math MAMMOTH

Grade 4-A Worktext

A ddition, subtraction, patterns, and graphs

L arge numbers

M ulti-digit multiplication

T ime and measuring

Light Blue Series

By Maria Miller

Contents

Chapter 3: Multi-Digit Multiplication

Chapter 4: Time and Measuring

Foreword

Math Mammoth Grade 4-A and Grade 4-B worktexts comprise a complete math curriculum for the fourth grade mathematics studies, aligned to the Common Core Standards.

In the fourth grade, students focus on multi-digit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and they get started in studying fractions and decimals. This is of course accompanied by studies in geometry and measuring.

The year starts out with a review of addition and subtraction, patterns and graphs. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations, and statistical graphs.

Next come large numbers—up to millions, and the place value concept. At first the student reviews thousands and some mental math with them. Next are presented numbers up to one million, calculations with them, the concept of place value and comparing. In the end of the chapter we find out more about millions and an introduction to multiples of 10, 100, and 1000.

The third chapter is all about multiplication. After briefly reviewing the concept and the times tables, the focus is on learning multi-digit multiplication (multiplication algorithm). The children also learn why it works when they multiply in parts. We also study the order of operations again, touch on proportional reasoning, and do more money and change related word problems.

The last chapter in part A is about time, temperature, length, weight, and volume. Students will learn to solve more complex problems using various measuring units and to convert between measuring units.

In part B, we first study division. The focus is on learning long division and using division in word problems. In geometry, we first review area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple angle problems, and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last in the school year. These two chapters practice only some of the basic operations with fractions and decimals. The focus is still on conceptual understanding and on building a good foundation towards 5th grade math, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a “framework”, but you do have some liberty in organizing the study schedule. Chapters 1, 2, and 3 should be studied in this order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry) and schedule them somewhat earlier or later if you so wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year; however finding parts with division should naturally be studied only after mastering division.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 1: Addition, Subtraction, Patterns, and Graphs

Introduction

The first chapter of *Math Mammoth Grade 4* covers addition and subtraction topics, problem solving, patterns, graphs, and money.

At first, we review the "technical aspects" of adding and subtracting: mental math techniques and adding and subtracting in columns. We also study some patterns. The lesson on Pascal's triangle is intended to be fun and fascinating—after all, Pascal's triangle is full of patterns!

In the next lesson, we study the connection between addition and subtraction and bar models. Bar models help students write addition and subtraction sentences with unknowns, and solve them. This is teaching the students *algebraic thinking*: how to write and solve simple equations.

The lesson on the order of operations contains some review, but we also study connecting the topic with real-life situations (such as shopping). Here, the student writes the mathematical expression (number sentence) for word problems, which again, practices algebraic thinking.

Going towards applications of math, the chapter then contains straightforward lessons on bar graphs, line graphs, rounding, estimating, and money problems.

The Lessons in Chapter 1

Helpful Resources on the Internet

Calculator Chaos
Most of the keys have fallen off the calculator but you have to make certain numbers using the keys that are left.
http://www.mathplayground.com/calculator_chaos.html

ArithmeTiles
Use the four operations and numbers on neighboring tiles to make target numbers.
http://www.primarygames.com/math/arithmetiles/index.htm

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

MathCar Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations at the same time.
http://www.funbrain.com/osa/index.html

Fill and Pour
Fill and pour liquid with two containers until you get the target amount. A logical thinking puzzle.
http://nlvm.usu.edu/en/nav/frames_asid_273_g_2_t_4.html

Division and Order of operations and
Division and Addition - Order of Operations
Two mystery picture games.
http://www.dositey.com/2008/math/m/mystery2MD.htm and
http://www.dositey.com/2008/math/m/mystery2AD.htm

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parenthesis in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, —, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.htm

Order of Operations Practice
A simple online quiz of 10 questions. Uses parenthesis and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate
Practice your arithmetic of all four operations plus the order of operations.
http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Estimate Addition Quiz
Scroll down the page to find this quiz plus some others. Fast loading.
http://www.quiz-tree.com/Math_Practice_main.html

Shop 'Til You Drop
Get as many items as you can and be left with the least amount of change, and practice your addition skills. The prices are in English pounds and pennies.
http://www.channel4.com/learning/microsites/P/puzzlemaths/shop.shtml

Change Maker
Determine how many of each denomination you need to make the exact change. Good and clear pictures! Playable in US, Canadian, Mexican, UK, or Australian money.
http://www.funbrain.com/cashreg/index.html

Cash Out
Give correct change by clicking on the bills and coins.
http://www.mrnussbaum.com/cashd.htm

Piggy bank
When coins fall from the top of the screen, choose those that add up to the given amount, and the piggy bank fills.
http://fen.com/studentactivities/Piggybank/piggybank.html

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool.
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness and scale of the graph are adjustable. You can put in your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Create a Graph
A neat online tool for creating a graph from your own data.
http://nces.ed.gov/nceskids/createagraph/

Math Mahjong
A Mahjong game where you need to match tiles with the same value. It uses all four operations and has three levels.
http://www.sheppardsoftware.com/mathgames/mixed_mahjong/mahjongMath_Level_1.html

Pop the Balloons
Pop the balloons in the order of their value. You need to use all four operations.
http://www.sheppardsoftware.com/mathgames/numberballoons/BalloonPopMixed.htm

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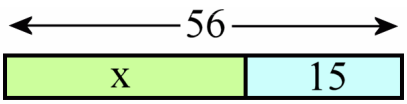
Bar Models in Addition and Subtraction

Think of this **bar model** as a long board, cut into two pieces. It is 56 units long in total (you can think of inches, for example), and the two parts are 15 and *x* units long.

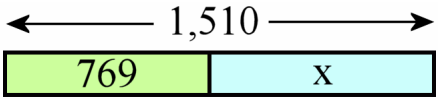


From the bar model, we can write TWO addition and TWO subtraction sentences—a *fact family*.

$x + 15 = 56$	$56 - x = 15$
$15 + x = 56$	$56 - 15 = x$

The *x* stands for a number, too. We just do not know what it is yet. It is an *unknown*.

From this bar model, we can write a **missing addend** problem.



It means that a number to be added is "missing" or unknown.

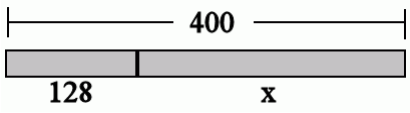
We can solve it by **subtracting** the one part (769) from the total (1,510).

$769 + x = 1{,}510$

$x = 1{,}510 - 769 = 741$

1. Write a missing addend problem that matches the bar model. Then solve it by subtracting.

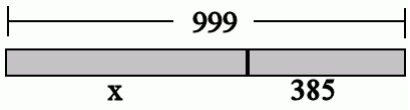
a.



_____ + _____ = _____

x = _____ – _____ = _____

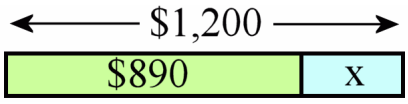
b.



_____ + _____ = _____

x = _____ – _____ = _____

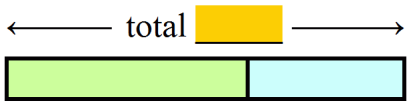
c. A car costs $1,200. Dad has $890.
How much more does he need to buy it?



_____ + _____ = _____

x = _____ – _____ = _____

d. The school has 547 students, of which 265 are girls. How many are boys?



_____ + _____ = _____

x = _____ – _____ = _____

2. Write the numbers and *x* to the model. Remember, *x* is the unknown, or what the problem asks for. Write an addition using the numbers and *x*. Lastly solve.

a. Of their 1,200-mile trip, the Jones family traveled 420 miles yesterday and 370 miles today. How many miles do they have left to travel?

Addition:

Solution: *x* = _____

b. The store is expecting a shipment of 4,000 blank CDs. Three boxes of 400 arrived. How many CDs are yet to come?

Addition:

Solution: *x* = _____

c. A 250-cm board is divided into three parts: two 28-cm parts at the ends and a part in the middle. How long is the middle part?

Addition:

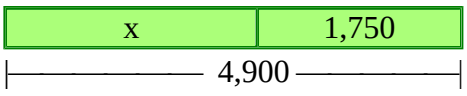
Solution: *x* = _____

d. After traveling 56 miles, Dad said, "Okay, in 9 miles we will be at Kensville, and from there we will have 118 miles left." How many total miles is the trip?

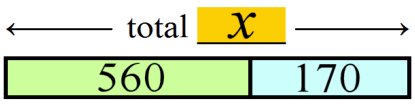
Addition:

Solution: *x* = _____

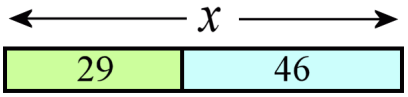
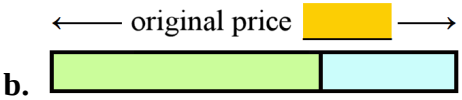
3. Make a word problem that matches the model. Then solve for *x*.



x = _____

In this problem: $x - 170 = 560$, the TOTAL is unknown. Remember, subtraction problems start with the total. Look at the bar model. We can solve x by adding.	 $x - 170 = 560$ $x = 170 + 560 = 730$

4. Write a subtraction problem that matches the bar model. Then solve it by adding.

a.  $x - \underline{\quad 29 \quad} = \_\_\_\_\_$ $x = \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_$	**b.**  $x - \_\_\_\_\_ = \_\_\_\_\_$ $x = \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_$

5. The number you subtract from is missing! Solve.

a. $\_\_\_\_\_ - 4 = 20$	**b.** $\_\_\_\_ - 15 = 17$	**c.** $\_\_\_\_ - 22 - 7 = 70$
Still, the number you subtract from is missing. But this time, it is denoted by x, not by an empty line.		
d. $x - 8 = 7$ $x = \_\_\_\_\_$	**e.** $x - 24 = 48$ $x = \_\_\_\_\_$	**f.** $x - 300 - 50 = 125$ $x = \_\_\_\_\_$

6. Here, the number you subtract is the unknown. Write the numbers and x into the bar model. Notice carefully what number is the <u>*total*</u>. Then write another matching subtraction that helps you solve x.

a.  $52 - x = 28$ $x = \_\_\_\_\_ - \_\_\_\_\_ = \_\_\_\_\_$	**b.**  $97 - x = 54$ $x = \_\_\_\_\_ - \_\_\_\_\_ = \_\_\_\_\_$

7. The number you subtract is still the unknown. Solve.

a. $20 - \_\_\_\_ = 12$	**b.** $55 - \_\_\_\_ = 34$	**c.** $234 - \_\_\_\_ = 100$
d. $61 - x = 43$ $x = \_\_\_\_\_$	**e.** $100 - x = 72$ $x = \_\_\_\_\_$	**f.** $899 - x = 342$ $x = \_\_\_\_\_$

8. Circle the number sentence that fits the problem. Then solve for x.

a. Jane had \$15. After Dad gave Jane her allowance (x), Jane had \$22. \$15 + x = \$22 OR \$15 + \$22 = x x = _____	**b.** Mike had many drawings. He put 24 of them in the trash. Then he had 125 left. 125 – 24 = x OR x – 24 = 125 x = _____
c. Jill had 120 marbles, but some of them got lost. Now she has 89 left. 120 – x = 89 OR 120 + 89 = x x = _____	**d.** Dave gave 67 of his stickers to a friend and now he has 150 left. 150 – 67 = x OR x – 67 = 150 x = _____

9. Write a number sentence (addition or subtraction) with x. Solve it.

a. A school's teachers and students filled a 450-seat auditorium. If the school had 43 teachers, how many students did it have?	_____ + _____ = _____ x = _____
b. Mom went shopping with \$250 and came back home with \$78. How much did she spend?	originally – spent = left _____ – _____ = _____ x = _____
c. Janet had \$200. She bought an item for \$54 and another for \$78. How much money is left?	_____ – _____ – _____ = _____ x = _____
d. Jean bought one item for \$23 and another for \$29, and she had \$125 left. How much did she have initially?	_____ – _____ – _____ = _____ x = _____

Puzzle Corner Find the missing numbers.

a. 200 − 45 − _____ − 70 = 25	**b.** _____ − 5 − 55 − 120 = 40
c. 23 + 56 + x = 110 x = _____	**d.** x + 15 + 15 + 15 + 15 = 97 x = _____

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Chapter 2: Large Numbers and Place Value

Introduction

The second chapter of *Math Mammoth Grade 4* covers large numbers (up to 1 million) and place value.

The first lessons only deal with thousands, or numbers with a maximum of four digits. These are for review and for deepening the student's understanding of place value. It is crucial that the student understands place value with four-digit numbers before moving on to larger numbers. Then, larger numbers will be very easy to study.

Then we go on to study numbers up to one million, or numbers that have tens or hundreds of thousands. Students write them in expanded form, compare them, add and subtract them, and learn more about rounding.

Lastly we study briefly the multiples of 10, 100, and 1000. This lesson prepares the way for some very important ideas in the next chapter (multi-digit multiplication).

The Lessons in Chapter 2

Helpful Resources on the Internet

Place Value Payoff
Match numbers written in standard form with numbers written in expanded form in this game.
http://www.quia.com/mc/279741.html

Keep My place
Fill in the big numbers in this cross-number puzzle.
http://www.counton.org/magnet/kaleidoscope2/Crossnumber/index.html

Can you say really big numbers?
Enter a really big number, try to say it out loud, and see it written.
http://www.mathcats.com/explore/reallybignumbers.html

Megapenny Project
Visualizes big numbers with pictures of pennies.
http://www.kokogiak.com/megapenny/default.asp

Place value puzzler
Place value or rounding game. Click on the asked place value in a number, or type in the rounded version of the number.
http://www.funbrain.com/tens/index.html

Rounding Sharks
You'll be asked to round numbers in the thousands to the nearest hundred. Click on the shark that has the correctly rounded number.
http://www.free-training-tutorial.com/rounding/sharks.html

Rounding Master
A Mathionare-type game where you answer rounding questions, and try to become a Rounding Master Math Millionaire.
http://www.mrnussbaum.com/roundingmaster.htm

Estimation at AAA Math
Exercises about rounding whole numbers and decimals, front-end estimation, estimating sums and differences. Each page has an explanation, interactive practice, and games.
http://www.aaamath.com/B/est.htm

Maximum Capacity
Drag as many gorillas as you can into the elevator without exceeding the weight capacity. You will have to use your quick addition, estimation, and number sense skills.
http://www.mrnussbaum.com/maximumcapacity.htm

Home Run Derby Math
Estimate answers to math problems. The closer you get, the further your ball will fly at-bat. In addition and subtraction, the numbers are in the thousands. In multiplication, the numbers are in the hundreds.
http://www.mrnussbaum.com/derby.htm

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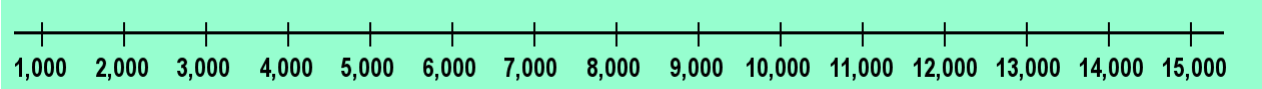
More Thousands



On this number line you see whole thousands from one thousand to fifteen thousand.

The colored digits are the "thousands period" and count as the whole thousands. Read the colored digits as its own number. Say the word "thousand" for the comma.

We continue with whole thousands until reaching *a thousand* thousands.

That number has a new name: *one million.*

78,000	*Read:* 78 thousand
153,000	*Read:* 153 thousand
802,000	*Read:* 802 thousand
990,000	*Read:* 990 thousand
999,000	*Read:* 999 thousand
1,000,000	Thousand thousand = 1 million

The rest of the digits tell us our hundreds, tens, and ones just like you have learned.

17,544	*Read:* 17 thousand five hundred forty four
609,230	*Read:* 609 thousand two hundred thirty
70,080	*Read:* 70 thousand eighty
902,005	*Read:* 902 thousand five

1. Place a comma in the number. Fill in the missing parts.

a. 164000	**b. 92000**	**c. 309000**	**d. 34000**	**e. 780000**
_____ thousand	_____ thousand	_____ thousand	_____ thousand	_____ thousand

2. Place a comma in the number. Fill in missing parts. Read the numbers aloud.

a. 164,453	**b. 92908**	**c. 329033**	**d. 14004**
164 thousand *453*	_____ thousand _____	_____ thousand _____	_____ thousand _____
e. 550053	**f. 72001**	**g. 800004**	**h. 30036**
_____ thousand _____	_____ thousand _____	_____ thousand _____	_____ thousand _____

3. Read these numbers aloud.

a. 456,098 **b.** 950,050 **c.** 23,090 **d.** 560,008

e. 78,304 **f.** 266,894 **g.** 1,000,000 **h.** 306,700

4. Think in whole thousands and add!

a. 30,000 + 5,000 =
think: 30 thousand + 5 thousand

b. 200,000 + 1,000 =

c. 400,000 + 30,000 =

d. 710,000 + 40,000 =

e. 300,000 + 700,000 =

f. 700,000 + 70,000 =

5. Add and subtract, thinking in whole thousands.

a. 35,000 + 5,000 =

b. 711,000 + 10,000 =

c. 420,000 + 30,000 =

d. 700,000 – 70,000 =

e. 300,000 – 60,000 =

f. 1,000,000 – 200,000 =

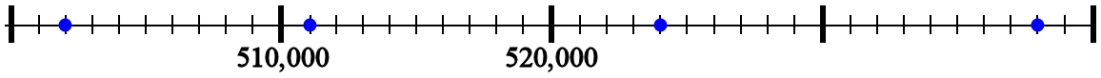
g. 30,000 – 5,000 =

h. 200,000 – 6,000 =

i. 723,000 – 400,000 =

j. 500,000 – 1,000 =

6. On the number line below, 510,000 and 520,000 are marked (at the "posts").
Write the numbers that correspond to the dots.



7. Make a number line from 320,000 to 340,000 with tick-marks at every whole thousand, similar to the one above. Then mark the following numbers on the number line:
323,000 328,000 335,000 329,000 330,000

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Chapter 3: Multi-Digit Multiplication
Introduction

The third chapter of *Math Mammoth Grade 4* covers multi-digit multiplication and some related topics.

The first lessons briefly review the multiplication concept and the times tables. The next lesson, where students solve scales or pan balance problems, is intended to be somewhat fun and motivational. The balance problems are actually equations in disguise.

Then, the focus is on multi-digit multiplication (also called algorithm of multiplication, or multiplying in columns). We start out by multiplying by whole tens and hundreds (such as 20×4 or 500×6). After this is mastered, we study a very important concept of **multiplying in parts** (also called partial products algorithm). It means that 4×63 is done in two parts: 4×60 and 4×3, and the results are added.

This principle underlies all other multiplication algorithms, so it is important to master. We do not want children to learn the multiplication algorithm "blindly", without understanding what is going on with it. Multiplying in parts is also tied in with an area model, which, again, is very important to understand.

Before showing the traditional form of multiplication, the lesson *Multiply in Columns—the Easy Way* shows a simplified form of the same, which is essentially just multiplying in parts. You may skip that lesson at your discretion or skim through it quickly if your child is ready to understand the standard form of the algorithm, which is taught next.

Students also study estimation, the order of operations, and multiplying with money. Many kinds of word problems abound in all of the lessons. Students are supposed to practice writing a number sentence for the word problems—essentially writing down the calculating they are doing.

The lesson "*So Many of the Same Thing*" could be entitled "Proportional Reasoning" but I wanted to avoid scaring parents and children with such a high-sounding phrase. The idea in that lesson is really simple, but it does prepare for proportions as they are taught in 7th grade and in algebra.

After that, we multiply two-digit numbers by two-digit numbers. Again, we first study partial products and tie that in with an area model. The lesson *Multiplying in Parts: Another Way* is optional. After that, the standard algorithm for multiplying a two-digit number by a two-digit number is taught, and the chapter ends.

The Lessons in Chapter 3

Helpful Resources on the Internet

Multiplication Games
A list of times tables games and activities to practice multiplication facts.
http://www.homeschoolmath.net/online/multiplication.php

Math FROG MultipliACTION
Online practice of 2 by 2 digit multiplication. You enter one digit in each box.
http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/mult5.shtml

Math Playground
Learn how to think algebraically with these clever weighing scales.
http://www.mathplayground.com/algebraic_reasoning.html

Thinking Blocks
Thinking Blocks is an engaging, interactive math tool that helps students learn how to solve multi-step word problems. Scroll down to Multiplication and Division.
http://www.mathplayground.com/thinkingblocks.html

Rectangle Multiplication
An interactive tool that illustrates multiplying in parts using the area model. Choose the "common" option for multiplying in parts.
http://nlvm.usu.edu/en/nav/frames_asid_192_g_2_t_1.html

One-Digit by Two-Digits Multiplication Game
Students will multiply one-digit numbers by two-digit whole numbers, and then get to try shoot a basket.
http://www.math-play.com/one-digit-by-two-digit-multiplication-game.html

Multiplication Jeopardy Game
You get to solve multi-digit multiplication questions of 1-digit by 1-digit, 1-digit by 2-digit, and 1-digit by 3-digit numbers in this game.
http://www.math-play.com/Multiplicaton-Jeopardy/Multiplication-Jeopardy.html

Interactive Pan Balance
Each of the four shapes is assigned a certain weight. Place shapes on either side of the pan balance and figure out their relationships.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=131

Balance Beam Activity
A virtual balance that provides balance puzzles where the student has to find the weights of various figures, practicing algebraic thinking. Includes three levels.
http://mste.illinois.edu/users/pavel/java/balance/

Choose Math Operation
Choose the mathematical operation(s) so that the number sentence is true. Practice the role of zero and one in basic operations or operations with negative numbers. Helps develop number sense and logical thinking.
http://www.homeschoolmath.net/operation-game.php

Order of Operations Quiz
A 10-question online quiz that includes two different operations and possibly parenthesis in each question. You can also modify the quiz parameters yourself.
http://www.thatquiz.org/tq-1/?-j8f-la

The Order of Operations Millionaire
Answer multiple-choice questions that have to do with the order of operations, and win a million. Can be played alone or in two teams.
http://www.math-play.com/Order-of-Operations-Millionaire/order-of-operations-millionaire.html

Exploring Order of Operations (Object Interactive)
The program shows an expression, and you click on the correct operation (either +, —, ×, ÷ or exponent) to be done first. The program then solves that operation, and you click on the *next* operation to be performed, etc., until it is solved. Lastly the resource includes a game where you click on the falling blocks in the order that order of operations would dictate.
http://www.learnalberta.ca/content/mejhm/html/object_interactives/order_of_operations/use_it.htm

Order of Operations Practice
A simple online quiz of 10 questions. Uses parenthesis and the four operations.
http://www.onlinemathlearning.com/order-of-operations-practice.html

Quick Calculate
Practice the arithmetic of all four operations plus order of operations.
http://themathgames.com/arithmetic-games/addition-subtraction-multiplication-division/quick-calculate-game.php

Multiplication Tool
This online tool lets you illustrate and/or practice multi-digit multiplication using the standard algorithm, partial products algorithm, or the lattice method.
http://www.multiplicationtool.org

Mental Math Tricks for Multiplication
Includes some very basic common-sense ones such as multiplying by 9 or multiplying by doubling and halving.
http://wildaboutmath.com/2007/11/11/impress-your-friends-with-mental-math-tricks

Mental math multiplication guide
Rules of thumb and other "tricks" for mental multiplication of two-digit or bigger numbers, conveniently in one place. (This is not about single-digit multiplication; you are supposed to know those by heart of course.)
http://arscalcula.com/mental_math_multiplication_guide.shtml

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Multiplying by Whole Tens and Hundreds

We have studied the SHORTCUTS for multiplying any number by 10, 100, or 1,000:

To multiply any number **by 10**, just **tag ONE zero** on the end.

To multiply any number **by 100**, just **tag TWO zeros** on the end.

To multiply any number **by 1,000**, just **tag THREE zeros** on the end.

1**0** × 481 = 4,81**0**	1**00** × 47 = 4,7**00**	1**000** × 578 = 578**,000**

Note especially what happens when the number you multiply already ends in a zero or zeros. The rule works the same; you still have to tag the zero or zeros.

1**0** × 800 = 800**0**	1**00** × 6,600 = 660,0**00**	1**000** × 40 = 40**,000**

1. Multiply.

a. 10 × 315 = _____	**b.** 100 × 6,200 = _____	**c.** 1,000 × 250 = _____
3,560 × 10 = _____	10 × 1,200 = _____	38 × 1,000 = _____
35 × 100 = _____	100 × 130 = _____	10 × 5,000 = _____

SHORTCUT for multiplying by 20 or 200 (You can probably guess this one!)

What is 20 × 14?	**What is 200 × 31?**
Imagine the problem without the zero. Then it becomes 2 × 14 = 28. Then, just tag a zero to the 28 you got, so it becomes 280. So, 20 × 14 = 280.	Imagine the problem without the zeros. Then it becomes 2 × 31 = 62. Then, just tag *two* zeros to the result you got, so you get 6,200. In other words, 200 × 31 = 6,200.

2. Now try it! Multiply by 20 and 200.

a.	**b.**	**c.**	**d.**
20 × 8 = _____	200 × 7 = _____	20 × 12 = _____	20 × 16 = _____
4 × 20 =_____	5 × 200 = _____	35 × 20 = _____	42 × 200 = _____
20 × 5 = _____	11 × 200 = _____	200 × 9 = _____	54 × 20 = _____

Why does the shortcut work? It is based on the fact that you can multiply in any order.

When multiplying by 20, we can change the 20 into 10 × 2. For example:

$$20 \times 14 = 10 \times 2 \times 14$$

In that problem, first multiply 2 × 14 = 28. Then the problem becomes 10 × 28, which we know is 280.

$$20 \times 14 = 10 \times 2 \times 14$$

$$= 10 \times 28$$

$$= 280$$

That's it!

Let's try the same with 200. For example,

$$200 \times 31 = 100 \times 2 \times 31$$

In that problem, first multiply 2 × 31 = 62. The problem now becomes 100 × 62, which is 6,200:

$$100 \times 2 \times 31$$

$$= 100 \times 62$$

$$= 6{,}200$$

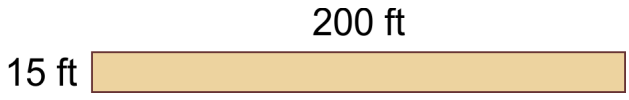
3. Try it yourself! Fill in.

a. 20 × 7	**b.** 20 × 5	**c.** 200 × 8	**d.** 200 × 25
= _____ × 2 × 7	= _____ × 2 × 5	= _____ × 2 × 8	= _____ × 2 × 25
= 10 × _____	= 10 × _____	= 100 × _____	= 100 × _____
= _____	= _____	= _____	= _____

4. Mark's shed measures 20 ft by 15 ft. What is its area? Write a number sentence. A means area.

A = _____

5. Write a number sentence, and find the area of Mark's driveway.



A = _____

6. Mark was told he needed four truckloads of gravel to cover his driveway. One truckload costs 5 × $20 plus $30 for the delivery. How much will it cost him to cover the driveway with gravel?

SHORTCUT for multiplying by whole tens and whole hundreds			
The same principle works if you multiply by whole tens (30, 40, 50, 60, 70, 80, or 90): simply multiply by 3, 4, 5, 6, 7, 8, or 9, and then tag a zero to the end result. Similarly, if you multiply by some whole hundred, FIRST multiply without those two zeros, and then tag the two zeros to the end result.			
50 × 8 = 400	90 × 11 = 990	300 × 8 = 2,400	12 × 800 = 9,600

7. Multiply.

a. 40 × 3 = _____ 8 × 20 = _____	**b.** 70 × 6 = _____ 50 × 11 = _____	**c.** 80 × 9 = _____ 30 × 15 = _____
d. 60 × 11 = _____ 12 × 40 = _____	**e.** 200 × 9 = _____ 7 × 400 = _____	**f.** 700 × 6 = _____ 600 × 11 = _____
g. 200 × 12 = _____ 15 × 300 = _____	**h.** 3 × 1100 = _____ 8 × 900 = _____	**i.** 11 × 120 = _____ 8 × 300 = _____

It even works this way:		
To multiply 40 × 70, simply multiply 4 × 7, and tag two zeros to the result: 40 × 70 = 2,800	To multiply 600 × 40, simply multiply 6 × 4, and tag three zeros to the result: 600 × 40 = 24,000	To multiply 700 × 800, simply multiply 7 × 8, and tag four zeros to the result. 700 × 800 = 560,000

8. Multiply.

a. 20 × 90 = _____ 70 × 300 = _____	**b.** 60 × 80 = _____ 30 × 900 = _____	**c.** 400 × 50 = _____ 200 × 200 = _____
d. 80 × 800 = _____ 200 × 500 = _____	**e.** 100 × 100 = _____ 40 × 30 = _____	**f.** 800 × 300 = _____ 90 × 1100 = _____

Write a number sentence for each question.

9. One hour has _____ minutes.
How many minutes are in 12 hours? _____

How many minutes are in 24 hours? _____

10. One hour has _____ minutes, and one minute has _____ seconds.

How many seconds are there in one hour? _____

11. Ed earns $30 per hour.

a. How much will he earn in a 8-hour workday? _____

b. How much will he earn in a 40-hour workweek? _____

c. How many days will he need to work in order to earn more than $1,000?

_____

12. Find the missing factor. Think "backwards"! How many zeros do you need?

a. _____ × 3 = 360 _____ × 50 = 450	**b.** 40 × _____ = 320 5 × _____= 600	**c.** _____ × 40 = 400 _____ × 2 = 180
d. _____ × 30 = 4,800 _____ × 200 = 1,800	**e.** 40 × _____ = 2,000 6 × _____= 4,200	**f.** _____ × 800 = 56,000 _____ × 20 = 12,000

Puzzle Corner

John wanted to prove that 40 × 70 is indeed 2,800 by breaking the multiplication into smaller parts. He wrote 40 as 4 × 10 and 70 as 7 × 10, and then multiplied in a different order:

$$40 \times 70 = 4 \times 10 \times 7 \times 10$$

$$= 10 \times 10 \times (4 \times 7) = 100 \times 28 = 2{,}800.$$

You do the same, and prove that 600 × 50 is indeed 30,000.

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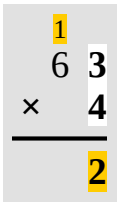
Multiplying in Columns, the Standard Way

The standard algorithm of multiplication is based on the principle that you already know: **multiplying in parts** (partial products): simply multiply ones and tens separately, and add.

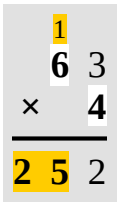
However, in the standard way the *adding* is done at the same time as multiplying. The calculation looks more compact and takes less space than the "easy way to multiply" you have learned.

The standard way to multiply



Multiply the ones: 4 × 3 = 12

Place 2 in the ones place, but write the tens digit (1) above the tens column as a little memory note. You are *regrouping* (or carrying).



Then multiply the tens, ***adding*** the 1 ten that was regrouped.

4 × 6 + 1 = 25

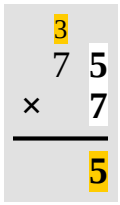
Write 25 in front of the 2. **Note** that 25 tens means 250!

"The easy way"

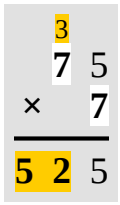
$$\begin{array}{r} 63 \\ \times\ \ 4 \\ \hline 12 \\ +\ 240 \\ \hline 252 \end{array}$$

In the "easy way," we multiply in parts, and the adding is done separately.

The standard way to multiply



Multiply the ones: 7 × 5 = 35
Regroup the 3 tens.



Multiply & add the tens:
7 × 7 + 3 = 52

"The easy way"

$$\begin{array}{r} 75 \\ \times\ \ 7 \\ \hline 35 \\ +\ 490 \\ \hline 525 \end{array}$$

1. Multiply using both methods: the standard one and the easy one.

a.

$$\begin{array}{r} 53 \\ \times\ \ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 53 \\ \times\ \ 8 \\ \hline \end{array}$$

b.

$$\begin{array}{r} 88 \\ \times\ \ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 88 \\ \times\ \ 3 \\ \hline \end{array}$$

2. Multiply using both methods: the standard one and the easy one.

a.

	7	9
x		3

	7	9
x		3

b.

	1	8
x		5

	1	8
x		5

3. Multiply. Be careful with the regrouping.

a.

	5	1
x		6

b.

	1	9
x		3

c.

	6	2
x		2

d.

	4	6
x		7

e.

	6	6
x		6

f.

	3	9
x		9

g.

	8	7
x		3

h.

	6	7
x		2

i.

	2	0
x		9

j.

	5	4
x		8

k.

	3	4
x		6

l.

	4	6
x		2

4. Solve. Also, write number sentences (additions, subtractions, multiplications) on the empty lines.

a. What is the cost of buying three chairs for $48 each?

_____

And the cost for six chairs? _____

x		

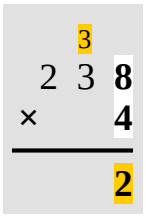
b. You earn $77 a day. How many days do you need to work in order to have $600 or more? Guess and check.

_____

_____

x		

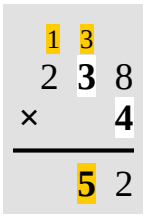
With a 3- or 4-digit number you have to regroup many times.



Multiply the ones first.

$4 \times 8 = 32$

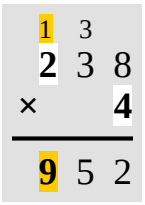
Write 2 in the ones place and regroup the 3 tens to the tens column.



Then multiply the tens, adding the 3 regrouped tens.

$4 \times 3 + 3 = 15$

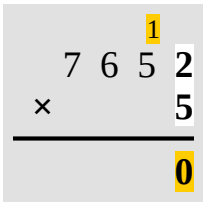
Write 5 in the tens place and regroup the 1 hundred.



Then multiply the hundreds, adding the regrouped hundred.

$4 \times 2 + 1 = 9$

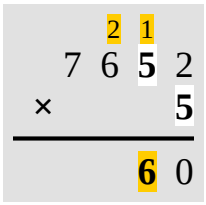
Write 9 in the hundreds place.



Multiply the ones:

$5 \times 2 = 10$

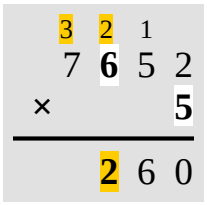
Write 0 in the ones place and regroup the 1 ten.



Then the tens. Add the regrouped ten:

$5 \times 5 + 1 = 26$

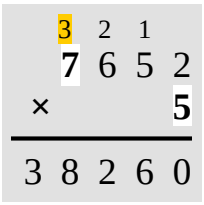
Write 6 in the tens place and regroup the 2 hundreds.



Multiply the hundreds.

$5 \times 6 + 2 = 32$

Write 2 in the hundreds place, and regroup the 3 thousands.



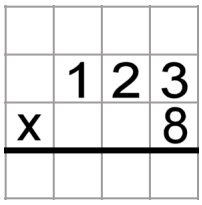
Multiply the thousands:

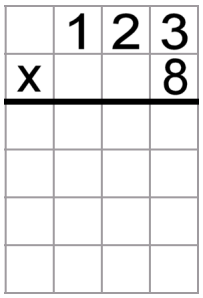
$5 \times 7 + 3 = 38$

Write 38 in front of the 260.

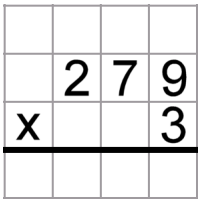
5. Multiply using both methods: the standard one and the easy one.

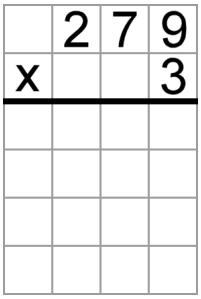
a.



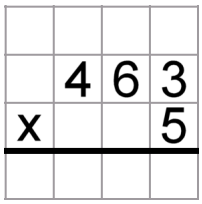


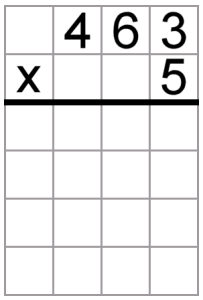
b.



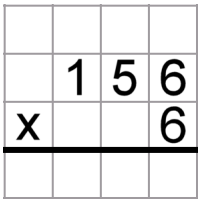


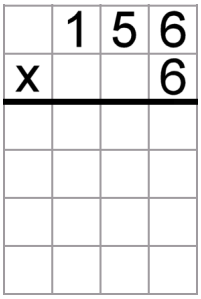
c.





d.





6. Multiply using the standard method.

a. 462 × 2	**b.** 506 × 7	**c.** 278 × 5	**d.** 319 × 7
e. 288 × 3	**f.** 809 × 9	**g.** 287 × 3	**h.** 367 × 2
i. 1208 × 9	**j.** 2514 × 3	**k.** 6177 × 4	**l.** 5330 × 9

7. Solve the word problems. Also, write number sentences (additions, subtractions, multiplications) on the empty lines to show what you calculate.

a. The school has 304 students. To go to the museum, they hired buses which can each seat 43 passengers. How many buses did they need?

Hint: Guess and check.

_____

_____

_____

b. The school also has 24 teachers. How many seats were left empty in those buses when all the students and all the teachers joined the trip?

_____

_____

_____

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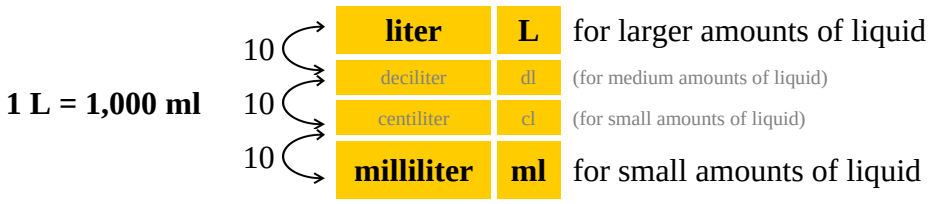
Chapter 4: Time and Measuring
Introduction

The fourth chapter of *Math Mammoth Grade 4* includes lessons on time, temperature, length, weight, and volume. The focus is no longer the actual act of measuring, but on conversions between the units and on word problems that involve conversions.

Students may have difficulty with the conversions, and that is why they will also be studied in 5th grade. At this point, students should be able to easily convert from a bigger unit to a smaller unit (such as converting 3 feet into 36 inches, or 2 kg into 2,000 grams).

And while the Common Core standards do not include them for 4th grade, I have also included some problems where we convert from a smaller unit to a bigger unit (such as 4,500 ml into 4 L 500 ml or 12 feet into 4 yards), because I feel most children are capable of doing these in 4th grade. If you feel your child has difficulty with these types of conversions (from a smaller unit to a bigger unit), feel free to omit those particular exercises. They are intermixed though, and not marked in any special way.

There are separate lessons for customary units and for metric units. These lessons include a table that lists the units and the conversion factors. For metric units, those tables always include all the units, even when they are not in common usage. For example, for metric units of volume, the chart looks like this:



The lesson only deals with milliliters and liters. However, the chart *also* shows the two other units (deciliters and centiliters) in order to help familiarize the students with these two basic ideas of the metric system:

1. The units always differ by a factor of ten;
2. The units are *named* consistently with the same prefixes (milli-, centi-, deci-, deka-, hecto-, and kilo-). These prefixes and their meanings are not yet studied in detail in fourth grade. You may, of course, at your discretion, explain them to the student.

The Lessons in Chapter 4

Helpful Resources on the Internet

The Ruler Game
Choose between whole inches, half-inches, quarters, eighths, or sixteenth parts of an inch to measure. Click on the given measurement on a ruler. Timed or not timed versions available.
http://www.rickyspears.com/rulergame

Measure It!
Practice measuring lines with either centimeters or inches. Multiple choice questions.
http://www.funbrain.com/measure

Sal's Sub Shop
Customers order subs, and you need to cut them to the given measurements - sometimes in metric units, sometimes in inches.
http://www.mrnussbaum.com/sal.htm

Reading a Tape Measure Worksheets
Worksheet generator - you can choose to which accuracy to measure, in inches, or inches and feet.
http://themathworksheetsite.com/read_tape.html

Measurement Game for Kids
Measure the length and weight of various parcels using the interactive scales and ruler so you can give them a stamp with the correct postage rate. Uses grams and centimeters.
http://www.kidsmathgamesonline.com/geometry/measurement.html

Reading Scales
You can illustrate a variety of measuring devices, such as scales, measuring cup, thermometer, and speedometer, and how to read them. Generate examples using different scales on different devices at the press of a button.
http://www.teacherled.com/2008/01/28/reading-scales

Reading Scales
Weigh objects on this virtual balance scale, using weights of 10 g, 50 g, 250 g, and 500 g.
http://www.teacherled.com/resources/oldscales/oldscalesload.html

Measures
An online activity about metric measuring units and how to read scales, a measuring cup, and a ruler. Uses British spelling.
http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/measures

Hours vs Minutes Game BBC SkillsWise
An online quiz to practice minutes versus hours. You have to tell whether, for example, 76 minutes or 1 hour is more.
http://www.bbc.co.uk/skillswise/game/ma25time-game-hours-vs-minutes

24 hour snap game
Two times are given, one using the 24-hour clock, and another using the am/pm system. Snap or don't snap the two times together.
http://www.bbc.co.uk/skillswise/game/ma25time-game-24-hour-snap

A Dictionary of Units of Measurement
Explains the common measuring systems and has lots of background information on their history.
http://www.unc.edu/~rowlett/units/

Bitesize Measures
Facts, problems, and quizzes about measuring length, mass, and capacity (in metric units).
http://www.bbc.co.uk/bitesize/ks2/maths/shape_space/measures/read/1/

Measurements
Online lessons with interactive exercises on metric prefixes, symbols, number values, metric mass, length, volume, US length and volume, and temperature conversions.
http://www.aaamath.com/B/mea.htm

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Metric Units for Measuring Length

The **basic unit** for measuring length in the metric system is **the meter**. All the other units for measuring length have the word "meter" in them.

Each unit is 10 times the smaller unit. For example, 1 kilometer is 10 hectometers. But we do not commonly use hectometers, dekameters, or decimeters. You only need to learn the bolded units in the chart.

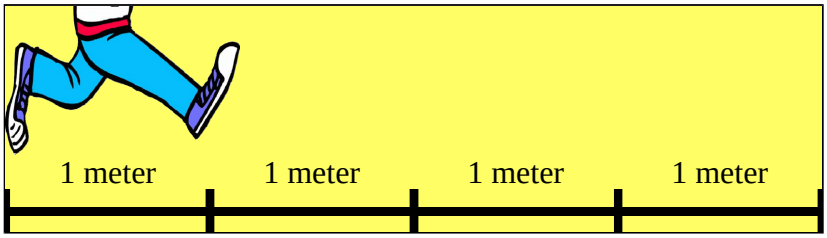
Units of length in the metric system

10 ↷	**kilometer**	**km**	"Kilo" means 1,000.
10 ↷	hectometer	hm	(not used)
10 ↷	dekameter	dam	(not used)
10 ↷	**meter**	**m**	the basic unit
10 ↷	decimeter	dm	(not used much)
10 ↷	**centimeter**	**cm**	100 of these make a meter.
10 ↷	**millimeter**	**mm**	This is 1/10 of a centimeter.

Remember also that 1 meter is very close to 1 yard. One meter is a tiny bit longer than 1 yard.

1. Outside, or in a long corridor or room, draw two lines that start at the same place.

a. Using a measuring tape, mark on the one line 1 m, 2 m, 3 m, and 4 m. Can you take "hops" 1 meter long?



b. Mark on the second line marks from 1 foot to 13 feet. Make 1-yard hops. Compare: do the two kinds of hops feel about the same?

2. Measure how tall you and other people are in centimeters. Write it also using whole meters and centimeters.

Name	How tall
	_____ *cm* = *1* *m* _____ *cm.*

Conversions between units

Remember what millimeters look like on your ruler. 10 millimeters make 1 cm.

And 100 centimeters is 1 meter. "Centi" actually means a hundred (from the Latin word *centum*).

Lastly, 1 kilometer means one thousand meters, because "kilo" means 1,000!

1 km = 1,000 m	**1 m = 100 cm**	**1 cm = 10 mm**

3. One meter is 100 cm. Convert between meters and centimeters.

a. 5 m = _____ cm	**b.** 4 m 6 cm = _____ cm	**c.** 800 cm = _____ m
8 m = _____ cm	9 m 19 cm = _____ cm	239 cm = ____ m _____ cm
12 m = _____ cm	10 m 80 cm = _____ cm	407 cm = ____ m _____ cm

4. One centimeter is 10 mm. Convert between centimeters and millimeters.

a. 5 cm = _____ mm	**b.** 2 cm 8 mm = _____ mm	**c.** 50 mm = ____ cm _____ mm
8 cm = _____ mm	7 cm 5 mm = _____ mm	72 mm = ____ cm _____ mm
14 cm = _____ mm	10 cm 4 mm = _____ mm	145 mm = ____ cm _____ mm

5. One kilometer is 1,000 m. Convert between kilometers and meters.

a. 5 km = _____ m	**b.** 2 km 800 m = _____ m	**c.** 2,000 m = _____ km
23 km = _____ m	6 km 50 m = _____ m	4,300 m = ____ km _____ m
1 km 200 m = _____ m	13 km 579 m = _____ m	18,700 m = ____ km _____ m

6. Calculate. Give your answer using whole kilometers and meters.

a. 5 km 200 m + 8 km 900 m

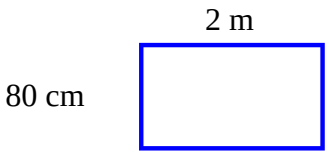
b. 3 km 600 m + 2 km 800 m

c. 1,500 m + 2 km 600 m

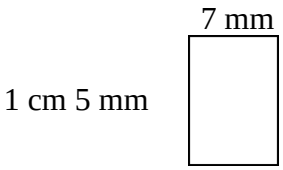
d. 6 × 700 m

7. Solve.

a. Find the perimeter of this rectangle.



b. Find the perimeter of this rectangle.



c. One side of a square measures 5 cm 6 mm. What is its perimeter?

d. *A challenge.* A square has a perimeter of 6 cm. How long is its side?

8. Solve the problems.

a. How many millimeters are in a *meter*?

b. John jogs around a track 1 km 800 m long twice a day, five days a week.
How long a distance does he jog in a day?

In a week?

c. Gary is 1 m 34 cm tall and Jared is 142 cm tall.
How much taller is Jared?

d. Kathy's wallpaper has butterflies that are 8 cm wide. She will put the wallpaper in her room. How many complete butterflies can she have on a wall that is 1 meter long?

How about if the wall is 3 meters long?

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math MAMMOTH

Grade 4-B Worktext

Division

Geometry

Fractions

Decimals

Light Blue Series

By Maria Miller

Edition 3.0

Contents

Chapter 7: Fractions

Chapter 8: Decimals

Foreword

Math Mammoth Grade 4-A and *Grade 4-B* worktexts comprise a complete math curriculum for the fourth grade mathematics studies, aligned to the Common Core Standards.

In the fourth grade, students focus on multi-digit multiplication and division, learning to use bigger numbers, solving multi-step word problems that involve several operations, and they get started in studying fractions and decimals. This is of course accompanied by studies in geometry and measuring.

The year starts out with a review of addition and subtraction, patterns and graphs. We illustrate word problems with bar diagrams and study finding missing addends, which teaches algebraic thinking. Children also learn addition and subtraction terminology, the order of operations, and statistical graphs.

Next come large numbers—up to millions, and the place value concept. At first the student reviews thousands and some mental math with them. Next are presented numbers up to one million, calculations with them, the concept of place value and comparing. In the end of the chapter we find out more about millions and an introduction to multiples of 10, 100, and 1000.

The third chapter is all about multiplication. After briefly reviewing the concept and the times tables, the focus is on learning multi-digit multiplication (multiplication algorithm). The children also learn why it works when they multiply in parts. We also study the order of operations again, touch on proportional reasoning, and do more money and change related word problems.

The last chapter in part A is about time, temperature, length, weight, and volume. Students will learn to solve more complex problems using various measuring units and to convert between measuring units.

In part B, we first study division. The focus is on learning long division and using division in word problems. In geometry, we first review area and perimeter, and then concentrate on the topic of angles. Students measure and draw angles, solve simple angle problems, and classify triangles according to their angles. They also study parallel and perpendicular lines.

Fractions and decimals are presented last in the school year. These two chapters practice only some of the basic operations with fractions and decimals. The focus is still on conceptual understanding and on building a good foundation towards 5th grade math, where fractions and decimals will be in focus.

When you use these books as your only or main mathematics curriculum, they can be like a “framework”, but you do have some liberty in organizing the study schedule. Chapters 1, 2, and 3 should be studied in this order, but you can be flexible with chapters 4 (Time and Measuring) and 6 (Geometry) and schedule them somewhat earlier or later if you so wish. Chapter 3 (Multiplication) needs to be studied before long division in Chapter 5. Many topics from chapters 7 and 8 (Fractions and Decimals) can also be studied earlier in the school year; however finding parts with division should naturally be studied only after mastering division.

I wish you success in your math teaching!

Maria Miller, the author

Chapter 5: Division
Introduction

The fifth chapter of *Math Mammoth Grade 4* includes lessons on division, long division, remainder, part problems, average, and problem solving. It is a long chapter, as division and long division are “in focus” in fourth grade.

We start out reviewing basic division by single-digit numbers. Then students study division terms and dividing by whole tens and hundreds.

The lesson *Finding Fractional Parts with Division* shows an important relationship between fractions and division. For example, we can find 3/4 of a number by first finding 1/4 (divide by 4), then multiplying that result by 3.

Next we briefly study order of operations again, this time including divisions in the problems.

In the lesson *The Remainder, Part 1*, we study the concept of remainder, first using pictures and small numbers. In the second lesson on remainder, we still use small numbers, but students work the problems using the long division symbol or "corner", as I like to call it. That is of course preparing them for long division.

Next, long division is taught in several small steps over many lessons. We start with the situation where each of the thousands, hundreds, tens, and ones can be divided evenly by the divisor. Then is introduced the remainder in the ones. Next comes the situation where we have a remainder in the tens. Finally, when we have a remainder in the hundreds, and so on. We also have lots of word problems to solve.

After long division is mastered, we study the concept of average and problem solving involving a fractional part of a whole. I have included many bar diagrams and pictorial representations of these problems to help the students.

The last section deals with elementary number theory topics. We study some basic divisibility rules (though not all of them), prime numbers, and find all factors of a given two-digit number.

The Lessons in Chapter 5

Helpful Resources on the Internet

Long division

MathFrog Dividerama!
Interactive long division practice. Guided help available optionally.
http://cemc2.math.uwaterloo.ca/mathfrog/english/kidz/div5.shtml

Snork's Long Division Game
Interactive and guided long division practice that only accepts correct answers and truly guides the student step-by-step through long division problems.
http://www.kidsnumbers.com/long-division.php

Mr. Martini's Classroom: Long Division
An interactive long division tool.
http://www.thegreatmartinicompany.com/longarithmetic/longdivision.html

Double-Division.org
Another form of long division algorithm - takes the guesswork away from estimating how many times the divisor goes into what needs to be divided. Also called 1-2-4-8 division.
http://www.doubledivision.org/

Short Division
A page that explains short division in detail. Short division is the same algorithm as long division, but some steps are only done in one's head, not written down.
http://www.themathpage.com/ARITH/divide-whole-numbers.htm

Factors and primes

Arrays and factors
Drag rectangles to show the factorizations of a given number.
http://www.shodor.org/interactivate/activities/factors2/index.html

Factor Game
Interactive game to practice divisibility among numbers 1-100. Play against the computer or a friend.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=12

Factor Feeder
Eat factors of the given number, and avoid numbers that are not factors of the given number in this Pacman-style game. Use Arrow Keys to move.
http://hoodamath.com/games/factorfeeder.php

Sliding Tile Factorization Game
Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.
http://www.visualmathlearning.com/Games/sliding_factors.html

Octopus Factors
Move counters up the legs of an octopus but only when the number on the circle is a multiple of the number on the card.
http://www.counton.org/games/map-numbers/octopus/

Factors Millionaire Game
A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.
http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Not a Factor
Choose a number that is NOT a factor of the given number.
http://www.helpingwithmath.com/resources/games/target_factors01/not_factor.html

Factors and Remainders
An interactive animation demonstrating factors and remainders. Choose a number and its possible divisor. The animation shows boxes (as given by the number) arranged into rows of (possible divisor), and you can SEE if there is any remainder.
http://www.absorblearning.com/media/item.action?quick=ml

Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.pompuzzle.com/Snake

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Primes, Factors and Divisibility—Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes

The following games can be used to practice basic division facts, if the student hasn't mastered them.

A+ math games
Practice all four basic operations with math bingo (matho), hidden picture games, or concentration games.
http://www.aplusmath.com/games/

Math Magician games
Flashcard problems in all 4 operations. Answer 20 questions in 1 minute.
http://www.oswego.org/ocsd-web/games/Mathmagician/cathymath.html

Division Practice at AAAMath
Learn or practice basic division facts, and more.
http://www.aaastudy.com/div39hx3.htm

Cross the Swamp
Help Little Ron move from log to log across the swamp and practice multiplication/division or addition/subtraction.
http://www.bbc.co.uk/schools/starship/maths/crosstheswamp.shtml

Math Car Racing
Keep ahead of the computer car by thinking logically, and practice any of the four operations.
http://www.funbrain.com/osa/index.html

Arithmetic Game
Find numbers to fit an equation that may use all four operations.
http://www.primarygames.com/math/arithmeticgame/index.htm

Primary Games
A collection of games. The following links open the evaluation versions of some division-related games. The game collections themselves are sold at
http://www.primarygames.co.uk/

- **Eggs on Legs**
  http://www.primarygames.co.uk/PG5/Eggs/Div/eggsdiv.html
- **DiviPods**
  http://www.primarygames.co.uk/pg4/Divipods/divipods.html
- **Division Divers**
  http://www.primarygames.co.uk/pg3/ddivers/ddivers.html
- **Sum Sense - Division**
  http://www.primarygames.co.uk/pg2/sumsense/sumdiv.html

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Long Division 1

Divide hundreds, tens, and ones separately.

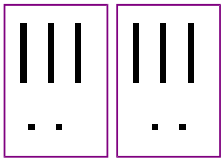
Write the dividend inside the long division "corner", and the quotient on top.

64 ÷ 2 = ?

Divide tens and ones separately:

6 tens ÷ 2 = 3 tens (t)

4 ones ÷ 2 = 2 ones (o)



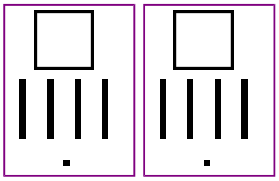
	t	o
2)	3	2
	6	4

282 ÷ 2 = ?

2 hundreds ÷ 2 = 1 hundred (h)

8 tens ÷ 2 = 4 tens (t)

2 ÷ 2 = 1. (o)



	h	t	o
2)	1	4	1
	2	8	2

1. Make groups. Divide. Write the dividend inside the "corner" if it is missing.

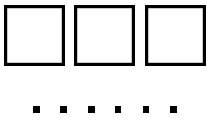
a. Make 2 groups

2) 6 2

b. Make 3 groups

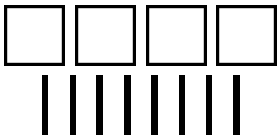
3)

c. Make 3 groups



3)

d. Make 4 groups



4)

2. Divide thousands, hundreds, tens, and ones separately.

a. 4) 8 4

b. 3) 3 9 3

c. 3) 6 6 0

d. 4) 8 0 4 0

e. 3) 6 6

f. 6) 6 0 3 6

g. 3) 3 3 0

h. 4) 4 8 0 4

h t o 0 4) 2 4 8 h t o 0 6 2 4) 2 4 8 4 does not go into 2. You can put zero in the quotient in the hundreds place or omit it. But 4 does go into 24, six times. Put 6 in the quotient.	th h t o 0 5) 3 5 0 5 th h t o 0 7 0 1 5) 3 5 0 5 5 does not go into 3. You can put zero in the quotient. But 5 does go into 35, seven times.
Explanation: The 2 of 248 is of course 200 in reality. If you divided 200 by 4, the result would be less than 100, so that is why the quotient won't have any whole hundreds. But then you combine the 2 hundreds with the 4 tens. That makes 24 tens, and you CAN divide 24 tens by 4. The result 6 tens goes as part of the quotient. Check the final answer: 4 × 62 = 248.	**Explanation:** 3,000 ÷ 5 will not give any whole thousands to the quotient because the answer is less than 1,000. But 3 thousands and 5 hundreds make 35 hundreds together. You can divide 3,500 ÷ 5 = 700, and place 7 as part of the quotient in the hundreds place. Check the final answer: 5 × 701 = 3,505.
If the divisor does not "go into" the first digit of the dividend, look at the first two digits of the dividend.	

3. Divide. Check your answer by multiplying the quotient and the divisor.

a. $3\overline{)123}$

b. $4\overline{)284}$

c. $6\overline{)360}$

d. $8\overline{)248}$

e. $2\overline{)184}$

f. $7\overline{)427}$

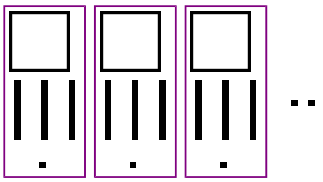
g. $3\overline{)1833}$ (quotient: 06)

h. $4\overline{)2404}$

i. $7\overline{)4970}$

j. $5\overline{)4505}$

Ones division is not even. There is a remainder.



395 ÷ 3 = 131 R2

h t o
1 3
3) 3 9 5

3 goes into 3 one time.
3 goes into 9 three times.

h t o
1 3 1 R2
3) 3 9 5

3 goes into 5 one time, but not evenly.
Write the remainder 2 after the quotient.

h t o
0 4 1 R1
4) 1 6 5

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

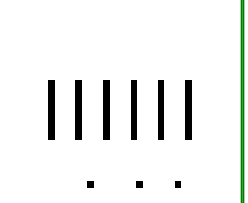
4 goes into 5 once, leaving a remainder of 1.

th h t o
0 4 0 0 R7
8) 3 2 0 7

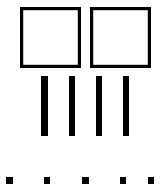
8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

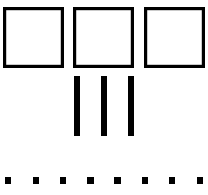
4. Divide into groups. Find the remainder.



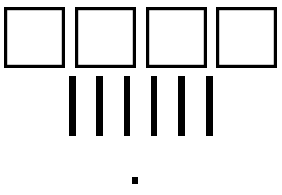
a. 2) 6 3



b. 2)



c. 3)



d. 2)

5. Divide. Indicate the remainder if any.

a. 4) 8 4 7

b. 2) 6 9

c. 3) 3 6 7

d. 4) 8 9

e. 2) 1 2 1

f. 6) 1 8 0 5

g. 7) 2 1 5

h. 8) 2 4 8 2

In the problems before, you just wrote down the remainder of the ones. Usually, we write down the subtraction that actually finds the remainder. Look carefully:

h t o 0 6 1 4) 2 4 7 − 4 3	th h t o 0 4 0 2 4) 1 6 0 9 − 8 1
When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3. Check: 4 × 61 + 3 = 247	When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1. Check: 4 × 402 + 1 = 1,609

6. Practice some more. Subtract to find the remainder in the ones. Check your answer by multiplying the divisor times the quotient, and then adding the remainder. You should get the dividend.

a. $3\overline{)128}$

b. $3\overline{)95}$

c. $6\overline{)4267}$

d. $4\overline{)2845}$

e. $5\overline{)5507}$

f. $2\overline{)8063}$

7. Divide these numbers mentally. Remember, you can always check by multiplying!

a. 440 ÷ 4 = 820 ÷ 2 =	**b.** 3600 ÷ 400 = 369 ÷ 3 =	**c.** 824 ÷ 2 = 560 ÷ 90 =

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Divisibility

A number ***a*** is ***divisible*** by another number ***b*** if the division ***a*** ÷ ***b*** is exact (no remainder).

For example, 18 ÷ 3 = 6. So, 18 is divisible by 3. Also, 18 is divisible by 6, because we can write the other division 18 ÷ 6 = 3. So, 18 is divisible by both 6 and 3.

We say 6 and 3 are ***divisors*** or ***factors*** of 18.

You can use long division to check if a number is divisible by another.

67 ÷ 4 = 16, R3. There is a remainder, so 67 is *not* divisible by 4.

Also, from this we learn that neither 4 nor 16 is a factor (divisor) of 67.

```
  16
4 67
 -4
  27
 -24
   3
```

1. Divide and determine if the numbers are divisible by the given number.

a. 21 ÷ 3 = _____ Is 21 divisible by 3?	**b.** 40 ÷ 6 = _____ Is 40 divisible by 6?	**c.** 17 ÷ 5 = _____ Is 5 a divisor of 17?	**d.** 84 ÷ 7 = _____ Is 7 a factor of 84?

2. Answer the questions. You may need long division.

a. Is 98 divisible by 4?	**b.** Is 603 divisible by 7?	**c.** Is 3 a factor of 1,256?

In any multiplication, the numbers that are multiplied are called ***factors*** and the result is called a ***product***.

factor		factor		product
7	×	6	=	42

So, since 6 × 7 = 42, 6 and 7 are ***factors*** of 42.

From this multiplication fact we can write two divisions: 42 ÷ 6 = 7 and 42 ÷ 7 = 6. So, this also means that 42 is divisible by both 6 and 7.

Yet one more new word that ties in with all of this: ***multiple***.

We say ***42 is a multiple of 6***, because 42 is some number times 6, namely 7 × 6.

And of course 42 is also a multiple of 7, because it is some number times 7!

3. Fill in.

Here's a multiplication fact: 8 × 9 = 72. So, 8 is a _____ of 72, and so is 9.
Also, 72 is a _____ of 8, and also 72 is a _____ of 9.
And, 72 is _____ by 8 and also by 9.

4. Fill in.

a. Is 5 a factor of 55? Yes, because ____ × ____ = _____.	**b.** Is 8 a divisor of 45? No, because ____ ÷ ____ = _____.
c. Is 36 a multiple of 6? _____, because ____ × ____ = _____.	**d.** Is 34 a multiple of 7? _____, because ____ ÷ ____ = _____.
e. Is 7 a factor of 46? _____, because _____.	**f.** Is 63 a multiple of 9? _____, because _____.

Multiples of 6 are all those numbers we get when we multiply 6 by other numbers. For example, we can multiply 0 × 6, 7 × 6, 11 × 6, 109 × 6, and so on, and the resulting numbers are all multiples of six.

In fact, the skip-counting pattern of 6 gives us a list of multiples of 6:

0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, and so on.

5. **a.** Make a list of multiples of 11, starting at 0 and at least till 154.

b. Make a list of multiples of 111, starting at 0. Make it as long as you can in this space!

Divisibility by 2

Numbers that are divisible by 2 are called ***even*** numbers.
Numbers that are NOT divisible by 2 are called ***odd*** numbers.

Even numbers end in 0, 2, 4, 6, or 8. Every second number is even.

Divisibility by 5

Numbers that end in 0 and 5 are divisible by 5.

For example, 10, 35, 720, and 3,675 are such numbers.

6. Mark with "x" if the numbers are divisible by 2 or 5.

number	divisible		number	divisible		number	divisible		number	divisible	
	by 2	by 5		by 2	by 5		by 2	by 5		by 2	by 5
750			755			760			765		
751			756			761			766		
752			757			762			767		
753			758			763			768		
754			759			764			769		

Divisibility by 10

Numbers that end in 0 are divisible by 10.

For example, 10, 60, 340, and 2,570 are such numbers.

7. Mark an "x" if the numbers are divisible by 2 or 5 or 10.

number	divisible			number	divisible			number	divisible		
	by 2	by 5	by 10		by 2	by 5	by 10		by 2	by 5	by 10
860				865				870			
861				866				871			
862				867				872			
863				868				873			
864				869				874			

If a number is divisible by 10, it ends in zero, so it is ALSO divisible by ____ and ____.

8. **a.** Write a list of numbers divisible by 2, from 0 to 60.

_____

This is also a list of _____ of 2.

b. In the list above, *underline* those numbers that are divisible by 4.
What do you notice?

c. In the list above, *color* those numbers that are divisible by 6.
What do you notice?

d. Which numbers are divisible by both 4 and 6?

9. **a.** Write a list of numbers divisible by 3, from 0 to 60.

_____

This is also a list of _____ of 3.

b. In the list above, *underline* those numbers that are divisible by 6.
What do you notice?

c. In the list above, *color* those numbers that are divisible by 9.
What do you notice?

10. Use the lists you made in (7) and (8). Find numbers that are divisible by *both* 2 and 9.

11. What number is a factor of every number?

12. Twenty is a multiple of 4. It is also a multiple of 5. It is also a multiple of four other numbers. Which ones?

Who am I?
(Hint: I am less than 50.)

Divided by 9, I leave a remainder of 6.
Divided by 4, I leave a remainder of 1.
Divided by 10, I leave a remainder of 3.

Who am I?
(Hint: I am less than 100.)

I am a multiple of 3, 4, 5, and 6.
I am a factor of 120.
Divided by 7, I leave a remainder of 4.

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Chapter 6: Geometry
Introduction

We start fourth grade geometry by reviewing the concepts of area and the perimeter of rectangles (from third grade). Students get to apply these concepts in problem solving, including problems where they write simple equations and explore possible perimeters for a given fixed area.

The focus of this chapter is angles. Students learn about lines, rays and angles, and about acute, right, obtuse, and straight angles. Next they learn how to measure and draw angles with a protractor. We also study angle problems where students write simple equations, and estimate some common angles.

The lesson *Parallel and Perpendicular Lines* ties in with the topic of angles, because perpendicular means to be at a right angle. Next we study parallelograms and other quadrilaterals in more detail, paying attention to the angles and side lengths in them.

We study triangles, and classify them according to the angles. Classifying triangles according to their sides (equilateral vs. isosceles triangles) is left for the 5th grade. The last topic for this chapter (an easy one) is line symmetry.

The study of geometry is full of strange-sounding words to learn. I encourage you to let the student(s) keep a *geometry notebook*, where they will write every new concept or term, and draw a picture or pictures and text to explain the term. The students could also do the drawing exercises from this chapter in this notebook. It will then become their very own geometry book, and while working with it, it helps them to learn and remember the terms and concepts better.

The Lessons in Chapter 6

Helpful Resources on the Internet

Area and perimeter

Shape explorer
Find the perimeter and area of odd shapes on a rectangular grid.
http://www.shodor.org/interactivate/activities/perimeter/index.html

Math Playground: Measuring the Area and Perimeter of Rectangles
Amy and her brother, Ben, explain how to find the area and perimeter of rectangles and show you how changing the perimeter of a rectangle affects its area. After the lesson, you will use an interactive ruler to measure the length and width of 10 rectangles, and to calculate the perimeter and area of each.
http://www.mathplayground.com/area_perimeter.html

Math Playground: Party Designer
You need to design areas for the party, such as crafts table, food table, seesaw, and so on, so that they have the given perimeters and areas.
http://www.mathplayground.com/PartyDesigner/PartyDesigner.html

Geometry Area/Perimeter Quiz from ThatQuiz.org
An online quiz, about the area and perimeter of rectangles, triangles, and trapezoids. You can modify the quiz parameters to your liking, for example to omit a certain shape, or instead of solving for perimeter/area, you solve for an unknown side when perimeter/area is given.
http://www.thatquiz.org/tq-4/?-j1200b-lc-p0

Perimeter Game from Cyram.org
A simple online quiz for finding the perimeter of rectangles, triangles, or compound rectangles where not all side lengths are given.
http://www.cyram.org/Projects/perimetergame/index.html

FunBrain: Shape Surveyor Geometry Game
A simple & easy game that practices finding either the perimeter or area of rectangles.
http://www.funbrain.com/poly/index.html

Angles

Turtle Pond
Guide a turtle to a pond using commands, which include turning him in certain angles, or moving him a specific distance.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=83

Banana hunt at Primary Games
Help the monkey to find bananas and learn to estimate angles.
http://www.primarygames.co.uk/pg2/bhunt/bhunt.html

Ladybug Leaf
Guide the ladybug by giving her commands to turn 90° or 45°, right or left, or to move forward/backward.
http://nlvm.usu.edu/en/nav/frames_asid_287_g_2_t_3.html

LadyBug Mazes
Similar to the Ladybug Leaf, but this time you guide the ladybug through the maze.
http://nlvm.usu.edu/en/nav/frames_asid_141_g_2_t_3.html

Shapes/Polygons

Interactive Quadrilaterals
See all the different kinds of quadrilaterals "in action". You can drag the corners, see how the angles change, and observe what properties do not change.
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Dynamic Rectangle and Parallelogram
Drag the sides of a dynamic parallelogram or a rectangle to explore these concepts.
http://standards.nctm.org/document/eexamples/chap5/5.3/index.htm

Polygon Matching Game
Learn all the common polygons by playing this fun, timed matching game.
http://www.mathplayground.com/matching_shapes.html

Polygon Vocabulary
A matching game.
http://www.quia.com/cc/2758.html

Shapes Identification Quiz from ThatQuiz.org
An online quiz in a multiple-choice format, asking to identify common two-dimensional shapes. You can modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-f/math/shapes/

General

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!
http://www.shodor.org/interactivate/activities/Tessellate

Patch Tool
An online activity where the student designs a pattern using geometric shapes.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=27

Polygon Playground
Drag various colorful polygons to the work area to make your own creations!
http://www.mathcats.com/explore/polygons.html

Interactive Tangram Puzzle
Place the tangram pieces so they form the given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Tangram set
Cut out your Tangram set by folding paper
http://tangrams.ca/fold-set

Logic Tangram game
Note: this uses four pieces only. Use logic and spatial reasoning skills to assemble the four pieces into the given shape.
http://www.mathplayground.com/tangrams.html

Geometry worksheets & quizzes
Worksheets about complementary and supplementary angles, parallel, perpendicular, and intersecting lines, types of angles, basic shapes, area & perimeter of rectangles, and parts of a circle.
http://www.dadsworksheets.com/v1/Worksheets/Basic Geometry.html

Geometry worksheets & quizzes
A bunch of PDF worksheets on geometry topics for elementary level, plus online quizzes.
http://www.math4children.com/Topics/Geometry

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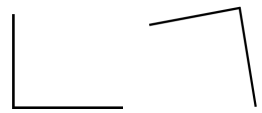
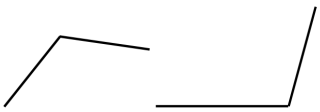
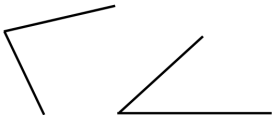
Triangles



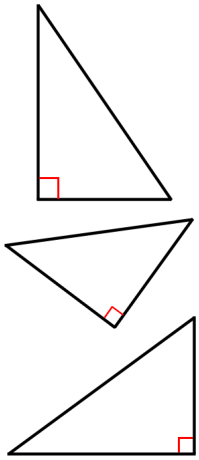
right angles
(exactly 90°)



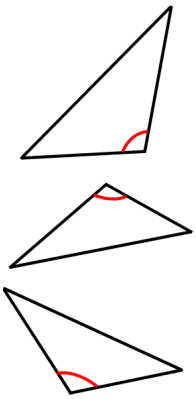
obtuse angles
(more than 90°, less than 180°)



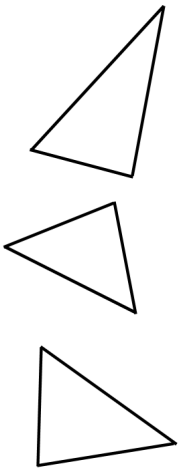
acute angles
(less than 90°)



Right triangles have exactly one right angle.



Obtuse triangles have exactly one obtuse angle.



Acute triangles have three acute angles. In other words, ALL the angles are acute.

1. **a.** Draw a right *angle*. Then make it into a right *triangle* by drawing in the third side.

b. Draw another, different right triangle.

c. A right triangle has one right angle. Are the other two angles in a right triangle acute, right, or obtuse?

A right triangle has one right angle. The other two angles are _____.

2. **a.** Draw an obtuse angle. Then make it into an obtuse triangle by drawing in the third side.

b. Draw another, different obtuse triangle.

c. An obtuse triangle has one obtuse angle. Are the other two angles in a obtuse triangle acute, right, or obtuse?

An obtuse triangle has one obtuse angle. The other two angles are _____.

3. **a.** Draw an acute triangle. The side lengths can be any.

b. Measure its angles.

They measure _____°,

_____°, and _____°.

4. Observe all you have done thus far in this lesson, and fill in.

Right triangles have exactly 1 _____ _____,

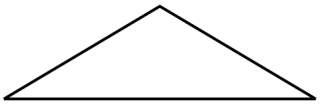
and the other two angles are _____ .

Obtuse triangles have exactly 1 _____ _____,

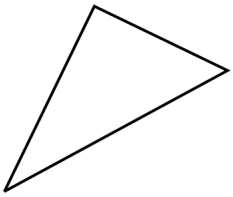
and the other two angles are _____ .

Acute triangles have ___ _____ angles.

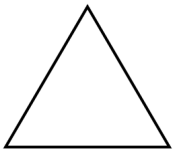
5. Label the triangles in the pictures as right, acute, or obtuse.



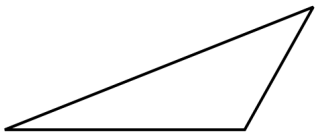
a.



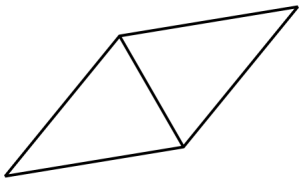
b.



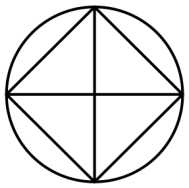
c.



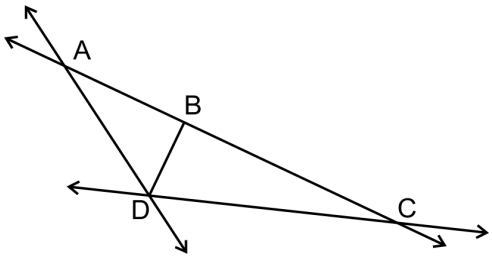
d.



e.



f.

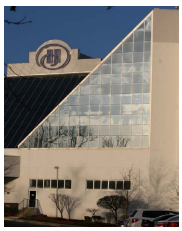


g.

6. Label the triangles in the pictures as right, acute, or obtuse.

a. National Museum of Marine Corps
Photo by Ron Cogswell

_____

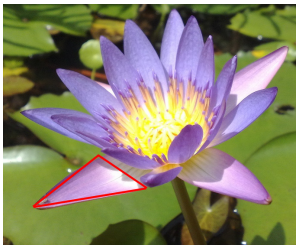


b. The City Center of Woburn, MA
Photo by Randy Robertson

_____

c. The White House
Photo by Glyn Lower

_____



d. Waterlily
Photo by Francis Chung

_____

e. Warning of a steep hill ahead
Photo by www.flickr.com/photos/shirokazan/

The black triangle is _____

The red triangle is _____

When you go outside, find acute, obtuse, and right triangles in nature, in buildings, in signs, and so on!

7. **a.** Draw a triangle with 85° and 40° angles.

   Hint: First draw a 85° angle. Then, mark a point anywhere on one side of that angle to be the second vertex of the triangle. Use that point as a vertex for the 40° angle, and draw the 40° angle.

   b. Measure the third angle. It is _____ degrees.

   c. What kind of triangle is it? (acute, right, obtuse)

   d. What is the angle sum?

8. **a.** Draw a triangle with 125° and 40° angles.

   b. Measure the third angle. It is _____ degrees.

   c. What kind of triangle is it? (acute, right, obtuse)

   d. What is the angle sum?

9. **a.** Draw a triangle with 55° and 35° angles.

   b. Measure the third angle. It is _____ degrees.

   c. What kind of triangle is it? (acute, right, obtuse)

   d. What is the angle sum?

New Terms	
• *an acute triangle*	• *an obtuse triangle*
• *a right triangle*	• *a diagonal*

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Chapter 7: Fractions
Introduction

In the third grade, children studied the concept of a fraction, equivalent fractions, and compared some easy fractions. In fourth grade, it is time to expand the fraction topics. We study

- mixed numbers
- adding and subtracting like fractions and mixed numbers with like fractional parts (the denominators are the same)
- equivalent fractions
- comparing fractions
- multiplying a fraction by a whole number

Then in fifth grade, students tackle *all* of the four operations with fractions. Our studies here are still laying groundwork for that, emphasizing conceptual understanding and using visual models a lot.

These lessons are also important because they are the basis for understanding decimal numbers, the topic of the next chapter. Decimals are just another way of writing fractions with denominators 10, 100, 1,000 etc.

The topics in this chapter are studied with the help of visual models in order to emphasize the concepts. We must avoid presenting fraction math as a list of computational rules. Children easily confuse the various fraction rules, because there are so many, such as:

- a rule for converting a mixed number to a fraction, and vice versa
- a rule for adding like fractions
- a rule for finding a common denominator
- a rule for changing fractions to like fractions
- a rule for adding unlike fractions
- a rule for simplifying fractions
- a rule for finding equivalent fractions
- a rule for multiplying fractions
- a rule for dividing fractions
- a few rules for doing the four operations with mixed numbers

There is a place for the rules, as *shortcuts* for ideas that are already understood, but we do not start with them. The goal is to let the big ideas sink in conceptually first, followed by some shortcuts.

The Lessons in Chapter 7

Helpful Resources and Games on the Internet

General

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com/

Conceptua Math Fraction Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop conceptual understanding of fractions. Free registration required.
https://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software
Games for addition & subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.
http://www.sheppardsoftware.com/math.htm#fractions

Who Wants pizza?
This site explains the concept of fractions, addition, and multiplication with a pizza example, then has some interactive exercises.
http://math.rice.edu/~lanius/fractions/index.html

Fractioncity
Make “fraction streets” and help children with comparing fractions, equivalent fractions, addition of fractions of like and unlike denominators while they drive toy cars on the streets. This is not an online activity but has instructions of how to do it at home or at school.
http://www.teachnet.com/lesson/math/fractioncity.html

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

Fractions and mixed numbers

Identifying Fractions at Conceptua Fractions
A tool that shows fractions or mixed numbers using a pie, a bar, dots, and a number line. A free registration required.
https://www.conceptuamath.com/app/tool/identifying-fractions

Visualizing Fractions
The computer shows a fraction, and you divide the pie and color the pieces.
http://nlvm.usu.edu/en/nav/frames_asid_103_g_2_t_1.html

Pattern Blocks—Parts as Wholes
Click on the "Activities" in the top menu, and click on arrows until you find Parts as Wholes activity.
http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html

Fraction Model
Adjust the the numerator and the denominator, and the applet shows the fraction as a pie/rectangle/set model, as a decimal and as a percent.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=44

Clara Fraction's Ice Cream Shop
Convert improper fractions to mixed numbers and scoop the right amount of ice cream flavors onto the cone.
http://www.mrnussbaum.com/icecream/index.html

Addition and subtraction

MathSplat
Click on the right answer for addition problems or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Action Fraction
A racing game with several levels where you answer questions about adding and subtraction fractions. The levels advance from using like fractions to using unlike fractions and eventually subtraction.
http://funschool.kaboose.com/formula-fusion/number-fun/games/game_action_fraction.html

Fraction Worksheets: Addition and Subtraction
Create custom-made worksheets for the four operations with fractions and mixed numbers. Choose "Like Fractions" for this level.
http://www.homeschoolmath.net/worksheets/fraction.php

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Ordering Fractions at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/fractions.html#OrderingFractions

Fractional Hi Lo
The computer has selected a fraction. You guess and it tells you if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Equivalent fractions

Equivalent Fractions from National Library of Virtual Manipulatives (NLVM)
See the equivalency of two fractions as the applet divides the whole into more pieces.
http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html

Equivalent Fractions
Draw two equivalent fractions for the given fraction. Choose either a square or a circle for the shape.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=80

Fraction Frenzy
Click on pairs of equivalent fractions, as fast as you can. See how many levels you can get!
http://www.learningplanet.com/sam/ff/index.asp

Fresh Baked Fractions
Practice equivalent fractions by clicking on a fraction that is not equal to others.
http://www.funbrain.com/fract/index.html

Free Equivalent Fractions Worksheets
Create custom-made worksheets for equivalent fractions that can either include pie images or not.
http://www.homeschoolmath.net/worksheets/equivalent_fractions.php

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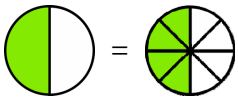
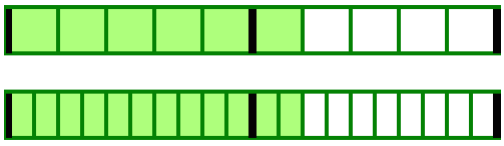
Equivalent Fractions

If you eat half of a pizza, or 4/8 of a pizza, you have eaten the same amount.



$\frac{1}{2} = \frac{4}{8}$

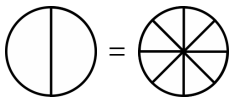
1/2 and 4/8 are ***equivalent fractions***.



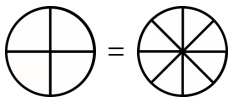
The two fraction strips show an equal amount. So, we can write an equal sign between the two mixed numbers:

$$1\frac{1}{5} = 1\frac{2}{10}$$

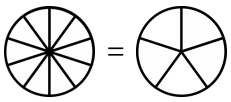
1. Color the first fraction. Shade the same *amount of pie* in the second picture. Write the second fraction.



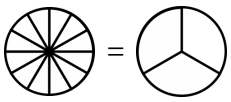
a. $\frac{1}{2} =$



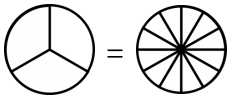
b. $\frac{3}{4} =$



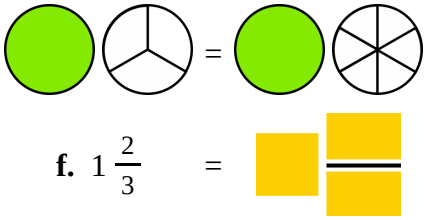
c. $\frac{6}{10} =$



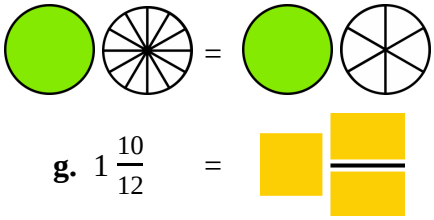
d. $\frac{8}{12} =$



e. $\frac{1}{3} =$

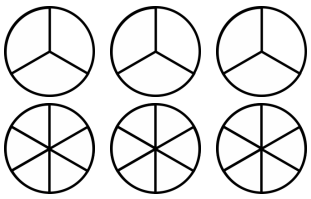


f. $1\frac{2}{3} =$



g. $1\frac{10}{12} =$

2. Write the fractions that have thirds using sixths instead. You can shade parts in the pictures.



a. $\frac{3}{3} =$

b. $\frac{4}{3} =$

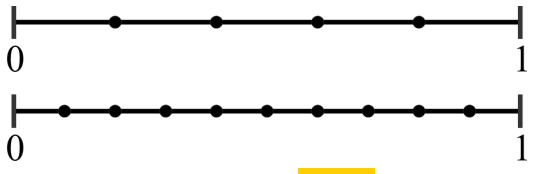
c. $\frac{7}{3} =$

d. $2\frac{1}{3} =$

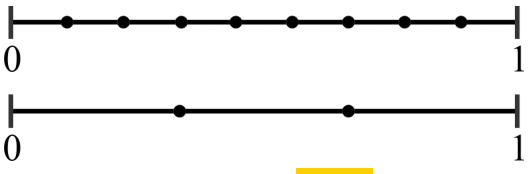
e. $1\frac{2}{3} =$

f. $2\frac{2}{3} =$

3. Mark the equivalent fractions on the number lines.

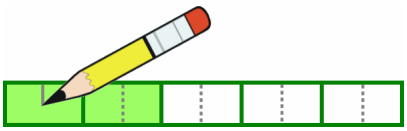


a. $\frac{4}{5} =$



b. $\frac{3}{9} =$

The fraction strip illustrates $\frac{2}{5}$. If you split each piece (both the colored and white pieces) into *two* new pieces, what fraction do you get?



You get $\frac{4}{10}$ − four colored pieces, and ten pieces total.

You have *two* times as many colored pieces, and *two* times as many total pieces as before.

4. Split both the colored and white pieces as instructed. Write the fraction after you change it.

a. Split all the pieces into two new ones.

$\frac{1}{2} = \frac{\quad}{\quad}$

b. Split all the pieces into four new ones.

$\frac{1}{2} = \frac{\quad}{\quad}$

c. Split all the pieces into three new ones.

$\frac{1}{4} = \frac{\quad}{\quad}$

d. Split all the pieces into three new ones.

$\frac{1}{3} = \frac{\quad}{\quad}$

e. Split all the pieces into two new ones.

$\frac{5}{6} = \frac{\quad}{\quad}$

f. Split all the pieces into three new ones.

$\frac{2}{5} = \frac{\quad}{\quad}$

Can you notice a *shortcut* for finding the second fraction without using a picture?

g. Split all the pieces into four new ones.

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

h. Split all the pieces into two new ones.

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

i. Split all the pieces into three new ones.

$\frac{\quad}{\quad} = \frac{\quad}{\quad}$

***If* you found the shortcut, explain how it works in these problems:**

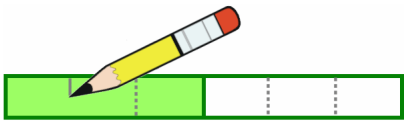
Split all the pieces into three new ones.

$\frac{1}{3} = \frac{\quad}{\quad}$

Split all the pieces into two new ones.

$\frac{3}{5} = \frac{\quad}{\quad}$

The fraction strip illustrates $\frac{1}{2}$. If we split each piece (both the colored and the white piece) into *three* new pieces, we get $\frac{3}{6}$.



We now have *three* times as many colored pieces, and *three* times as many total pieces as before. We can show this in writing this way:

We multiply both the top and bottom number in a fraction by 3. We get an equivalent fraction—it is the **same amount**, just cut into more pieces. *This does not mean we multiply the whole fraction by 3.*

$$\frac{1}{2} \overset{\times 3}{=} \frac{3}{6}$$

5. Split the pieces. Fill in the missing parts.

a. This is $\frac{3}{4}$. Make it $\frac{9}{12}$.

Each piece is split into ____ new ones.

$$\frac{3}{4} \overset{\times \_\_\_\_}{=} \frac{9}{12}$$

b. This is $\frac{1}{3}$. Make it $\frac{4}{12}$.

Each piece is split into ____ new ones.

$$\frac{1}{3} \overset{\times \_\_\_\_}{=} \frac{4}{12}$$

c. This is $\frac{1}{2}$. Make it $\frac{5}{10}$.

Each piece is split into ____ new ones.

$$\frac{1}{2} \overset{\times \_\_\_\_}{=} \frac{5}{10}$$

d. This is $\frac{1}{4}$. Make it $\frac{4}{16}$.

$$\frac{1}{4} \overset{\times \_\_\_\_}{=} \frac{4}{16}$$

e. This is $\frac{2}{3}$. Make it $\frac{6}{9}$.

$$\frac{2}{3} \overset{\times \_\_\_\_}{=} \frac{6}{9}$$

f. This is $\frac{2}{3}$. Make it $\frac{8}{12}$.

$$\frac{2}{3} \overset{\times \_\_\_\_}{=} \frac{8}{12}$$

g. 

$$\frac{4}{5} \overset{\times \_\_\_\_}{=} \frac{\square}{10}$$

h. 

$$\frac{2}{3} \overset{\times \_\_\_\_}{=} \frac{\square}{15}$$

i. 

$$\frac{2}{5} \overset{\times \_\_\_\_}{=} \frac{\square}{15}$$

6. Write the equivalent fraction. Use multiplication.

a. Split all the pieces into three new ones.	**b.** Split all the pieces into five new ones.	**c.** Split all the pieces into four new ones.	**d.** Split all the pieces into ten new ones.
$\frac{5}{6} = \frac{\square}{\square}$	$\frac{3}{4} = \frac{\square}{\square}$	$\frac{2}{5} = \frac{\square}{\square}$	$\frac{9}{10} = \frac{\square}{\square}$

7. Figure out how many new pieces the existing pieces were split into. Fill in the missing parts.

a. Pieces were split into ____ new ones.	**b.** Pieces were split into ____ new ones.	**c.** Pieces were split into ____ new ones.	**d.** Pieces were split into ____ new ones.
$\frac{1}{2} = \frac{\square}{6}$	$\frac{3}{10} = \frac{30}{\square}$	$\frac{2}{5} = \frac{\square}{30}$	$\frac{7}{8} = \frac{35}{\square}$
e. $\frac{2}{3} = \frac{\square}{6}$	**f.** $\frac{3}{5} = \frac{9}{\square}$	**g.** $\frac{5}{6} = \frac{\square}{12}$	**h.** $\frac{1}{3} = \frac{\square}{9}$

8. Write the fractions that have tenths with hundredths instead.

a. $\frac{1}{10} = \frac{\square}{100}$	**b.** $\frac{3}{10} =$	**c.** $\frac{6}{10} =$	**d.** $\frac{4}{10} =$	**e.** $\frac{13}{10} =$

9. Connect the equivalent fractions with a line.

a.	$\frac{2}{3}$	$\frac{1}{3}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{5}{10}$	$\frac{2}{8}$
	$\frac{2}{6}$	$\frac{6}{9}$

b.	$\frac{1}{2}$	$\frac{2}{10}$
	$\frac{3}{4}$	$\frac{1}{3}$
	$\frac{1}{5}$	$\frac{6}{12}$
	$\frac{4}{12}$	$\frac{9}{12}$

c.	$\frac{3}{6}$	$\frac{3}{12}$
	$\frac{1}{4}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{8}{12}$
	$\frac{2}{3}$	$\frac{4}{12}$

10. Write chains of equivalent fractions!

a. $\frac{1}{2} = \frac{\square}{4} = \frac{\square}{6} = \frac{\square}{8} = \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $\frac{1}{3} = \frac{\square}{6} = \frac{\square}{9} = \frac{\square}{12} = \frac{\square}{\square}$

We can use equivalent fractions to add fractions that have different denominators.

Example. Add $\frac{2}{10} + \frac{17}{100}$. First, write 2/10 as 20/100 (an equivalent fraction).

Then you can add, because the fractions now have the same denominator: $\frac{20}{100} + \frac{17}{100} = \frac{37}{100}$.

11. Add.

a. $\frac{1}{10} + \frac{8}{100}$ ↓ ↓ $\frac{\square}{100} + \frac{8}{100} =$	**b.** $\frac{7}{10} + \frac{3}{100}$ ↓ ↓ $\frac{\square}{100} + \frac{\square}{100} =$	**c.** $\frac{45}{100} + \frac{3}{10}$
d. $\frac{9}{10} + \frac{9}{100}$	**e.** $\frac{7}{10} + \frac{23}{100}$	**f.** $\frac{24}{100} + \frac{9}{10}$
g. $\frac{7}{100} + 1\frac{4}{10}$	**h.** $2\frac{28}{100} + 1\frac{5}{10}$	**i.** $\frac{6}{10} + \frac{35}{100} + \frac{7}{100}$

12. Draw a picture showing that 1/3 and 4/12 are equivalent fractions.

Puzzle Corner

Add. This is challenging. *Hint: You cannot simply add the top numbers and the bottom numbers. Use equivalent fractions.*

a. $\frac{3}{4} + \frac{1}{2}$	**b.** $\frac{1}{5} + \frac{3}{10}$	**c.** $\frac{2}{3} + \frac{2}{9}$

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Chapter 8: Decimals
Introduction

In fourth grade, we study decimal numbers with one or two decimal digits, and add and subtract them. It is important that the student grasps these simple topics well, because we are laying a groundwork towards fifth and sixth grade, where decimal operations and using decimals take a "center stage."

For now, the focus is first of all, understanding the fact that decimals are simply fractions with a denominator 10 or 100. Then with that in mind (decimals are fractions), we study comparing, adding, and subtracting them.

Notice:

- In the addition problem 0.5 + 0.9, we get 14 tenths, which is 1.4. A common student misconception is to add 0.5 + 0.9 = 0.14.
- In a problem such as 0.5 + 0.11, a common student misconception is to get 0.16. Such students are thinking of the decimal parts as if they were "whole numbers." To solve 0.5 + 0.11 correctly, students can rewrite 0.5 as 0.50, and then the problem becomes 0.50 + 0.11 = 0.61.

In the lesson Using Decimal Numbers, students use decimals with some metric measuring units, including converting between units. This topic will also be studied further in 5th grade.

The Lessons in Chapter 8

Helpful Resources on the Internet

Mathematical Interactivities
http://mathematics.hellam.net/
Find several games related to fractions and decimals in the **Number Puzzles** section, including:

- **Decimal Challenge** - Guess the decimal number between 0 and 10.
  Each time feedback tells whether your guess was too high or too low.
  http://www.interactivestuff.org/sums4fun/decchall.html
- **Switch** - Put the sequence of decimal numbers into ascending order by switching them around.
  Refresh the page from your browser to get another problem to solve.
  http://www.interactivestuff.org/sums4fun/switch.html
- **Scales** - Move the pointer to match the decimal number given to you.
  Refresh the page from your browser to get another problem to solve.
  http://www.interactivestuff.org/sums4fun/scales.html

A Decimal Puzzle
Make every circle add up to 3.
http://nlvm.usu.edu/en/nav/frames_asid_187_g_2_t_1.html?open=instructions&from=category_g_2_t_1.html

Fraction/Decimal Worksheets
Change fractions to decimal numbers or decimal numbers to fractions.
http://www.homeschoolmath.net/worksheets/fraction-decimal.php

Modeling Decimals (Area and Grid Models)
An interactive "gizmo" for modeling decimals in a grid or on a number. By subscription, but you can try the gizmo for free for 5 minutes.
http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1007

Adding Decimals (Base 10 Blocks)
An interactive "gizmo" for modeling decimal addition with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.
http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1023

Subtracting Decimals (Base 10 Blocks)
An interactive "gizmo" for modeling decimal subtraction with regrouping. By subscription, but you can try the gizmo for free for 5 minutes.
http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=1030

Beat the Clock
Type in the decimal to show much of the square is shaded in this timed game.
http://www.decimalsquares.com/dsGames/games/beatclock.html

Decimal Darts
Try to pop the balloons with darts by estimating at which height the balloons are.
http://www.decimalsquares.com/dsGames/games/darts.html

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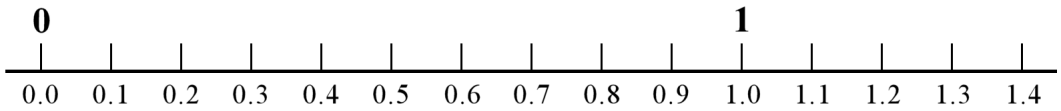
Adding and Subtracting with Tenths

You *already* know how to add or subtract decimals with tenths. They are just fractions with a denominator of 10. Compare these additions that are written with decimals or fractions.	$0.1 + 0.5 = 0.6$ $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$	$8.4 - 2.3 = 6.1$ $8\frac{4}{10} - 2\frac{3}{10} = 6\frac{1}{10}$
There is one tricky part though: 0.6 + 0.7 *is* ***NOT*** 0.13 !! To see why, add the fractions. Notice that six tenths and seven tenths makes more than one whole!	$0.6 + 0.7 = 1.3$ $\frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10}$	$1.5 + 0.9 = 2.4$ $1\frac{5}{10} + \frac{9}{10} = 2\frac{4}{10}$

1. Write an addition *or* subtraction sentence for each "number line jump."



a. You're at 0.7, and you jump *five tenths* to the right. _____

b. You're at 0.6, and you jump *eight tenths* to the right. _____

c. You're at 1.1, and you jump *eight tenths* to the left. _____

d. You're at 1.3, and you jump *four tenths* to the left. _____

e. You're at 0.2, and you jump *eleven tenths* to the right. _____

2. Solve the fraction additions, and then write them using decimals.

a. $\frac{2}{10} + \frac{7}{10} =$ *0.2 +*	**b.** $\frac{5}{10} + \frac{6}{10} =$	**c.** $\frac{9}{10} + \frac{8}{10} =$

3. Add and subtract.

a.	**b.**	**c.**	**d.**
0.9 + 0.2 = _____	0.5 + 0.7 = _____	0.8 + 0.7 = _____	1.8 − 0.9 = _____
1.9 + 0.2 = _____	3.5 + 0.7 = _____	0.8 + 2.7 = _____	5.8 − 0.9 = _____

4. Fill in the missing parts.

a.	**b.**	**c.**	**d.**
2.3 + 0.9 = _____	1.5 + 0.7 = _____	6.6 − 0.5 = _____	4.7 − 1.7 = _____

5. Write the numbers.

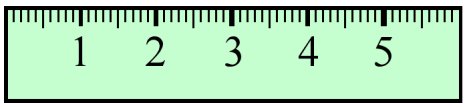
a. 3 tenths, 5 ones	**d.** Write the numbers in order. 9 8.9 9.1 9.0 9.9 1.9
b. 7 tens, 8 ones, 4 tenths	
c. 4 tenths, 3 ones, 6 tens	

6. Continue the patterns by adding or subtracting the same number repeatedly.

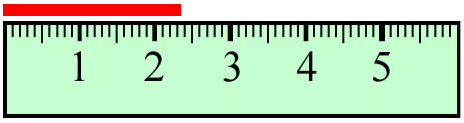
a. 0.1	**b.** 1.1	**c.** 2.5	**d.** 3.6
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____
+ 0.2 = _____	+ 0.5 = _____	+ 0.3 = _____	− 0.4 = _____

7. Remember? **1 millimeter is one-tenth of a centimeter.** Or, 1 mm = 0.1 cm.

a. Draw a line that is 4.7 cm long.



b. Measure the line in centimeters. Use a decimal.



8. Convert. In (c), add and give your answer in centimeters.

a. 0.5 cm = _____ mm

1.2 cm = _____ mm

b. 7 mm = _____ cm

35 mm = _____ cm

c. 5 mm + 0.9 cm = _____ cm

4 cm + 3.4 cm = _____ cm

9. The two sides of a rectangle measure 6.5 cm and 3.6 cm. Draw the rectangle on blank paper. What is its perimeter?