swim?

Puzzle Corner

The seven dwarfs could not divide a pizza into seven equal slices. The oldest suggested, "Let's cut it into eight slices, let each dwarf have one piece, and give the last piece to the dog."

Then another dwarf said, "No! Let's cut it into 12 slices instead, and give each of us 1 ½ of those pieces, and the dog gets the 1 ½ pieces left over."

Which suggestion would give more pizza to the dog?

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Chapter 7: Fractions: Multiply and Divide

Introduction

This is another long chapter devoted solely to fractions. It rounds out our study of fraction arithmetic. (If you feel that your student(s) would benefit from taking a break from fractions, you can optionally have them study chapter 8 on geometry in between chapters 6 and 7.)

We start out by simplifying fractions. Since this process is the opposite of making equivalent fractions, studied in chapter 6, it should be relatively simple for students to understand. We also use the same visual model, just backwards: This time the pie pieces are joined together instead of split apart.

Next comes multiplying a fraction by a whole number. Since this can be solved by repeated addition, it is not a difficult concept at all.

Multiplying a fraction by a fraction is first explained as taking a certain part of a fraction, in order to teach the concept. After that, students are shown the usual shortcut for the multiplication of fractions.

Simplifying before multiplying is a process that is not absolutely necessary for fifth graders. I have included it here because it prepares students for the same process in future algebra studies and because it makes fraction multiplication easier. I have also tried to include explanations of *why* we are allowed to simplify before multiplying. These explanations are actually *proofs*. I feel it is a great advantage for students to get used to mathematical reasoning and proof methods well before they start high school geometry.

Then, we find the area of a rectangle with fractional side lengths, and show that the area is the same as it would be found by multiplying the side lengths. Students multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Students also multiply mixed numbers, and study how multiplication can be seen as resizing or scaling. This means, for example, that the multiplication (2/3) × 18 km can be thought of as finding two-thirds of 18 km.

Next, we study division of fractions in special cases. The first one is seeing fractions *as* divisions; in other words recognizing that 5/3 is the same as 5 ÷ 3. This of course gives us a means of dividing whole numbers and getting fractional answers (for example, 20 ÷ 6 = 3 2/6).

Then students encounter sharing divisions with fractions. For example, if two people share equally 4/5 of a pizza, how much will each person get? This is represented by the division (4/5) ÷ 2 = 2/5. Another case we study is dividing unit fractions by whole numbers (such as (1/2) ÷ 4).

We also divide whole numbers by unit fractions, such as 6 ÷ (1/3). Students will solve these thinking how many times the divisor "fits into" the dividend.

The last lesson is an introduction to ratios, and is optional. Ratios will be studied a lot in 6th and 7th grades, especially in connection with proportions. We are laying the groundwork for that here.

The Lessons in Chapter 7

	page	*span*
Simplifying Fractions 1	100	4 pages
Simplifying Fractions 2	104	4 pages
Multiply Fractions by Whole Numbers	108	4 pages
Multiplying Fractions by Fractions	112	5 pages
Fraction Multiplication and Area	117	6 pages
Simplifying Before Multiplying	123	4 pages
Multiplying Mixed Numbers	127	5 pages
Multiplication as Scaling/Resizing	132	4 pages
Fractions Are Divisions	136	4 pages
Dividing Fractions 1: Sharing Divisions	140	4 pages
Dividing Fractions 2: Fitting the Divisor	145	4 pages
Introduction to Ratios	149	4 pages
Mixed Review	153	3 pages
Review	156	4 pages

Helpful Resources on the Internet

General

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
http://www.mathmammoth.com/videos/fractions_2.php

Visual Fractions
A great site for studying all aspects of fractions, including: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by a Java applet with either a number line or a circle. There are also a couple of games; for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Simplifying & Equivalent Fractions

Equivalent Fractions
Draw two other, equivalent fractions to the given fraction. Choose either square or circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Fraction Worksheets: Simplifying and Equivalent Fractions
Create custom-made worksheets for fraction simplification and equivalent fractions.
http://www.homeschoolmath.net/worksheets/fraction.php

Multiplication and Division

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Multiply and Reduce Fractions Battleship Game
When you hit the enemy's battleship, you need to solve a fraction multiplication problem.
http://www.quia.com/ba/57713.htm

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Number line bars
Fraction bars that illustrate visually how many times a fraction "fits into" another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

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Multiplying Fractions by Fractions

We have studied how to find a fractional part of a whole number using multiplication.

For example, $\frac{3}{5}$ of 80 is written as a multiplication: $\frac{3}{5} \times 80 = \frac{240}{5} = 48$.

NOTE: The word *OF* translates here into **MULTIPLICATION**.

We can use the same idea to find *a fractional part* ***of*** *a fraction*!

One-half of  is  .

As a multiplication, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

One-fourth of  is  .

As a multiplication, $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$.

1. Find a fractional part of the given fraction. You can think of a leftover pizza piece, which you must share equally with one, two, or three other people. Write a multiplication sentence.

a. Find $\frac{1}{2}$ of 

$\frac{1}{2} \times \frac{1}{4} =$

b. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

c. Find $\frac{1}{2}$ of 

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

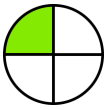
d. Find $\frac{1}{3}$ of 

e. Find $\frac{1}{3}$ of 

f. Find $\frac{1}{3}$ of 

g. Find $\frac{1}{4}$ of 

h. Find $\frac{1}{4}$ of 

i. Find $\frac{1}{4}$ of 

Did you notice a shortcut? If so, calculate $\frac{1}{5} \times \frac{1}{6} = \frac{\square}{\square}$

Shortcut: multiplying fractions of the type 1/*n*

To multiply fractions of the form 1/*n* where *n* is a whole number, simply multiply the denominators to get the new denominator →

$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ or $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

2. Multiply.

a. $\frac{1}{9} \times \frac{1}{2}$	**b.** $\frac{1}{13} \times \frac{1}{3}$	**c.** $\frac{1}{5} \times \frac{1}{20}$

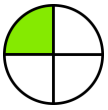
We have now studied how to find 1/2 or 1/3 or 1/5 of some fractions. What about finding some other kind of fractional part? Let's again compare this to finding fractional parts of whole numbers.

Review: To find $\frac{3}{4}$ of 16, or in other words $\frac{3}{4} \times 16$, you can first find $\frac{1}{4}$ of 16, which is 4.

Then just take that three times, which is 12. In other words, $\frac{3}{4} \times 16 = 12$.

We can use the same idea when finding a fractional part *of* another fraction.

Example. Find $\frac{2}{3}$ of $\frac{1}{4}$. First, we find $\frac{1}{3}$ of $\frac{1}{4}$, which is $\frac{1}{12}$. Then, $\frac{2}{3}$ of $\frac{1}{4}$ is double that much, or $\frac{2}{12}$.

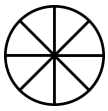
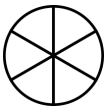
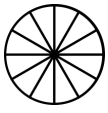
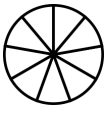
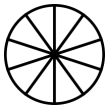
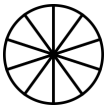
$\frac{2}{3}$ of  =

Example. Find $\frac{4}{5}$ of $\frac{1}{7}$.

First, we find $\frac{1}{5}$ of $\frac{1}{7}$, which is $\frac{1}{35}$. Then, $\frac{4}{5}$ of $\frac{1}{7}$ is four times that much, or $\frac{4}{35}$.

Multiplying a fraction by a fraction means taking that fractional part *of* the fraction. It is just like taking a certain part of the leftovers, when what is left over is a fraction.

3. The pictures show how much pizza is left, and you get a certain part of the leftovers. How much will you get? Color in a picture to show the answer.

a. $\frac{3}{4} \times$  = 	**b.** $\frac{2}{3} \times$  = 
c. $\frac{3}{4} \times$  = 	**d.** $\frac{2}{3} \times$  = 
e. $\frac{2}{5} \times$  = 	**f.** $\frac{4}{5} \times$  = 

4. Solve the multiplications by using two helping multiplications. Lastly, simplify if possible.

a. $\frac{2}{3} \times \frac{1}{8} =$

First find 1/3 of 1/8, then multiply the result by 2.

$\frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$ and $\frac{1}{24} \times 2 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

b. $\frac{3}{4} \times \frac{1}{10} =$

First find 1/4 of 1/10, then multiply the result by 3.

$\frac{1}{4} \times \frac{1}{10} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad}$

c. $\frac{3}{5} \times \frac{1}{6} =$

First find 1/5 of 1/6, then multiply the result by 3.

$\frac{1}{5} \times \frac{1}{6} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 3 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

d. $\frac{5}{6} \times \frac{1}{9} =$

First find 1/6 of 1/9, then multiply the result by 5.

$\frac{1}{6} \times \frac{1}{9} = \frac{\quad}{\quad}$ and $\frac{\quad}{\quad} \times 5 = \frac{\quad}{\quad}$

e. $\frac{2}{3} \times \frac{1}{7} =$

f. $\frac{3}{8} \times \frac{1}{4} =$

A shortcut for multiplying fractions

Multiply the numerators to get the numerator for the answer.
Multiply the denominators to get the denominator for the answer.

Study the examples on the right.

Remember always to give your final answer as a mixed number and in lowest terms (simplified).

$\frac{3}{7} \times \frac{4}{9} = \frac{3 \times 4}{7 \times 9} = \frac{12}{63} = \frac{4}{21}$

$\frac{4}{5} \times \frac{11}{8} = \frac{4 \times 11}{5 \times 8} = \frac{44}{40} = \frac{11}{10} = 1\frac{1}{10}$

5. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{9} \times \frac{2}{9}$

b. $\frac{11}{12} \times \frac{1}{6}$

c. $\frac{1}{3} \times \frac{3}{13}$

d. $9 \times \frac{2}{3}$

e. $\frac{2}{9} \times \frac{6}{7}$

f. $10 \times \frac{5}{7}$

COMPARE	
The roundabout way	**The shortcut**
$\frac{5}{6} \times \frac{1}{2} = ?$ First find 1/6 of 1/2, then multiply the result by 5. $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ and $\frac{1}{12} \times 5 = \frac{5}{12}$	$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$
$\frac{2}{8} \times \frac{3}{5} = ?$ Find 1/8 of 3/5, then multiply that result by 2. And to find 1/8 of 3/5, first find 1/8 of 1/5, and then multiply that by 3. $\frac{1}{8} \times \frac{1}{5} = \frac{1}{40}$. That multiplied by 3 is $\frac{1}{40} \times 3 = \frac{3}{40}$. Then, that multiplied by 2 is $\frac{3}{40} \times 2 = \frac{6}{40} = \frac{3}{20}$.	$\frac{2}{8} \times \frac{3}{5} = \frac{2 \times 3}{8 \times 5} = \frac{6}{40} = \frac{3}{20}$
In the "roundabout way," we do each multiplication separately. In the shortcut, we can just do them all at once.	

6. Multiply. Give your answers in the lowest terms (simplified) and as mixed numbers, if possible.

a. $\frac{3}{4} \times \frac{7}{8} =$	**b.** $\frac{7}{10} \times \frac{8}{5} =$
c. $\frac{9}{20} \times \frac{4}{5} =$	**d.** $\frac{2}{5} \times \frac{1}{3} =$
e. $\frac{1}{4} \times \frac{2}{7} =$	**f.** $\frac{9}{4} \times \frac{1}{3} =$
g. $\frac{2}{3} \times \frac{11}{8} =$	**h.** $\frac{2}{9} \times \frac{3}{10} =$

7. There was 1/4 of the pizza left. Marie ate 2/3 of that.

a. What part of the *original* pizza did she eat?

b. What part of the *original* pizza is left now?

8. Theresa has painted 5/8 of the room.

a. What part is still left to paint?

b. Now, Theresa has painted half of what was still left.
Draw a bar model of the situation.
What part of the room is still left to paint?

9. Ted has completed 2/3 of a job that his boss gave him.

a. What part is still left to do?

b. Now Ted has completed a third of what was still left to do.
Draw a bar model of the situation.
What (fractional) part of the original job is still left undone?

What part is completed?

10. Sally wants to make 1/3 of the recipe on the right.
How much does she need of each ingredient?

Carob Brownies

3 cups sweetened carob chips
8 tablespoons extra virgin olive oil
2 eggs
1/2 cup honey
1 teaspoon vanilla
3/4 cup whole wheat flour
3/4 teaspoon baking powder
1 cup walnuts or other nuts

11. For an upcoming get-together, Alison needs to multiply the coffee recipe. Assume that half of the guests drink one serving, and the other half drink two servings. Find how much coffee she will need, if she has:

a. 30 guests

b. 50 guests

c. 80 guests.

Coffee (5 servings)

3 1/2 cups water
1/4 cup coffee

Puzzle Corner Find the missing factors.

a. $\times \frac{6}{7} = \frac{1}{7}$	**b.** $\times \frac{1}{4} = \frac{5}{16}$	**c.** $\times \frac{3}{8} = \frac{1}{16}$	**d.** $\times \frac{2}{5} = \frac{3}{10}$

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Chapter 8: Geometry
Introduction

The problems in this chapter involve lots of drawing. Geometry is a hands-on subject, and many children like that. Moreover, drawing is an excellent means of achieving the conceptual understanding that geometry requires.

Exercises marked with the symbol “  ” are meant to be done in a notebook or on blank paper.

This chapter starts out with several lessons that review topics studied in previous grades, such as measuring angles, the vocabulary of basic shapes, and how to draw a perpendicular line through a given point on a line. Some fun is included, too, with star polygons.

In the lesson about circles, we learn the terms circle, radius, and diameter. Students draw circles and circle designs using a compass.

Then we go on to classify quadrilaterals and learn the seven different terms used for them. The focus is on understanding the classification, and understanding that attributes defining a certain quadrilateral also belong to all the “children” (subcategories) of that type of quadrilateral. For example, squares are also rhombi, because they have four congruent sides (the defining attribute of rhombus).

Next, we study and classify different triangles. Students are now able to classify triangles both in terms of their sides and also in terms of their angles. The lesson has several drawing problems and one easy compass-and-ruler construction of an equilateral triangle.

The last focus of this chapter is volume. Students learn that a cube with the side length of 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume. They find the volume of right rectangular prisms by “packing” them with unit cubes and by using formulas. They recognize volume as additive and solve both geometric and real-word problems involving volume of right rectangular prisms.

The Lessons in Chapter 8

	page	*span*
Review: Angles	164	2 pages
Review: Drawing Polygons	166	4 pages
Circles	170	3 pages
Quadrilaterals	173	4 pages
Equilateral, Isosceles, and Scalene Triangles	177	5 pages
Area and Perimeter Problems	182	3 pages
Volume	185	5 pages
Volume of Rectangular Prisms (Cuboids)	190	4 pages
A Little Bit of Problem Solving	194	2 pages
Mixed Review	196	3 pages
Review	199	3 pages

Helpful Resources on the Internet

General/Review/Fun things

Turtle Pond
Guide a turtle to a pond using commands that include turning him through certain angles and moving him specific distances.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Shape Explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Interactive Tangram Puzzle
Place the tangram pieces so that they form a given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation! Requires Java.
http://www.shodor.org/interactivate/activities/Tessellate

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry
A collection of interactive activities: fractals, geoboard activities, golden rectangle, ladybug leaf, ladybug mazes, tangrams, tessellations, transformations, and more.
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Quadrilaterals

Interactive Quadrilaterals
See all the different kinds of quadrilateral "in action". You can drag the corners, see how the angles change, and observe what properties do not change.
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Complete the Quadrilateral
Join the dots to complete these quadrilaterals. Where there are options, try to find the one on the grid with the largest possible area. The author, Fawn Nguyen, wishes, "If appropriate for your class, I hope you will consider doing this lesson — it will make the hours I spent recreating the handout and key worth it! :)" Great for 5th grader or whenever you study classifying quadrilaterals.
http://fawnnguyen.com/2013/02/08/don-stewards-complete-the-quadrilateral.aspx

Polygon Matching Game
Many of the polygons included are quadrilaterals.
http://www.mathplayground.com/matching_shapes.html

Classify Quadrilaterals Worksheets
Make free printable worksheets for classifying (idenfitying, naming) quadrilaterals. There are seven special types of quadrilaterals: square, rectangle, rhombus, parallelogram, trapezoid, kite, scalene, and these worksheets ask students to name the quadrilaterals among these seven types.
http://www.homeschoolmath.net/worksheets/classify_quadrilaterals.php

Quadrilaterals Quest
First, the quest asks you to choose all the quadrilaterals with the given properties. After several of those types of activities follows a quiz.
http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/quad_quest/quad_quest.html

Quadrilateral Properties
Investigate the properties of square, rectangle, rhombus, an isosceles trapezoid, and a non-isosceles trapezoid in this dynamic, online activity.
http://www.glencoe.com/sites/texas/student/mathematics/assets/interactive_lab/geometry/G_08/G_08

Quadrilateral Classification Game
A virtual manipulative that challenges students to "draw" quadrilaterals with specific characteristics by moving vertices on a coordinate grid. Includes some challenging vocabulary, which is explained below the activity, such as orthodiagonal quadrilateral, cyclic, or convex quadrilateral.
http://www.uff.br/cdme/jcq/jcq-html/jcq-en.html

Triangles

Classify Triangles Worksheets
Make free printable worksheets for classifying triangles by their sides, angles, or both.
http://www.homeschoolmath.net/worksheets/classify_triangles.php

Triangle Classification at Cut The Knot
A tutorial and an applet about classifying triangles by their sides and angles. In the applet, you can drag any of the vertices of the triangle, and the applet tells you whether your triangle is acute, obtuse, or right, or equilateral, isosceles, or scalene.
http://www.cut-the-knot.org/triangle/Triangles.shtml

Rags to Riches: Classify Triangles by Sides and Angles
Answer multiple-choice questions about the angles of a triangle and classification of triangles in a quest for fame and fortune.
http://www.quia.com/rr/457498.html

Identify Triangles Quiz
A simple multiple-choice quiz about identifying (classifying) triangles either by their sides or angles. You can modify some of the quiz parameters, such as the number of problems in it.
http://www.thatquiz.org/tq-A/?-j1-l34-p0

Triangles & Quadrilaterals Classification Game
Look at the shapes as they go past, and drag them into the right groups (equilateral, isosceles, or scalene triangles, and quadrilaterals with 4 congruent sides, 2 congruent sides, or no congruent sides).
http://www.bbc.co.uk/bitesize/ks2/maths/shape_space/shapes/play/

Classifying Triangles Game
A fast-paced game where you drag triangles into the correct basket as fast as you can (acute, obtuse, right).
http://www.math-play.com/classifying-triangles/classifying-triangles.html

Triangle Classification Exploration Tool
Line segment AB is drawn in the plane. Where should point C be placed so that ABC is a right triangle? …so that it is an isosceles triangle? …so that it is obtuse? This activity will allow you to explore these questions.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=142

Volume

Geometric Solids
Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

Cuboid Exploder and **Isometric Shape Exploder**
These interactive demonstrations let you see either various cuboids (a.k.a. boxes or rectangular prisms) or various shapes made of unit cubes, and then "explode" them to the unit cubes, illustrating volume.
www.teacherled.com/resources/cuboidexplode/cuboidexplodeload.html and
www.teacherled.com/resources/isoexplode/isoexplodeload.html

Geometry Volume/Surface Area Quiz from ThatQuiz.org
An online quiz, asking either the volume or surface area of cubes, prisms, spheres, cylinders, or cones. You can modify the quiz parameters to your liking, for example to omit some shapes, solve only for volume or surface area, or instead of solving for volume/surface area, you solve for an unknown dimension (side or radius) when the volume or surface area is given.
www.thatquiz.org/tq-4/?-j3vu0-lc-m2kc0-na-p0

Cubes
An online tool where you can explore filling a rectangular prism (a box) with unit cubes, rows of cubes, or layers of cubes. You can use this to let the student find the rule for finding the volume of a box if you know its width, depth, and height. Requires Java.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=6

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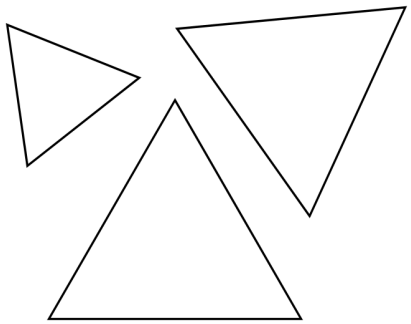
Equilateral, Isosceles, and Scalene Triangles

If all three sides of a triangle are congruent (the same length), it is called an **equilateral triangle**.

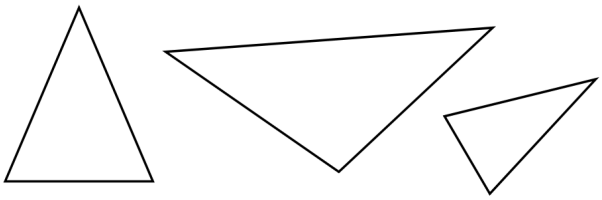
Equi- refers to things that are the "same" or "equal", and *lateral* means "sided." Think of it as a "same-sided" triangle.



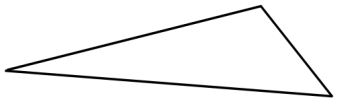
If just *two* of a triangle's sides are congruent, then it is called an **isosceles triangle**.

Think of it as a "same-legged" triangle, the "legs" being the two sides that are the same length.

Mark the two congruent sides of each isosceles triangle:

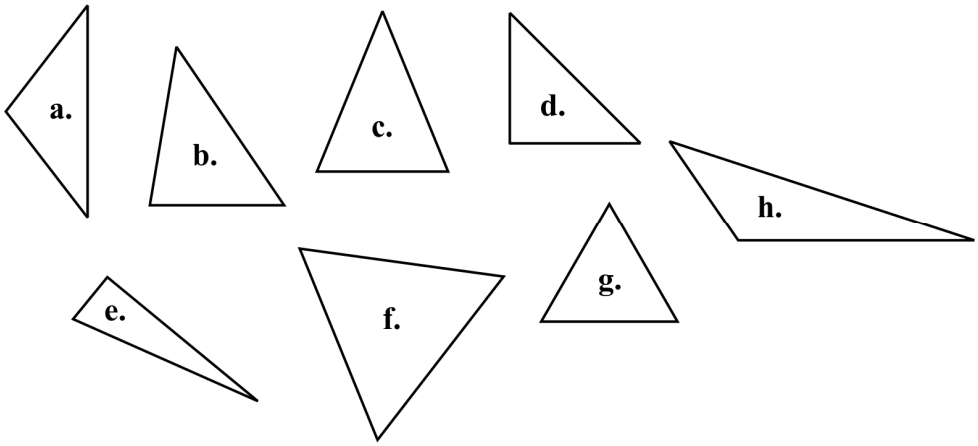


Lastly, if none of the sides of a triangle are congruent (all are different lengths), it is a **scalene triangle**.



1. Classify the triangles by the lengths of their sides as either equilateral, isosceles, or scalene.

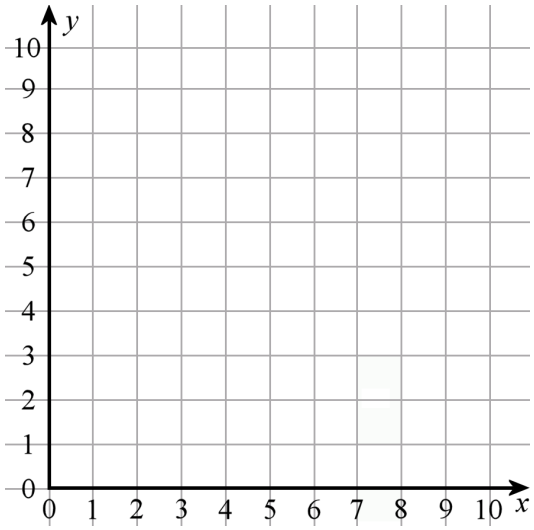
   You can mark each triangle with an "*e*," "*i*," or "*s*" correspondingly.



2. Fill in the table by classifying the triangles labeled as (a), (d), (e), and (g) above as "acute," "right," or "obtuse" (by their angles), and also as "equilateral," "isosceles," or "scalene" (by their sides).

Triangle	Classification by the sides	Classification by the angles
a		
d		
e		
g		

3. Plot the points, and connect them with line segments to form two triangles. Classify the triangles by their angles and sides.



Triangle 1: (0, 0), (4, 0), (0, 4)

_____ and

_____

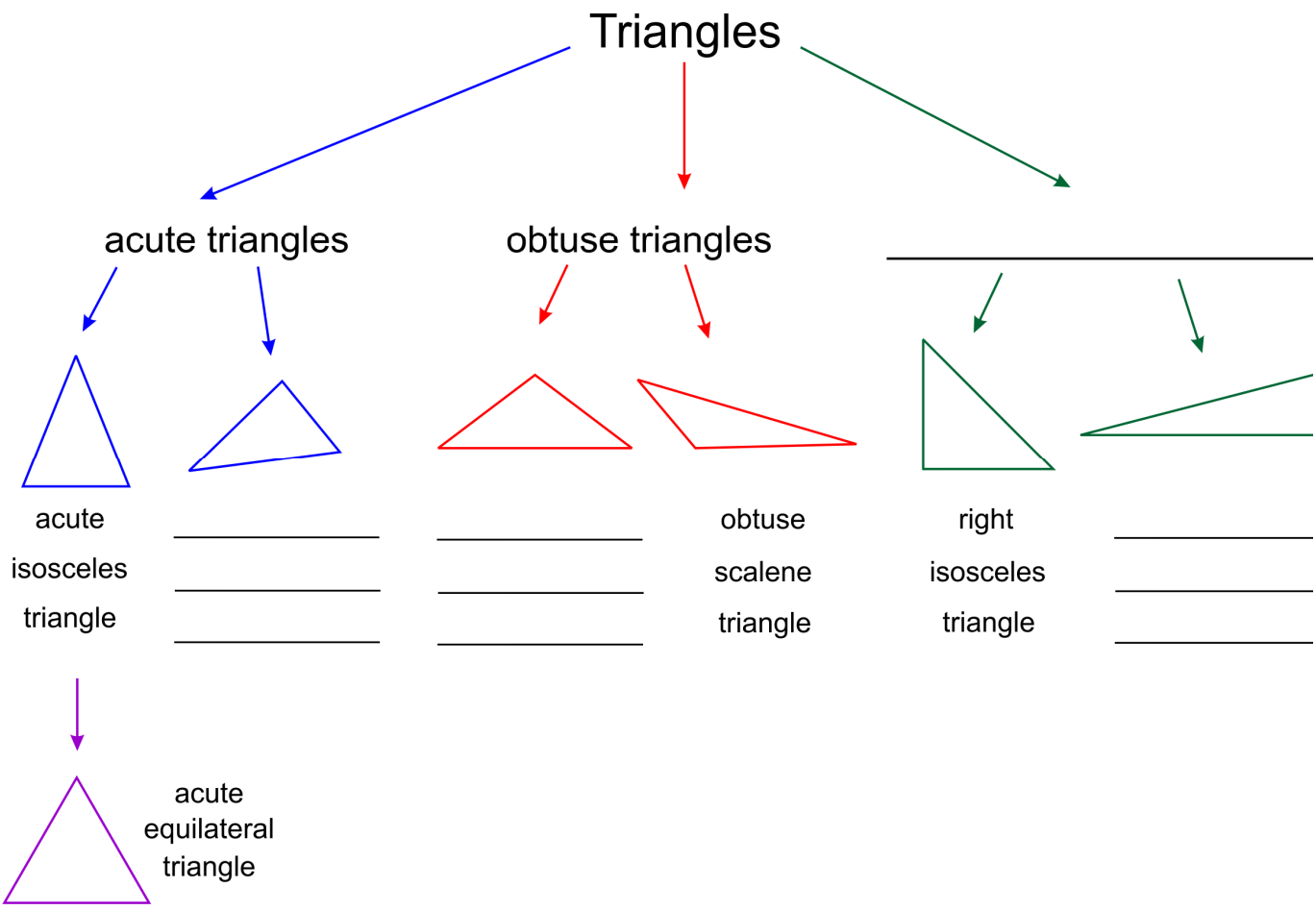
Triangle 2: (5, 5), (1, 8), (9, 4)

_____ and

_____

4. Plot in the coordinate grid an acute scalene triangle.

5. Fill in the missing parts in this tree diagram classification for triangles.



6. **a.** Draw a scalene obtuse triangle where one side is 3 cm and another is 7 cm.
Hint: Draw the 7-cm side first, then the 3-cm side forming any obtuse angle with the first side.

b. Measure the third side.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

7. **a.** Draw an isosceles right triangle whose two sides measure 5 cm.
Hint: Draw a right angle first. Then, measure off the 5-cm sides. Then draw in the last side.

b. Measure the third side. It is _____ cm.
Compare your triangle to those of your classmates, or draw another one yourself.
Can you draw several different-looking triangles with this information,
or are they all identical (congruent)?

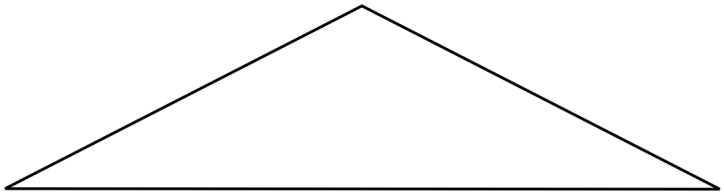
8. **a.** Draw any isosceles triangle.
Hint: Draw any angle. Then, measure off the two congruent sides, making sure they have the same length. Then draw the last side.

b. Measure the angles of your triangle. They measure _____ °, _____ °, and _____ ° .

The angle sum is _____ ° .

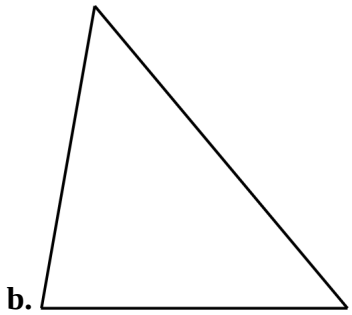
9. Measure all the angles in the isosceles triangles (a) and (b).
Continue their sides, if necessary.

a.



_____ °, _____ °, and _____ ° .

The angle sum is _____ ° .



b.

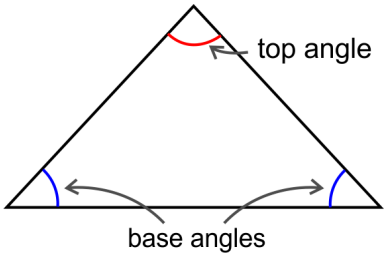
_____ °, _____ °, and _____ °.

The angle sum is _____ ° .

What do you notice?

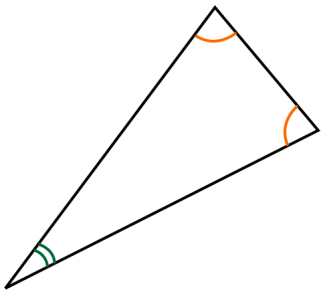
_____

_____



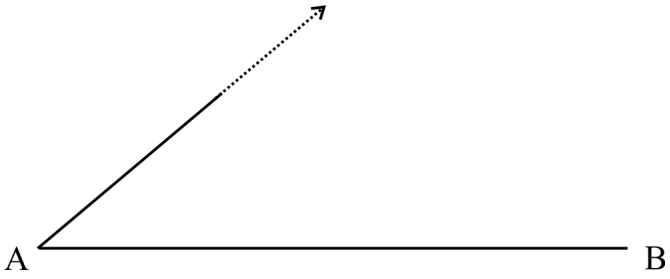
There are two angles in an isosceles triangle that have the same angle measure. They are called the **base angles.**

The remaining angle is called the **top angle**.



Can you find the top angle and the base angles in this isosceles triangle?

10. The angle at A measures 40°. Draw another angle of 40° at B, and then continue its side so that you get an isosceles triangle with 40° base angles.



Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

11. **a.** Draw an isosceles triangle with 75° base angles. (The length of the sides can be anything.)
Hint: start by drawing the base side (of any length). Then, draw the 75° angles.

b. Measure the top angle. It is _____ ° . The three angle measures add up to _____ ° .

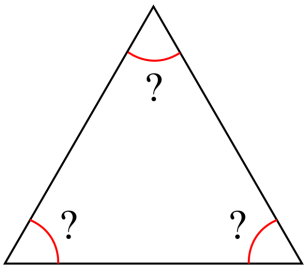
c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

12. **a.** Draw an isosceles triangle with a 50° top angle.
Hint: start by drawing a 50° angle. The two sides of the angle you drew are the two congruent sides of the triangle, so choose how long those sides should be, measure, and mark them. Then draw in the third side.

b. The base angles are _____ ° each. The three angle measures add up to _____ ° .

c. Compare your triangle to those of your classmates, or draw another one yourself. Can you draw several different-looking triangles with this information, or are they all identical?

13. Make a guess about the angle measures in an equilateral triangle: _____°
Measure to check.



14. **a.** Could an equilateral triangle be a right triangle?
If yes, sketch an example. If not, explain why not.

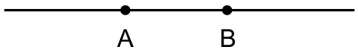
b. Could a scalene triangle be obtuse?
If yes, sketch an example. If not, explain why not.

c. Could an acute triangle be scalene?
If yes, sketch an example. If not, explain why not.

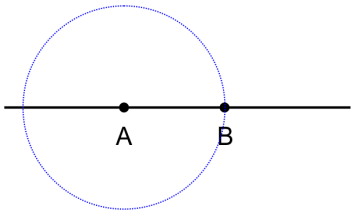
d. Could a right triangle be scalene?
If yes, sketch an example. If not, explain why not.

e. Could an obtuse triangle be equilateral?
If yes, sketch an example. If not, explain why not.

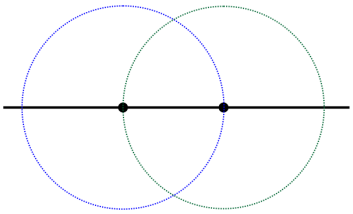
Draw an equilateral triangle with a compass and a ruler (optional)



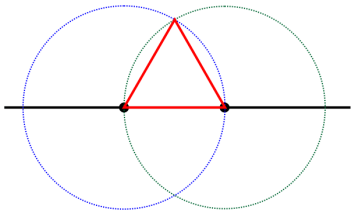
Draw a line segment and mark two points on it. These points mark one of the sides of the triangle.



Draw a circle using point A as the center point and the length AB as the radius. The third vertex of the triangle MUST lie on this circle. Can you see why?



Can you see what was done in this picture?



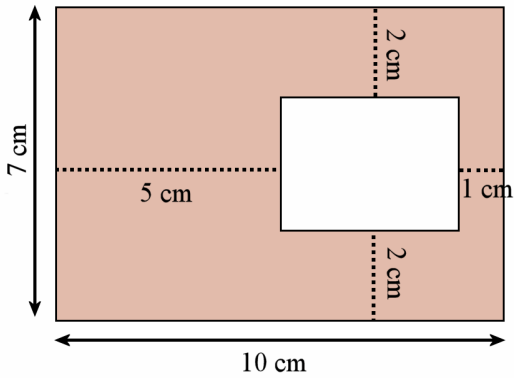
The triangle is finished!

15. Draw at least three different equilateral triangles in your notebook using the method above.
Can you explain why this method works?

Important Terms
• *equilateral triangle* • *isosceles triangle* • *scalene triangle*

Area and Perimeter Problems

Find the area of the shaded figure.



The easiest way to do this is:
(1) Find the area of the larger outer rectangle,
(2) find the area of the white inner rectangle, and
(3) subtract.

1. The area of the large rectangle is 7 cm × 10 cm = 70 cm^2.

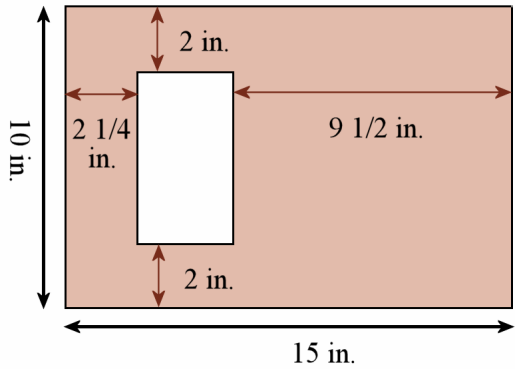
2. We find the *sides* of the white rectangle by subtracting.

   The longer side of the white rectangle is
   10 cm − 5 cm − 1 cm = 4 cm.
   The shorter side is 7 cm − 2 cm − 2 cm = 3 cm.

   So, the area of the white rectangle is 4 cm × 3 cm = 12 cm^2.

3. Now we subtract to find the shaded area: 70 cm^2 − 12 cm^2 = 58 cm^2.

1. **a.** Find the area of the white rectangle.
   All lines meet at right angles.



b. Find the area of the shaded figure.

2. The image on the right shows a picture frame.
   Find the area of the actual frame (that is, of the shaded part).
   All lines meet at right angles.