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Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Some of them, such as fractions and decimals, students are familiar with, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers).

The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: summarizing distributions using measures of center and variability.

This year starts out, in chapter 1 of part 6-A, with a review of the four operations with whole numbers (including long division), place value, and rounding. Students are also introduced to exponents and do some problem solving.

Chapter 2 starts the study of algebra topics, delving first into expressions and equations. Students practice writing expressions in different ways, and use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple equations. We also briefly study inequalities and using two variables.

Chapter 3 has to do with decimals. This is a long chapter, as we revise all of decimal arithmetic, just using more decimal digits than in 5th grade Students also convert measuring units in this chapter.

Ratios (chapter 4) is a new topic. Students are already familiar with finding fractional parts, and now it is time to advance that knowledge into the study of ratios, which arise naturally from dividing a quantity into many equal parts. We study such topics as rates, unit rates, equivalent ratios, and problem solving using bar models.

Percent (chapter 5) is an important topic because of its many applications in real life. The goal of this chapter is to develop a basic understanding of percent, to see percentages as decimals, and to learn to calculate discounts.

In part 6-B, students study number theory, fractions, integers, geometry and statistics.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the "Introduction" of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the "blue teaching boxes" to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn't a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!
- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your student's needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.
- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.
- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.
- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.
- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!
- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.
- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2	10
6-B	157	88	10	78	2	10
Grade 6 total	323	180	20	160	2	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional review before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional review. Then, to get a count of “pages/day”, divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let’s assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems (“drill”) or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + _____ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

Chapter 1: Review of the Basic Operations
Introduction

The goal of the first chapter in year 6 is to review the four basic operations with whole numbers, place value and rounding, as well as to learn about exponents and problem solving.

A lot of this chapter is review, and I hope this provides a gentle start for 6th year math. In the next chapter, we will then delve into some beginning algebra topics.

Special notes for this chapter: problem solving

This chapter doesn't have many new concepts – only the concept of exponents and powers. Besides reviewing how to perform the four basic operations with pencil and paper, students also get some practice for problem solving.

Solving (word) problems in math works much the same way as solving problems in real life. You may start out one way, come to a "dead end", and have to take an entirely different approach. Good problem solvers monitor their progress as they work, and change course if necessary.

Here is a list of general tips and strategies for solving mathematical problems that you can share with your student(s).

- If you cannot solve the original problem, try to **solve an easier, related problem first**. This may help you find a way to solve the original. For example, if the numbers in the problem seem intimidating, change them temporarily to really easy numbers and see if you can solve the problem then. Or reduce the details mentioned in the problem to make it simpler, solve the simpler problem, then go back to the original. You can also try special cases of the problem at hand at first.
- Drawing a sketch, a diagram (e.g. a bar model), or making a table can be very helpful.
- **Check your final answer** if at all possible, using a different method. For example, division problems can be checked by multiplication and subtractions by addition. Multi-step problems can often be solved in different ways or in a different order.

  At the very least, **check that your answer is reasonable** and actually makes sense. If the problem is asking how many days of vacation someone might get in a year, and you get an answer in the thousands, you can tell something went really wrong. And, once you find your answer is wrong – maybe it doesn't make sense – it is NOT time to cry and give up. Do you know how many times Thomas Alva Edison tried and failed, until he finally found a way to make a commercially viable electric light bulb? Thousands of times.

  Perseverance is something that is very necessary when you encounter problems in real life, and I don't mean math problems. Everyone fails, but it is those who keep trying who will ultimately succeed. Every successful entrepreneur can tell you that. Failing is *not* a sign of being stupid. It is a sign of being a human. ALL of us make mistakes and fail. ALL of us improve as we keep trying.
- Often, it is easier and neater to perform paper-and-pen calculations (long addition, subtraction, multiplication, division) on a grid paper.
- The space in the worktext may not be enough. Use as much scrap paper (extra paper) as necessary.
- Remember to include a unit (if applicable) in the answers to word problems.

General principles in using the curriculum

Please note that it is not recommended to assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

The specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about two pages daily or 10 pages a week in order to finish the curriculum in about 36 weeks.

See the user guide at in the beginning of this book or online at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

The Lessons in Chapter 1

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch1

SCAN ME

Warm-up: Mental Math

To **multiply** 2,000 × 120, simply multiply 2 × 12, and place four zeros on the end of the answer: 2,000 × 120 = 240,000	Solve **division** by thinking of multiplication "backwards": 5,600 ÷ 70 = ? Think what number times 70 will give you 5,600. Since 70 × 80 = 5,600, then 5,600 ÷ 70 = 80.	You can **add in parts**. 76 + 120 + 65 = ? First add 70 + 120 + 60 = 250. Then, 6 + 5 = 11. Lastly, 250 + 11 = 261.

The **order of operations** is:
1. Parentheses 2. Exponents; 3. Multiplication and division; 4. Addition and subtraction.

To calculate 9 × 80 − 10 × 70, first solve 9 × 80 and 10 × 70. Subtract only after those calculations. 9 × 80 − 10 × 70 = 720 − 700 = 20	In the expression 4,500 ÷ (5 + 45) × 80, solve 5 + 45 first. Then, divide. 4,500 ÷ (5 + 45) × 80 = 4,500 ÷ 50 × 80 = 90 × 80 = 7,200

1. Solve in your head.

a. 410 + 2 × 19 =	**b.** 3 × 50 + 4 × 150 =	**c.** 70 × 80 − 40 × 50 =
d. 14 + (530 − 440) =	**e.** 45 + 56 + 35 =	**f.** 300 ÷ 5 − 400 ÷ 10 =

2. Solve in your head.

a. 17 + _________ = 110 **b.** 345 + ____________ = 1,000 **c.** 3 × 40 + ___________ = 500

3. Divide. Remember that division can also be written using a fraction line.

a. $\frac{240}{4}$ = **c.** $\frac{72}{9}$ = **e.** $\frac{5,600}{10}$ = **g.** $\frac{420}{20}$ = **i.** $\frac{420}{70}$ =

b. $\frac{7,200}{100}$ = **d.** $\frac{450}{9}$ = **f.** $\frac{8,000}{200}$ = **h.** $\frac{10,000}{50}$ = **j.** $\frac{7,200}{800}$ =

4. Solve. Notice carefully which operation(s) are done first.

a. $500 - 40 - 3 \times 50$ = ___________	**b.** $1{,}020 - (40 - 10) \times 20$ = ___________
c. $42{,}000 - 12{,}000 + 3 \times 5{,}000$ = ___________	**d.** $(70 - 20) \times 70$ = ___________
e. $\dfrac{210}{2} + 3 \times 15$ = ___________	**f.** $250 \times 4 + \dfrac{6{,}300}{70}$ = ___________

5. Find a number that fits in place of the unknown.

a. $x \div 70 = 40$	**b.** $20 \times M = 1{,}200$	**c.** $500 - y = 320$

6. Find the rule that is used in the table and fill in the missing numbers.

n	130	250	360	410	775	820	1,000
n − ____		215		375			

7. Find the rule that is used in the table and fill in the missing numbers.

n	3	5	12	15	25	35	60
		200			1,000		

8. Rick cut off a 50-cm piece from a 6-meter board, and then he divided the rest of the board into five equal pieces. How long was each piece?

9. **a.** Evelyn works 8 hours a day and earns $104 daily. What is her hourly wage?

b. How much does Evelyn earn in a five-day work week?

How much does she earn in three months (which is 13 weeks)?
(You may use paper and pencil for this one.)

10. Alexis and Mia baked biscuits for a bake sale. They used this recipe, but they needed to triple it:

2 1/4 cups of flour
3 teaspoons of baking powder
1/3 cup of honey
1/2 cup of butter
3/4 teaspoon of nutmeg
1 1/2 teaspoons of cinnamon
1/2 teaspoon of ground cloves
3/4 cup of walnuts
Makes 2 1/2 dozen biscuits.

a. Triple the recipe for them.

b. How many biscuits did they bake?

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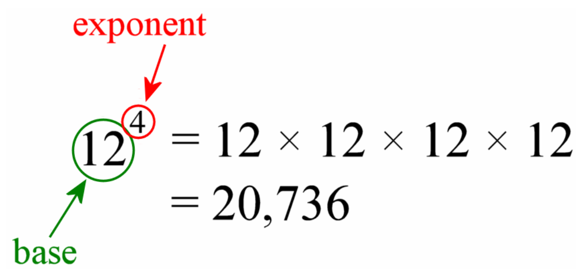
Powers and Exponents

Exponents are a "shorthand" for writing repeated multiplications by the same number.

For example, $2 \times 2 \times 2 \times 2 \times 2$ is written 2^5.
$5 \times 5 \times 5 \times 5 \times 5 \times 5$ is written 5^6.

The tiny raised number is called the **exponent**. It tells us how many times the *base* number is multiplied by itself.



The expression 2^5 is read as "two to the fifth power," "two to the fifth," or "two raised to the fifth power."

Similarly, 7^9 is read as "seven to the ninth power," "seven to the ninth," or "seven raised to the ninth power."

The "powers of 6" are simply expressions where 6 is raised to some power: For example, 6^3, 6^4, 6^{45} and 6^{99} are powers of 6. What would powers of 10 be?

Expressions with the exponent 2 are usually read as something "**squared.**" For example, 11^2 is read as "**eleven squared.**" That is because it gives us *the area of a square* with the side length of 11 units.

Similarly, if the exponent is 3, the expression is usually read using the word "**cubed.**" For example, 31^3 is read as "**thirty-one cubed**" because it gives the *volume of a cube* with the edge length of 31 units.

1. Write the expressions as multiplications, and then solve them in your head.

a. $3^2 = \underline{3 \times 3 = 9}$ **b.** 1^6

c. 4^3 **d.** 10^4

e. 5^3 **f.** 10^2

g. 2^3 **h.** 8^2

i. 0^5 **j.** 10^5

k. 50^2 **l.** 100^3

2. Rewrite the expressions using an exponent, then solve them. You may use a calculator.

a. $2 \times 2 \times 2 \times 2 \times 2 \times 2$ **b.** $8 \times 8 \times 8 \times 8 \times 8$

c. 40 squared **d.** $10 \times 10 \times 10 \times 10$

e. nine to the eighth power **f.** eleven cubed

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Place Value

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
2	**0**	**9**	**3**	**5**	**6**	**0**	**7**	**5**	**8**	**5**	**5**	**4**	**0**	**2**
trillions period			billions period			millions period			thousands period			ones period		

The letters "h t o" stand for hundreds, tens, ones.

Read this number as:

"Two hundred nine trillion, three hundred fifty-six billion, seventy-five million, eight hundred fifty-five thousand, four hundred and two."

To write this number in its *expanded form*, take each digit's value, and write them all as a sum:

200,000,000,000,000 + 9,000,000,000,000 + 300,000,000,000 + 50,000,000,000 + 6,000,000,000 + 70,000,000 + 5,000,000 + 800,000 + 50,000 + 5,000 + 400 + 2

This is easier to write using exponents:

$2 \times 10^{14} + 9 \times 10^{12} + 3 \times 10^{11} + 5 \times 10^{10} + 6 \times 10^{9}$

$+ 7 \times 10^{7} + 5 \times 10^{6} + 8 \times 10^{5} + 5 \times 10^{4} + 5 \times 10^{3} + 4 \times 10^{2} + 2 \times 10^{0}$

Remember that in powers of 10, the exponent tells you how many zeros are in the number.
For example, 10^{11} = 100,000,000,000 has eleven zeros.
Notice especially: $\mathbf{10^0 = 1}$ (the number 1 has no zeros!).

The number system we use is based on *place value*. This means that a digit's *value* depends on its position or *place* within the number.

Our number system is called a *decimal*, or *base-ten*, system (from the Latin word `decima`, *a tenth part*). The value of each position or place is *one-tenth* of the value of the previous place.

h	t	o	h	t	o	h	t	o	h	t	o	h	t	o
0	**0**	**0**	**6**	**3**	**0**	**9**	**5**	**7**	**8**	**1**	**2**	**4**	**9**	**8**
trillions period			billions period			millions period			thousands period			ones period		

The digit "6" is in the hundred billions place.
Its value is 6 × a hundred billion, or 600 billion.

The digit "5" is in the ten millions place.
Its value is 5 × ten million, or 50 million.

1. Write the numbers in the place value chart. Answer the questions.

a. 89 million, 2 thousand, 4 hundred

What is the value of the digit "9"?

____

trillions period			billions period			millions period			thousands period			ones period		

b. 142 billion, 2 million, 139 thousand

What is the value of the digit "3"?

____

trillions period			billions period			millions period			thousands period			ones period		

c. 5 trillion, 47 million, 260

What is the value of the digit "4"?

____

trillions period			billions period			millions period			thousands period			ones period		

Chapter 2: Expressions and Equations
Introduction

In this chapter students start learning *algebra* – in a nutshell, the way to "do arithmetic with variables". Algebra enables us to solve real-life problems abstractly, in terms of variable(s) instead of numbers, and it is a very powerful tool.

Special notes for this chapter: algebra

The chapter focuses on two important basic concepts: **expressions** and **equations**. We also touch on inequalities and graphing on a very introductory level. In order to make the learning of these concepts easier, the expressions and equations in this chapter do not involve negative numbers (as they typically do when studied in pre-algebra and algebra). Integers are introduced in part 6-B, and then Math Mammoth grade 7 deals with algebraic concepts including with negative numbers.

We start out by reviewing the order of operations. Then the lessons focus on algebraic expressions. Students encounter the exact definition of an expression, a variable, and a formula, and practice writing expressions in many different ways. They study equivalent expressions and simplifying expressions. Length and area are two simple contexts I have used extensively for students to learn to write and simplify expressions.

In these lessons, students have opportunities to **write real-world scenarios in terms of variables**. In other words, they *decontextualise* – they abstract a given situation and represent it symbolically. Then, as they learn algebra, they learn to manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and to reason abstractly about those quantities represented by the variables.

The other major topic of the chapter is equations. Students learn some basics, such as, the solutions of an equation are the values of the variables that make the equation true. They use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. I have also included a few easy two-step equations.

Next, students solve and graph simple inequalities, again practicing the usage of variables to represent quantities. Lastly, they are introduced to the usage of *two* variables in algebra, including how to graph that relationship on a coordinate plane. This is an important topic, as so many real-life situations involve a relationship between two quantities, and graphing that relationship is an important tool in mathematical modeling.

You will find free videos covering many topics of this chapter of the curriculum at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 2

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch2

SCAN ME

The Order of Operations

The Order of Operations (PEMDAS)

1) Solve what is within the **parentheses (P)**.

2) Solve **exponents (E)**.

3) Solve **multiplication (M)** and **division (D)** from left to right.

4) Solve **addition (A)** and **subtraction (S)** from left to right.

Note: From now on, we will use the raised dot · for the multiplication symbol. This is because we will be studying algebra, and × can be confused with the letter *x*, often used in algebra.

So, for example, we will write $5 \cdot 2$ to signify five times two.

Example 1. Solve $200 - (10 - 4 + 5)^2$.

1. Solve what is within the parentheses: $10 - 4 + 5$. Since subtractions and additions are on the same level, solve them from left to right: $10 - 4 + 5 = 11$. The expression is now $200 - 11^2$.
2. Next, solve the exponent: $11^2 = 121$. The expression is now $200 - 121$.
3. Lastly, subtract. $200 - 121 = 79$.

Example 2. $\dfrac{10 + 50}{12 - 6}$.

This expression is *not* the same as $10 + 50 \div 12 - 6$. Instead, the fraction line works as a grouping symbol, grouping together what is above and below the line, so that the division is to be done *last*. The expression is actually $(10 + 50) \div (12 - 6)$.

First, solve the expressions above and below the line (as if they were grouped using parentheses), and lastly divide:

$$\frac{10 + 50}{12 - 6} = \frac{60}{6} = 10$$

Example 3a. Here is an expression that has only *multiplications* and *divisions*: $20 \cdot 2 \div 4 \cdot 10$.

Those operations are on the SAME level in the order of operations, but that does *not* mean that multiplications are solved before divisions. Instead, they are solved in order from left to right.

$$\begin{aligned} & 20 \cdot 2 \div 4 \cdot 10 \\ = \ & 40 \div 4 \cdot 10 \\ = \ & 10 \cdot 10 = 100 \end{aligned}$$

Example 3b. Let's rewrite the expression from 3a. using the fraction line for division—it will become easier!

Notice, there is a division by 4:

$$20 \cdot 2 \div 4 \cdot 10$$

This means that 4 needs to be in the denominator.

The expression can be written as $20 \cdot \dfrac{2}{4} \cdot 10$ or

as $\dfrac{20 \cdot 2}{4} \cdot 10$ (either is correct).

Comparing to the original expression $20 \cdot 4 \div 4 \cdot 10$, it looks quite different, but it is now easier to see what needs done. Verify that you get the same answer as in example 3a.

1. Put parentheses into the equations to make them true.

a. $100 - 50 - 50 = 100$ **b.** $200 \div 10 + 10 + 5 = 15$ **c.** $50 + 50 \cdot 4 - 10 = 390$

2. Rewrite each expression using the fraction line, then solve. Compare each expression in the top row of boxes to the one below it. *Hint: Only whatever comes right after the ÷ sign needs to be in the denominator.*

a. $64 \div 8 \cdot 4$	**b.** $64 \div (8 \cdot 4) \cdot 2$	**c.** $4 \cdot 8 \div 4 \cdot 2$
d. $64 \div (8 \cdot 4)$	**e.** $64 \div 8 \cdot 4 \cdot 2$	**f.** $(4 \cdot 8) \div (4 \cdot 2)$

3. Find the value of these expressions.

a. $150 + 2 \cdot 10$	**b.** $5^2 \cdot 2^3$	**c.** $3^2 \cdot (150 + 900) \div 3$
d. $\dfrac{12 + 9}{4 + 1}$	**e.** $\dfrac{5^2}{3^2}$	**f.** $\dfrac{2^3}{8} + 10^3$
g. $(6 + 6)^2 \cdot (15 - 5)^2$	**h.** $40 + 80 \div 2 \cdot 4 - 15$	**i.** $\dfrac{7^2}{7} \cdot 7$

4. Write the expressions in a shorter way, using multiplication. Find their values.

a. $20\,000 - 500 - 500 - 500 - 500 - 500 - 500 - 500$

b. $70 + 70 + 70 + 70 + 70 + 70 + 120 + 120 + 120 + 120 + 120$

5. Write the expressions in a shorter way, using exponents. Find their values.

a. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 + 5 \cdot 5 \cdot 5$

b. $5 \cdot 100 \cdot 100 \cdot 100 - 2 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

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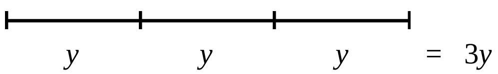
Writing and Simplifying Expressions 1: Length and Perimeter

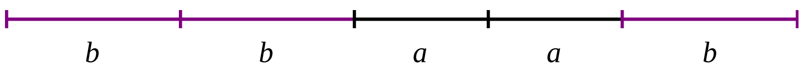


= $3y$

If the length of each line segment is y, then the total length of the line segments is $y + y + y$.

As you know, the shortcut for repeated addition is *multiplication*. So, we can *simplify* the sum $y + y + y$, and write $\mathbf{3y}$ in its place.

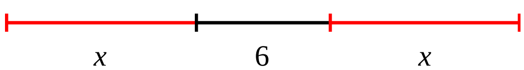
The expressions $y + y + y$ and $3y$ are **equivalent expressions**. This means that they have the same value no matter what value y has.



= $2a + 3b$

Here, we can write the total length as $b + b + a + a + b$, which is equivalent to $b + b + b + a + a$, which simplifies to $3b + 2a$. We can also write it as $2a + 3b$, because you can add in any order.

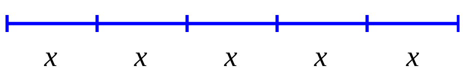
However, we *cannot* simplify the sum $2a + 3b$ any farther! The a and the b are not the same! Trying to add them would be like trying to add 2 meters and 3 liters. The expression is now as simple as it can get.



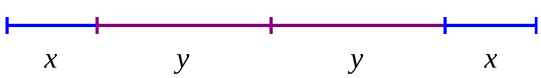
The total length here is $x + 6 + x$, which simplifies to $2x + 6$.

1. Write an expression for the total length of the line segments in simplified form.

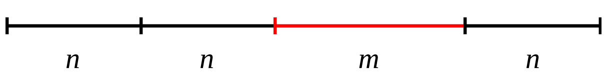
a.



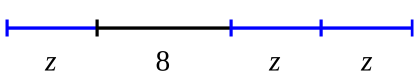
b.



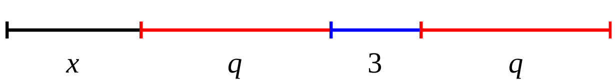
c.



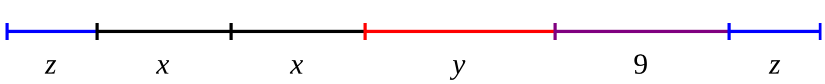
d.



e.



f.



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The Distributive Property

The **distributive property** states that $a(b + c) = ab + ac$

It may look like a meaningless or difficult equation to you now, but don't worry, it will become clearer!

The equation $\boldsymbol{a(b + c) = ab + ac}$ means that you can *distribute* the multiplication (by a) over the sum $b + c$ so that you multiply the numbers b and c separately by a, and add last.

You have already used the distributive property! When you separated $3 \cdot 84$ into $3 \cdot (80 + 4)$, you then multiplied 80 and 4 *separately* by 3, and added last: $3 \cdot 80 + 3 \cdot 4 = 240 + 12 = 252$.
We called this using "partial products" or "multiplying in parts."

Example 1. Using the distributive property, we can write the product $2(x + 1)$ as $2x + 2 \cdot 1$, which simplifies to $2x + 2$.

Notice what happens: Each term in the sum $(x + 1)$ gets multiplied by the factor 2! Graphically:

$$2(x + 1) = \underline{2x} + \underline{2 \cdot 1}$$

Example 2. To multiply $s \cdot (3 + t)$ using the distributive property, we need to multiply *both* 3 and t by s:

$s \cdot (3 + t) = s \cdot 3 + s \cdot t$, which simplifies to $3s + st$.

1. Multiply using the distributive property.

a. $3(90 + 5) = 3 \cdot$ ___ $+ 3 \cdot$ ___ $=$	**b.** $7(50 + 6) = 7 \cdot$ ___ $+ 7 \cdot$ ___ $=$
c. $4(a + b) = 4 \cdot$ ___ $+ 4 \cdot$ ___ $=$	**d.** $2(x + 6) = 2 \cdot$ ___ $+ 2 \cdot$ ___ $=$
e. $7(y + 3) =$	**f.** $10(s + 4) =$
g. $s(6 + x) =$	**h.** $x(y + 3) =$
i. $8(5 + b) =$	**j.** $9(5 + c) =$

Example 3. We can use the distributive property also when the sum has three or more terms. Simply multiply *each term* in the sum by the factor in front of the parentheses:

$5(x + y + 6) = 5 \cdot x + 5 \cdot y + 5 \cdot 6$, which simplifies to $5x + 5y + 30$

2. Multiply using the distributive property.

a. $3(a + b + 5) =$	**b.** $8(5 + y + r) =$
c. $4(s + 5 + 8) =$	**d.** $3(10 + c + d + 2) =$

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Using Two Variables

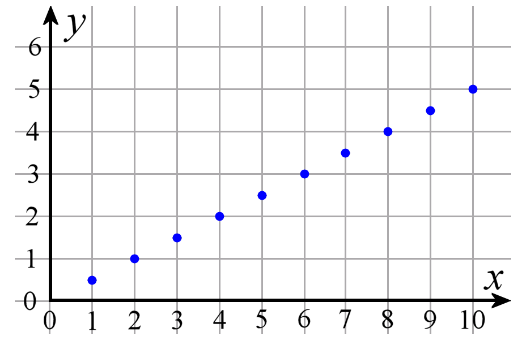
Often in mathematics—and in real life—we study the relationship between two variables.

Example 1. The equation $y = \frac{1}{2}x$ has two variables, y and x.

There are many values of x and y that make that equation true. For example, when x is 4, then y is $(1/2) \cdot 4 = 2$. Some of the values of x and y are listed below.

x	1	2	3	4	5
y	½	1	1 ½	2	2 ½

x	6	7	8	9	10
y	3	3 ½	4	4 ½	5



We can plot or graph these (x, y) pairs as points in the coordinate grid.
These ordered pairs actually are a **function**. We will not study the exact definition of a function here, but you can think of a function as a relationship between two variables.

In this lesson, you will study only **linear functions**. The word "linear" comes from the fact that the graphs of those functions look like a *line*. There exist many other, different kinds of functions as well.

Example 2. One towel costs \$4. If you buy 17 towels, the cost is 17 · \$4 = \$68.
In this situation, we are interested in <u>two</u> <u>variables</u> whose values can change:

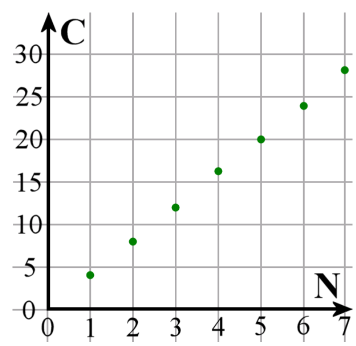
1. **The number of towels** a person buys is a variable. (It can vary!) Let's denote the number of towels by N.
2. **The total** cost varies according to how many towels are bought. Let C be the cost.

There is a very simple relationship between N and C: **C = N · \$4**
(This means the total cost *is* the number of towels times \$4.)

This is normally written as **C = 4N** because in algebra we write the number in front of the variable (not vice versa), and we omit the multiplication sign between a number and a variable.

The table below shows some *possible* values of C and N.

(x)	N	1	2	3	4	5	6	7	10	15	20
(y)	C	4	8	12	16	20	24	28	40	60	80



From this table, we get lots of number pairs. Some of them are plotted on the coordinate grid you see on the right.

You may have seen coordinate grids that have x and y-axis. This time we will label our axes N and C, according to the names of the variables. If this seems confusing, think of the variable N as the "x", and the variable C as the "y".

In this situation, we think of the variable N as the *independent variable*, and the variable C as the *dependent variable*, because its value *depends* on the value of N <u>according to the given equation</u> (C = 4N). In other words, we let the value of N vary (sort of independently), and the values of C are what we calculate or "observe," noticing how they depend on the value of N.

The independent variable is *always* plotted on the <u>horizontal axis</u>.

We *could* look at this situation just the opposite way also: let the cost be the independent variable, and study how the number of towels depends on that. Then, we would plot C on the horizontal axis, and calculate N using an equation that depends on C (it would be N = C/4).

Chapter 3: Decimals
Introduction

In this chapter we study all four operations of decimals, the metric system and using decimals with measuring units. Most of these topics have already been studied in 5th grade, but in 5th grade we were using numbers with a maximum of three decimal digits. This time there is no such restriction, and the decimals used can have many more decimal digits than that.

However, since the topics are the same, consider assigning only one-fourth to one-half of the exercises initially. Monitor the student's progress and assign more if needed. The skipped problems can be used for review later.

We start out by studying place value with decimals and comparing decimals up to six decimal digits. The next several lessons contain a lot of review, just using longer decimals than in fifth grade: adding and subtracting decimals, rounding decimals, multiplying and dividing decimals, fractions and decimals, and multiplying and dividing decimals by the powers of ten.

Since the chapter focuses on restudying the mechanics of decimal arithmetic, it is a good time to stress to your student(s) the need for **accurate calculations** and for **checking one's final answer**. Notice how the lessons often ask students to estimate the answer before calculating the exact answer. Estimation can be used as a type of check for the final answer: if the final answer is far from the estimation, there is probably an error in the calculation. It can also be used to check if an answer calculated with a calculator is likely correct.

In the lessons about multiplication and division of decimals, students work both with mental math and with standard algorithms. The lessons that focus on mental math point out various patterns and shortcuts for students, helping them to see the structure and logic in math. I have also explained why the common rules (or shortcuts) for decimal multiplication and decimal division actually work, essentially providing a mathematical proof on a level that sixth graders can hopefully understand.

The last lessons deal with measuring units and the metric system, rounding out our study of decimals.

Consider mixing the lessons here with lessons from some other chapter. For example, the student could study decimals and some other topic on alternate days, or study a little of each topic each day. Such somewhat spiral usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

As a reminder, check out this list of resources for challenging problems:
https://l.mathmammoth.com/challengingproblems

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 3

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch3

SCAN ME

Place Value with Decimals

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
4	**5**	**7**	**3.**	**9**	**1**	**6**	**0**	**7**	**2**

The various places on the place value chart are positioned symmetrically around the ONES place.

From the ones place, we have tens to the left, and tenths to the right. Two places away are hundreds to the left, and hundredths to the right. Three places away are thousands to the left, and thousandths to the right and so on.

In expanded form:

$$4 \cdot 1{,}000 + 5 \cdot 100 + 7 \cdot 10 + 3 \cdot 1 + 9 \cdot \frac{1}{10} + 1 \cdot \frac{1}{100} + 6 \cdot \frac{1}{1{,}000} + 7 \cdot \frac{1}{100{,}000} + 2 \cdot \frac{1}{1{,}000{,}000}$$

Using decimals: 4 000 + 500 + 70 + 3 + 0.9 + 0.01 + 0.006 + 0.00007 + 0.000002

Example 1.

6 hundred-thousandths is $\frac{6}{100{,}000}$ or 0.00006.

It has five decimal places, the same as one hundred thousand (100,000) has five zeros.

Example 2.

123 ten-thousandths is $\frac{123}{10{,}000}$ or 0.0123.

There are four decimal places, the same as ten thousand (10,000) has four zeros.

Example 3. 7 millionths is 0.000007. It has six decimal places, the same as one million has six zeros.

0.000007 *also* happens to have six zeros, if you count the zero in the ones place. However, think of it as having six *decimal places*, instead, because that allows you to easily convert, for example, 453 millionths or 6,795 millionths into decimals: 0.000453 and 0.006795. They do not have six zeros, but they *do* have six decimal places.

Example 4. 465 hundredths is $\frac{465}{100}$.

As a decimal, it needs to have two decimal places because it is so many hundredths. (You can remember that because 100 has two zeros.) So it is 4.65.

Example 5.

2,180, 964 ten-thousandths is $\frac{2{,}180{,}964}{10{,}000}$.

As a decimal, it needs to have four decimal places because it is so many ten-thousandths (and 10,000 has four zeros). So it is 218.0964.

1. Draw lines to match the expressions that have the same value.

0.00006	6 parts per thousand	$\frac{6}{100{,}000}$	$\frac{15}{100{,}000}$
0.0015	15 hundred-thousandths	$\frac{6}{1{,}000}$	$\frac{15}{1{,}000}$
0.000006	15 ten-thousandths	$\frac{6}{100{,}000}$	$\frac{15}{10{,}000}$
0.00015	6 hundred-thousandths		$\frac{15}{1{,}000{,}000}$
0.006	15 parts per million		
0.000015	15 thousandths		
0.015	6 millionths		

2. Write as decimals.

a. three thousandths

b. 34 tenths

c. 1 and 1934 millionths

d. 34 ten-thousandths

e. 907 millionths

f. 837 hundred-thousandths

g. 52 hundredths

h. 8 hundred-thousandths

i. 3 and 17 thousandths

j. 91 millionths

k. 1 and 56 thousandths

l. 2 and 28 319 millionths

m. 291 ten-thousandths

n. 4 and 5 millionths

3. Write as pure fractions, *not* as mixed numbers—that is, the numerator (the number on the top) can be greater than the denominator (the number on the bottom).

a. 0.09

b. 0.005

c. 0.045

d. 0.00371

e. 0.02381

f. 3.0078

g. 2.9302

h. 2.003814

i. 5.3925012

j. 0.0000031

k. 3.294819

l. 45.00032

4. Write in expanded form, as a sum of fractions. Follow the example.

a. $2.67 = 2 \cdot 1 + 6 \cdot \frac{1}{10} + 7 \cdot \frac{1}{100}$

b. 0.594

c. 45.6

d. 0.004923

e. 0.00000506

5. Write as decimals.

a. $60 + 5 + \frac{2}{10} + \frac{8}{100} + \frac{6}{1{,}000}$	**b.** $5 + \frac{5}{100} + \frac{5}{1{,}000} + \frac{9}{1{,}000{,}000}$
c. $700 + \frac{1}{1{,}000} + \frac{3}{100{,}000} + \frac{7}{100}$	**d.** $\frac{1}{100} + \frac{3}{10{,}000} + \frac{4}{1{,}000{,}000}$
e. $\frac{9}{100} + 6 + \frac{3}{10{,}000} + \frac{5}{10}$	**f.** $\frac{2}{100} + 2 + \frac{1}{1{,}000} + \frac{1}{100{,}000}$

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Rounding Decimals

Let's review the rounding rules for decimals once again.
This time you will practice with "longer" decimals!

1. Find the digit that you are rounding to. You can draw a "cut-off line" after that digit to help you.
2. Look at the *next smaller* place (the digit after that). If that digit is 4 or less, round down. If it is 5 or more, round up.
3. If you round *up*, the digit in the place that you are rounding to will go up by 1. If you round *down*, that digit stays the same.
4. All the digits *after* the place you are rounding to become zeros... BUT, **if those are decimal digits, we do not write them!** We simply cut off those decimal digits.

   For example, 0.274|91 ≈ 0.275.

Rounding to the nearest ten:
Look at the ones digit.

↓
32|**5**.067248 ≈ 330

Rounding to the nearest hundredth:
Look at the thousandths digit.

↓
325.06|**7**248 ≈ 325.07

Rounding to the nearest ten-thousandth:
Look at the hundred-thousandths digit.

↓
325.0672|**4**8 ≈ 325.0672

1. Round to the place (digit) just before the dashed line.

a. 2.6|72 ≈	**b.** 3.055|23	**c.** 2.26|54	**d.** 0.048|97

2. Round to the nearest hundredth.

a. 7.248	**b.** 0.02499	**c.** 1.358	**d.** 4.97611

3. Round to the nearest thousandth.

a. 7.249392	**b.** 0.02684	**c.** 1.39452	**d.** 4.908472

4. Jack bought coffee for $1.80, rolls for $0.95, a meal for $6.75 and two "kids' meals" for $6.15 each.

   a. Estimate the total cost by rounding the numbers to the nearest dollar.

   b. Estimate Jack's change from $30.

   c. Find the exact cost and the error of estimation.

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Chapter 3 Review

1. Write as decimals.

a. three ten-thousandths

b. 39,234 hundred-thousandths

c. 4 millionths

d. 2 and 5 thousandths

2. Write as fractions.

a. 0.00039

b. 0.0391

c. 4.0032

3. Write as decimals.

a. $\frac{3}{4}$	**b.** $1\frac{2}{5}$	**c.** $\frac{17}{20}$	**d.** $\frac{11}{25}$

4. Fill in the table, noting that 1 micrometer is 1 millionth of a meter ($\frac{1}{1{,}000{,}000}$ of a meter).

Organism	Size (fraction)	Size (micrometers)	Size (decimal)
amoeba proteus	$\frac{600}{1{,}000{,}000}$ meters	_______ micrometers	0.0006 m
protozoa	from $\frac{10}{1{,}000{,}000}$ to $\frac{50}{1{,}000{,}000}$ m	from <u>10</u> to <u>50</u> micrometers	from _____ to _____ m
bacteria	from $\frac{1}{1{,}000{,}000}$ to $\frac{5}{1{,}000{,}000}$ m	from ______ to ______ micrometers	from _____ to _____ m

5. Write in order from the smallest to the largest.

a. 0.0256 0.000526 0.0062	**b.** 0.000087 0.000007 0.00008

6. Round to...

	0.37182	0.04828384	0.39627	0.099568
the nearest hundredth				
the nearest ten-thousandth				

7. Calculate in your head.

a. $0.02 + \frac{4}{1{,}000}$	**b.** $0.7 + \frac{5}{100}$	**c.** $3.021 + \frac{22}{1{,}000}$

8. Calculate. Remember to line up the decimal points.

a. 2.1 – 1.09342

b. 17 + 93.1 + 0.0483

9. Find the value of the expression $y + 0.04$ when

a. $y = 0.1$	**b.** $y = 0.01$	**c.** $y = 0.0001$

10. Divide in your head. For each division, write a corresponding multiplication.

a. 0.48 ÷ 6 =	**b.** 1.5 ÷ 0.3 =	**c.** 0.056 ÷ 0.008 =

11. Multiply in your head.

a. 3 · 0.006 =	**b.** 0.2 · 0.6 =	**c.** 0.9 · 0.0007 =

12. 327 · 4 is 1 308. Based on that, figure out the answer to 32.7 · 0.004.

13. **a.** Estimate the answer to 8.9 · 0.061.

b. Calculate the exact answer.

14. Solve the equations by thinking logically.

a. 3 · ________ = 0.09	**b.** 0.2 · ________ = 0.024	**c.** 0.03 · ________ = 0.0015

15. Solve the equations. If necessary, round your answers to three decimals.

a. $0.4p = 90$	**b.** $0.03x = 5.2$	**c.** $y + 0.056 = 0.38$

16. Jim cut seven 0.56-meter pieces out of a 4-meter board.
How much is left?

17. Multiply or divide the decimals by the powers of ten.

a. $10^6 \cdot 21.7 =$	**b.** $100 \cdot 0.00456 =$
c. $2.3912 \div 1{,}000 =$	**d.** $324 \div 10^5 =$
e. $10^5 \cdot 0.003938 =$	**f.** $0.7 \div 10^4 =$

18. Find the value of the expression $\dfrac{a}{b} + 1$
when $a = 2.068$ and $b = 0.8$.

19. Divide, giving your answer as a decimal. If necessary, round the answers to three decimal digits.

a. $28.2 \div 2$	**b.** $0.11 \div 15$
c. $\dfrac{4}{9}$	**d.** $\dfrac{5}{11}$

20. Fill in the entries missing from this table.

Prefix	Meaning	Units - length	Units - mass	Units - volume
			centigram (cg)	
deci-				deciliter (dl)
	ten = 10		decagram (dag)	
				hectoliter (hl)

21. Change into the basic unit (meter, liter, or gram). Think of the meaning of the prefix.

a. 34 dl   **b.** 89 cg   **c.** 16 kl

22. Convert the measurements into the given units.

a. 2.7 L = _______ dl = ________ cl = ________ ml

b. 5,600 m = ________km = _________ dm = _________ cm

c. 676 g = _______ dg = ________ cg = ________ mg

23. You have eleven empty soda bottles. Six are 350 ml, two are 2 liters and three are 9 dl. What is the total amount of water that you can put into them?

24. Convert into the given units. Round your answers to 2 decimals if needed.

a. 56 m = ____________ km	**c.** 2.7 L = ___________ ml	**e.** 0.48 km = ___________ m
b. 134 g = ____________ kg	**d.** 0.391 kg = ___________ g	**f.** 2.45 m = _____m _____ cm

25. For a parade, each of 230 children needs a ribbon that is at least 60 cm long. If you buy a 150-m roll of ribbon, how long will the ribbons be if you divide the roll equally?

26. A scientist measured the length of some tadpoles caught from a pond. The recorded lengths are below, in centimeters. Find the average length of the tadpoles.

3.2 3.1 3.4 3.1 3.5 2.9 2.7 2.7 3.0 3.0 3.1
3.4 3.2 2.8 2.8 2.9 3.6 3.4 2.9 3.4 3.1

Chapter 4: Ratios
Introduction

In this chapter we concentrate on the concept of ratio and various applications involving ratios and rates.

The chapter starts out with the basic concepts of ratio, rate and unit rate. We also connect the concept of rates (specifically, tables of equivalent rates) with ordered pairs, use equations (such as $y = 3x$) to describe these tables, and plot the ordered pairs in the coordinate plane.

Next, we study various kinds of word problems involving ratios and use a bar model to solve these problems in two separate lessons. These lessons tie ratios in with the student's previous knowledge of bar models as a tool for problem solving.

Lastly, students encounter the concept of aspect ratio, which is simply the ratio of a rectangle's width to its height or length, and they solve a variety of problems involving aspect ratio.

This chapter contains lots of opportunities for problem solving, once again. In the lessons that use bar models, encourage your student(s) to communicate their thinking and explain (justify) how they solved the problems. It doesn't have to be fancy. All we are looking for is some explanation of what the student did and why. The bar models provide an excellent way for the students to demonstrate their reasoning here. Essentially, they are practicing constructing a **mathematical argument**.

Once again, there are some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 4

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch4

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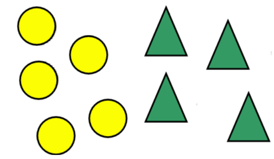
Ratios and Rates



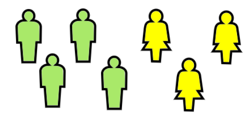
A **ratio** is simply a *comparison* of two numbers or other quantities.

To compare the circles to the triangles in the picture, we say that the *ratio of circles to triangles* is 5:4 (read "five to four").

We can write this ratio (in text) in many different ways:

- The ratio of circles to triangles is 5:4 (read "5 to 4").
- The ratio of circles to triangles is 5 to 4.
- The ratio of circles to triangles is $\frac{5}{4}$.
- For each five circles, there are four triangles.

The two numbers in the ratio are called the **first term** and the **second term** of the ratio.

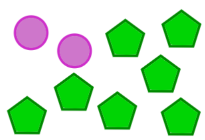


In this picture, the ratio of males to females is 4:3. However, the ratio of *females to males* is **3:4**. The order in which the terms are mentioned does matter!

We can also compare a part to the whole. The ratio of males to everyone is 4:7.

Also, we can use fractions to describe the same image: $\frac{4}{7}$ of the people are males, and $\frac{3}{7}$ are females.

1. Describe the images using ratios and fractions.



a.

The ratio of circles to pentagons is ____ : ____

The ratio of pentagons to all shapes is ____ : ____

$\frac{\square}{\square}$ of the shapes are pentagons.

b.

The ratio of hearts to stars is ____ : ____

The ratio of stars to all shapes is ____ : ____

$\frac{\square}{\square}$ of the shapes are stars.

2. **a.** Draw a picture: There are hearts and circles, and the ratio of hearts to all the shapes is 1:3.

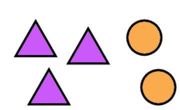
   b. What is the ratio of hearts to circles?

3. Look at the picture of the triangles and circles. If we drew more triangles and circles in the same ratio, how many circles would there be …

   a. … for 9 triangles?

   b. … for 15 triangles?

   c. … for 300 triangles?



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Using Equivalent Rates

Example 1. If Jake can ride his bike to a town that is 33 kilometers away in 45 minutes, how far can he ride in 1 hour?

Let's form some equivalent rates, starting with 33 kilometers per 45 minutes and hoping to arrive at so many kilometers per 60 minutes.

However, it is not easy to go directly from 45 minutes to 60 minutes (1 hour). So, first you figure the rate for <u>15 minutes</u>, which *is* easy.

Why? Because to get from 45 minutes to 15 minutes you simply divide both terms of the rate by 3.

$$\frac{33 \text{ kilometers}}{45 \text{ minutes}} \overset{\div 3}{=} \frac{11 \text{ kilometers}}{15 \text{ minutes}} \overset{\cdot 4}{=} \frac{44 \text{ kilometers}}{60 \text{ minutes}}$$

Then from 15 minutes, we can easily get to 60 minutes: Just multiply both terms by 4. We find that he can ride 44 kilometers in one hour.

1. Write the equivalent rates.

a. $\frac{15 \text{ km}}{3 \text{ hr}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{45 \text{ min}}$	**b.** $\frac{\$6}{45 \text{ min}} = \frac{\quad}{15 \text{ min}} = \frac{\quad}{1 \text{ hr}} = \frac{\quad}{1 \text{ hr } 45 \text{ min}}$
c. $\frac{3 \text{ cm}}{8 \text{ m}} = \frac{\quad}{2 \text{ m}} = \frac{\quad}{12 \text{ m}} = \frac{\quad}{20 \text{ m}}$	**d.** $\frac{115 \text{ words}}{2 \text{ min}} = \frac{\quad}{1 \text{ min}} = \frac{\quad}{3 \text{ min}}$

2. **a.** James can ride 10 kilometers in 16 minutes. How long will it take him to ride 55 kilometers? Use the equivalent rates.

$$\frac{10 \text{ kilometers}}{16 \text{ minutes}} = \frac{5 \text{ kilometers}}{\square \text{ minutes}} = \frac{55 \text{ kilometers}}{\square \text{ minutes}}$$

b. How many kilometers can James ride in 40 minutes?

3. An automobile can go 80 kilometers on 8 liters of gasoline.

a. How many liters of gas would the automobile need for a trip of 95 kilometers? Use the equivalent rates below.

$$\frac{80 \text{ kilometers}}{8 \text{ liters}} = \frac{10 \text{ kilometers}}{\square \text{ liters}} = \frac{95 \text{ kilometers}}{\square \text{ liters}}$$

b. How far can the automobile travel on 15 liters of gas?

Example 2. You get 20 erasers for $5.00. How much would 22 erasers cost?

You can solve this problem in several ways. Let's use a table of rates this time.

Cost (C)			$2.50	$5.00	
Erasers (E)	1	2	10	20	22

First, find the cost for 10 erasers, and then the cost for 2. After that, you can get the cost for 22 by adding.

Ten erasers will cost half of $5.00. Two erasers will cost one-fifth of that (divide by 5 to find it!). Lastly, add the cost of 20 erasers to the cost of 2 erasers to get the cost for 22 erasers.

Note 1: In the table, each pair of numbers is a rate. For example, $5.00 for 20 erasers (or $5.00/20 erasers) is a rate, and so is $2.50 for 10 erasers.

Note 2: Let's write an equation relating the Cost (C) and the number of Erasers (E). You will find that easily from the unit rate (the price for one): $C = 0.25E$. In other words, the cost is 0.25 times the number of erasers.

4. Finish solving the problem in the example above.

5. How many erasers would you get for $1.75?

6. On average, Scott makes a basket nine times out of twelve shots when he is practicing. How many baskets can he expect to make with 200 shots? A table of rates can help you solve this.

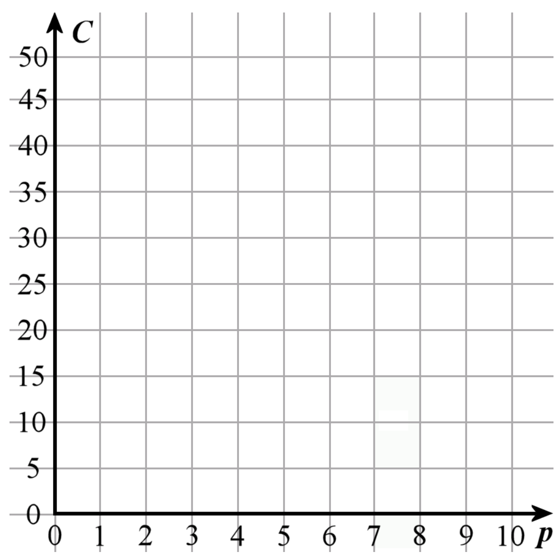
baskets						
shots						

7. **a.** Three pairs of socks cost $9. Fill in the table of rates. The variable *C* stands for cost, and *p* for pairs of socks.

C			9							
p	1	2	3	4	5	6	7	8	9	10

b. Each number pair in the table *is* a rate, but we can also view them as points with two coordinates. Plot the number pairs in the coordinate grid.

c. Write an equation relating the cost (C) and the number of pairs of socks (p).



8. **a.** You get 30 pencils for $4.50. How much would 52 pencils cost?

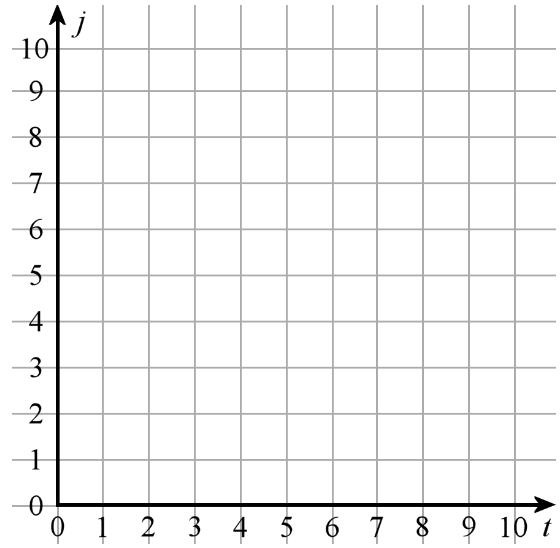
Cost						
Pencils						

b. Write an equation relating the cost (C) and the number of pencils (P).

9. When Kate makes 4 liters of tea (a pot full), she needs five jars for the tea. From this, we get the rate of 4 liters / 5 jars.

a. Fill in the table. The variable *t* stands for the amount of tea, and *j* for the number of jars.

t					4					
j	1	2	3	4	5	6	7	8	9	10



b. Plot the number pairs from the table in this coordinate grid.

c. How many jars will Kate need for 20 liters of tea?

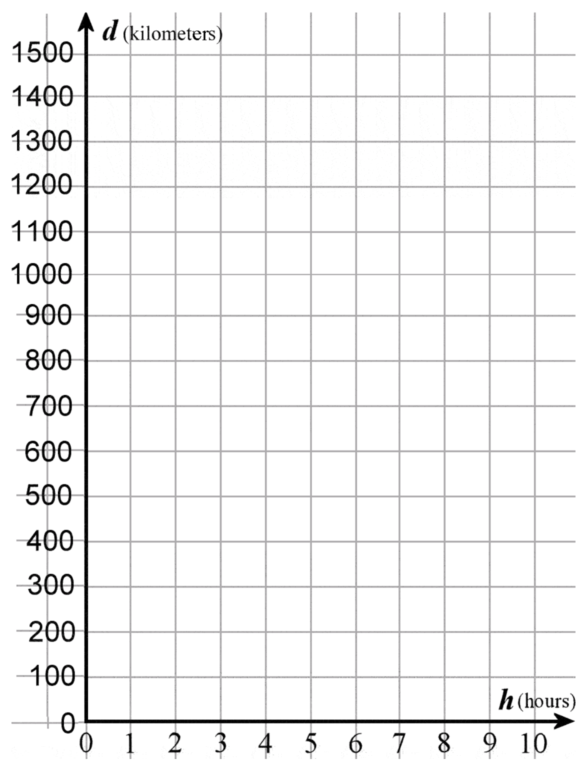
d. If Kate has 16 jars full of tea, how many liters of tea is in them?

10. **a.** A train travels at a constant speed of 130 kilometers per hour. Fill in the table of rates.

d										
h	1	2	3	4	5	6	7	8	9	10

b. Write an equation relating the distance (*d*) and the number of hours (*h*).

c. Plot the points in the grid on the right. The variable *h* stands for hours, and *d* for distance.



11. Another train travels at the constant speed of 96 km per hour. Fill in the table of rates. Then, plot the points in the same coordinate grid as for the train in #10.

d					
h	1	2	3	4	5

d					
h	6	7	8	9	10

12. *How* can you see <u>from the graph</u> which train travels faster?

13. The plot shows the walking speed for two people (t is in minutes, d is in kilometers). Your task is to fill in the two ratio tables below. To make that easier, first find the dots that are at places where the lines cross, so that you can easily read the coordinates.
(Hint: For some of the points, you will need to use decimals and whole numbers.)

Person 1 (red dot)

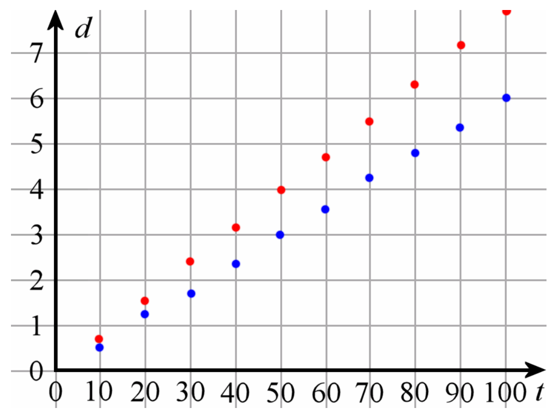
d **(kilometers)**										
t **(minutes)**										

Person 2 (blue dot)

d **(kilometers)**										
t **(minutes)**										

a. What is the speed of the first person in kilometers per hour?

b. What is the speed of the second person in kilometers per hour?



14. Train 1 travels at a constant speed of 165 miles in three hours. Train 2 travels 315 miles in seven hours. Which train is faster?

15. Find which is a better deal by comparing the unit rates: \$45 for eight bottles of shampoo, or \$34 for six bottles of shampoo?

16. In a poll of 1,000 people, 640 said they liked blue.

a. Simplify this ratio to lowest terms:

640 people *out of* 1,000 people = ________ people *out of* _______ people

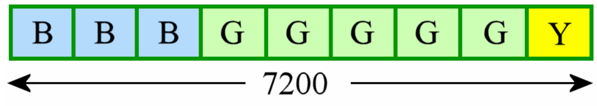
b. Assuming the same ratio holds true in another group of 100 people, how many of those people can we expect to like blue?

c. Assuming the same ratio holds true in another group of 225 people, how many of those people can we expect to like blue?

Ratio Problems and Bar Models 1

Often, ratio problems become easy by drawing a **bar model**.

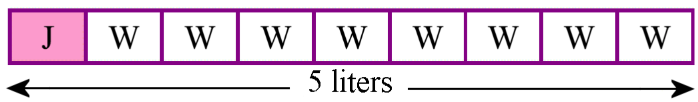
The ratio of blue shirts to green shirts to yellow shirts is 3 to 5 to 1. If there are 7,200 shirts in all, how many of them are of each color?



Look at the bar model. There are a total of 7,200 shirts.
We draw 3 “blocks” for the blue shirts, 5 “blocks” for the green shirts and 1 “block” for the yellow shirts to show the ratio of **3 : 5 : 1**. It is obvious one “block” means 7,200 ÷ 9 = 800 shirts. So there are a total of 2,400 blue shirts, 4,000 green shirts and 800 yellow shirts.

Juice concentrate is mixed with water in a ratio of 1:8. If you want to make 5 liters of juice, how much concentrate and how much water do you need?

Let’s draw a bar model. (In reality, of course, the juice and water mix, but for the purpose of calculating, this model is helpful.)



There are a total of 9 equal parts, so we simply divide 5 liters by 9. First, change 5 liters to 5,000 milliliters, and then divide: 5,000 ml ÷ 9 ≈ 555.56 ml.

However, that is way too accurate. Measuring cups do not normally let us measure to the nearest milliliter, and not even to the nearest 10 milliliters, so let’s round this to the nearest 50 ml to get 550 ml.

So we need 550 ml of juice concentrate and 5,000 ml − 550 ml = 4,450 ml of water.

1. A factory makes shirts in a ratio of 1:3:3:1 for the sizes S, M, L and XL, respectively.

   a. Draw a bar model. What is the ratio of small (S) shirts to the total number of shirts?

   b. In a batch of 1,000 shirts, how many of them are of each size?

2. The instructions on a box of juice concentrate say to mix 2 parts of concentrate to 5 parts of water.

   a. If you want to make 3 liters of juice, how much concentrate and how much water do you need?

   b. Let’s say that you have 1/2 liter of concentrate left. According to the instructions, how much water would you need to add to that?

   How much diluted juice does this make?

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Chapter 5: Percent
Introduction

This chapter is all about the basics of the concept of percent—a very important topic in regards to real life. We focus on how to calculate percentages (e.g. what percentage is $20 of $50) and how to find a certain percentage of a given number or quantity (e.g. what is 20% of 80 km). In seventh grade, students learn about percent of change and how to make comparisons with percent.

The lessons emphasize the connection between percentages and fractions and decimals in various ways. After all, percentages *are* fractions: the word percent simply means “a hundredth part,” and the concept of percent builds on the student’s previous understanding of fractions and decimals.

Specifically, the student should be very familiar with the idea of finding a fractional part of a whole (such as finding 3/4 of $240). Students using Math Mammoth have been practicing that concept since fourth grade, and one reason why I have emphasized finding a fractional part of a whole in the earlier grades is specifically to lay a groundwork for the concept of percent. Assuming the student has mastered that, and can easily convert fractions to decimals, then studying the concept of percent should not be difficult.

In this context of thinking of percentages as fractions, students learn how to find a percentage of a given number or quantity using **mental math techniques**. For example, students find 10% of $400 by thinking of it as 1/10 of $400, and thus dividing $400 by 10. They also learn to find a percentage of a quantity using *decimal* multiplication, both manually and with a calculator. For example, students find 17% of 45 km by multiplying 0.17 × 45 km.

In fact, in cases where mental math is not a good option, I prefer teaching students to calculate percentages of quantities using decimals, instead of using percent proportion or fractions. That is because using decimals is simpler and quicker. Also, this method is often superior later on in algebra courses, when students need to write equations from verbal descriptions, and symbolically represent situations that involve percentages.

The last lesson of the chapter teaches students how to find the total when the percentage and the partial amount are known. For example: “Three-hundred twenty students, which is 40% of all students, take PE. How many students are there in total?” Students solve these with the help of the visual bar models, which they are already familiar with.

As the lessons constantly refer back to fractions and decimals, students can relate calculations with percentages to their earlier knowledge, and thus see **the logical structure of mathematics**. It will also prevent students from memorizing calculations with percentages without understanding what is going on.

As a reminder, it is not recommended that you assign all the exercises by default. Use your judgment, and strive to vary the number of assigned exercises according to the student’s needs. Some students might only need half or even less of the available exercises, in order to understand the concepts.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/**.

The Lessons in Chapter 5

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch5

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Percentage of a Number (Mental Math)

100% of something means *all* of it.   1% of something means 1/100 of it.

Since one percent means “a hundredth part,” calculating a percentage of a quantity is the same thing as finding a fractional part of it. So **percentages are really fractions**!

How much is 1% of 200 kg? This means how much is 1/100 of 200 kg? It is simply 2 kg.

To find 1% of something (1/100 of something), divide by 100.

Do you remember how to divide by 100 in your head? Just move the decimal point two places to the left. For example, 1% of 540 is 5.4, and 1% of 8.30 is 0.083.

To find 2% of some quantity, first find 1% of it, and double that.

For example, let’s find 2% of $6. Since 1% of $6 is $0.06, then 2% of $6 is $0.12.

To find 10% of some quantity, divide by 10.

Why does that work? It is because 10% is 10/100, which equals 1/10. So 10% is 1/10 of the quantity!

For example, 10% of $780 is $78. And 10% of $6.50 is $0.65.
(To divide by 10 in your head, just move the decimal point one place to the left.)

Can you think of a way to find 20% of a number?

____

1. Find 10% of these numbers.

a. 700 _______   **b.** 321 _______   **c.** 60 _______   **d.** 7 _______

2. Find 1% of these numbers.

a. 700 _______   **b.** 321 _______   **c.** 60 _______   **d.** 7 _______

3. One percent of Mother’s paycheck is $22. How much is her total paycheck?

4. Fill in the table. Use mental math.

percentage ↓ number →	**1,200**	**80**	**29**	**9**	**5.7**
1% of the number					
2% of the number					
10% of the number					
20% of the number					

5. Fill in this guide for using mental math with percentages:

Mental Math and Percentage of a Number	
50% is $\frac{1}{2}$. To find 50% of a number, divide by _______. **10% is $\frac{1}{}$. To find 10% of a number, divide by ________.** **1% is $\frac{1}{}$. To find 1% of a number, divide by ________.**	50% of 244 is _________. 10% of 47 is __________. 1% of 530 is __________ .
To find 20%, 30%, 40%, 60%, 70%, 80%, or 90% of a number, • **First find __________% of the number, and** • **then multiply by 2, 3, 4, 6, 7, 8, or 9.**	10% of 120 is __________. 30% of 120 is __________. 60% of 120 is __________.

6. Find the percentages. Use mental math.

a. 10% of 60 kg __________ 20% of 60 kg __________	**b.** 10% of $14 __________ 30% of $14 __________	**c.** 10% of 5 m __________ 40% of 5 m __________
d. 1% of $60 __________ 4% of $60 __________	**e.** 10% of 110 cm __________ 70% of 110 cm __________	**f.** 1% of $1,330 __________ 3% of $1,330 __________

7. David pays a 20% income tax on his $2,100 salary.

a. How many dollars is the tax?

b. How much money does he have left after paying the tax?

c. What percentage of his salary does he have left?

8. Nancy pays 30% of her $3,100 salary in taxes. How much money does she have left after paying the tax?

9. Identify the errors that these children made. Then find the correct answers.

a. Find 90% of $55. Peter's solution: 10% of $55 is $5.50 So, I subtract 100% − $5.50 = $94.50	**b.** Find 6% of $1,400. Patricia's solution: 1% of $1,400 is $1.40. So, 6% is six times that, or $8.40.

Some more mental math "tricks"	
90% of a quantity First find 10% of the quantity and then subtract that from 100% of it.	**25% of a quantity** 25% is the same as 1/4. So, to find 25% of a quantity, divide it by 4.
12% of a quantity First find 10% of it. Then find 1% of it, and use that 1% to find 2% of it. Then add the 10% and the 2%.	**75% of a quantity** 75% is 3/4. First find 1/4 of the quantity and multiply that by 3.

10. Find percentages of the quantities.

a. 50% of 26 cm ____________	**b.** 25% of 40 mm __________	**c.** 80% of 45 m ____________
d. 75% of $4.40 ____________	**e.** 90% of 1.2 m ____________	**f.** 25% of 120 kg ____________

11. Fill in the mental math method for finding 12% of $65.

10% of $65 is $__________. 1% of $65 is $___________. 2% of $65 is $___________.

Now, add to get 12% of $65: $___________ + $___________ = $___________

12. Fill in the mental math shortcut for finding 24% of 44 kg.

25% of 44 kg is __________ kg. 1% of 44 kg is ___________ kg.

Subtract __________ kg − __________ kg = __________ kg

13. From her cell phone bill, Hannah sees that of the 340 text messages she sent last month, 15% were sent during the night at a cheaper rate. How many messages did Hannah send at night? During the day?

14. A herd of 40 horses had some bay, some chestnut and some white horses. Thirty percent of them are bay, and 45% are chestnut. How many horses are white?

15. A college has 1,500 students, and 12% of them ride the bus. Another 25% walk to the college. How many students do not do either?

Percentage of a Number: Using Decimals

You have learned that finding 1% of a number means finding 1/100 of it. Similarly, finding 60% of a number means finding 60/100 (or 6/10) of it.

In these types of expressions, the word "of" translates into **multiplication**:

$$1\% \text{ of } 90 \;\downarrow\; 1\% \cdot 90 \quad \text{OR} \quad 60\% \text{ of } \$700 \;\downarrow\; 60\% \cdot \$700$$

Next, let's write those percentages as *decimals*. We get:

$$1\% \text{ of } 90 \;\downarrow\; 0.01 \cdot 90 \quad \text{OR} \quad 60\% \text{ of } \$700 \;\downarrow\; 0.6 \cdot \$700$$

This gives us another way to calculate a certain percentage of a number (or a percentage of some quantity):

To calculate a percentage of a number, you need to make TWO simple changes:

1. **Change the percentage into a decimal.**
2. **Change the word "of" into multiplication.**

Example 1. Find 70% of 80.

Making the two changes, we write this as 0.7 · 80.

(Remember, in decimal multiplication, you multiply just as if there were no decimal points, and the answer will have as many decimal digits as the total number of decimal digits in all of the factors.)

So, when you multiply 0.7 · 80, think of multiplying 7 · 80 = 560. Since 0.7 has one decimal digit, and 80 has none, the answer has one decimal digit. Thus, 0.7 · 80 = 56.0 or just 56.

You can also use common sense and estimation: 0.7 · 80 must be less than 80, yet more than 1/2 of 80, which is 40. Since 7 · 8 = 56, you know that the answer must be 56—not 5.6 or 560.

Example 2. Find 3% of $4,000.

First, write this as 0.03 · $4,000. Next, multiply without decimal points: 3 · $4,000 = $12,000.
Lastly, put the decimal point so that the answer will have two decimal digits: $120.00.

Example 3. Find 23% of 5,500 km.

Write this as 0.23 · 5,500 km and use a calculator. The answer is 1,265 km. This makes sense, because 10% of 5,500 km is 550 km, and 20% of it is 1,100 km. Therefore, 1,265 km as 23% of 5,500 km is reasonable.

1. "Translate" the expressions into multiplications by a decimal. Solve, using mental math.

a. 20% of 70 _______ · _______ = _______	**b.** 90% of 50 _______ · _______ = _______	**c.** 80% of 400 _______ · _______ = _______
d. 60% of $8 _______ · _______ = _______	**e.** 9% of 3,000 _______ · _______ = _______	**f.** 7% of 40 L _______ · _______ = _______
g. 150% of 44 kg _______ · _______ = _______	**h.** 200% of 56 students _______ · _______ = _______	**i.** 2% of 1,500 km _______ · _______ = _______

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Contents

Chapter 9: Geometry

Chapter 10: Statistics

Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Some of them, such as fractions and decimals, students are familiar with, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers).

The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: summarizing distributions using measures of center and variability.

This book, 6-B, covers number theory topics (chapter 6), fractions (chapter 7), integers (chapter 8), geometry (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 6-A worktext.

Chapter 6 first reviews prime factorization and then applies those principles to using the greatest common factor to simplify fractions and the least common multiple to find common denominators. Chapter 7 provides a thorough review of the fraction operations from fifth grade, and includes ample practice in solving problems with fractions.

Chapter 8 introduces students to integers. Students plot points in all four quadrants of the coordinate plane, reflect and translate simple figures, and learn to add and subtract with negative numbers. (Multiplication and division of integers will be studied in 7th grade.)

The next chapter, Geometry, focuses on calculating the area of polygons. The final chapter is about statistics. Beginning with the concept of a statistical distribution, students learn about measures of center and measures of variability. They also learn how to make dot plots, histograms, and boxplots, as ways to summarize and analyze distributions.

I heartily recommend that you read the full user guide in the following pages.

I wish you success in teaching math!

Maria Miller, the author

User Guide

Note: You can also find the information that follows online, at **https://www.mathmammoth.com/userguides/** .

Basic principles in using Math Mammoth Complete Curriculum

Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. The two books (parts A and B) are like a “framework”, but you still have a lot of liberty in planning your child’s studies. You can even use it in a *spiral* manner, if you prefer. Simply have your student study in 2-3 chapters simultaneously. In sixth grade, chapters 1 and 2 should be studied before the other chapters, but you can be flexible with all the other chapters and schedule them earlier or later.

Math Mammoth is not a scripted curriculum. In other words, it is not spelling out in exact detail what the teacher is to do or say. Instead, Math Mammoth gives you, the teacher, various tools for teaching:

- **The two student worktexts** (parts A and B) contain all the lesson material and exercises. They include the explanations of the concepts (the teaching part) in blue boxes. The worktexts also contain some advice for the teacher in the “Introduction” of each chapter.

  The teacher can read the teaching part of each lesson before the lesson, or read and study it together with the student in the lesson, or let the student read and study on his own. If you are a classroom teacher, you can copy the examples from the “blue teaching boxes” to the board and go through them on the board.

- There are hundreds of **videos** matched to the curriculum available at https://www.mathmammoth.com/videos/ . There isn’t a video for every lesson, but there are dozens of videos for each grade level. You can simply have the author teach your child or student!

- Don’t automatically assign all the exercises. Use your judgment, trying to assign just enough for your student’s needs. You can use the skipped exercises later for review. For most students, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.

- For each chapter, there is a **link list to various free online games** and activities. These games can be used to supplement the math lessons, for learning math facts, or just for some fun. Each chapter introduction (in the student worktext) contains a link to the list corresponding to that chapter.

- The student books contain some **mixed review lessons**, and the curriculum also provides you with additional **cumulative review lessons**.

- There is a **chapter test** for each chapter of the curriculum, and a comprehensive end-of-year test.

- The **worksheet maker** allows you to make additional worksheets for most calculation-type topics in the curriculum. This is a single html file. You will need Internet access to be able to use it.

- You can use the free online exercises at https://www.mathmammoth.com/practice/
  This is an expanding section of the site, so check often to see what new topics we are adding to it!

- Some grade levels have **cut-outs** to make fraction manipulatives or geometric solids.

- And of course there are answer keys to everything.

How to get started

Have ready the first lesson from the student worktext. Go over the first teaching part (within the blue boxes) together with your child. Go through a few of the first exercises together, and then assign some problems for your child to do on their own.

Repeat this if the lesson has other blue teaching boxes. Naturally, you can also use the videos at https://www.mathmammoth.com/videos/

Many children can eventually study the lessons completely on their own — the curriculum becomes self-teaching. However, children definitely vary in how much they need someone to be there to actually teach them.

Pacing the curriculum

The lessons in Math Mammoth complete curriculum are NOT intended to be done in a single teaching session or class. Sometimes you might be able to go through a whole lesson in one day, but more often, the lesson itself might span 3-5 pages and take 2-3 days or classes to complete.

Therefore, it is not possible to say exactly how many pages a student needs to do in one day. This will vary. However, it is helpful to calculate a general guideline as to how many pages per week you should cover in the student worktext in order to go through the curriculum in one school year (or whatever span of time you want to allot to it).

The table below lists how many pages there are for the student to finish in this particular grade level, and gives you a guideline for how many pages per day to finish, assuming a 180-day school year.

Example:

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166	92	10	82	2	10
6-B	157	88	10	78	2	10
Grade 6 total	323	180	20	160	2	10

The table below is for you to fill in. First fill in how many days of school you intend to have. Also allow several days for tests and additional review before the test — at least twice the number of chapters in the curriculum. For example, if the particular grade has 8 chapters, allow at least 16 days for tests & additional review. Then, to get a count of "pages/day", divide the number of pages by the number of available days. Then, multiply this number by 5 to get the approximate page count to cover in a week.

Grade level	Lesson pages	Number of school days	Days for tests and reviews	Days for the student book	Pages to study per day	Pages to study per week
6-A	166					
6-B	157					
Grade 6 total	323					

Now, let's assume you determine that you need to study about 2 pages a day, 10 pages a week in order to get through the curriculum. As you study each lesson, keep in mind that sometimes most of the page might be filled with blue teaching boxes and very few exercises. You might be able to cover 3 pages on such a day. Then some other day you might only assign one page of word problems. Also, you might be able to go through the pages quicker in some chapters, for example when studying graphs, because the large pictures fill the page so that one page does not have many problems.

When you have a page or two filled with lots of similar practice problems ("drill") or large sets of problems, feel free to **only assign 1/2 or 2/3 of those problems**. If your child gets it with less amount of exercises, then that is perfect! If not, you can always assign him/her the rest of the problems some other day. In fact, you could even use these unassigned problems the next week or next month for some additional review.

In general, 1st-2nd graders might spend 25-40 minutes a day on math. Third-fourth graders might spend 30-60 minutes a day. Fifth-sixth graders might spend 45-75 minutes a day. If your child finds math enjoyable, he/she can of course spend more time with it! However, it is not good to drag out the lessons on a regular basis, because that can then affect the child's attitude towards math.

Working space, the usage of additional paper and mental math

The curriculum generally includes working space directly on the page for students to work out the problems. However, feel free to let your students to use extra paper when necessary. They can use it, not only for the "long" algorithms (where you line up numbers to add, subtract, multiply, and divide), but also to draw diagrams and pictures to help organize their thoughts. Some students won't need the additional space (and may resist the thought of extra paper), while some will benefit from it. Use your discretion.

Some exercises don't have any working space, but just an empty line for the answer (e.g. 200 + ______ = 1,000). Typically, I have intended that such exercises to be done using MENTAL MATH.

However, there are some students who struggle with mental math (often this is because of not having studied and used it in the past). As always, the teacher has the final say (not me!) as to how to approach the exercises and how to use the curriculum. We do want to prevent extreme frustration (to the point of tears). The goal is always to provide SOME challenge, but not too much, and to let students experience success enough so that they can continue enjoying learning math.

Students struggling with mental math will probably benefit from studying the basic principles of mental calculations from the earlier levels of Math Mammoth curriculum. To do so, look for lessons that list mental math strategies. They are taught in the chapters about addition, subtraction, place value, multiplication, and division. My article at https://www.mathmammoth.com/lessons/practical_tips_mental_math also gives you a summary of some of those principles.

Using tests

For each chapter, there is a **chapter test**, which can be administered right after studying the chapter. **The tests are optional.** Some families might prefer not to give tests at all. The main reason for the tests is for diagnostic purposes, and for record keeping. These tests are not aligned or matched to any standards.

In the digital version of the curriculum, the tests are provided both as PDF files and as html files. Normally, you would use the PDF files. The html files are included so you can edit them (in a word processor such as Word or LibreOffice), in case you want your student to take the test a second time. Remember to save the edited file under a different file name, or you will lose the original.

The end-of-year test is best administered as a diagnostic or assessment test, which will tell you how well the student remembers and has mastered the mathematics content of the entire grade level.

Using cumulative reviews and the worksheet maker

The student books contain mixed review lessons which review concepts from earlier chapters. The curriculum also comes with additional cumulative review lessons, which are just like the mixed review lessons in the student books, with a mix of problems covering various topics. These are found in their own folder in the digital version, and in the Tests & Cumulative Reviews book in the print version.

The cumulative reviews are optional; use them as needed. They are named indicating which chapters of the main curriculum the problems in the review come from. For example, "Cumulative Review, Chapter 4" includes problems that cover topics from chapters 1-4.

Both the mixed and cumulative reviews allow you to spot areas that the student has not grasped well or has forgotten. When you find such a topic or concept, you have several options:

1. Check if the worksheet maker lets you make worksheets for that topic.
2. Check for any online games and resources in the Introduction part of the particular chapter in which this topic or concept was taught.
3. If you have the digital version, you could simply reprint the lesson from the student worktext, and have the student restudy that.
4. Perhaps you only assigned 1/2 or 2/3 of the exercise sets in the student book at first, and can now use the remaining exercises.
5. Check if our online practice area at https://www.mathmammoth.com/practice/ has something for that topic.
6. Khan Academy has free online exercises, articles, and videos for most any math topic imaginable.

Concerning challenging word problems and puzzles

While this is not absolutely necessary, I heartily recommend supplementing Math Mammoth with challenging word problems and puzzles. You could do that once a month, for example, or more often if the student enjoys it.

The goal of challenging story problems and puzzles is to **develop the student's logical and abstract thinking and mental discipline**. I recommend starting these in fourth grade, at the latest. Then, students are able to read the problems on their own and have developed mathematical knowledge in many different areas. Of course I am not discouraging students from doing such in earlier grades, either.

Math Mammoth curriculum contains lots of word problems, and they are usually multi-step problems. Several of the lessons utilize a bar model for solving problems. Even so, the problems I have created are usually tied to a specific concept or concepts. I feel students can benefit from solving problems and puzzles that require them to think "out of the box" or are just different from the ones I have written.

I recommend you use the free Math Stars problem-solving newsletters as one of the main resources for puzzles and challenging problems:

Math Stars Problem Solving Newsletter (grades 1-8)
https://www.homeschoolmath.net/teaching/math-stars.php

I have also compiled a list of other resources for problem solving practice, which you can access at this link:

https://l.mathmammoth.com/challengingproblems

Another idea: you can find puzzles online by searching for "brain puzzles for kids," "logic puzzles for kids" or "brain teasers for kids."

Frequently asked questions and contacting us

If you have more questions, please first check the FAQ at https://www.mathmammoth.com/faq-lightblue

If the FAQ does not cover your question, you can then contact us using the contact form at the Math Mammoth.com website.

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Prime Factorization, GCF and LCM
Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we review prime factorization and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over into algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as 27 + 45. For example, a sum like 27 + 45 factors into 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is in finding the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures," and you can let students read more about them by accessing the Internet resources mentioned below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list for Internet resources.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs. Please see the user guide at **https://www.mathmammoth.com/userguides/** for more guidance on using and pacing the curriculum.

You can find some free videos for the topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 6

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch6

SCAN ME

The Sieve of Eratosthenes and Prime Factorization

Remember? A number is a **prime** if it has no other factors besides 1 and itself.

For example, 13 is a prime, since the only way to write it as a multiplication is $1 \cdot 13$. In other words, 1 and 13 are its only factors.

And, 15 is not a prime, since we can write it as $3 \cdot 5$. In other words, 15 has other factors besides 1 and 15, namely 3 and 5.

To find all the prime numbers less than 100 we can use the *sieve of Eratosthenes.*

Here is an online interactive version: https://www.mathmammoth.com/practice/sieve-of-eratosthenes

1. Cross out 1, as it is not considered a prime.
2. Cross out all the even numbers except 2.
3. Cross out all the multiples of 3 except 3.
4. You do not have to check multiples of 4. Why?
5. Cross out all the multiples of 5 except 5.
6. You do not have to check multiples of 6. Why?
7. Cross out all the multiples of 7 except 7.
8. You do not have to check multiples of 8 or 9 or 10.
9. The numbers left are primes.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

List the **primes between 0 and 100** below:

2, 3, 5, 7, ____

Why do you not have to check numbers that are bigger than 10? Let's think about multiples of 11. The following multiples of 11 have already been crossed out: $2 \cdot 11$, $3 \cdot 11$, $4 \cdot 11$, $5 \cdot 11$, $6 \cdot 11$, $7 \cdot 11$, $8 \cdot 11$ and $9 \cdot 11$. The multiples of 11 that have not been crossed out are $10 \cdot 11$ and onward... but they are not on our chart! Similarly, the multiples of 13 that are less than 100 are $2 \cdot 13$, $3 \cdot 13$, ..., $7 \cdot 13$, and all of those have already been crossed out when you crossed out multiples of 2, 3, 5 and 7.

1. You learned this in 4th and 5th grades... find all the factors of the given numbers. Use the checklist to help you keep track of which factors you have tested.

a. 54 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________	**b.** 60 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________
c. 84 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________	**d.** 97 Check 1 2 3 4 5 6 7 8 9 10 factors: ________________________________

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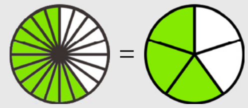
Using Factoring When Simplifying Fractions

You have seen the process of **simplifying fractions** before.

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the "same amount of pie" as it was before. It is just cut differently.



<ins>Every four slices</ins> have been joined together.

$$\frac{12}{20} \overset{\div 4}{=} \frac{3}{5} \quad (\div 4)$$

Why does this work?

It is based on finding common factors and on how fraction multiplication works. In our example above, the fraction $\frac{12}{20}$ can be written as $\frac{4 \cdot 3}{4 \cdot 5}$. Then we can **cancel out** those fours: $\frac{\cancel{4} \cdot 3}{\cancel{4} \cdot 5} = \frac{3}{5}$.

The reason this works is because $\frac{4 \cdot 3}{4 \cdot 5}$ is equal to the fraction multiplication $\frac{4}{4} \cdot \frac{3}{5}$. And in that, 4/4 is equal to 1, which means we are only left with 3/5.

Example 1. Often, the simplification is simply written or indicated this way →
Notice that here, the 4's that were cancelled out do ***not*** get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

Example 2. Here, 35 and 55 are both divisible by 5. This means we can cancel out those 5's, but notice this is not shown in any way. We simply cross out 35 and 55, think of dividing them by 5, and write the division result above and below.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

1\. Simplify the fractions, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2\. Leah simplified various fractions like you see below. She did not get them right though. Explain to her what she is doing wrong.

$$\frac{24}{84} = \frac{20}{80} = \frac{1}{4} \qquad \frac{27}{60} = \frac{7}{40} \qquad \frac{14}{16} = \frac{10}{12} = \frac{6}{8} = \frac{3}{4}$$

Using factoring when simplifying

Carefully study the example on the right where we simplify the fraction 144/96 to **lowest terms --** in other words, where the numerator and the denominator have no common factors.

- First we factor (write) 144 as 12 · 12 and 96 as 8 · 12.
- Then we simplify in two steps:
  1. 12 and 8 are both divisible by 4, so they simplify into 3 and 2.
  2. 12 and 12 are divisible by 12, so they simplify into 1 and 1. Essentially, they cancel each other out.
- Lastly we write the improper fraction 3/2 as a mixed number.

$$\frac{144}{96} = \frac{\overset{3}{\cancel{12}} \cdot \overset{1}{\cancel{12}}}{\underset{2}{\cancel{8}} \cdot \underset{1}{\cancel{12}}} = \frac{3}{2} = 1\frac{1}{2}$$

For a comparison, here is another way to write the simplification in several steps, and that you've seen in earlier grades in Math Mammoth:

$$\frac{144}{96} \underset{\div 12}{\overset{\div 12}{=}} \frac{12}{8} \underset{\div 4}{\overset{\div 4}{=}} \frac{3}{2} = 1\frac{1}{2}$$

Let's study some more examples. (Remember that they don't show the number that you divide by.)

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{5}{\cancel{35}} \cdot \underset{1}{\cancel{3}}} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\overset{3}{\cancel{9}} \cdot \overset{1}{\cancel{5}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

3. Simplify. Write the simplified numerator above and the simplified denominator below the old ones.

a. $\frac{14}{16}$	**b.** $\frac{33}{27}$	**c.** $\frac{12}{26}$	**d.** $\frac{9}{33}$	**e.** $\frac{42}{28}$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify. Also, give your final answer as a mixed number, if applicable.

a. $\frac{56}{84} = \frac{7 \cdot 8}{21 \cdot 4} =$	**b.** $\frac{54}{144} = \frac{6 \cdot 9}{12 \cdot 12} =$	**c.** $\frac{120}{72} = \frac{10 \cdot \square}{\square \cdot 9} =$
d. $\frac{80}{48} = \frac{\square \cdot 8}{\square \cdot 8} =$	**e.** $\frac{36}{90} = \underline{\qquad\qquad} =$	**f.** $\frac{28}{140} = \underline{\qquad\qquad} =$

5. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	**b.** $\frac{66}{82}$	**c.** $\frac{69}{99}$
d. $\frac{120}{600}$	**e.** $\frac{38}{52}$	**f.** $\frac{72}{84}$

Simplify "criss-cross"

These examples are from the previous page. This time the 45 in the numerator has been written as 5 · 9 instead of 9 · 5. We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

This same concept can be applied to make multiplying fractions easier.

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \cdot \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \cdot \underset{1}{\cancel{5}}} = \frac{3}{10}$$

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \cdot \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \cdot \underset{5}{\cancel{35}}} = \frac{2}{5}$$

6. Simplify. Give your final answer as a mixed number, if applicable.

a. $\frac{14}{84} = \frac{2 \cdot 7}{21 \cdot 4} =$	**b.** $\frac{54}{150} = \frac{9 \cdot \square}{10 \cdot \square} =$	**c.** $\frac{138}{36} = \frac{2 \cdot \square}{\square \cdot 4} =$
d. $\frac{27}{20} \cdot \frac{10}{21} =$	**e.** $\frac{75}{90} = \frac{\quad}{\quad} =$	**f.** $\frac{48}{45} \cdot \frac{55}{64} =$

Example 3. Here, the simplification is done in two steps.
In the first step, 12 and 2 are divided by 2, leaving 6 and 1.
In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot 69} = \frac{\overset{2}{\cancel{6}} \cdot 4}{1 \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

$$\frac{48}{138} = \frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \cdot 4}{\underset{1}{\cancel{2}} \cdot \underset{23}{\cancel{69}}} = \frac{8}{23}$$

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

7. Simplify the fractions to lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	**b.** $\frac{84}{102}$	**c.** $\frac{85}{105}$
d. $\frac{8}{5} \cdot \frac{8}{20} =$	**e.** $\frac{72}{120}$	**f.** $\frac{104}{240}$
g. $\frac{35}{98}$	**h.** $\frac{5}{7} \cdot \frac{17}{15} =$	**i.** $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9 and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15 and 30.

Their common factors are 1, 2, 3 and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4 and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

Chapter 7: Fractions
Introduction

This chapter begins with a review of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, “How many times does this number fit into that number?” For example, we can write a division from the question, “How many times does 1/3 fit into 1?” The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \cdot (1/3) = 1$. These two numbers, 3/1 and 1/3, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. Thinking of fitting the divisor into the dividend (measurement division) also gives us a tool to check whether the answer to a division problem is reasonable.

Naturally, the lessons also present the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the “invert (flip)-and-multiply” rule. However, that “rule” is just a shortcut. It is necessary to memorize it, but memorizing a shortcut doesn’t help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are review in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

Also, don’t forget to use the resources for challenging problems:
https://l.mathmammoth.com/challengingproblems
I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 7

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch7

SCAN ME

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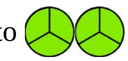
Dividing Fractions: Reciprocal Numbers

One interpretation of division is **measurement division**, where we think: *How many times does one number go into another?* For example, to solve how many times 11 fits into 189, we divide 187 ÷ 11 = 17.

(The other interpretation is equal sharing; we will come to that later.)

Let's apply that to fractions. How many times does  go into  ?

We can solve this just by looking at the pictures: three times. We can write the division: $2 \div \frac{2}{3} = 3$.

To check the division, we multiply: $3 \cdot \frac{2}{3} = \frac{6}{3} = 2$. Since we got the original dividend, it checks.

We can use measurement division to check whether an answer to a division is reasonable.

For example, if I told you that $7 \div 1\frac{2}{3}$ equals $14\frac{1}{3}$, you can immediately see it doesn't make sense:

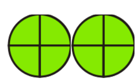
1 2/3 surely does not fit into 7 that many times. Maybe three to four times, but not 14!

You could also multiply to see that: 14-*and-something* times 1-*and-something* is way more than 14, and closer to 28 than to 14, instead of 7.

1. Find the answers that are unreasonable without actually dividing.

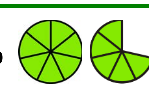
a. $\frac{4}{5} \div 6 = \frac{2}{15}$	**b.** $2\frac{3}{4} \div \frac{1}{4} = \frac{7}{12}$	**c.** $\frac{7}{9} \div 2 = \frac{7}{18}$	**d.** $8 \div 2\frac{1}{3} = 18\frac{1}{3}$	**e.** $5\frac{1}{4} \div 6\frac{1}{2} = 3\frac{1}{8}$

2. Solve with the help of the visual model, checking how many times the given fraction fits into the other number. Then write a division. Lastly, write a multiplication that checks your division.

a. How many times does  go into  ?

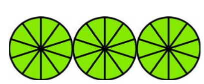
$2 \div \frac{3}{4} =$

Check: ____ $\cdot \frac{3}{4} =$

b. How many times does  go into  ?

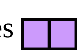
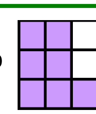
$\square\,\frac{\square}{\square} \div \frac{\square}{\square} =$

Check:

c. How many times does  go into  ?

$3 \div \frac{\square}{\square} =$

Check:

d. How many times does  go into  ?

$\frac{\square}{\square} \div \frac{\square}{\square} =$

Check:

3. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$$\begin{array}{ccc} 5 & \div & \frac{1}{4} \\ \downarrow & & \downarrow \\ 5 & \cdot & 4 = 20 \end{array} \qquad \begin{array}{ccc} 3 & \div & \frac{1}{8} \\ \downarrow & & \downarrow \\ 3 & \cdot & 8 = 24 \end{array} \qquad \begin{array}{ccc} 9 & \div & \frac{1}{7} \\ \downarrow & & \downarrow \\ 9 & \cdot & 7 = 63 \end{array}$$

Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 reciprocal numbers, or just reciprocals. So the shortcut is: multiply by the reciprocal of the divisor.

Does the shortcut make sense to you? For example, consider the problem 5 ÷ (1/4).
Since 1/4 goes into 1 exactly four times, it must go into 5 exactly 5 · 4 = 20 times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \cdot 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \cdot \frac{n}{m} = \frac{n \cdot m}{m \cdot n} = \frac{m \cdot n}{m \cdot n} = 1$.

To find the reciprocal of a mixed number or a whole number, first write it as a fraction, then "flip" it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$. And since $28 = \frac{28}{1}$, its reciprocal number is $\frac{1}{28}$.

4. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

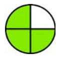
a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\frac{7}{8}$	**d.** 32	**e.** $2\frac{1}{8}$
$\frac{5}{8} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$	$32 \cdot \frac{\square}{\square} = 1$	$\frac{\square}{\square} \cdot \frac{\square}{\square} = 1$

5. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**c.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**d.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$	**e.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does  go into  ?

$$\frac{3}{4} \div \frac{1}{3}$$

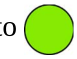
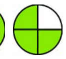
$$\downarrow \; \downarrow$$

$$\frac{3}{4} \cdot 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes,  fits into  a little more than two times.

How many times does  go into   ?

$$\frac{7}{4} \div \frac{2}{5}$$

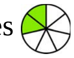
$$\downarrow \; \downarrow$$

$$\frac{7}{4} \cdot \frac{5}{2} = \frac{35}{8} = 4\ \frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes.  goes into 1 3/4 over four times.

How many times does  go into  ?

$$\frac{2}{9} \div \frac{2}{7} =$$

$$\downarrow \; \downarrow$$

$$\frac{\cancel{2}}{9} \cdot \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because  does not go into  even one full time!

Remember: There are ***two*** **changes** in each calculation:

1. **Change the division into multiplication**.
2. **Use the reciprocal of the divisor**.

6. Solve these division problems using the shortcut. Remember to check to make sure your answer makes sense.

a. $\frac{3}{4} \div 5$

$\downarrow \; \downarrow$

$\frac{3}{4} \cdot \frac{1}{5} =$

b. $\frac{2}{3} \div \frac{6}{7}$

c. $\frac{4}{7} \div \frac{3}{7}$

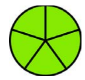
d. $\frac{2}{3} \div \frac{3}{5}$

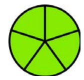
e. $4 \div \frac{2}{5}$

f. $\frac{13}{3} \div \frac{1}{5}$

Now let's try to **make some sense <u>visually</u>** out of how reciprocal numbers fit into the division of fractions.

Example 1. We can think of the division **1 ÷ (2/5)** as asking, **"How many times does 2/5 fit into 1?"**

Using pictures: How many times does  go into  ? (From the looks of it, at least two times!)

From the picture we can see that  goes into  two times, and then we have 1/5 left over.

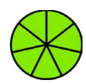
But how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

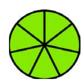
That is like trying to fit a TWO-part piece into a hole that holds just ONE part.
Only 1/2 of the two-part piece fits! So, 2/5 fits into 1/5 exactly half a time.

So we found that, in total, 2/5 fits into 1 exactly **2 ½ times**. We can write the division $1 \div \frac{2}{5} = 2\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \cdot \frac{2}{5} = 1$. Reciprocals!

Example 2. We can think of the division **1 ÷ (5/7)** as, **"How many times does 5/7 fit into 1?"**

Using pictures: How many times does  go into  ? (It looks like, a bit over one time.)

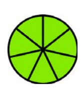
From the picture we can see that  goes into  just once, and then we have 2/7 left over.

But how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The five-part piece fits into a hole that is only big enough for two parts just 2/5 of the way.

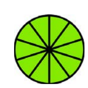
So 5/7 fits into one exactly 1 2/5 times—and this makes sense because, as we noted at first, it looked like 5/7 fit into one a little over one time. The division is $1 \div \frac{5}{7} = 1\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$. Reciprocals again!

7. Write a division.

a. How many times does  go into  ?

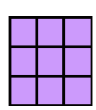
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

b. How many times does  go into  ?

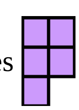
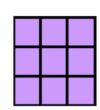
$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

c. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

d. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

Check: Does your answer make sense visually?

8. Fill in the answers and complete the patterns. You will be able to do a lot of these in your head!

a.	**b.**	**c.**	**d.**
$3 \div \frac{1}{5} =$	$6 \div \frac{1}{4} =$	$1 \div \frac{1}{4} =$	$8 \div \frac{1}{2} =$
$3 \div \frac{2}{5} =$	$6 \div \frac{2}{4} =$	$2 \div \frac{1}{4} =$	$8 \div \frac{2}{2} =$
$3 \div \frac{3}{5} =$	$6 \div \frac{3}{4} =$	$3 \div \frac{1}{4} =$	$8 \div \frac{3}{2} =$
$3 \div \frac{4}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$
$3 \div \frac{5}{5} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$	$\square \div \frac{\square}{\square} =$

Epilogue (optional)

The lesson didn't go into full details as to why multiplication by the reciprocal always gives us the answer to a division problem. Let's continue that discussion a bit.

Any division can be turned into a multiplication. For example, from the division $2 \div \frac{3}{4} =$ ______ , we can write the multiplication $\frac{3}{4} \cdot$ ______ $= 2$.

To find what goes on the empty line, we can first of all put there the reciprocal of ¾ : $\frac{3}{4} \times \underline{\frac{4}{3} \times \qquad\quad} = 2$.

Notice the multiplication of the two fractions above equals 1 (since they are reciprocals). To make the left side of the equation equal 2, we place 2 in the empty line: $\frac{3}{4} \times \underline{\frac{4}{3} \times \ 2\ } = 2$

And thus, the answer to the original division is $\frac{4}{3} \cdot 2$ (or $2 \cdot \frac{4}{3}$) — which is the original dividend times the reciprocal of the divisor.

Let's take another example: $2\frac{1}{6} \div 5\frac{3}{4} =$ ______

We turn it around and make a multiplication problem: $5\frac{3}{4} \cdot$ ______ $= 2\frac{1}{6}$.

Now, 5 ¾ = 23/4. So first, we insert the reciprocal of 23/4, which is 4/23: $5\frac{3}{4} \cdot \frac{4}{23} \cdot$ ______ $= 2\frac{1}{6}$.

Since 5 ¾ · (4/23) = 1, then the number we still need to put on the empty line must be 2 1/6, and thus the answer to the original division problem is 4/23 · (2 1/6), or (2 1/6) · 4/23 — the original divided times the reciprocal of the divisor.

We could use a similar argument to show in the general case that the answer to any division problem $a \div b$ is always going to be a times the reciprocal of b.

Divide Fractions

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.		
This shortcut works *in every case*, whether the numbers involved are whole numbers, fractions, or mixed numbers.		
$\frac{2}{5} \div \frac{7}{9}$ $\downarrow \; \downarrow$ $\frac{2}{5} \cdot \frac{9}{7} = \frac{18}{35}$ Check: $\frac{18}{35} \cdot \frac{7}{9} = \frac{2}{5}$	$7 \div \frac{9}{10}$ $\downarrow \; \downarrow$ $7 \cdot \frac{10}{9} = \frac{70}{9} = 7\frac{7}{9}$ Check: $\frac{70}{9} \cdot \frac{9}{10} = \frac{7}{1} = 7$	$1\frac{10}{11} \div 5$ $\downarrow \; \downarrow$ $\frac{21}{11} \cdot \frac{1}{5} = \frac{21}{55}$ Check: $\frac{21}{55} \cdot 5 = \frac{21}{11} = 1\frac{10}{11}$
Notice: when you check the problems, you will need to use the *original* divisor, not the "flipped" one.		

1. Solve. Change mixed numbers to fractions before dividing. Check each division by multiplication.

a. $\frac{9}{10} \div \frac{2}{5}$ Check:	**b.** $\frac{3}{7} \div \frac{4}{3}$ Check:	**c.** $\frac{2}{11} \div \frac{2}{3}$ Check:
d. $1\frac{7}{8} \div \frac{3}{4}$ Check:	**e.** $2\frac{1}{15} \div 1\frac{3}{5}$ Check:	**f.** $5\frac{10}{11} \div 6$ Check:

2. How many 2/3 cup servings can you get out of 5 cups of ice cream?

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Problem Solving with Fractions 1

1. **a.** Anna needs to make 30 servings of spiced coffee for a party.
   Calculate the amount of each ingredient she needs.

Spiced Coffee – 4 servings
1 1/2 teaspoons of ground cinnamon
1/2 of a teaspoon of ground nutmeg
2 tablespoons of sugar
1 cup of heavy cream
3 cups of coffee
4 teaspoons of chocolate syrup

   b. Next week she wants to make just *one* serving for herself.
   Calculate the amount of each ingredient she needs.

2. Sam planted tomatoes in his garden, which is a rectangle with an area of 2 ½ m^2.
   If one side of the garden measures 5 m, how long is the other side?

3. Which is a better deal: a book that costs $45.55 at 1/5 off
   or a book that costs $52.80 at 1/4 off?

Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants and integer addition and subtraction. The multiplication and division of integers will be studied next year.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules", for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number-line and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

You will find free videos covering many topics of this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 8

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

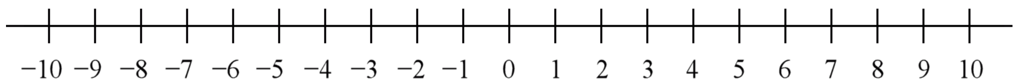
We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch8

SCAN ME

Integers

When we continue the number-line towards the left from zero, we come to the **negative numbers**.



The **negative whole numbers** are −1, −2, −3, −4 and so on.
The **positive whole numbers** are 1, 2, 3, 4 and so on. You can also write them as +1, +2, +3, *etc*.
Zero is neither positive nor negative.
All of the negative and positive whole numbers and zero are called **integers**.

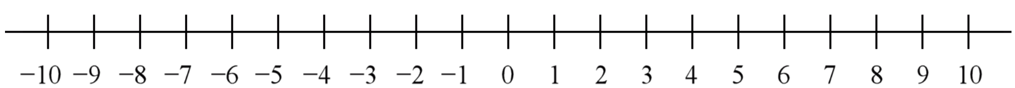
Read −1 as "negative one" and −5 as "negative five". Some people read −5 as "minus five".
That is very common, and it is not wrong, but be sure that you do not confuse it with subtraction.

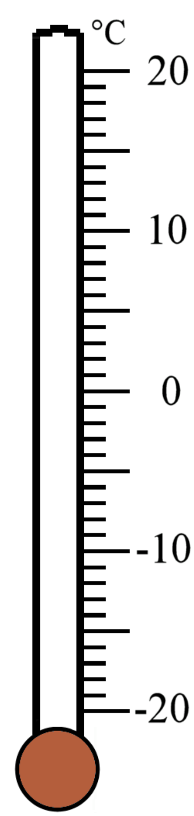
Put a "−" sign in front of negative numbers. This sign can also be elevated: ⁻5 is the same as −5.

Often, we need to put brackets around negative numbers in order to avoid confusion with other symbols. Therefore, ⁻5, −5 and (−5) all mean "negative five".

Negative numbers are commonly used with temperature. They are also used to express debt. If you owe $5, you write that as −$5. Another use is with elevation below sea level. For example, just as 200 m can mean an elevation of 200 meters above sea level, −100 m would mean 100 meters *below* sea level.

1. Plot the integers on the number-line. **a.** −7 **b.** +6 **c.** −4 **d.** −2





2. Write an integer appropriate to each situation.

a. Daniel owes $23.

b. Mary earned $250.

c. The airplane flew at the altitude of 8 800 meters.

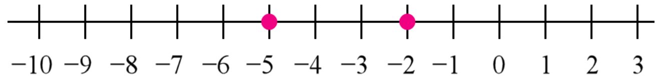
d. The temperature in the freezer is 18 degrees Celsius below zero.

e. A dolphin dove 9 m below sea level.

3. The temperature changed from what it was before. Find the new temperature.
You can draw the mercury on the thermometer to help you.

before	1°C	2°C	−2°C	−4°C	−12°C	−8°C
change	drops 3°C	drops 7°C	drops 1°C	rises 5°C	rises 4°C	rises 3°C
now						

Which is more, −5 or −2?

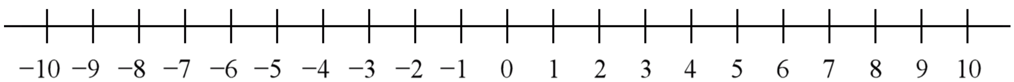


Which is *warmer*, −5°C or −2°C? Clearly −2°C is. Temperatures just get colder and colder the more you move towards the negative numbers. We can write a comparison: −2°C > −5°C.

Which is the *better* money situation, to have −\$5 (owe \$5) or to have −\$2 (owe \$2)? Clearly, it is better to owe only \$2 because you can pay that off easier. We can write: −\$5 < −\$2.

Which is the *higher* elevation, −5 m or −2 m? Of course, 2 m below sea level, or −2m, is higher.

On the number line, the number that is ***farther to the right*** is the **greater** number. So, −5 < −2.



4. Compare. Write **<** or **>** between the numbers. You can plot the integers on the number line to help you.

a. −2 □ −3	**b.** 8 □ −8	**c.** −3 □ 0	**d.** 4 □ −3	**e.** −5 □ −9
f. −10 □ −30	**g.** −4 □ 1	**h.** 0 □ −13	**i.** −2 □ −7	**j.** −11 □ −14

5. You can use the number line to help you. Which integer is ...

a. 2 more than −4　　**b.** 5 more than −3　　**c.** 3 less than 1　　**d.** 6 more than −11

6. Find the number that is 5 less than ...　　**a.** 0　　**b.** −3　　**c.** 3

7. Express the situations using integers. Then write > or < to compare them.

a. Shelly owes \$10 and Mary owes \$8.

b. One fish was swimming 3 m below the surface of the water, and another fish was swimming 4 m below the surface of the water.

c. The temperature this morning was 10°C below zero. Now it is 6°C below zero.

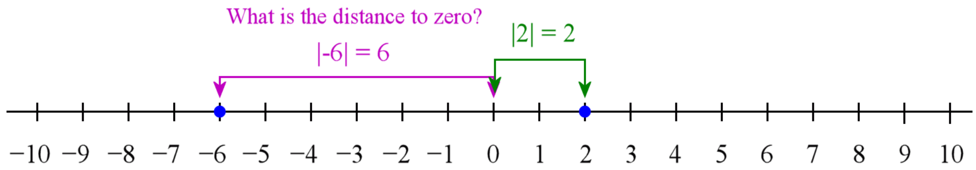
d. Henry has \$5. Emma owes \$5.

e. The temperature during the day was 10°C but at night it was 2°C below zero.

8. Write the numbers in order from the least to the greatest.

a. −2　0　−4　4	**b.** −3　−6　5　3
c. −20　−10　−14　−9	**d.** −3　0　−6　−8

The **absolute value** of a number is its distance from zero.



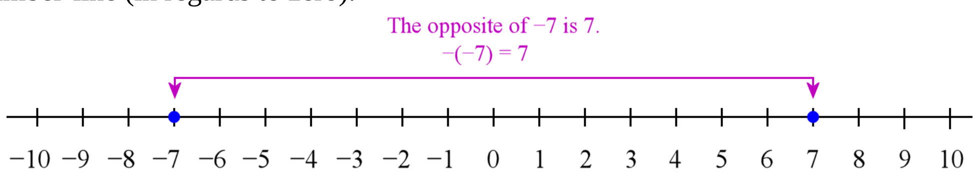
We denote the absolute value of a number using vertical bars around the number.

So, $|-4|$ means "the absolute value of 4", which is 4. Similarly, $|87| = 87$.

9. Find the absolute values of these numbers.

a. $|-5|$ **b.** $|-12|$ **c.** $|7|$ **d.** $|0|$ **e.** $|68|$

The **opposite** of a number is the number that is at the same distance from zero as it is, but on the *opposite* side of the number-line (in regards to zero).



We denote the opposite of a number using the minus sign. For example, −4 means the opposite of 4, which is −4. Or, −(−2) means the opposite of negative two, which is 2.

The opposite of zero is zero itself. In symbols, $-0 = 0$.

"But wait," you might ask, "doesn't −4 mean negative four, not the 'opposite of four'?"

It can mean either. Sometimes the context will help you to differentiate between the two (to tell which is which). Other times it's unnecessary to differentiate because, after all, the opposite of four is negative four: $-4 = -4$. ☺

So there are three different meanings for the minus sign:

1. To indicate subtraction: $7 - 2 = 5$.
2. To indicate negative numbers: "negative 7" is written −7.
3. To indicate the opposite of a number: −(−14) is the opposite of negative 14.

10. Think of the minus sign as signifying "the opposite of". Simplify.

a. -5 **b.** $-(-9)$ **c.** -10 **d.** -0 **e.** $-(-100)$

11. Write using mathematical symbols, and simplify (solve) if possible.

a. the opposite of 6

b. the opposite of the absolute value of 6

c. the absolute value of negative 6

d. the absolute value of the opposite of 6

e. the opposite of −6.

f. the absolute value of the opposite of −6

Coordinate Grid

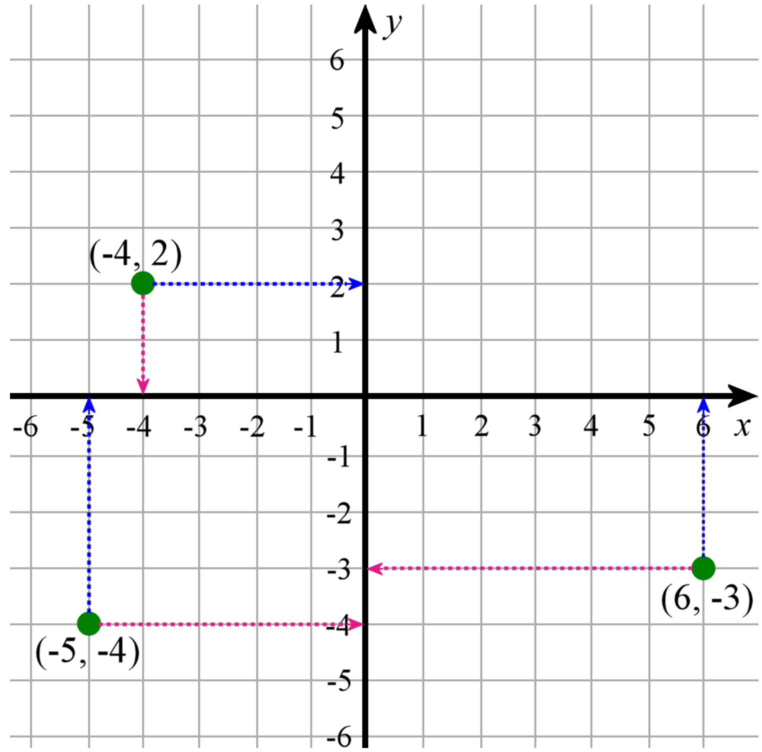
This is the *coordinate grid* or *coordinate plane*. We have extended the *x*-axis and the *y*-axis to include negative numbers now. The axes cross each other at the *origin*, or the point (0, 0).

The axes divide the coordinate plane into four parts, called *quadrants*. Previously you have worked in only the so-called first quadrant, but now we will use all four quadrants.

The coordinates of a point are found in the same manner as before. Draw a vertical line (either up or down) from the point towards the *x*-axis. Where this line crosses the *x*-axis tells you the point's *x*-coordinate.

Similarly, draw a horizontal line (either right or left) from the point towards the *y*-axis. Where this line crosses the y-axis tells you the point's *y*-coordinate.

We list first the point's *x*-coordinate and then the *y*-coordinate. Look at the examples in the picture.



1. Write the *x*- and *y*-coordinates of the points.

A (____ , ____)

B (____ , ____)

C (____ , ____)

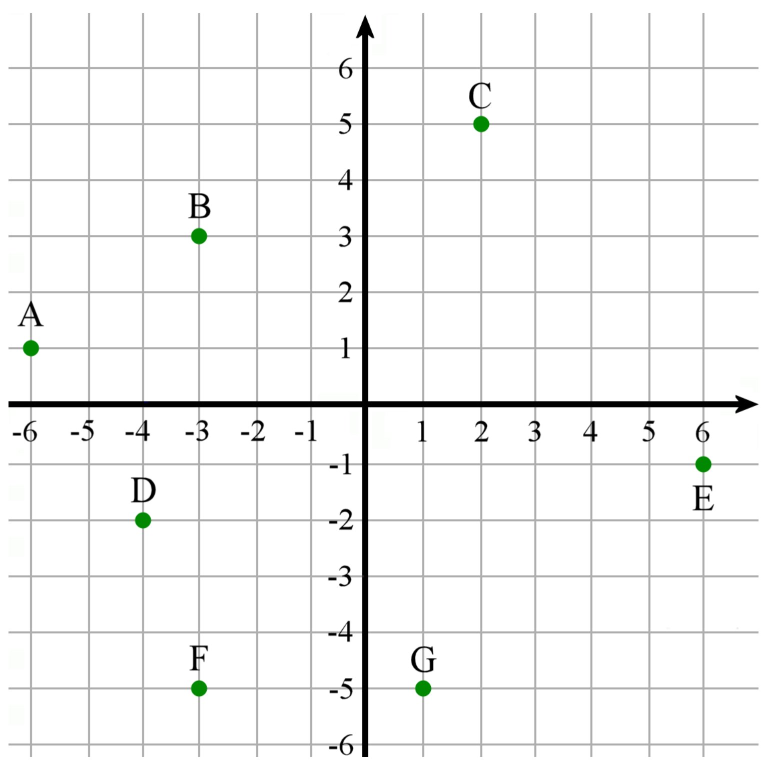
D (____ , ____)

E (____ , ____)

F (____ , ____)

G (____ , ____)

Self-check: Add the *x*-coordinates of all points. You should get −7.



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Graphing

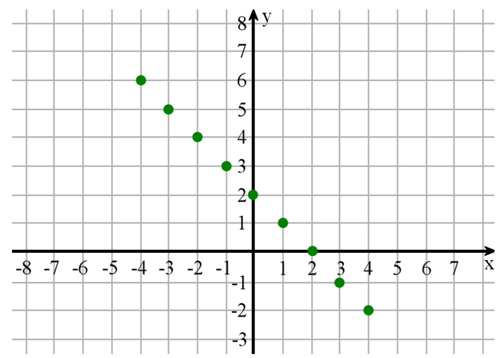
Remember? When an equation has two variables, there are many values of x and y that make that equation true.

Example. Note the equation $y = 2 - x$. If $x = 0$, then we can calculate the value of y using the equation: $y = 2 - 0 = 2$.

So, when $x = 0$ and $y = 2$, that equation is true. We can plot the number pair (0, 2) on the coordinate grid.

Some of the other (x, y) values that make the equation true are listed below, and they are plotted on the right.

x	−4	−3	−2	−1	0	1	2	3	4
y	6	5	4	3	2	1	0	−1	−2

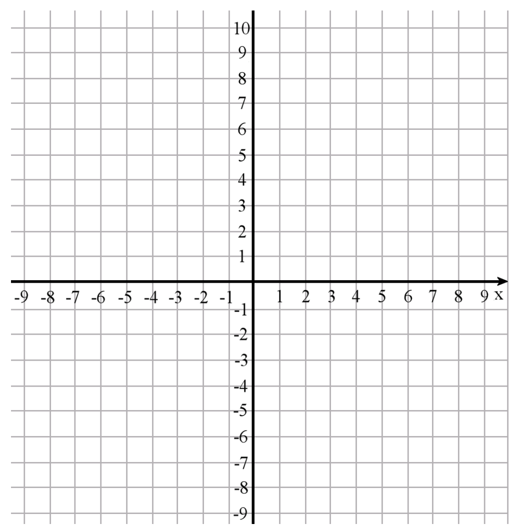


1. Plot the points from the equations. Graph both (b) and (c) in the same grid.

a. $y = x + 4$

x	−9	−8	−7	−6	−5	−4	−3	−2
y								

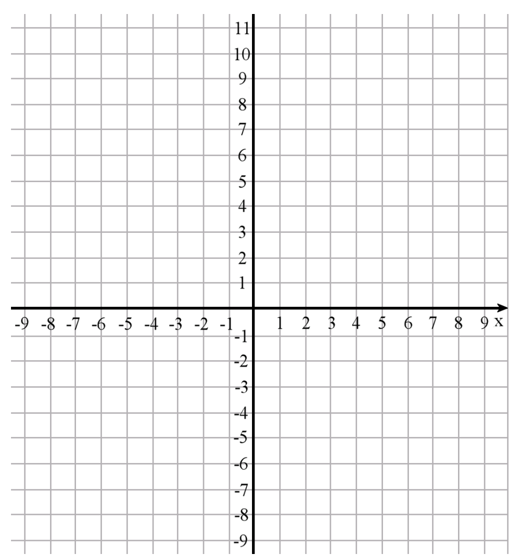
x	−1	0	1	2	3	4	5	6
y								



b. $y = 6 - x$

x	−3	−2	−1	0	1	2	3
y							

x	4	5	6	7	8	9
y						



c. $y = x - 2$

x	−5	−4	−3	−2	−1	0	1	2
y								

x	3	4	5	6	7	8	9
y							

Chapter 9: Geometry
Introduction

The main topics in this chapter include:

- area of triangles
- area of polygons
- nets and surface area of prisms and pyramids
- volume of rectangular prisms with sides of fractional length

However, the chapter starts out with some review of topics from earlier grades, as we review the different types of quadrilaterals and triangles and students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalize finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those and summing them. Students also practice their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges ½ or ⅓ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at **https://www.mathmammoth.com/videos/** (choose 6th grade).

Also, don't forget to use the resources for challenging problems: **https://l.mathmammoth.com/challengingproblems**

I recommend that you at least use the first resource listed, Math Stars Newsletters.

The Lessons in Chapter 9

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch9

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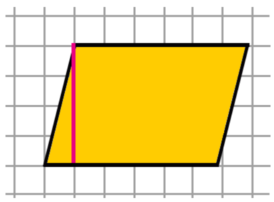
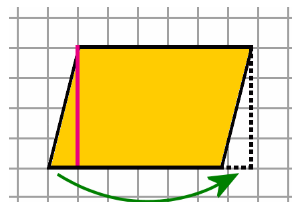
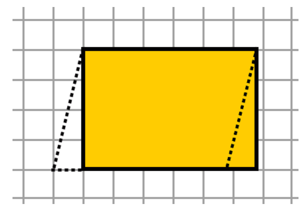
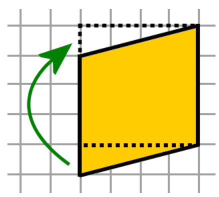
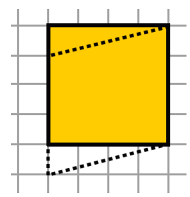
Area of Parallelograms

We draw a line from one vertex of the parallelogram in order to form a *right triangle*. Then we move the triangle to the other side, as shown. Look! We get a *rectangle*!

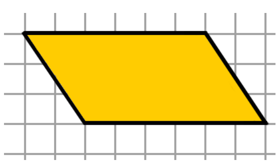
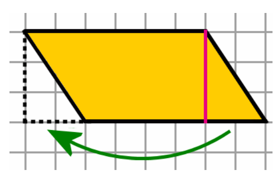
The rectangle's area is 6 · 4 = 24 square units, and that is *also the area of the original parallelogram*.

It works here, as well. The area of the rectangle and of the parallelogram are the same: both have the area of 4 · 4 = 16 square units.

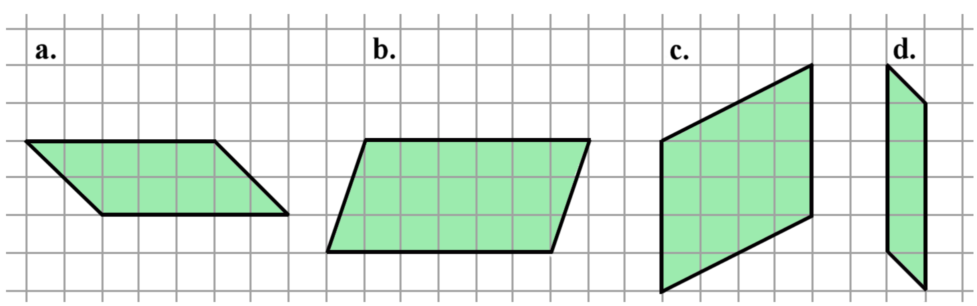



The area of a parallelogram is the same as the area of the corresponding rectangle.
You construct the rectangle by moving a right triangle from one side of the parallelogram to the other.

1. Imagine moving the marked triangle to the other side as shown. What is the area of the original parallelogram?

2. Draw a line in each parallelogram to form a right triangle. Imagine moving that triangle to the other side so that you get a rectangle, like in the examples above. Find the area of the rectangle, thereby finding the area of the original parallelogram.



a. __________ square units **b.** __________ square units

c. __________ square units **d.** __________ square units

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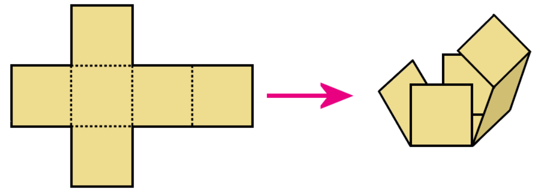
Nets and Surface Area 1



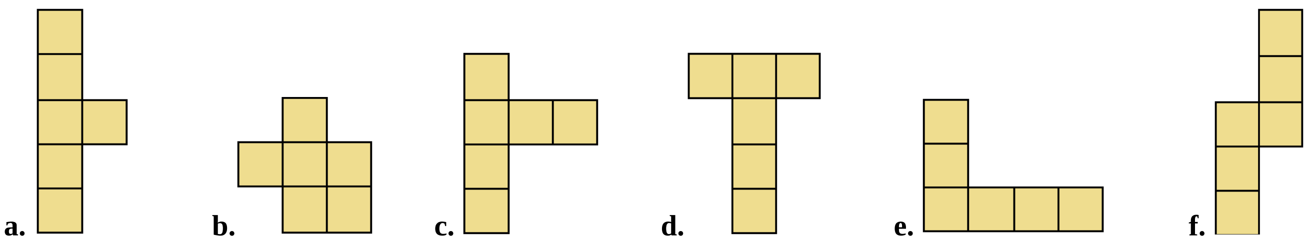
This picture shows a flat figure, called a **net**, that can be folded up to form a solid, in this case a cube.

Each face of a cube is a square. If we find the total area of its faces, we will have found the **surface area** of the cube.

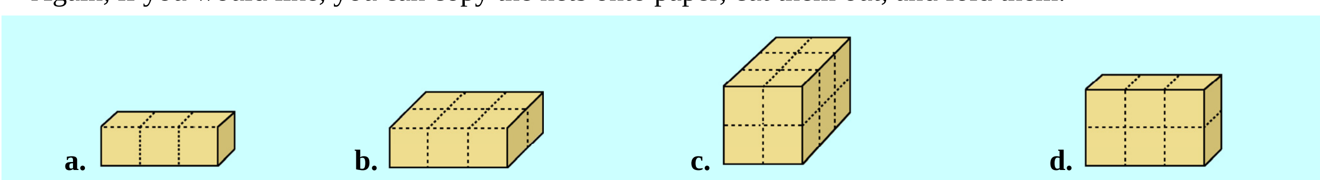
Let's say that each edge of this cube measures 2 cm. Then one face would have an area of $2 \text{ cm} \cdot 2 \text{ cm} = 4 \text{ cm}^2$, and the total surface area of the six faces of the cube would be $6 \cdot 4 \text{ cm}^2 = 24 \text{ cm}^2$.

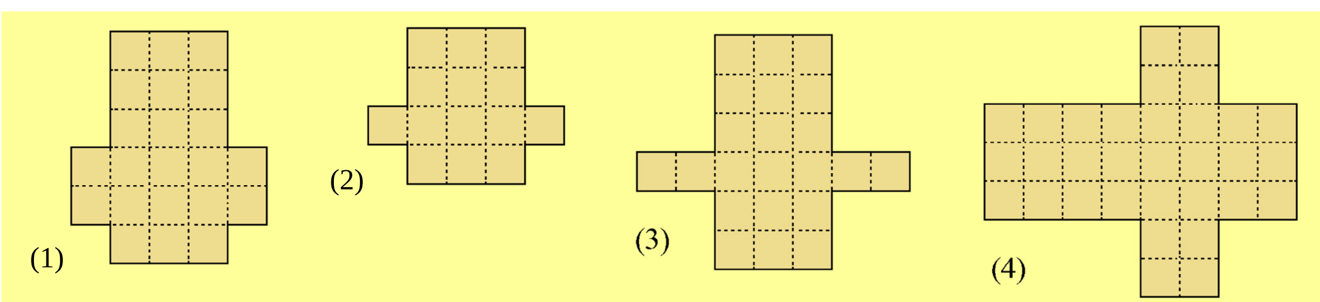
What is its volume? Remember, **volume** has to do with how much space a figure takes up, and not with "flat" area. Volume is measured in *cubic* units, whereas area is measured in *square* units. The volume of this cube is $2 \text{ cm} \cdot 2 \text{ cm} \cdot 2 \text{ cm} = (2 \text{ cm})^3 = 8 \text{ cm}^3$.

1. Which of these patterns are nets of a cube? In other words, which ones can be folded into a cube? You can copy the patterns on paper, cut them out and fold them.



2. Match each rectangular prism (a), (b), (c) and (d) with the correct net (1), (2), (3) and (4). Again, if you would like, you can copy the nets onto paper, cut them out, and fold them.





3. Find the surface area (A) and volume (V) of each rectangular prism in problem #2 if the edges of the little cubes are 1 cm long.

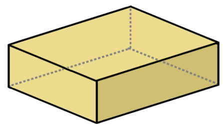
a. A = ________ cm^2 **b.** A = ________ cm^2 **c.** A = ________ cm^2 **d.** A = ________ cm^2

V = ________ cm^3 V = ________ cm^3 V = ________ cm^3 V = ________ cm^3

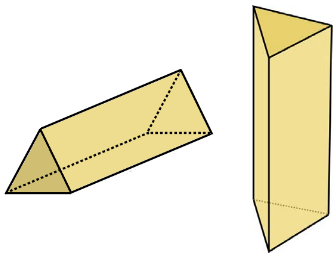
A **prism** has two identical polygons as its top and bottom faces. These polygons are called the *bases* of the prism. The bases are connected with faces that are parallelograms (and often rectangles).

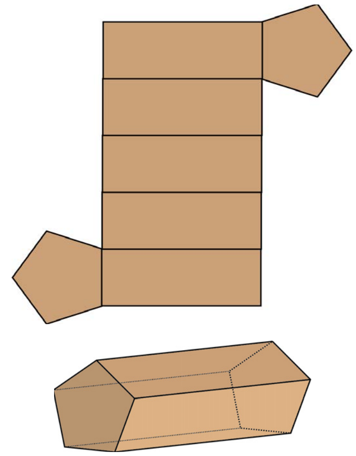
Prisms are named after the polygon used as the bases.

A rectangular prism.
The bases are rectangles.



Two **triangular prisms**.
One is lying down, where the base is facing you.
The other is "standing up".

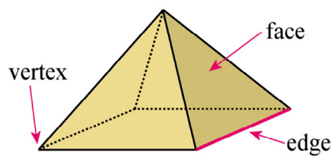




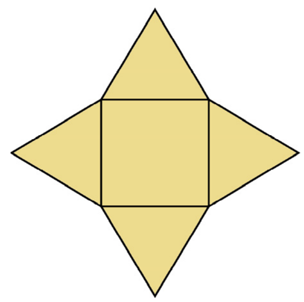
A pentagonal prism and its net. The bases are pentagons. Again, the base is not "on the bottom" but facing you.

A **pyramid** has a polygon as its bottom face (the base), and triangles as other faces.

Pyramids are named after the polygon at the base: a triangular pyramid, square pyramid, rectangular pyramid, pentagonal pyramid, and so on.

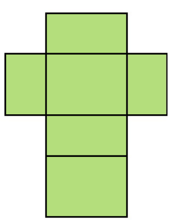


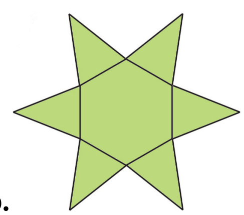
A **square pyramid** and its net.

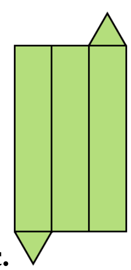


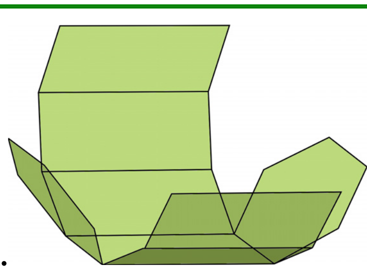
See interactive solids and their nets at the link below:
https://www.mathsisfun.com/geometry/polyhedron-models.html

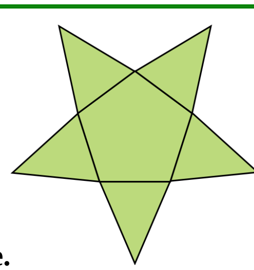
4. Name the solid that can be built from each net.

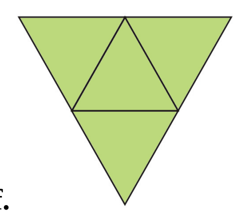
a.


b.


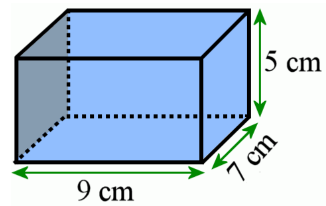
c.


d.


e.


f.


5. Which expression, (1), (2), or (3), can be used to calculate the surface area of this prism correctly? (You do *not* have to actually calculate the surface area.)

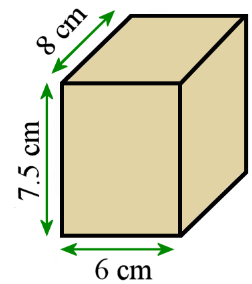


1. $2 \cdot 35\text{ cm}^2 \ + \ 2 \cdot 63\text{ cm}^2 \ + \ 2 \cdot 45\text{ cm}^2$

2. $5\text{ cm} \cdot 9\text{ cm} \cdot 7\text{ cm}$

3. $5\text{ cm} \cdot 7\text{ cm} + 9\text{ cm} \cdot 7\text{ cm} + 9\text{ cm} \cdot 5\text{ cm}$

6. Ryan organized the calculation of the surface area of this prism into three parts. Write down the intermediate calculations, and solve. This way, your teacher (or others) can follow your work. Remember also to include the units (cm or cm^2)!



Top and bottom:

Back and front:

The two sides:

Total:

7. The surface area of a cube is 96 square inches.

a. What is the area of one face of the cube?

b. How long is each edge of the cube?

c. Find the volume of the cube.

Puzzle Corner

Consider the rectangular prisms in problem #2. If the edges of the little cubes were double as long, how would that affect the surface area? Volume?

You can use the table below to investigate the situation.

Prism a.	Prism b.	Prism c.	Prism d.
A = _________ cm^2	A =_________ cm^2	A =_________ cm^2	A =_________ cm^2
V = _________ cm^3	V = _________ cm^3	V = _________ cm^3	V = _________ cm^3

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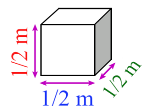
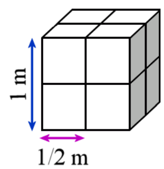
Volume of a Rectangular Prism with Sides of Fractional Length

Example 1. Let's imagine that the edges of this little cube each measure ½ m.



If we stack eight of them so that we get a bigger cube… we get this:



The bigger cube has 1 m edges, so its volume is 1 cubic meter.

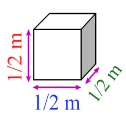
If eight identical little cubes make up this bigger cube, and its volume is 1 cubic meter, then the volume of *one* little cube is 1/8 cubic meter.

Notice: this is the same result that we get if we multiply the height, width and depth of the little cube:

$$\frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} \cdot \frac{1}{2}\text{ m} = \frac{1}{8}\text{ m}^3$$

1. The edges of each little cube measure ½ m. What is the total volume, in cubic meters, of these figures?

a.



width = 1/2 m

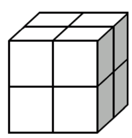
height = ________ m

depth = ________ m

1 little cube, 1/8 m^3

V = __________ m^3

b.



width = ________ m

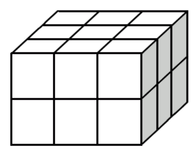
height = ________ m

depth = ________ m

8 little cubes, each 1/8 m^3

V = __________ m^3

c.



width = ________ m

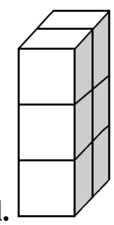
height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3

d.



width = ________ m

height = ________ m

depth = ________ m

________ little cubes, each 1/8 m^3

V = __________ m^3

2. Write a multiplication (width · depth · height) to calculate the volume of the figures (c) and (d) above, and verify that you get the same result as above.

a. V = ________ m · ________ m · ________ m

=

b. V = ________ m · ________ m · ________ m

=

Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its center, spread and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarizing and analyzing distributions are its center and its variability. First we study the center, in the lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of center to the shape of the data distribution and the type of data.

Next, we study measures of variation, starting with range and interquartile range. Students use these measures in the following lesson, as they both read and draw boxplots.

The lesson *Mean Absolute Deviation* introduces students to this measure of variation. It takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program (such as Excel or LibreOffice Calc).

Next, students learn to make histograms. They will also continue summarizing distributions by describing their shape, and giving a measure of center and a measure of variability. The lesson on stem-and-leaf plots is optional.

There are some videos available for these topics at **https://www.mathmammoth.com/videos/** (choose 6th grade).

The Lessons in Chapter 10

Helpful Resources on the Internet

We have compiled a list of Internet resources that match the topics in this chapter. This list of links includes web pages that offer:

- **online practice** for concepts;
- online **games**, or occasionally, printable games;
- **animations** and interactive **illustrations** of math concepts;
- **articles** that teach a math concept.

We heartily recommend you take a look at the list. Many of our customers love using these resources to supplement the bookwork. You can use the resources as you see fit for extra practice, to illustrate a concept better, and even just for some fun. Enjoy!

https://l.mathmammoth.com/gr6ch10

SCAN ME

Understanding Distributions

A **statistical question** is a question where we expect a range of *variability* in the answers to the question.

For example, "How old am I?" is *not* a statistical question (there is only one answer), but "How old are the students in my school?" *is* a statistical question because we expect the students' ages not to be all the same.

"How much does this TV cost?" is *not* a statistical question because we expect there to be just one answer.

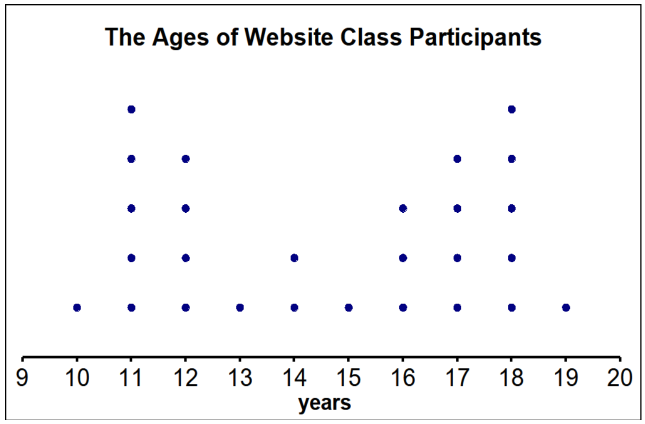
"How much does this TV cost in various stores around town?" *is* a statistical question, because we expect a number of different answers: the prices in different stores will vary.

To answer a statistical question we collect a set of **data** (many answers). The data can be displayed in some kind of a graph, such as a bar graph, a histogram, or a dot plot.

This is a **dot plot** showing the ages of the participants in a website-building class. Each dot in the plot signifies one observation. For example, we can see there was one 13-year old and two 14-year olds in the class.

The dot plot shows us the **distribution** of the data: it shows how many times (the frequency) each particular value (age in this case) occurs in the data.

This distribution is actually **bimodal,** or "double-peaked". This means it has two "centers": one around 11-12 years, and another around 17-18 years.



1. Are these statistical questions? If not, change the question so that it becomes a statistical question.

a. What color are my teacher's eyes?

b. How much money do the students in this university spend for lunch?

c. How much money do working adults in Romania earn?

d. How many children in the United States use a cell phone regularly?

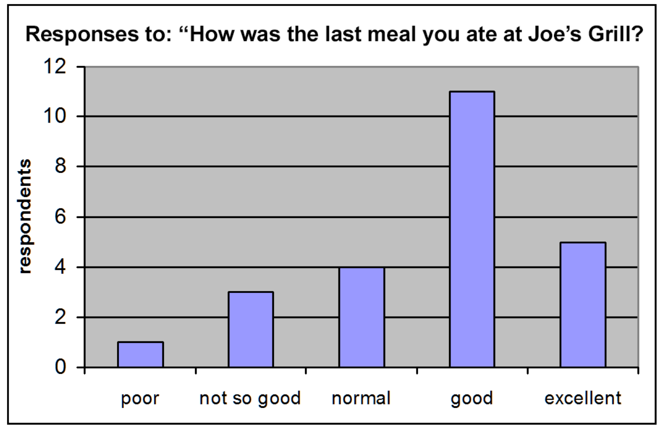
e. What is the minimum wage in Ohio?

f. How many sunny days were there in August, 2020, in London?

g. How many pets does my friend have?

2. Is this graph based on a statistical question?

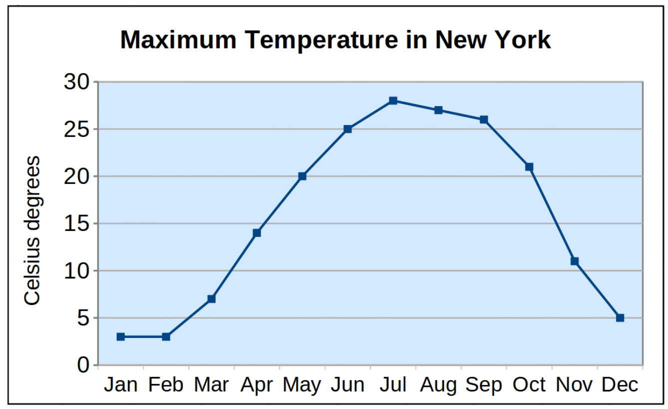
   Why or why not?



3. The line graph shows the maximum temperatures in New York for each month of a certain year.

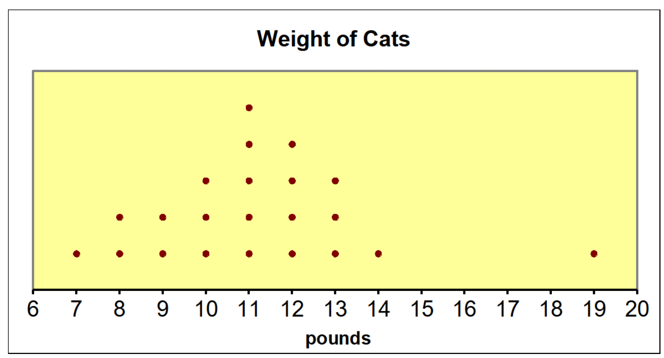
   Is this graph based on a statistical question?

   Why or why not?



4. The title of this dot plot is not the best. But, could the plot be based on a statistical question?

   If yes, give it a better, more specific, title. Imagine what situation and what question might have produced the data.



5. Change each question from a non-statistical question to a statistical question, and vice versa.

   a. What shampoo do you use?

   b. How cold was it yesterday where you live?

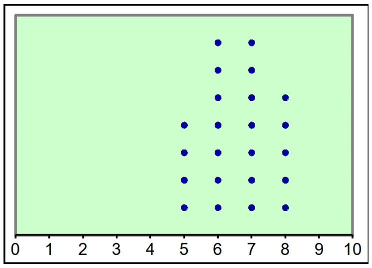
   c. How old are people in Germany when they marry (the first time)?

   d. How long does it take for our company's packages to reach the customers?

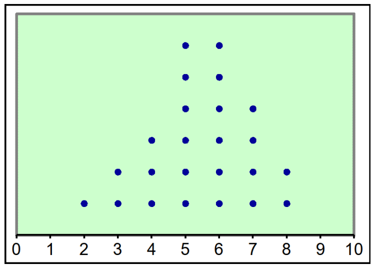
We are often interested in the **center**, **spread** and **overall shape** of the distribution. Those three things can summarize for us what is important about the distribution.

The **center** of a distribution has to do with where its peak is. We can use mean, median and mode to characterize the central tendency of a distribution. We will study those in detail in the next lesson.

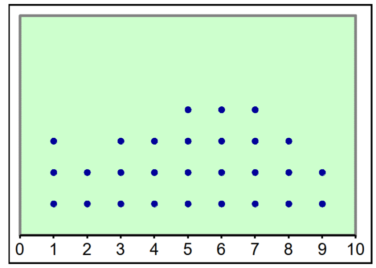
These three dot plots show how the **spread** of a distribution can vary. This means how the data items themselves are spread — whether they are "spread" all over, or tightly concentrated near some value, or somewhat concentrated around some value. We will study more about spread in another lesson.



little spread

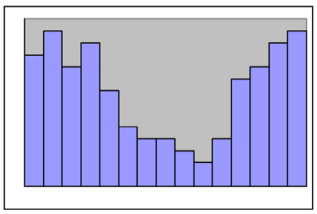


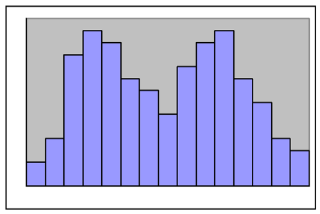
medium spread

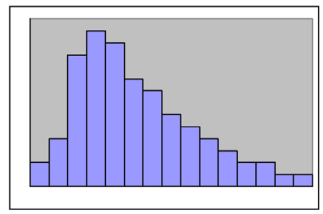


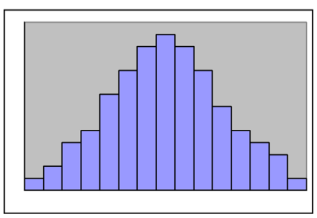
large spread

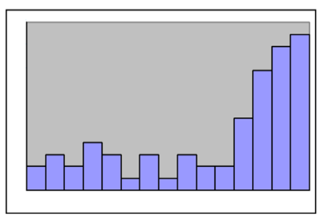
The distribution can have many varying overall **shapes.** For example:

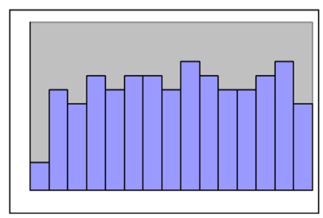

U-shaped


double-peaked (bimodal)


asymmetrical, right-tailed
(a.k.a. right-skewed)

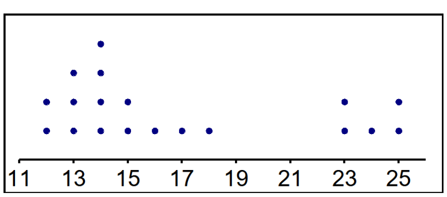

bell-shaped


J-shaped
(can also be mirrored where
most of the values are at the left)

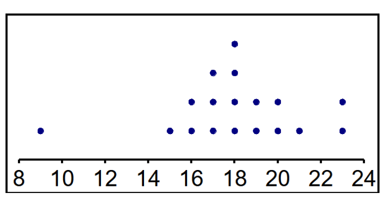

rectangular

In addition to its overall shape, a distribution may have a gap, an outlier, or a cluster:

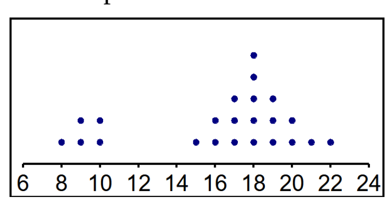
This distribution has a **gap** from 19 to 22:



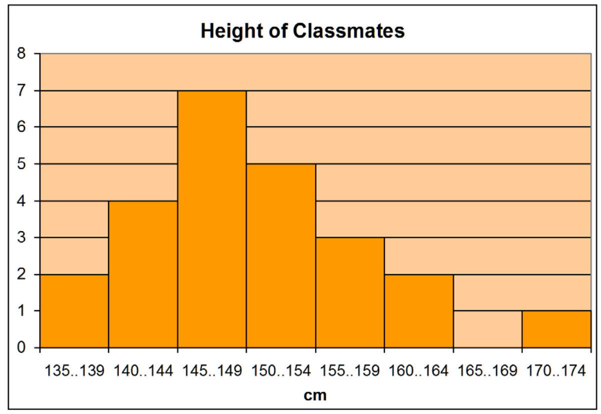
In this distribution, 9 is an **outlier —** a data item whose value is considerably less or more than all the others.



This distribution has a bell shape overall (with a peak at 18), but also a **cluster** or a smaller peak at 8-10.



6. Anne asked her classmates the question, “How tall are you?” The histogram shows the distribution of her data.



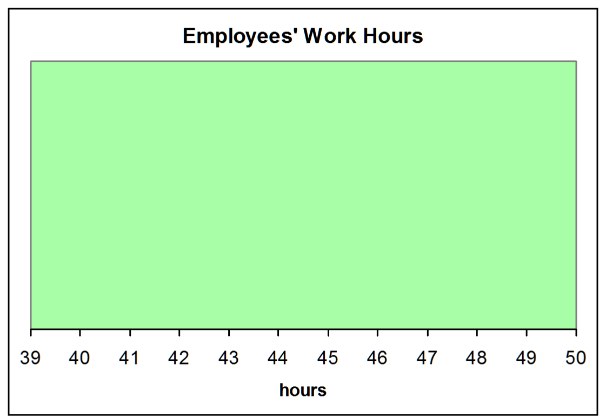
a. Describe the overall shape of the distribution, and also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

7. Make a dot plot from this data (weekly work hours of a restaurant’s employees). You need to place a dot for each observation.

48	45	46	41	42	42	43	43	42	42	41
41	45	49	40	41	41	42	46	47	42	40

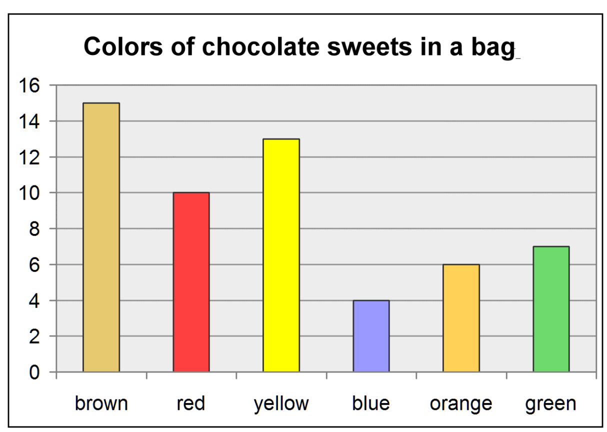


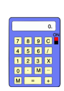
a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

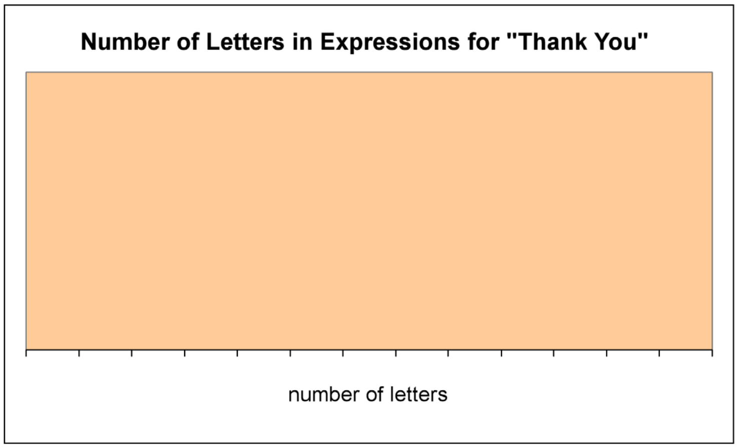
8. **a.** Does this graph show a statistical distribution? Why or why not?





b. Calculate what percentage of the candies are red and what percentage are green.

9. First, count the number of letters in these expressions for “Thank You” from various languages and fill in the empty column in the table. Next, label the number line below the dot plot so that all of the data will fit. Finally, plot the data.



Language	Spelling	Number of letters
Africaans	dankee	
Arabic	shukran	
Chinese, Cantanese	do jeh	
Chinese, Mandarin	xie xie	
Czech	dêkuji	
Danish	tak	
English	thank you	
Finnish	kiitos	
French	merci	
German	danke	
Greek	efharisto	
Hawaiian	mahalo	
Hebrew	toda	
Hindi	sukria	
Italian	grazie	
Japanese	arigato	
Korean	kamsa hamnida	
Norwegian	takk	
Philippines (Tagalog)	salamat po	
Polish	dziekuje	
Portuguese	obrigado	
Russian	spasibo	
Spanish	gracias	
Sri Lanka (Sinhak)	istutiy	
Swahili	asante	
Swedish	tack	
Thai	khop khun krab	
Turkish	tesekkür ederim	
Vietnamese	ca’m on	

a. Describe the overall shape of the distribution. Also include if there are any striking deviations from the overall pattern (gaps, outliers, or clusters).

b. Where is the peak of the distribution?

c. How many observations are there?

Mean, Median and Mode

Mean, median and **mode** are all measures for the *center* of a data set. In other words, each of them gives us a *single number* that indicates a "middle point" of the distribution.

The **mode** is the most commonly occurring data item within the data set.

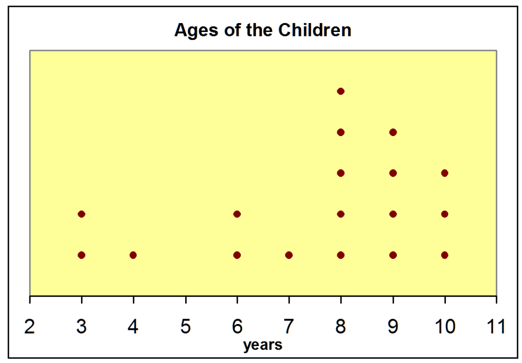
- If no item occurs more often than others, there is no mode.
  Example 1. The data set {*bear, parrot, cat, dog, lizard*} has no mode.
- If two (or three, four, *etc.*) items occur equally often, there are that many modes.
  Example 2. The data set {3, 3, 6, 6, 7, 8, 8, 10} has three modes: 3, 6 and 8.

The **median** is the *middle* item after the data is organized from the least to the greatest. Exactly half of the data is before the median, and the other half is after.

- If there is an even number of data items, the median is the average of the two items in the middle.

Example 3. Find the median of the ages of a group of children:
3, 3, 4, 6, 6, 7, 8, 8, **8**, **8**, 8, 9, 9, 9, 9, 10, 10, 10

There are 18 data items and they are already in order.
The median is the "middle item", in this case the average of the 9th and 10th ages, which are both 8. So the median is 8.
It matches well with the peak in the plot of the distribution.



What is the mode in this example?

The **mean**, or the **average**, is calculated by adding all the data items, then dividing by the number of items.

Example 4. Mia's scores on her spelling tests were 80%, 72%, 88%, 92% and 79%.
What was her average score?

We calculate the mean by adding the five scores and dividing by 5: $\dfrac{80 + 72 + 88 + 92 + 79}{5} = 82.2\%$

1. Find the median and mode of these data sets.

a. 20, 25, 21, 30, 29, 24, 18, 32, 25, 26, 25 (ages of participants in a parenting class)

median ________ mode ________

b. 1, 1, 0, 2, 2, 2, 3, 1, 2, 2, 1 (the number of cars per household, for 11 households on Meadow Street)

median ________ mode ________

c. 80, 85, 80, 90, 70, 75, 90, 85, 100, 80 (Alice's quiz scores in algebra class)

median ________ mode ________

d. sandals, crocs, tennis shoes, crocs, dress shoes, sandals (types of shoes Emma keeps on her shoe rack)

mode

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Stem-and-Leaf Plots

This lesson is optional.

A stem-and-leaf plot is made using the numbers in the data, and it looks a little bit like a histogram or a dot plot turned sideways.

In this plot, the tens digits of the individual numbers become the **stems**, and the ones digits become the **leaves**. For example, the second row 2 | 1 2 5 8 actually means 21, 22, 25, and 28. Notice how the leaves are listed in order from the smallest to the greatest.

Ages of the participants in the County Fair Karaoke Contest:

14 18 21 22 25 28 30 30
31 33 33 36 37 40 45 58

Stem	Leaf
1	4 8
2	1 2 5 8
3	0 0 1 3 3 6 7
4	0 5
5	8

4 | 5 means 45

Since stem-and-leaf plots show not only the *shape* of the distribution but also the individual *values*, they can be used to get a quick overview of the data. This distribution has a central peak and is somewhat skewed to the right.

You can also find the median fairly easily because you can follow the individual values from the smallest to the largest, and find the middle one.

Stem-and-leaf plots are most useful for numerical data sets that have 15 to 100 individual data items.

1. **a.** Complete the stem-and-leaf plot for this data:

   19 20 34 25 21 34 14 20 37 35 20 24 35 15 45 42 55

   (prices of hair dryers in three stores)

   b. What is the median?

Stem	Leaf
1	
2	
3	
4	
5	

5 | 4 means 54

2. **a.** Complete the stem-and-leaf plot for this data. This time, the stems are the first two digits of the numbers, and the leaves are the last digits.

   709 700 725 719 750 740 757 745 786 770 728 755

   (monthly rent, in dollars, for one-bedroom apartments in Houston, Texas)

   b. Find the median monthly rent.

   c. Find the interquartile range.

   d. Describe the spread of the distribution (is the data spread out a lot, a medium amount, a little, etc.)

Stem	Leaf
70	
71	
72	
73	
74	
75	
76	
77	
78	
79	

71 | 9 means 719