MATH MAMMOTH

Grade 5-A

Complete Worktext

- The four operations
- Problem solving
- Large numbers and the calculator
- Decimals
- Graphing and statistics

By Maria Miller

www.MathMammoth.com

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Chapter 1: The Four Operations
Introduction

We start out fifth grade by studying: the order of operations, multiplication, long division, equations, problem solving, and ratios.

The main line of thought throughout the chapter is that of *equations* and *problem solving*. Students encounter the exact definition of an *equation* and an *expression*. They practice the order of operations with problems that also reinforce the idea of the equal sign ("=") as denoting equality of the right and left sides of an equation. These kind of exercises are needed because children may think that an equal sign signifies *the act of finding the answer* to a problem (as in 134 + 23 = ?, for example), which is not so.

Students solve addition and subtraction equations both with the help of diagrams (a.k.a. bar models) and also without. Diagrams are also used for simple multiplication and division equations and for mixture equations, such as $4x + 38 = 128$.

In the middle of the chapter, we present lessons on multi-digit multiplication (multiplying in columns). These lessons go farther than just reviewing the well-known algorithm. We study in detail: multiplying in parts (partial products), how those partial products can be seen in the algorithm itself, and how multi-digit multiplication can be visualized geometrically.

Students practice long division, including two-digit divisors, as a review from 4th grade. And just in case they haven't already learned it, this review gives them a "second chance."

The last lessons concentrate on problem solving with the help of diagrams, including a lesson on the concept of *ratio*. Word problems involving fractional parts should now gradually become easy, routine problems.

The "Introduction to Ratios" is an important lesson. It shows the connection between fractional parts, ratios, and bar diagrams. We also study ratios further in other chapters, such as the one on fractions, so they will not be forgotten.

Although the chapter is named, "The Four Operations," please notice that the idea is not to practice each of the four operations separately, but rather to see how they are used together in solving problems and in simple equations. We are trying to develop students' *algebraic thinking*, including the abilities to: translate problems into mathematical operations, comprehend the many operations needed to yield an answer to a problem, "undo" operations, and so on. Many of the ideas in this chapter are preparing them for algebra in advance.

The Lessons in Chapter 1

	page	span
Warm Up: Mental Math	10	*2 pages*
The Order of Operations and Equations	12	*2 pages*
Addition and Subtraction Review....................................	14	*3 pages*
Multiplication and Division	17	*3 pages*
Multiplying in Parts and the Multiplication Algorithm	20	*4 pages*
A Three-Digit Multiplier, Plus Zeros	24	*3 pages*
Multiplication and Area	27	*2 pages*

	page	span
Long Division	29	*3 pages*
Long Division Practice Puzzle	32	*1 page*
Long Division Practice Puzzle	32	*1 page*
A Two-Digit Divisor	33	*4 pages*
Balance Problems and Equations	37	*5 pages*
More Equations	42	*3 pages*
Problem Solving 1: Finding a Fractional Part of the Whole	45	*2 pages*
Problem Solving 2: Problem Solving with Diagrams	47	*2 pages*
Problem Solving 3: One part is a multiple of the other	49	*2 pages*
Problem Solving 4	51	*2 pages*
Introduction to Ratios	53	*4 pages*
Chapter 1 Review	57	*2 pages*

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Order of Operations and Equations

Solve multiplications and divisions before additions and subtractions.

Solve multiplications and divisions "on the same level," from left to right.

Solve additions and subtractions "on the same level," from left to right.

Parentheses ( ) change the order. *First* solve whatever is inside parentheses.

1. Solve in the right order!

a. $12 \times 5 + 8 =$ _____	**b.** $10 \times 2 + 9 \times 8 =$ _____	**c.** $(8 + 16) \div 3 =$ _____
$45 + 5 \times 7 =$ _____	$10 + 2 \times 9 + 8 =$ _____	$120 - 2 \times 11 =$ _____
$8 \times 5 \div 2 =$ _____	$10 + 2 \times (9 + 8) =$ _____	$2 \times (100 - 80 + 20) =$ _____

Which expression(s) match each problem?

2. Mark bought three light bulbs for $8 each, and paid with $50. What was his change?

a. 3 × $8 − $50 **b.** $50 − $8 + $8 + $8

c. $50 − 3 × $8 **d.** $50 − ($8 − $8 − $8)

3. Andy buys a salad for $8 and a pizza for $13, and shares them evenly with his friend. How many dollars is Andy's share of the cost?

a. $8 + $13 ÷ 2 **b.** $2 ÷ ($8 + $13)

c. 2 × $8 + 2 × $13 **d.** ($8 + $13) ÷ 2

4. Melissa shares equally the cost of a new fence with three other neighbors and the cost of road repair with two other neighbors. The fence cost $600 and the road repair cost $1,200. What is Melissa's share of the costs?

a. $600 ÷ 4 + $1,200 ÷ 3

b. ($600 + $1,200) ÷ 3 ÷ 2

c. $600 ÷ 3 + $1,200 ÷ 2

d. ($600 + $1,200) ÷ 5

5. Division can also be written with a line. Solve in the right order.

a. $6 + \dfrac{24}{2} =$ **b.** $\dfrac{32}{2} - 6 =$ **c.** $\dfrac{54}{6} - 6 - 2 =$

In this case, what we do first is the operation that is *above* the line, as though it were written in parentheses:

d. $\dfrac{6 + 24}{2} =$ **e.** $\dfrac{32 - 6}{2} =$ **f.** $\dfrac{54 - 6}{6} - 2 =$

A Three-Digit Multiplier, Plus Zeros

The multiplication algorithm works the same with 3-digit numbers. We simply have three partial products to do, and so the multiplication process takes three lines. Lastly add.

2 6	1	1	
4 2 9	**4 2 9**	**4 2 9**	4 2 9
× 2 2 **7**	× 2 **2** 7	× **2** 2 7	× 2 2 7
3 0 0 3	3 0 0 3	3 0 0 3	3 0 0 3
	8 5 8 0	8 5 8 0	8 5 8 0
		8 5 8 0 0	+ 8 5 8 0 0
			9 7 3 8 3
First you multiply the number by the ones.	Then by the tens. Here you need to put a zero in the ones place.	Then by the hundreds. Here you need to put a zero in the ones AND in the hundreds place.	Lastly add.

1. Multiply.

a. 191×245

b. 409×228

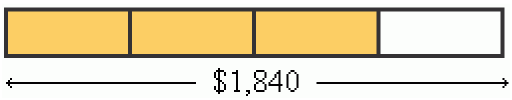
c. 246×137

d. 815×723

e. 207×803

f. 125×662

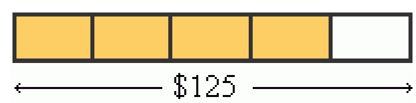
Problem Solving with Diagrams, Part 1
Finding a Fractional Part of the Whole

Problem	Solution
Jackie earns $1,840 monthly and Jessie earns 3/4 as much. How much does Jessie earn?  	In the diagram, Jackie's salary is divided into four equal parts. To find 3/4 of it, *first find 1/4 of it*, and then multiply that by 3. $1,840 ÷ 4 = $460. 3 × $460 = $1,380. Jessie earns $1,380.

Solve the problems. You may draw a diagram to help.

1. A pizza that weighs 680 g is divided into five equal pieces.
   How much do two pieces weigh?

2. A $125 camera was discounted by 1/5 of its price.
   What is its new price?



3. A bottle of water costs 2/3 as much as a $1.50 juice.
   How much do two bottles of water and two juices cost?

4. A T-shirt is discounted by 2/5 of its price. The original price is $10.50.
   How much would ten shirts cost with the discounted price?

Chapter 2: Large Numbers and the Calculator
Introduction

In this chapter we study place value up to the billions—that is, numbers of up to 12-digits, rounding them and using a calculator.

This is the first time the calculator is introduced in the Math Mammoth LightBlue Series books. I have delayed introducing the use of a calculator (as compared to most math curricula) for good reasons. On my HomeschoolMath.net web site I have received numerous comments on the harm that indiscriminate calculator usage can cause. In a nutshell, if children are allowed to use calculators freely, their minds get "lazy," and they will start relying on calculators even for simple things. It's just human nature!

As a result, students enter college without even knowing their multiplication tables by heart. Then they have tremendous trouble if they are required to use mental math to solve simple problems.

So we educators need to *limit* calculator usage until the students are much older. Children can *not* decide this for themselves, and definitely not in fifth grade.

However, I realize that the calculator is extremely useful, and students do need to learn to use it. In this curriculum, I strive to show the students not only *how* to use a calculator, but also *when* to use it and when *not* to use it.

This chapter includes many problems where calculator usage is appropriate. We also practice estimating the result before calculating it with calculator. In the last lesson, students need to choose whether mental math or a calculator is the best "tool" for the calculation.

The Lessons in Chapter 2

	page	span
Place Value Up to Billions	61	*3 pages*
Counting and Adding Large Numbers	64	*3 pages*
Rounding	67	*3 pages*
Calculator	70	*3 pages*
Multiples, Estimation and Calculator	73	*3 pages*
Review	76	*2 pages*

Place Value up to Billions

h t o h t o h t o h t o 4 0 9,3 8 2,0 4 3,5 5 9 billions period millions period thousands period ones period This number is read: “four hundred [and] nine billion, three hundred [and]eighty-two million, forty-three thousand, five hundred and fifty-nine”	We separate the digits in large numbers in groups of three. These groupings are called “periods.” Learn their names from the chart. The letters “h t o” over the top of the digits stand for “hundreds, tens, ones.” Once you remember the names for these groupings, it is very easy to read large numbers. Simply read each three digits as if it were a number by itself, and when you come to the comma, say the word “billion,” “million,” or “thousand.”
h t o h t o h t o h t o **2 7 3**,5 1 3,4 0 0,0 2 1 **billions period** millions period thousands period ones period	Look at the billions period. The digit 2 is in the *hundred billions* place. The digit 7 is in the *ten billions* place. The digit 3 is in the *billions* place.
h t o **h t o** h t o h t o 2 7 3,**5 1 3**,4 0 0,0 2 1 billions period **millions period** thousands period ones period	Look at the millions period. The digit 5 is in the *hundred millions* place. The digit 1 is in the *ten millions* place. The digit 3 is in the *millions* place.
h t o h t o **h t o** h t o 2 7 3,5 1 3,**4 0 0**,0 2 1 billions period millions period **thousands period** ones period	Look at the thousands period. The digit 4 is in the *hundred thousands* place. The digit 0 is in the *ten thousands* place. The digit 0 is in the *thousands* place.
h t o h t o h t o **h t o** 2 7 3,5 1 3,4 0 0,**0 2 1** billions period millions period thousands period **ones period**	Lastly look at the ones period. The digit 0 is in the *hundreds* place. The digit 2 is in the *tens* place. The digit 1 is in the *ones* place.

1. Study the number 85,359,204,031. Read it aloud.

a. Write the digit in the hundred thousands place. _____

b. Write the digit in the ten billions place. _____

c. Write the digit in the millions place._____

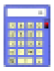
d. Write the digit in the billions place. _____

Calculator

A calculator has buttons for each of the numbers from 0 to 9. The button with the plus ("+") sign is used for addition. Similarly, the minus ("−") button for subtraction, the times ("×") button for multiplication, and divide ("÷") button for division. To get an answer, push "= ".

For example, to calculate 34 × 2,492, press [3] [4] [x] [2] [4] [9] [2] [=]

and the calculator should show you 84728.


In this lesson, use your calculator for *every exercise*. Otherwise, use a calculator only if you see the little calculator image next to the exercise.

1. Use a calculator to calculate these products.

a. 1,000 × 5,000	**b.** 3,000 × 7,000	**c.** 8,000 × 9,000	**d.** 3,000 × 6,000

Do you notice a *pattern* in the above results? This time, calculate the answer *mentally* first. Then use a calculator to check to see if you were right.

e. 3,000 × 5,000	**f.** 4,000 × 4,000	**g.** 7,000 × 6,000	**h.** 9,000 × 3,000

2. First estimate the answer by using rounded numbers. Then calculate the exact answer with a calculator. Lastly, find the error of estimation with a calculator.

Remember? **The error of estimation** is the *difference* between the exact answer and the estimated answer.

a. 54,395 + 89,302 (round to thousands) My estimation: ________________ Exact answer: ________________ Error of estimation: ________________	**b.** 9,807,520 − 1,532,392 (round to millions) My estimation: ________________ Exact answer: ________________ Error of estimation: ________________
c. 1,224,845 (to millions) ÷ 995 (to thousands) My estimation: ________________ Exact answer: ________________ Error of estimation: ________________	**d.** 2,873 × 3,204 (round to thousands) My estimation: ________________ Exact answer: ________________ Error of estimation: ________________

Chapter 3: Decimals
Introduction

In this chapter we study decimal place value and apply the four operations to decimal numbers.

The chapter starts with a short review of previously learned concepts: place value with tenths and hundredths, and adding and subtracting decimals that have tenths and hundredths.

The rest of the chapter is spent applying the four operations to numbers that have up to three decimal digits, and especially concentrating on decimal multiplication and division. We start by learning place value, comparing, and rounding decimals (tenths, hundredths, and thousandths). After that follow addition and subtraction, and then various multiplication and division topics.

I've strived to emphasize mental calculations based on conceptual understanding of decimals, and for that end the text also often includes little tips or “tricks” that help with mental calculations. Along with all that, the chapter naturally has lessons on long multiplication and long division with decimals. Problems that show a little calculator picture are meant to be solved with the help of a calculator. Otherwise, a calculator should not be allowed.

You might wonder why the *Math Mammoth Grade 5 Complete Worktext* presents decimals before fractions. The traditional way is to teach fractions first, because fractions are more general, and then to show that decimals are simply a specific instance of fractions, with denominators that are multiples of ten.

I have reversed the traditional order of presentation to avoid overwhelming the student with too much new information at once. In this chapter students learn the mechanics of *how* to multiply and divide decimals as an extension of the concept of place value. Then, when we get to fractions, we will carefully compare the multiplication and division of fractions to the multiplication and division of decimals, using the principles behind the methods for calculating fractions to show why the decimal calculations are done in the way that they are.

Since students will already be familiar with decimal multiplication and division, they won't have as many totally new concepts to comprehend at once. At that point, the comparison of fractions to decimals will not only review and reinforce what they learn in this chapter, but it will also tie together the important underlying mathematical concepts involved to fix them more clearly in the students’ comprehension.

The Lessons in Chapter 3

	page	span
Review: Tenths and Hundredths............................	81	*3 pages*
More Decimals: Thousandths	84	*3 pages*
Comparing Decimals	87	*2 pages*
Rounding	89	*2 pages*
Add and Subtract Decimals	91	*3 pages*
Multiplying Decimals	94	*3 pages*
Dividing Decimals	97	*4 pages*
Long Division with Decimals	101	*4 pages*
Decimals in Measuring Units and More	105	*2 pages*

Rounding and Estimating	107	*2 pages*
Multiplying Decimals by Decimals	109	*3 pages*
More Decimal Multiplication	112	*2 pages*
Multiply and Divide by 10, 100 and 1000	114	*4 pages*
Long Multiplication	118	*1 pages*
Divide Decimals by Decimals	119	*2 pages*
Number Rule Puzzles	121	*1 pages*
Problems to Solve	122	*2 pages*
Lessons in Problem Solving	124	*5 pages*
Review	129	*2 pages*
Review 2	131	*2 pages*

Helpful Resources on the Internet

Place Value Strategy
Place the 3 or 4 digits given by the spinner to make the largest number possible.
www.decimalsquares.com/dsGames/games/placevalue.html

Decimal Darts
Try to pop balloons with darts by estimating the balloons' height.
www.decimalsquares.com/dsGames/games/darts.html

Estimate
Estimate the decimal number that the arrow is pointing to on the number line. The game has the words "Evaluation version" across the screen, but it's still playable.
www.interactiveresources.co.uk/mathspack1/estimate/estimate.html

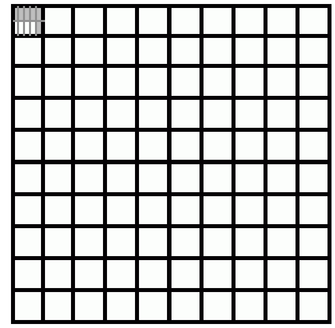
Decimal Challenge
Try to guess a decimal number between 0 and 10. Each time feedback tells you whether your guess was too high or too low.
www.interactivestuff.org/sums4fun/decchall.html

Beat the Clock
Type in the decimal number for the part of a square that is shaded in this timed game.
www.decimalsquares.com/dsGames/games/beatclock.html

Scales
Move the pointer to match the decimal number given to you. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/scales.html

Switch
Put the sequence of decimal numbers into ascending order by switching them around. Refresh the page from your browser to get another problem to solve.
www.interactivestuff.org/sums4fun/switch.html

More Decimals: Thousandths



This square illustrates *one whole*. It is divided into hundred parts or hundredths. The top left square is divided into ten new parts. Those are *thousandths*.

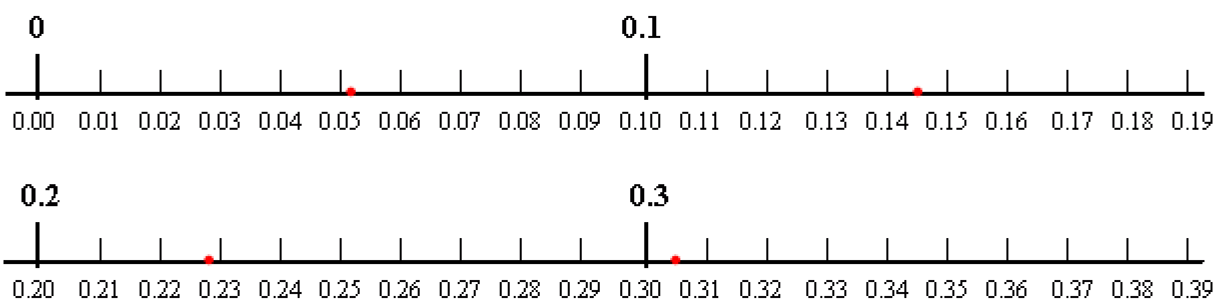
The 6 shaded parts represent $\frac{6}{1000}$, or 0.006 - six thousandths.

The third decimal digit from the decimal point is the thousandths digit. For example, 0.008 is eight thousandths.

Read the whole set of three decimal digits as a number, and say "thousandths."

0.391 is read "391 thousandths," and it is equal to $\frac{391}{1000}$.

0.047 is read "47 thousandths" and is equal to $\frac{47}{1000}$.



This number line has tick marks at every hundredth. For thousandths, we would need to divide each such interval into ten new intervals. Imagine that in between each two tick marks there are nine little lines. Those would represent thousandths.

The numbers 0.052, 0.145, 0.228, and 0.304 are marked on the number line. Can you find them?

Reminder: You can "tag" zeros to the end of a decimal number, and its value will not change:

O		t	h	th
0	**.**	**7**		
0	**.**	**7**	**0**	
0	**.**	**7**	**0**	**0**

0.7 = 0.70 = 0.700

Seven tenths = 70 hundredths = 700 thousandths.

$\frac{7}{10} = \frac{70}{100} = \frac{700}{1000}$.

1. Fill in.

When you divide one whole into ten equal parts, you get ________________________________.

When you divide one tenth into ten equal parts, you get ________________________________.

When you divide one hundredth into ten equal parts, you get ________________________________.

Add and Subtract Decimals

This "trick" will help you a lot in adding or subtracting decimals:

"Tag" zeros to the end of the decimal numbers so that all addends have the same amount of digits after the decimal. Then the answer will also have that same number of digits.

Look how the problem 0.2 + 0.05 is done (on the right). We "tag" a zero onto the end of 0.2 to make it have *two* digits after the decimal point! That way *both* addends have hundredths. Notice how it's just like adding fractions using a common denominator.

Note that 0.2 + 0.05 is *not* 0.7 or 0.07!

When adding in columns, write the numbers under each other and align the decimal points. You can write a zero in the empty "spot." Then add.

$$\begin{array}{ccc} 0.2 & + & 0.05 \\ \downarrow & & \downarrow \\ 0.20 & + & 0.05 = 0.25 \end{array}$$

$$\begin{array}{ccccc} \frac{2}{10} & + & \frac{5}{100} & & \\ \downarrow & & \downarrow & & \\ \frac{20}{100} & + & \frac{5}{100} & = & \frac{25}{100} \end{array}$$

$$\begin{array}{r} 0.20 \\ +\ 0.05 \\ \hline 0.25 \end{array}$$

0.09 + 0.5

= 0.09 + 0.50 = 0.59

$$\begin{array}{r} 0.09 \\ +\ 0.50 \\ \hline 0.59 \end{array}$$

Both addends have hundredths, and so does the answer.

1.007 + 2.02

= 1.007 + 2.020 = 3.027

$$\begin{array}{r} 1.007 \\ +\ 2.020 \\ \hline 3.027 \end{array}$$

Both addends have thousandths, and so does the answer.

1. Write the decimal that is one tenth, one hundredth, or one thousandth more than the given decimal.

a.

O		t	h	th
0	**.**	**2**	**8**	**5**

1 tenth more: __________

1 hundredth more: __________

1 thousandth more: __________

b.

O		t	h	th
0	**.**	**0**	**1**	**6**

2 tenths more: __________

2 hundredths more: __________

2 thousandths more: __________

c.

O		t	h	th
1	**.**	**0**	**7**	

5 tenths more: __________

2 hundredths more: __________

6 thousandths more: __________

2. Add.

a. 0.009 + 0.006

b. 0.009 + 0.06

c. 0.009 + 0.6

d. 0.8 + 0.6

e. 0.8 + 0.06

f. 0.8 + 0.006

g. 0.5 + 0.7

h. 0.05 + 0.07

i. 0.005 + 0.007

3. **a.** Write a number that is 5 thousandths, 2 tenths, and 8 hundredths more than 1.004.

b. Write a number that is 3 thousandths and 3 tenths less than 3.411.

Dividing Decimals

Do you remember that multiplication and division are opposite operations? From any multiplication equation we can write two corresponding division equations. Look at the example to the right: →

$3 \times 0.9 = 2.7$

$2.7 \div 3 = 0.9$

$2.7 \div 0.9 = 3$

When a decimal is divided by a whole number, as for example in the problem 4.5 ÷ 5, you can think of multiplication: What number multiplied by 5 will give you 4.5? The answer is 0.9.

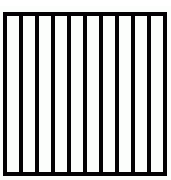
Remember that you can also think in terms of tenths, hundredths, and thousandths:

For example, 200 thousandths divided by 5 is 40 thousandths. (You could replace the word “thousandths” in that sentence with “dollars,” “millions,” “kilograms,” or anything else, and it stays true!)

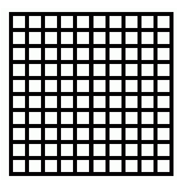
Written as fractions, this is $\frac{200}{1000} \div 5 = \frac{40}{1000}$. Written as decimals, it is $0.200 \div 5 = 0.040 = 0.04$.

When a decimal is divided by another decimal, think about how many times the divisor goes into the dividend. For 0.24 ÷ 0.03, you can think, “How many times does 0.03 go into 0.24?” Just as 3 apples goes into 24 apples 8 times, 3 hundredths goes into 24 hundredths 8 times

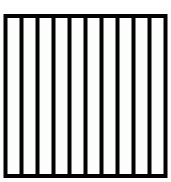
1. First shade the parts. Then divide and write a division sentence.



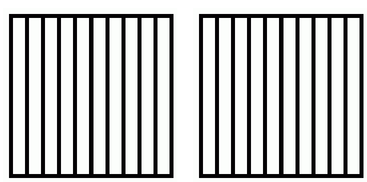
a. Shade 0.3.
Divide it into 3 parts.



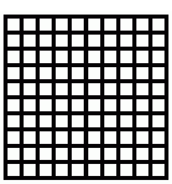
b. Shade 0.64.
Divide it into 2 parts.



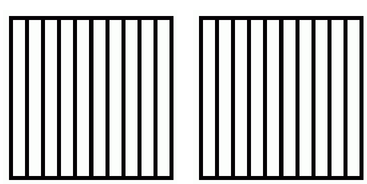
c. Shade 0.1.
Divide it into 10 parts.



d. Shade 1.6.
Divide it into 4 parts.



e. Shade 0.30.
Divide it into 10 parts.



f. Shade 1.8.
Divide it into 3 parts.

2. Write the division problems with numbers, and solve.

a. 54 hundredths divided by 6 equals ...

b. 72 thousandths divided by 9 equals ...

Multiplying Decimals by Decimals

You have learned to think of multiplication by a whole number, such as 3 × 4 or 8 × 0.6, as repeated addition. However, this concept doesn't work when neither of the factors is a whole number, as in 0.83 × 1.43 or 2/3 × 7/11. Instead, when you multiply decimals or fractions, think of it as finding "a certain part of" the other factor. In this sense, the symbol "×" translates to "of."

For example, think of 0.4 × 80 as finding four-tenths "of" 80. Since 0.1 of 80 is 8, then 0.4 of 80 is 32. Similarly, think of 0.21 × 700 as finding 21/100 of 700. Since 0.01 of 700 is 7, then 0.21 of 700 is 147.

1. Solve and compare the questions in each box:

a. Find 0.1 × 30.	**c.** Find 0.1 × 400.	**e.** Find 0.01 × 800.
b. Find 0.4 × 30.	**d.** Find 0.6 × 400.	**f.** Find 0.16 × 800.

Scaling a "stick"

Scaling means expanding or shrinking something by some factor.
Scaling is a useful model for multiplication. Let's look at scaling a "stick" (a line segment).

This red stick is 40 pixels long.
Let's scale it to be four times as long:

→

We can write a multiplication "equation":

4 × =

Using pixels, 4 × 40 px = 160 px.

Now let's scale the red stick to be 0.4 times as long as it is at first:

→

And we write the multiplication equation:

0.4 × =

In pixels, 0.4 × 40 px = 16 px.

The number we multiply by (4 and 0.4 above) is called the **scaling factor**.

If the scaling factor is more than 1, the resulting stick is *longer* than the original one.
If the scaling factor is less than 1, such as 0.5 or 0.66, the resulting stick is *shorter*.

2. The red stick is 40 pixels long. It is being scaled. Complete the corresponding multiplication sentence:

a. 0.1 × → 0.1 × 40 px = ______ px	**c.** 0.3 × → 0.3 × 40 px = ______ px	**e.** 0.6 × → 0.6 × 40 px = ______ px
b. 0.2 × → 0.2 × 40 px = ______ px	**d.** 0.5 × → 0.5 × 40 px = ______ px	**f.** 0.9 × → 0.9 × 40 px = ______ px

Towards a shortcut

Half of 5 is 2.5, or 0.5 × 5 = 2.5. This resembles the familiar multiplication 5 × 5 = 25!

One-tenth of 6 is 0.6, or 0.1 × 6 = 0.6. Therefore, seven-tenths of 0.6 is 4.2, or 0.7 × 6 = 4.2. This resembles the familiar multiplication 7 × 6 = 42!

The **shortcut** to decimal multiplication is:

1) Multiply as if there were no decimal points.

2) Place the decimal point in the answer.

But where? We will explore that in the next exercise.

3. Multiply first as if there was no decimal point. Then add the decimal point to the number. Think how BIG the answer should be.

Examples: 0.8 × 0.8 has to be *slightly smaller* than 0.8, because scaling anything by 0.8 is close to the original, but somewhat smaller. So, 0.8 × 0.8 can't be 64, and it can't be 6.4, but it is 0.64! 0.1 × 5.6 has to be 1/10 of the size of 5.6. So, 0.1 × 5.6 = 0.56. 0.06 × 0.4 has to be very much smaller than 0.4. It also has to be smaller than 0.06! It can't be 24, nor 2.4, nor 0.24. So it is 0.024. 0.9 × 0.04 would be just slightly less than 0.04. So 0.9 × 0.04 = 0.036.	**a.** 0.5 × 0.3 = **b.** 0.9 × 0.6 = **c.** 0.4 × 0.7 =	**d.** 0.4 × 0.08 = **e.** 0.5 × 0.09 = **f.** 0.7 × 0.02 =
	g. 0.1 × 0.3 = **h.** 0.1 × 0.8 = **i.** 0.1 × 2.7 =	**j.** 0.2 × 0.12 = **k.** 0.3 × 0.21 = **l.** 0.8 × 0.11 =
	m. 0.9 × 0.01 = **n.** 9 × 0.001 = **o.** 0.9 × 0.1 =	**p.** 7 × 0.3 = **q.** 0.7 × 0.3 = **r.** 7 × 0.03 =

4. **a.** Check your answers above with a calculator or the answer key.

b. Look carefully at the problems you did in Exercise #3. We're still thinking about where to put the decimal point. Look at the *number* of decimal digits (decimal places) in the factors, and the *number* of decimal digits in the answer. Do you notice a pattern?

Chapter 4: Statistics and Graphing
Introduction

The fourth chapter starts out with a study of the coordinate grid, but only the first quadrant. I have also included a very gentle *Introduction to Functions* lesson, where students plot ordered pairs from number rules.

Practicing the use of the coordinate grid is a natural "prelude" to the study of line graphs, which follow next. The goals are that the student will be able to:

- read line graphs, including double line graphs, and answer questions about data already plotted;
- draw line graphs from a given set of data.

To achieve these goals I have provided plenty of exercises, with a lot of variety in topics.

The goals for the study of bar graphs are similar to those for the study of line graphs, in that the student will need to both:

- read bar graphs, including double bar graphs, and answer questions about data already plotted; and
- draw bar graphs from a given set of data.

In order to make bar graphs, it is necessary to understand how to group the data into categories. The lesson *Making Bar Graphs 2* explains the method we use to make categories if the numerical data is not already categorized.

Toward the end of the chapter, we study the mean and the mode and how these two concepts relate to line and bar graphs. Other math curricula commonly introduce the median, too, but I decided to omit it from 5th grade. There is still plenty of time to learn that concept in 6th, 7th, and 8th grades. Introducing all three concepts at the same time tends to jumble the concepts together and confuse them—and all that many students are able to grasp out of that jumble is often just fairly meaningless calculation procedures. I feel it is better to introduce and contrast initially just the two concepts, the mean and the mode, in order to give the student a solid foundation to study them later in more depth when the median is introduced and compared and contrasted with them.

This chapter also includes an optional statistics project, in which the student can develop investigative skills.

The Lessons in Chapter 4

	page	span
Coordinate Grid	135	*3 pages*
Introduction to Functions	138	*4 pages*
Lines Graphs	142	*4 pages*
Reading Line Graphs	146	*2 pages*
Double and Triple Line Graphs	148	*2 pages*
Making Bar Graphs	150	*2 pages*
Making Bar Graphs 2	152	*2 pages*

Double Bar Graphs	154	*2 pages*
Average (Mean)	156	*3 pages*
Mean, Mode and Bar Graphs	159	*2 pages*
Statistics Project (optional)	161	*1 page*
Review	162	*2 pages*

Helpful Resources on the Internet

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool:
nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness, and scale of the graph are adjustable. You can input your own data, or you can use or alter pre-made data sets.
illuminations.nctm.org/ActivityDetail.aspx?ID=63

Create a Graph
Create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.
nces.ed.gov/nceskids/createagraph/default.aspx

Mode of a Set of Data
This is a very simple and clear lesson with examples and interactive quiz questions.
www.mathgoodies.com/lessons/vol8/mode.html

Using and Handling Data
Simple explanations for finding the mean, the median, or the mode.
www.mathsisfun.com/probability

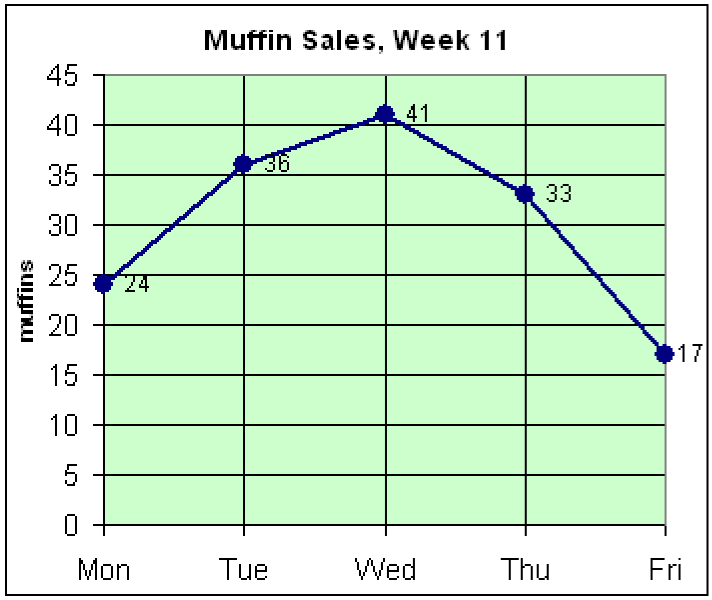
Finding the Mean, Median, and Mode
This is a great lesson, with interactive quiz questions at the end. It also explains briefly the different uses for mean, median, and mode. After all, why do we have three different numbers to describe the central tendency of a data set?
www.algebralab.org/lessons/lesson.aspx?file=Algebra_StatMeanMedianMode.xml

Mean, Median, and Mode
How to calculate the mean, the median, and the mode for sets of data given in different ways. There are also interactive exercises.
www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Line Graphs

Mary sold muffins every day at 2 pm in the school cafeteria. She recorded her sales in the table.

Muffin Sales, Week 11	
Day	*Muffins sold*
Mon	24
Tue	36
Wed	41
Thu	33
Fri	17



We can draw a line graph out of this data because the data is organized by *time* (days of the week). To do that, we first plot the individual data points in a coordinate grid. Then we draw lines to connect neighboring points.

Besides that, the line graph also needs:

- **a title** on top for the whole graph
- **labels** for the **tick marks** on the two axes
- a **label** for the **vertical axis** (the y-axis)
- a label for the horizontal axis (the x-axis) unless it is very clear what it is about.
  In the graph on the right, the labels "Mon," "Tue," and so on show very clearly that they are days of the week. So we don't necessarily need a title, "Days of the week," for the horizontal axis.

Use a **line graph** for data that is organized by some *unit of time* (hours, days, weeks, years, *etc*.)

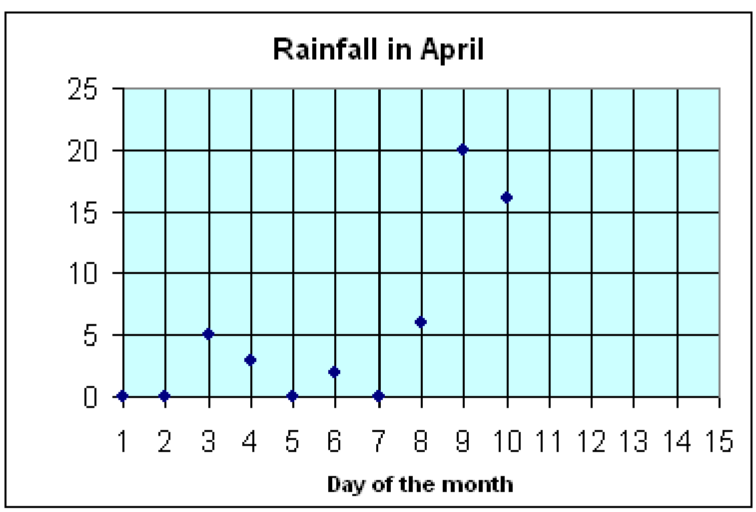
1. **a.** Add a label for the vertical axis that says "Rainfall (mm)".
The "mm" stands for millimeters.

b. Add five more data points to the graph according to this data:

Day	11	12	13	14	15
rainfall (mm)	9	0	0	13	2

c. Draw a line between each two consecutive points.

d. How many "dry" days were there in the first half of April?

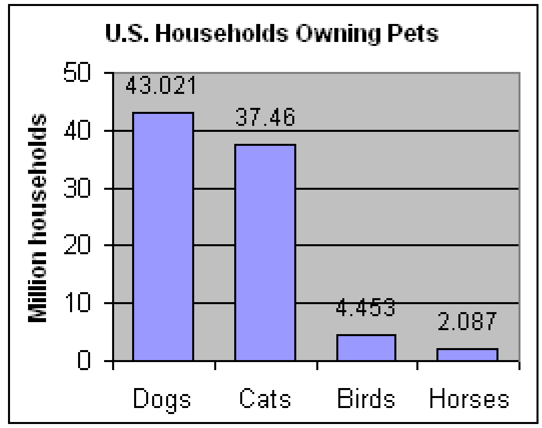


Making Bar Graphs 1

Bar graphs are used if the data can be separated into distinct groupings or categories. For example, if you study children's eye color, the categories are "blue," "green," "brown," "hazel," *etc*.

The graph on the right shows the number of U.S. households that own a dog, cat, bird, or a horse. A household owning, say, both a dog and a cat would be included in both numbers. Note that the vertical axis scale is in million households.

Note how the data values are recorded above each bar. To get the true number, multiply that by 1,000,000.



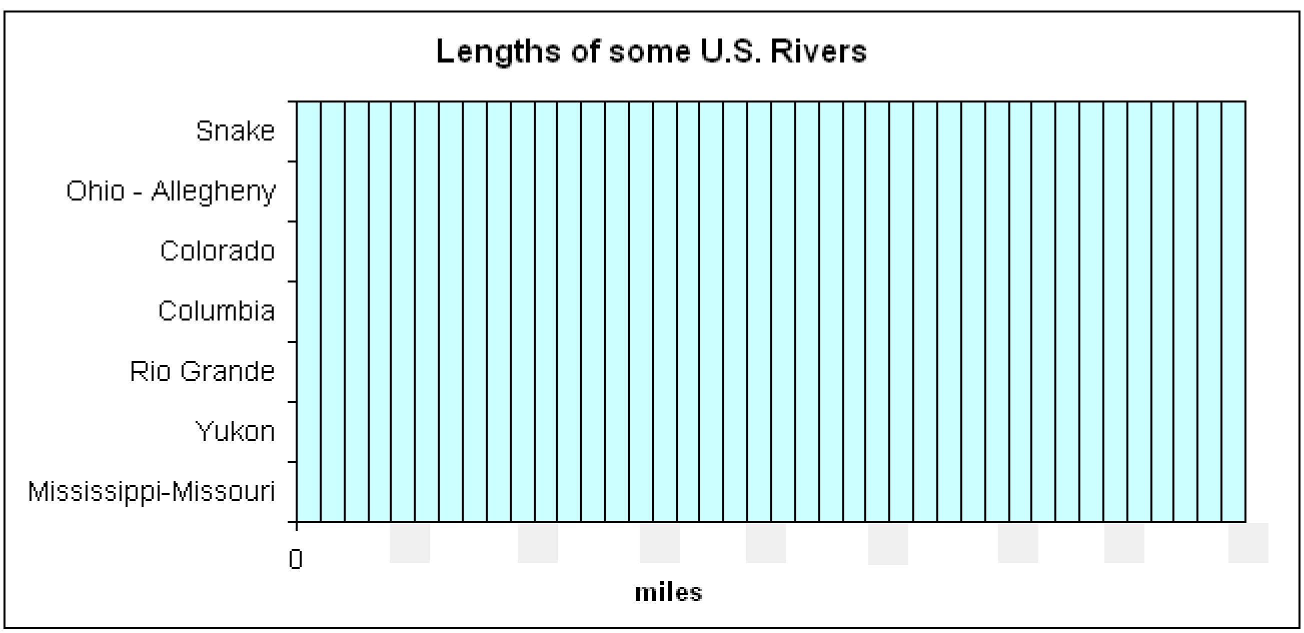
1. According to the graph above, how many U.S. households own a cat? A horse?

2. **a.** Draw a bar graph from the data on the right.
Notice that *you* need to figure out the scale on the horizontal axis (miles).
Hint: make sure the largest number in the river lengths fits on the grid, and that there isn't lots of "empty space" left over beyond that.

b. About how many times longer is the Mississippi-Missouri than the Ohio-Allegheny?

c. About how many times longer is the Mississippi-Missouri than the Yukon?

River	Length (miles)
Mississippi-Missouri	3,902
Yukon	1,980
Rio Grande	1,900
Columbia	1,450
Colorado	1,450
Ohio - Allegheny	1,306
Snake	1,038

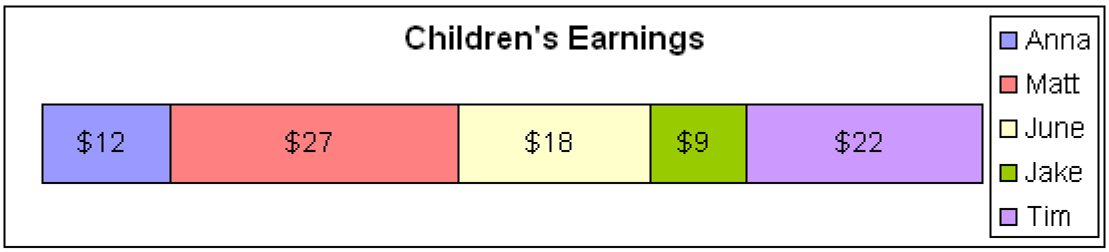


Average (Mean)

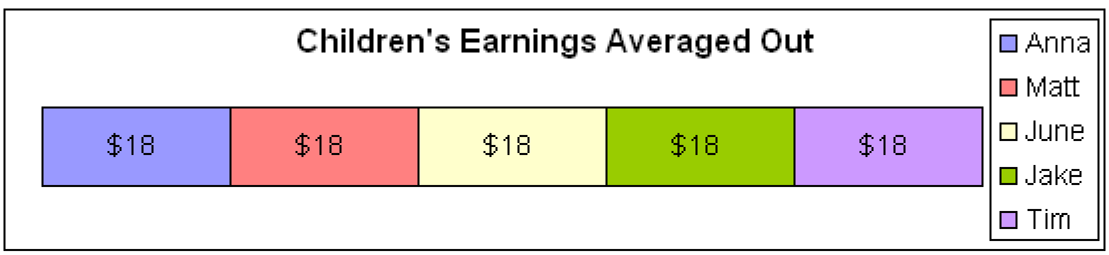
Example. Five children earned these amounts of money for a job: \$12, \$27, \$18, \$9, and \$22. The graph below shows visually how much each child earned.

Together, they earned \$88. If this \$88 had been divided equally among the children, each child would have gotten \$18. (Of course it wasn't, because the children got paid according to how much they worked.)

This \$18 is the ***average*** pay. Average = $\frac{\$12 + \$27 + \$18 + \$9 + \$22}{5} = \frac{\$88}{5} = \$18$.



The graph on the bottom shows the situation IF each child had received the *average* earning (\$18). Notice that \$18 is sort of in the "middle" or in between the lowest and highest earnings.



- You calculate the *average* of a data set by summing all the numbers in the data set, and dividing the sum by the number of entries in the data set. In other words

$$\text{average} = \frac{\text{sum of all data entries}}{\text{number of data entries}}$$

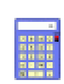
- The average is always somewhere in the "middle" of the data set's numbers.
- The average is also called the *mean*. We will use both terms in this lesson so you get used to both of them.

1. Calculate the average (the mean) of the data sets. Do not use a calculator.

a. 2, 4, 5, 9, 0, 4, 1, 7

b. 13, 16, 20, 22, 16, 13, 17, 12, 15

2. Calculate the mean of the data sets to the nearest tenth. This time use a calculator.



a. 2, 4.3, 5, 9, 4.7, 9.4, 3.7, 5.1

b. 312, 288, 284, 329, 293, 302