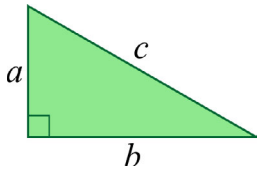
The Pythagorean Theorem

You will now learn a very famous mathematical result, the Pythagorean Theorem, which has to do with the lengths of the sides in a right triangle. First, we need to study some terminology.

In a right triangle, the two sides that are perpendicular to each other are called **legs**. The third side, which is always the longest, is called the **hypotenuse**.

In the image on the right, the sides a and b are the legs, and c is the hypotenuse.



Note: We don't use the terms "leg" and "hypotenuse" to refer to the sides of an acute or obtuse triangle — this terminology is restricted to *right* triangles.

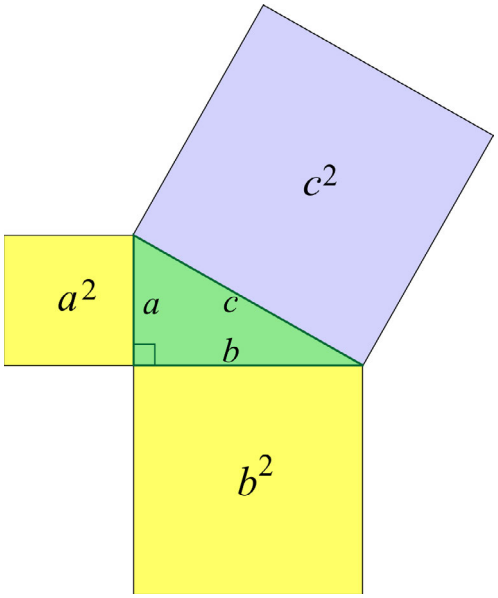
The Pythagorean Theorem states that **the sum of the squares of the legs equals the square of the hypotenuse.**

In symbols it looks much simpler:

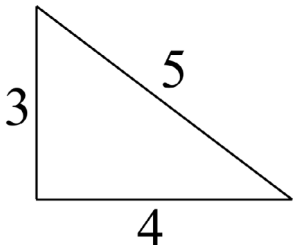
$$a^2 + b^2 = c^2$$

The picture shows squares drawn on the legs and on the hypotenuse of a right triangle. Verify visually that the total area of the two yellow squares drawn on the legs looks about equal to the area of the blue square on the hypotenuse.

We will prove this theorem in another lesson.
For now, let's get familiar with it and learn how to use it.



1. This is the famous 3-4-5 triangle: its sides measure 3, 4, and 5 units. It is a right triangle. Check that the Pythagorean Theorem holds for it by filling in the numbers below.



$\square^2 + \square^2 \stackrel{?}{=} \square^2$

$\square + \square \stackrel{?}{=} \square$

$\square = \square$

2. **a.** Check that the Pythagorean Theorem holds for a triangle with sides 6, 8, and 10 units long by filling in the numbers at the right.

b. Use a compass and a ruler to draw a triangle with sides 6, 8, and 10 cm long. You can review the box, "A Triangle with Three Given Sides," on page 127. Measure its angles: did you get a right triangle?

$\square^2 + \square^2 \stackrel{?}{=} \square^2$

$\square + \square \stackrel{?}{=} \square$

$\square = \square$

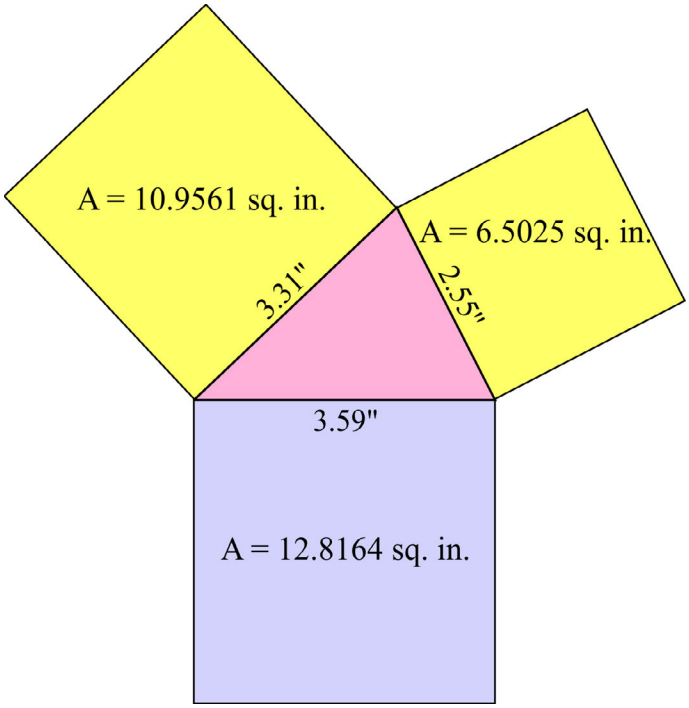
Example 1. This triangle is *not* a right triangle, so the Pythagorean Theorem does *not* hold:

$$2.55^2 + 3.31^2 \stackrel{?}{=} 3.58^2$$

$$6.5025 + 10.9561 \stackrel{?}{=} 12.8164$$

$$17.4586 > 12.8164$$

The sum of the areas of the squares drawn on the two shortest sides is more than the area of the square drawn on the longest side. As you can see, the triangle is acute.



Example 2. Is a triangle with sides 4 cm, 5 cm, and 7 cm a right triangle?

We check if 4, 5, and 7 fulfill the Pythagorean Theorem (on the right). They don't. In fact, $4^2 + 5^2 < 7^2$ and the triangle is obtuse. (You can check that by drawing it.)

$$4^2 + 5^2 \stackrel{?}{=} 7^2$$

$$16 + 25 \stackrel{?}{=} 49$$

$$41 < 49$$

This triangle is obtuse.

3. For each set of lengths, determine whether they form a right triangle using the Pythagorean Theorem. Notice carefully which length is the hypotenuse.

a. 6, 9, 13

b. 12, 13, 5