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Chapter 9: Geometry

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Foreword

Math Mammoth Grade 6 comprises a complete math curriculum for the sixth grade mathematics studies. The curriculum meets and exceeds the Common Core standards.

In sixth grade, we have quite a few topics to study. Some of them, such as fractions and decimals, students are familiar with, but many others are introduced for the first time (e.g. exponents, ratios, percent, integers). The main areas of study in Math Mammoth Grade 6 are:

- An introduction to several algebraic concepts, such as exponents, expressions, and equations;
- Rational numbers: fractions, decimals, and percents;
- Ratios, rates, and problem solving using bar models;
- Geometry: area, volume, and surface area;
- Integers and graphing;
- Statistics: students learn to describe distributions using measures of center and variability.

This book, 6-B, covers number theory topics (chapter 6), fractions (chapter 7), integers (chapter 8), geometry (chapter 9), and statistics (chapter 10). The rest of the topics are covered in the 6-A worktext.

Some important points to keep in mind when using the curriculum:

- The two books (parts A and B) are like a "framework", but you still have a lot of liberty in planning your child's studies. For the most part, the chapters in the 6th grade curriculum don't have to be studied in the order presented, but you can choose, for example, to study integers before decimals, or statistics right after ratios.

  Math Mammoth is mastery-based, which means it concentrates on a few major topics at a time, in order to study them in depth. However, you can still use it in a *spiral* manner, if you prefer. Simply have your child study in 2-3 chapters simultaneously. This type of flexible use of the curriculum enables you to truly individualize the instruction for your child.

- Don't automatically assign all the exercises. Use your judgment, trying to assign just enough for your child's needs. You can use the skipped exercises later for review. For most children, I recommend to start out by assigning about half of the available exercises. Adjust as necessary.
- For review, the curriculum includes a worksheet maker (Internet access required), mixed review lessons, additional cumulative review lessons, and the word problems continually require usage of past concepts. Please see more information about review (and other topics) in the FAQ at **https://www.mathmammoth.com/faq-lightblue.php**

I heartily recommend that you view the full user guide for your grade level, available at **https://www.mathmammoth.com/userguides/**

Lastly, you can find free videos matched to the curriculum at **https://www.mathmammoth.com/videos/**

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Prime Factorization, GCF, and LCM

Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we review prime factorization and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over into algebra, where students will factor polynomials. In this chapter, we lay the groundwork for that by using the GCF to factor simple sums, such as 27 + 45. For example, a sum like 27 + 45 factors into 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is in finding the smallest common denominator for adding fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures," and you can let students read more about them by accessing the Internet resources listed below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list below for Internet resources.

Keep in mind that the specific lessons in the chapter can take several days to finish. They are not "daily lessons." Instead, use the general guideline that sixth graders should finish about 2 pages daily or 9-10 pages a week in order to finish the curriculum in about 40 weeks. Also, I recommend not assigning all the exercises by default, but that you use your judgment, and strive to vary the number of assigned exercises according to the student's needs.

Please see the user guide at https://www.mathmammoth.com/userguides/ for more guidance on using and pacing the curriculum.

You can find some free videos for the topics of this chapter at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 6

Helpful Resources on the Internet

PRIMES

Sieve of Eratosthenes
Explore the sieve of Eratosthenes with this virtual online chart.
http://www.visnos.com/demos/sieve-of-eratosthenes

Sieve of Eratosthenes till 400
Click on any number and all its proper multiples will be removed from the table. Requires java.
http://www.hbmeyer.de/eratosiv.htm

Primes, Factors and Divisibility – Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
https://web.archive.org/web/20180319072651/http://www.counton.org/explorer/primes/

Prime Number Calculator
This calculator tests to see if a number is a prime, and tells you its smallest divisor if it is not a prime.
http://www.basic-mathematics.com/prime-number-calculator.html

Prime Numbers as Building Blocks – Euclid's Greatest Discovery
A short video about the fundamental theorem of arithmetic: that each composite number has a unique prime factorization.
http://www.youtube.com/watch?v=5kl28hmhin0

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many there are, and more.
http://primes.utm.edu/

Primality of 1
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

http://primefan.tripod.com/Prime1ProCon.html

PRIME FACTORIZATION

Factorization Forest
For each number you factorize, you will get to grow a tree in your forest! Choose from 6 different trees.
http://mrnussbaum.com/forest/

Factor Trees at Math Playground
Factor numbers to their prime factors using an interactive factor tree, or find the GCF and LCM of numbers.
http://www.mathplayground.com/factortrees.html

MathGoodies Interactive Factor Tree Game
Type in a missing number from the factor tree, and the program will find the other factor and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Free Worksheets for Prime Factorization
Generate free, printable worksheets for prime factorization or for finding all the factors of a given number. Customize the worksheets in various ways (difficulty level, spacing, font size, number of problems.)
http://www.homeschoolmath.net/worksheets/factoring.php

Multiplying Fractions with Cross-Canceling
This fraction worksheet is great for working on multiplying fractions with cross-canceling. The problems may be selected for four different degrees of difficulty.
http://www.math-aids.com/Fractions/Multiplying_Fractions_Cross_Cancel.html

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. There are problems to solve at the end of each chapter, tips, and historical information on how cryptography has been used over the centuries.
http://www.amazon.com/gp/product/156881223X?tag=mathmammoth-20

FACTORS (FOR REVIEW)

Product Game
The players choose factors and the product of those gets colored in on the game board. The player who gets four products in a row wins. You can play against the computer or with a friend. This game can easily be adapted to be played offline, with paper and colored pencils.
http://illuminations.nctm.org/Activity.aspx?id=4213

Sliding Tile Factorization Game
Slide a number over another number to capture it, if it is a factor of the original number. Number 1 is only supposed to be used to capture a prime number.
http://www.visualmathlearning.com/Games/sliding_factors.html

Connect 4 Factors Game
Practice factors with this interactive game. It can be played by one or two players.
http://www.transum.org/Software/Game/Connect4/

GREATEST COMMON FACTOR AND LEAST COMMON MULTIPLE

Fruit Shoot—Greatest Common Factor
Shoot the fruit that has the greatest common factor of two given numbers. Three levels and two different speeds.
http://www.sheppardsoftware.com/mathgames/fractions/GreatestCommonFactor.htm

Fruit Shoot—Least Common Multiple
Shoot the fruit that has the least common multiple of two given numbers. Three levels and two different speeds.
http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonMultiple.htm

Factors and Multiples Jeopardy Game
A jeopardy game where the questions have to do with factors, multiples, prime factorization, GCF, and LCM.
http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html

Factors, LCM, and GCF: Activity from Math Playground
Choose "Find the prime factorization of two numbers, GCF, and LCM." First, you find the prime factorization of two different numbers, using the factor tree. Once that is done, the activity shows you a Venn diagram. Drag the factors of the two numbers into the correct areas, then figure out their GFC and LCM.
http://www.mathplayground.com/factortrees.html

Least Common Multiple Tutorial
An animated tutorial and exercises for the least common multiple from e-learning for Kids.
http://www.e-learningforkids.org/math/lesson/least-common-multiples/

Factors Millionaire Game
A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.
http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Greatest Common Factor at ThatQuiz.org
10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.
http://www.thatquiz.org/tq-r/?-j2-l5-p0

GCF and LCM Quiz
10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.
http://www.thatquiz.org/tq-r/?-j4-l5-p0

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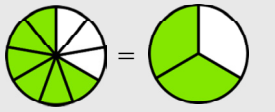
Using Factoring When Simplifying Fractions

On this page, we will review simplifying fractions. Let your teacher decide if you can skip this page.

You are used to seeing the process of **simplifying fractions** like this: →

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.



These slices have been joined together in threes.

$$\frac{6}{9} \overset{\div 3}{=} \frac{2}{3}$$

However, this does NOT change the actual value of the fraction.
It is the "same amount of pie" as it was before. It is just cut differently.

We can simplify a fraction only if its numerator and denominator are divisible by the same number:

We *can* simplify $\frac{25}{65}$ because both 25 and 65 are divisible by 5: →

$$\frac{25}{65} \overset{\div 5}{=} \frac{5}{13}$$

We *cannot* simplify $\frac{11}{20}$ because 11 and 20 do not have any common divisors except 1.

You can simplify in multiple steps. Just start somewhere, using the divisibility tests. The goal is to simplify the fraction to **lowest terms,** where the numerator and the denominator have no common factors.

$$\frac{42}{60} \overset{\div 2}{=} \frac{21}{30} \overset{\div 3}{=} \frac{7}{10}$$

$$\frac{180}{780} \overset{\div 10}{=} \frac{18}{78} \overset{\div 2}{=} \frac{9}{39} \overset{\div 3}{=} \frac{3}{13}$$

1. Simplify the fractions to the lowest terms, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	**b.** $\frac{66}{82}$	**c.** $\frac{69}{99}$
d. $\frac{120}{600}$	**e.** $\frac{38}{52}$	**f.** $\frac{72}{84}$

A new notation

Let's start using a new way to keep track of simplifying fractions. When we simplify a fraction, we'll just cross out the old numerator and denominator and write the new numerator above the fraction and the new denominator below it, as illustrated at the right.

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

The number you divide by (the 4) does ***not*** get indicated in any way! You only think it: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

You may not see any advantage over the "old" method yet, but this shortcut will come in handy soon.

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

3. Simplify the fractions. Write the simplified numerator above and the simplified denominator below the old ones.

a. $\frac{14}{16}$	**b.** $\frac{33}{27}$	**c.** $\frac{12}{26}$	**d.** $\frac{9}{33}$	**e.** $\frac{42}{28}$

Using factoring when simplifying

Carefully study the example on the right where we factor 96/144.

- First we factor (write) 96 as 8 × 12, and 144 as 12 × 12.
- Then we simplify in two steps:
  1. 8 and 12 are both divisible by 4, so they simplify into 2 and 3.
  2. 12 and 12 are divisible by 12, so they simplify into 1 and 1. Essentially, they cancel each other out.

$$\frac{96}{144} = \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{12}}}{\underset{3}{\cancel{12}} \times \underset{1}{\cancel{12}}} = \frac{2}{3}$$

For comparison, the "old" way looks like this:

$$\frac{96}{144} \overset{\div 12}{=} \frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$$

Let's study some more examples. (Remember that they don't show the number that you divide by.)

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \times \overset{2}{\cancel{6}}}{\underset{5}{\cancel{35}} \times \underset{1}{\cancel{3}}} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\overset{3}{\cancel{9}} \times \overset{1}{\cancel{5}}}{\underset{10}{\cancel{30}} \times \underset{1}{\cancel{5}}} = \frac{3}{10}$$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify.

a. $\frac{56}{84} = \frac{7 \times 8}{21 \times 4} =$	**b.** $\frac{54}{144} = \frac{6 \times 9}{12 \times 12} =$	**c.** $\frac{120}{72} = \frac{10 \times \square}{\square \times 9} =$
d. $\frac{80}{48} = \frac{\square \times 8}{\square \times 8} =$	**e.** $\frac{36}{90} = \frac{\quad}{\quad} =$	**f.** $\frac{28}{140} = \frac{\quad}{\quad} =$

Simplify "criss-cross"

These examples are from the previous page. This time the 45 in the numerator has been written as 5 × 9 instead of 9 × 5. We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \times \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \times \underset{1}{\cancel{5}}} = \frac{3}{10}$$

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \times \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \times \underset{5}{\cancel{35}}} = \frac{2}{5}$$

This same concept can be applied to make multiplying fractions easier.

5. Simplify.

a. $\frac{14}{84} = \frac{2 \times 7}{21 \times 4} =$	**b.** $\frac{54}{150} = \frac{9 \times \square}{10 \times \square} =$	**c.** $\frac{138}{36} = \frac{2 \times \square}{\square \times 4} =$
d. $\frac{27}{20} \times \frac{10}{21} =$	**e.** $\frac{75}{90} =$ _____ $=$	**f.** $\frac{48}{45} \times \frac{55}{64} =$

In this example, the simplification is done in two steps. In the first step, 12 and 2 are divided by 2, leaving 6 and 1. In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \times 4}{\underset{1}{\cancel{2}} \times 69} = \frac{\overset{2}{\cancel{6}} \times 4}{1 \times \underset{23}{\cancel{69}}} = \frac{8}{23}$$

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \times \overset{2}{4}}{\underset{1}{\cancel{2}} \times \underset{23}{\cancel{69}}} = \frac{8}{23}$$

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

6. Simplify the fractions to lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	**b.** $\frac{84}{102}$	**c.** $\frac{85}{105}$
d. $\frac{8}{5} \times \frac{8}{20} =$	**e.** $\frac{72}{120}$	**f.** $\frac{104}{240}$
g. $\frac{35}{98}$	**h.** $\frac{5}{7} \times \frac{17}{15} =$	**i.** $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's take two whole numbers. We can then list all the factors of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9, and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

Their common factors are 1, 2, 3, and 6. The greatest common factor is 6.

Here is a **method to find all the factors of a given number**.

Example 2. Find the factors (divisors) of 36.

We check if 36 is divisible by 1, 2, 3, 4, 5, and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us two factors of 36: both the smallest (**1**) and the largest (**36**).
- Next, 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has met the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

3. List all of the factors of the given numbers.

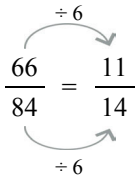
a. 44	**b.** 66
c. 28	**d.** 56
e. 100	**f.** 45

4. Find the greatest common factor of the given numbers. Your work above will help!

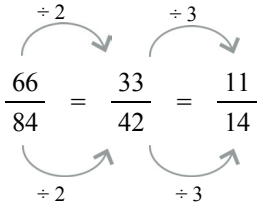
a. 44 and 66	**b.** 100 and 28	**c.** 45 and 100	**d.** 45 and 66
e. 28 and 44	**f.** 56 and 28	**g.** 56 and 100	**h.** 45 and 28

Example 3. What is the greatest common factor useful for?

It can be used to simplify fractions. For example, let's say you know that the GCF of 66 and 84 is 6. Then, to simplify the fraction 66/84 to lowest terms, you divide both the numerator and the denominator by 6. →



However, it is *not* necessary to use the GCF when simplifying fractions. You can always simplify in several steps. See the example at the right. →



Or, you can *simplify by factoring*, like we did in the previous lesson:

$$\frac{66}{84} = \frac{6 \cdot 11}{7 \cdot 6 \cdot 2} = \frac{11}{14}.$$

In fact, these other methods might be quicker than using the GCF.

5. Simplify these fractions, if possible. Your work in the previous exercises can help!

a. $\frac{48}{66}$ **b.** $\frac{42}{44}$ **c.** $\frac{42}{48}$ **d.** $\frac{99}{60}$

e. $\frac{48}{100}$ **f.** $\frac{100}{99}$ **g.** $\frac{56}{28}$ **h.** $\frac{44}{99}$

Using prime factorization to find the greatest common factor (optional)

Another, more efficient way to find the GCF of two or more numbers is to use the prime factorizations of the numbers to find *all* of the common prime factors. The product of those common prime factors forms the GCF.

Example 4. Find the GCF of 48 and 84.

The prime factorizations are: $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ and $84 = 2 \cdot 2 \cdot 3 \cdot 7$.

We see that the common prime factors are 2 and 2 and 3. Therefore, the GCF is $2 \cdot 2 \cdot 3 = 12$.

Example 5. Find the GCF of 75, 105, and 125.

The prime factorizations are: $75 = 3 \cdot 5 \cdot 5$, $105 = 3 \cdot 5 \cdot 7$, and $150 = 2 \cdot 3 \cdot 5 \cdot 5$.

The common prime factors for all of them are 3 and 5. Therefore, the GCF of these three numbers is $3 \cdot 5 = 15$.

6. Find the greatest common factor of the numbers.

a. 120 and 66	**b.** 36 and 136
c. 98 and 76	**d.** 132 and 72
e. 45 and 76	**f.** 64 and 120

7. Find the greatest common factor of the given numbers.

a. 75, 25, and 90	**b.** 54, 36, and 40
c. 18, 24, and 36	**d.** 72, 60, and 48

Find the greatest common factor of 187 and 264.

Puzzle Corner

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Chapter 7: Fractions
Introduction

This chapter begins with a review of fraction arithmetic from fifth grade—specifically, addition, subtraction, simplification, and multiplication of fractions. Then it focuses on the new topic: division of fractions.

The introductory lesson on the division of fractions presents the concept of reciprocal numbers and ties the reciprocity relationship to the idea that division is the appropriate operation to solve questions of the form, "How many times does this number fit into that number?" For example, we can write a division from the question, "How many times does 1/3 fit into 1?" The answer is, obviously, 3 times. So we can write the division $1 \div (1/3) = 3$ and the multiplication $3 \times (1/3) = 1$. These two numbers, 3/1 and 1/3, are reciprocal numbers because their product is 1.

Students learn to solve questions like that through using visual models and writing division sentences that match them. The eventual goal is to arrive at the shortcut for fraction division—that each division can be changed into a multiplication by taking the reciprocal of the divisor, which is often called the "invert (flip)-and-multiply" rule.

However, that "rule" is just a shortcut. It is necessary to memorize it, but memorizing a shortcut doesn't help students make sense conceptually out of the division of fractions—they also need to study the concept of division and use visual models to better understand the process involved.

In two lessons that follow, students apply what they have learned to solve problems involving fractions or fractional parts. A lot of the problems in these lessons are review in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but many involve the division of fractions, thus incorporating the new concept presented in this chapter.

Consider mixing the lessons from this chapter (or from some other chapter) with the lessons from the geometry chapter (which is a fairly long chapter). For example, the student could study these topics and geometry on alternate days, or study a little from both each day. Such, somewhat spiral, usage of the curriculum can help prevent boredom, and also to help students retain the concepts better.

The Lessons in Chapter 7

Helpful Resources on the Internet

ADDITION AND SUBTRACTION

Fraction Videos 1: Addition and Subtraction
My own videos that cover equivalent fractions, addition and subtraction of fractions and of mixed numbers.
http://www.mathmammoth.com/videos/fractions_1.php

Add Fractions Quiz
Use a pencil and paper to help you solve these fraction calculations involving the four operations.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Fractions.asp?Level=3

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure for adding two unlike fractions. A free registration is required.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators

Add and Subtract Mixed Numbers
Practice adding and subtracting mixed numbers with different denominators. Regrouping is required.
https://khanacademy.org/math/cc-fifth-grade-math/cc-5th-fractions-topic/tcc-5th-add-sub-mix-num-w-unlike-den/e/adding-and-subtracting-mixed-numbers-with-unlike-denominators-2

Drop Zone
Practice making a sum of one using fractions in this interactive online activity.
https://www.brainpop.com/games/dropzone/

Add and Subtract Fractions Word Problems
Practice solving fraction addition and subtraction word problems. The fractions in these problems have unlike denominators.
https://khanacademy.org/math/in-sixth-grade-math/fractions-1/addition-subtraction-fractions/e/adding-and-subtracting-fractions-with-unlike-denominators-word-problems

Old Egyptian Fractions
Puzzles to solve: Add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

MULTIPLICATION AND DIVISION

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
http://www.mathmammoth.com/videos/fractions_2.php

Multiply Fractions Jeopardy
A jeopardy-style game. Choose a question by clicking on the tile that shows the number of points you will win.
http://www.quia.com/cb/95583.html

Fraction of a Number
Practice finding fractional parts of various numbers in this interactive online exercise.
https://www.mathplayground.com/fractions_fractionof.html

Multiply Mixed Numbers Quiz
A self-check quiz requiring application of the four operations applied to mixed numbers.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Mixed_Numbers.asp?Level=3

Multiply Fractions Word Problems
Solve and interpret fraction multiplication word problems in this interactive online exercise.
https://www.khanacademy.org/math/in-seventh-grade-math/fractions-decimals/multiplicaiton-fractions/e/multiplying-fractions-by-fractions-word-problems

Interactive Area Model for the Multiplication of Fractions
In this interactive activity, you will learn how to use area models to multiply fractions.
https://www.learner.org/courses/learningmath/number/session9/part_a/try.html

Multiplying Fractions with Cross-Canceling Worksheets
Create customized worksheets for multiplication of fractions with cross-canceling.
http://www.math-aids.com/Fractions/Multiplying_Fractions_Cross_Cancel.html

Math Basketball - Dividing Fractions Game
First make a basket, and then you get to solve a fraction division problem with multiple-choice answers.
http://www.math-play.com/math-basketball-dividing-fractions-game/math-basketball-dividing-fractions-game.html

Soccer Math - Dividing Fractions Game
In order to kick the ball and score points, you first have to answer math problems correctly.
http://www.math-play.com/soccer-math-dividing-fractions-game/soccer-math-dividing-fractions-game.html

Divide Fractions "Strict"
Enter the values into the calculator that are shown in the illustrations to divide the mixed numbers.
http://www.visualfractions.com/DivideStrict/

Divide Fractions Quiz
Solve the problems in this online quiz with the help of a pencil and paper.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Fractions.asp?Level=8

Dividing Fractions Word Problems
Practice solving word problems by dividing fractions by fractions.
https://www.khanacademy.org/math/on-sixth-grade-math/on-number-sense-numeration/on-fractions/e/dividing-fractions-by-fractions-word-problems

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

WORD PROBLEMS

Thinking Blocks - Fractions
Model and solve fraction word problems. Choose "Remainders" or "¾ of ⅔" to practice the concepts studied in the lesson *Problem Solving with Fractions 1*.
http://www.mathplayground.com/tb_fractions/index.html

Thinking Blocks - Fractions
Model and solve fraction word problems. Choose "Find the total or part" to practice the concepts studied in the lesson *Problem Solving with Fractions 2*.
http://www.mathplayground.com/tb_fractions/index.html

GENERAL

Fraction Games
These fun fractions games reinforce ordering, identifying, converting and drawing fractions, as well as equivalent fractions and fractions operations.
http://mrnussbaum.com/fraction-games/

Fraction Games at Sheppard Software
Games for practicing adding and subtracting fractions, simplifying fractions, and finding equivalent fractions and the fraction of a set.
http://www.sheppardsoftware.com/mathgames/menus/fractions.htm

Numerate Game for Fractions
The object of this two-player game is to form equations using the available tiles. Drag tiles onto the board to form an equation and click the "confirm" button.
http://www.transum.org/Maths/Game/Numerate/Default.asp?Level=3

Fraction Calculations Quiz
Practice fraction arithmetic with all four operations in this interactive online quiz.
http://www.transum.org/software/SW/Starter_of_the_day/Students/Fractions.asp?Level=9

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages. The activity includes several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the representations of the fractions and the models.
http://illuminations.nctm.org/Activity.aspx?id=3519

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells you if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Fractions Workshop
Reinforce your fraction skills with this interactive activity. Choose which area you would like to practice and the number of problems.
http://mrnussbaum.com/fractions-workshop-2/

Conceptua Math Fractions Tools
Free and interactive tools for fractions: identify them, add or subtract them, estimate with them, compare them, find equivalent fractions, multiply or divide them, find common denominators, and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, *etc.* that allow students to develop a conceptual understanding of fractions. A free registration is required.
http://www.conceptuamath.com/app/tool-library

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos to explain all of the basic topics in fractions.
http://www.mathexpression.com/learning-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets students interact with those.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

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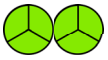
Dividing Fractions: Reciprocal Numbers

First, let's review a little.

How many times does one number go into another?

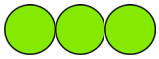
From this situation, you can always write a division, even if the numbers are fractions!

How many times does  go into  ?

Three times. We write the division: $2 \div \frac{2}{3} = 3.$

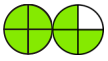
Then check the division: $3 \times \frac{2}{3} = \frac{6}{3} = 2.$

1. Solve. Write a division. Then write a multiplication that checks your division.

a. How many times does  go into  ?

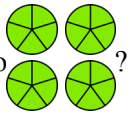
$3 \div \frac{1}{3} =$ _____

Check: ____ $\times \frac{1}{3} =$

b. How many times does  go into  ?

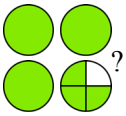
$\div$  = _____

Check: ____ $\times$  =

c. How many times does  go into  ?

$\div$  = _____

Check:

d. How many times does  go into  ?

$\div$  = _____

Check:

2. Solve. Think how many times the fraction goes into the whole number. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$$5 \div \frac{1}{4} \quad \downarrow \downarrow \quad 5 \times 4 = 20$$

$$3 \div \frac{1}{8} \quad \downarrow \downarrow \quad 3 \times 8 = 24$$

$$9 \div \frac{1}{7} \quad \downarrow \downarrow \quad 9 \times 7 = 63$$

That is, *multiply the number by the reciprocal of the divisor*. Notice that 1/4 inverted (upside down) is 4/1 or simply 4. We call 1/4 and 4 **reciprocal numbers,** or just **reciprocals**. So the shortcut is: *multiply by the reciprocal of the divisor*.

Does the shortcut make sense to you? For example, consider the problem $5 \div (1/4)$. Since 1/4 goes into 1 exactly four times, it must go into 5 exactly $5 \times 4 = 20$ times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.

$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$.

$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \times 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by inverting the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \times \frac{n}{m} = \frac{n \times m}{m \times n} = 1$.

To find the reciprocal of a mixed number, first write it as a fraction, then invert it.

Since $2\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$.

3. Find the reciprocal numbers. Then write a multiplication with the given number and its reciprocal.

a. $\frac{5}{8}$ $\quad \frac{5}{8} \times \frac{\square}{\square} = 1$

b. $\frac{1}{9}$ $\quad \frac{\square}{\square} \times \frac{\square}{\square} = 1$

c. $1\frac{7}{8}$ $\quad \frac{\square}{\square} \times \frac{\square}{\square} = 1$

d. 32 $\quad 32 \times \frac{\square}{\square} = 1$

e. $2\frac{1}{8}$ $\quad \frac{\square}{\square} \times \frac{\square}{\square} = 1$

4. Write a division sentence to match each multiplication above.

a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$

b. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$

c. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$

d. $\_\_\_ \div \frac{\square}{\square} = \frac{\square}{\square}$

e. $\_\_\_ \div \frac{\square}{\square} = \frac{\square}{\square}$

Read the following explanation and really try to understand it. It is important!

Now let's try to **make some sense visually** out of how reciprocal numbers fit into the division of fractions.

We can think of the division problem **1 ÷ (2/5)** as asking, **"How many times does 2/5 fit into 1?"**

Using pictures: How many times does  go into  ?

From the picture we can see that  goes into  two times, and then we have 1/5 left over.

But how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

That is like trying to fit a TWO-part piece into a hole that holds just ONE part.
Only 1/2 of the two-part piece fits! And 2/5 fits into 1/5 exactly half a time.

So we found that, in total, 2/5 fits into one exactly **2 1/2 times**. We can write the division $1 \div \frac{2}{5} = 2\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \times \frac{2}{5} = 1$. They are reciprocals!

One more example. Thinking of the division problem **1 ÷ (5/7)**, we ask **how many times does 5/7 fit into 1?**

Using pictures: How many times does  go into  ?

From the picture we can see that  goes into  just once, and then we have 2/7 left over.

But how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The five-part piece fits into a hole that is only big enough for two parts just 2/5 of the way.

So 5/7 fits into 1 exactly **1 2/5 times**. The division is $1 \div \frac{5}{7} = 1\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$. Reciprocals again!

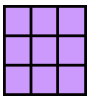
5. Solve. Think how many times the given fraction fits into one whole. Write a division.

a. How many times does  go into  ?

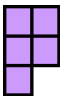
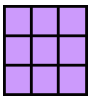
$1 \div \frac{\square}{\square} =$

b. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

c. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

d. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

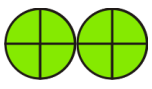
e. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

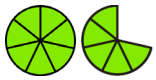
f. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

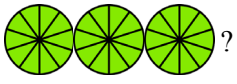
6. Solve. Think how many times the given fraction fits into the other number. Write a division.

a. How many times does  go into  ?

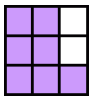
$2 \div \frac{\square}{\square} =$

b. How many times does  go into  ?

$\square\frac{\square}{\square} \div \frac{\square}{\square} =$

c. How many times does  go into  ?

$3 \div \frac{\square}{\square} =$

d. How many times does  go into  ?

$\frac{\square}{\square} \div \frac{\square}{\square} =$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does  go into  ?

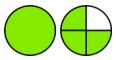
$$\frac{3}{4} \div \frac{1}{3}$$

$$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes,  fits into  a little more than two times.

How many times does  go into  ?

$$\frac{7}{4} \div \frac{2}{5}$$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes.  goes into 1 3/4 over four times.

How many times does  go into  ?

$$\frac{2}{9} \div \frac{2}{7} =$$

$$\frac{\cancel{2}}{9} \times \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because  does not go into  even one full time!

Remember: There are ***two*** **changes** in each calculation:

1. **Change the division into multiplication.**
2. **Use the reciprocal of the divisor.**

7. Solve these division problems using the shortcut. Remember to check to make sure your answer makes sense.

a. $\frac{3}{4} \div 5$

$$\frac{3}{4} \times \frac{1}{5} =$$

b. $\frac{2}{3} \div \frac{6}{7}$

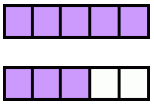
c. $\frac{4}{7} \div \frac{3}{7}$

d. $\frac{2}{3} \div \frac{3}{5}$

e. $4 \div \frac{2}{5}$

f. $\frac{13}{3} \div \frac{1}{5}$

8. **a.** Write a division to match the situation on the right.

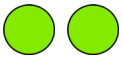
How many times does  fit into  ?

We have 8/5, which is eight pieces, trying to fit into five pieces... so they fit 5/8 of the way.

b. Check your division by multiplication.

9. Fill in.

$2 \div \frac{3}{4} = ?$

Or, how many times does  go into  ?

First, let's solve how many times  goes into  .

Since $1 \div \frac{3}{4} =$  , it goes into one  =  times.

If 3/4 fits into  _____ times, then it fits into  **double that many times**, or _____ times.

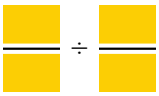
We get the same answer by using the shortcut:

$2 \div \frac{3}{4}$

$\downarrow \quad \downarrow$

$2 \times$  =

10. Fill in.

 $= ?$

Or, how many times does  go into  ?

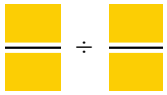
First, let's solve how many times  goes into  .

Since $1 \div$  =  , it goes into one  =  times.

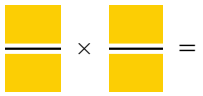
If 2/7 fits into  _____ times, then it fits into  exactly 5/6 as many times as it fits into 1,

which is $\frac{5}{6} \times$  =  =

We get the same answer by using the shortcut:



$\downarrow \quad \downarrow$



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Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants, and integer addition and subtraction. The multiplication and division of integers will be studied in seventh grade.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation, and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules," for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer from three different viewpoints (as a manipulation of counters, as movements on a number line, and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

Note: Addition and subtraction of integers are not included in the Common Core standards for sixth grade. I have included them because I feel students are ready to study them, at least to some extent, at the same time as they study the concepts of integers, absolute value, and ordering integers. In seventh grade, we will study all operations with integers.

The last topic in this chapter is graphing. Students will plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They will notice the patterns in the coordinates of the points and the pattern in the points drawn in the grid and also work through some real-life problems.

The Lessons in Chapter 8

Helpful Resources on the Internet

CONCEPT OF INTEGERS

Free Integers Video Lessons by Maria
A collection of free video lessons on beginner integers topics for grades 6-9.
http://www.mathmammoth.com/videos/integers/integer_lessons.php

Temperature Comparison
Click on two places on the map, and then compare their average temperatures for a certain month.
http://www.teacherled.com/resources/eurotemps/eurotempsload.html

Interpreting Negative Numbers
Practice explaining the meaning of negative numbers in different scenarios in this interactive exercise.
https://www.khanacademy.org/math/in-sixth-grade-math/integers-india/integers-in/e/negative_number_word_problems

Number Balls Game
Click on the rotating number balls in ascending order.
http://www.sheppardsoftware.com/mathgames/numberballs/numberballsAS2.htm

Order Negative Numbers
Drag and drop the numbers in the right order onto the ladder (scroll down the page a bit to see the activity).
http://www.bbc.co.uk/bitesize/ks3/maths/number/negative_numbers/revision/2/

Number Opposites
Practice finding the opposites of numbers in this online activity from Khan Academy.
https://www.khanacademy.org/math/ab-sixth-grade-math/ab-number/integers-negative-numbers/e/number-opposites

Number Opposites Challenge
Practice solving more challenging problems by finding the opposites of numbers in this online activity.
https://www.khanacademy.org/math/arithmetic-home/negative-numbers/number-opposites/e/opposites-on-the-number-line

Interpreting Absolute Value
Practice understanding the meaning of absolute value in real-world scenarios.
https://www.khanacademy.org/math/algebra-basics/basic-alg-foundations/alg-basics-absolute-value-new/e/absolute-value-word-problems

GRAPHING

Billy Bug Returns
Move Billy Bug to the feeding place with given co-ordinates.
http://coolsciencelab.com/graphing_coordinates_harder.html

Graph Mole
A fun game about plotting points in the coordinate plane. Plot the points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Catch the Fly
Wait for the fly to land on the coordinate grid, then type its coordinates, and a frog will eat it.
http://hotmath.com/hotmath_help/games/ctf/ctf_hotmath.swf

Coordinate Grid Quiz from ThatQuiz.org
This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0

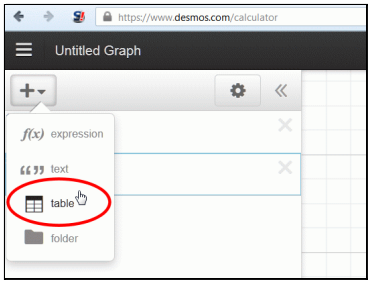
Looking for the Top Quark Game
You receive six quarks that you hide on a grid. Then, use coordinates to find your opponent's hidden quarks.
http://education.jlab.org/topquarkgame

Coordinate Plane Problems
Practice solving word problems by interpreting the meaning of points plotted on an xy co-ordinate system.
https://www.khanacademy.org/math/on-sixth-grade-math/on-geometry-spatial-sense/on-coordinate-plane/e/coordinate-plane-word-problems

Reflecting Points
Practice reflecting points across axes on the xy co-ordinate plane in this interactive online exercise.
https://www.khanacademy.org/math/on-sixth-grade-math/on-geometry-spatial-sense/on-coordinate-plane/e/reflecting-points

Distance Between Two Points
Practice finding the distance between two points on the co-ordinate plane that share the same x or y co-ordinate.
https://www.khanacademy.org/math/ab-sixth-grade-math/shape-space/ab-coordinate-plane/e/relative-position-on-the-coordinate-plane

Desmos Graphing Calculator
A versatile, easy-to-use, and free graphing calculator. To practice plotting points and lines as learned in this chapter, add an item from the "+" button and choose "table." Fill in x and y values, and Desmos will plot the points. You can then type the equation of the line in the form $y =$ (something), such as $y = 2x$, and check if the line goes through your points.
https://www.desmos.com/calculator



Meta-Calculator 2.0
Choose "Graphing Calculator". You can enter an equation to be graphed, or choose "plot points" from the stop menu to enter individual points.
http://www.meta-calculator.com/online/

Free worksheets for the Coordinate Grid
Generate printable worksheets for plotting points and shapes and for moving and reflecting shapes in the coordinate grid. Options include limiting to the first or all quadrants, scaling, image size, workspace, and border.
http://www.homeschoolmath.net/worksheets/coordinate_grid.php

Graphing Lines Quiz
Improve your graphing skills with this 10-question online quiz.
https://www.thatquiz.org/tq-0/?-j10g-l2-p0

ADDITION AND SUBTRACTION

Number Line Integer Addition
Click on the addition sentence on the fruit that matches the jumps shown on the number line. Choose level 3 to practice concepts studied in this chapter.
http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_integer.htm

Number Line Integer Subtraction
Click on the subtraction sentence on the fruit that matches the jumps shown on the number line. Choose level 3 to practice concepts studied in this chapter.
http://www.sheppardsoftware.com/mathgames/integers/FS_NumberLine_int_minus.htm

Number-Line Jump Maker
Use this interactive number-line jump maker tool to practice making jumps of all sizes. Move the slider up or down to change the number line.
http://www.ictgames.com/numberlineJumpMaker/

Temperature Map
Answer questions about temperature in various places using information given to you on a map.
http://mrnussbaum.com/temperature-map-play/

Add Integers Quiz
Practice adding negative and positive numbers in this 10-question online quiz.
https://www.thatquiz.org/tq-1/?-j4101-la-p0

Integers Conundrum
Can you solve the conundrum? Each number is the sum of the two numbers directly beneath it.
http://www.mathplayground.com/number_conundrum/number_conundrum_integers.html

Integer Tilt
Use the arrow keys to guide the falling blocks to the side that they need to land on in order to keep the bar balanced.
http://www.hoodamath.com/games/integertilt2.html

Integer War Card Game
In this card game, students work in pairs to compare integers, as well as add, subtract, or multiply, depending on which operation is being practiced.
http://www.mathfilefoldergames.com/integer-war/

Red and Black TripleMatch Game for Adding Integers
This is a fun card game for 2-5 people to practice adding integers.
http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html

Casey Runner
See if you can help Casey reach the finish line by correctly adding or subtracting negative and positive numbers.
http://www.mathsisfun.com/numbers/casey-runner.html

Subtract Integers Quiz
Practice subtracting integers in this interactive 10-question quiz.
https://www.thatquiz.org/tq-1/?-j4102-lc-p0

Missing Numbers Quiz
Find the missing numbers in this 10-question quiz that practices addition and subtraction of integers.
https://www.thatquiz.org/tq-1/?-j113-lc-p0

GENERAL

Create Integers Worksheets
Use the basic operations worksheet generator to make worksheets for integers within a certain range of negative to positive numbers.
http://www.homeschoolmath.net/worksheets/basic-operations-worksheets.php

Free Downloadable Integer Fact Sheets
http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf
http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf

The History of Negative Numbers
Although they seem normal to us now, in the past negative numbers have spurred controversy and been called "fictitious" or worse.
http://nrich.maths.org/5961
http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2

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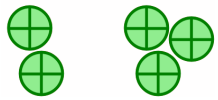
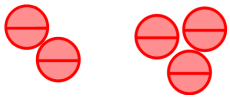
Adding Integers: Counters

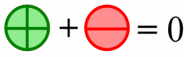
Addition of integers can be modeled using **counters.** We will use green counters with a "+" sign for positives and red counters with a "–" sign for negatives.



Here we have the sum 2 + 3. There is a group of 2 positives and another of 3 positives.



This picture shows the sum (−2) + (−3). We *add* negatives and negatives. In total, there are five negatives, so the sum is −5.



$1 + (-1) = 0$

One positive counter and one negative counter *cancel* each other. In other words, their sum is zero!

$2 + (-2) = 0$

Two negatives and two positives also cancel each other. Their sum is zero.

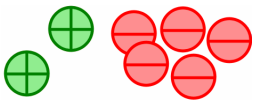
$3 + (-1) = 2$

Here, one "positive-negative" pair is canceled (you can cross it out!). We are left with 2 positives.

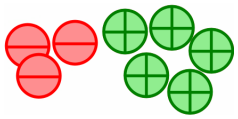
$(-4) + 3 = -1$

Now the negatives outweigh the positives. Pair up three negatives with three positives. Those cancel out. There is still one negative left.

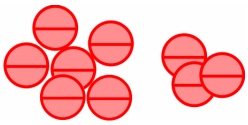
1. Refer to the pictures and add. Remember each "positive-negative" pair is canceled.



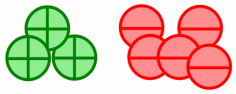
a. 2 + (−5) = _____



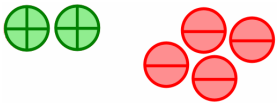
b. (−3) + 5 = _____



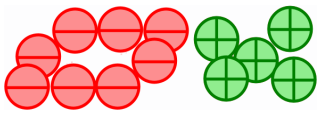
c. (−6) + (−3) = _____



d. 3 + (−5) = _____

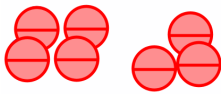


e. 2 + (−4) = _____

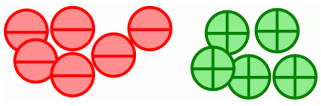


f. (−8) + 5 = _____

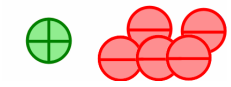
2. Write addition sentences (equations) to match the pictures.



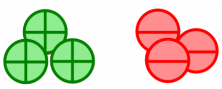
a.



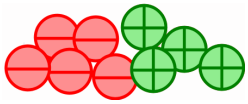
b.



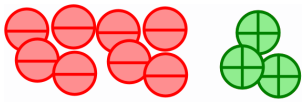
c.



d.

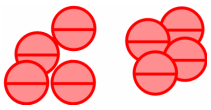


e.



f.

A note on notation



We can write an elevated minus sign to indicate a negative number: $^{-}4$.
Or we can write it with a minus sign and parentheses: (-4).
We can even write it without the parentheses if the meaning is clear: -4.

So $^{-}4 + {}^{-}4 = {}^{-}8$ is the same as $(-4) + (-4) = (-8)$, which is the same as $-4 + (-4) = -8$.

You ***should*** write the parentheses if you have + and −, or two − signs, next to each other.

So, do *not* write "8 + − 4"; write "8 + (−4)." And do not write "3 − −3"; write "3 − (−3)."

3. Think of the counters. Add.

a. $7 + (-8) =$ $(-7) + 8 =$	**b.** $(-7) + (-8) =$ $7 + 8 =$	**c.** $5 + (-7) =$ $7 + (-5) =$	**d.** $50 + (-20) =$ $10 + (-40) =$
e. $^{-}2 + {}^{-}4 =$ $^{-}6 + 6 =$	**f.** $10 + {}^{-}1 =$ $^{-}10 + {}^{-}1 =$	**g.** $^{-}8 + 2 =$ $^{-}8 + {}^{-}2 =$	**h.** $^{-}9 + {}^{-}1 =$ $9 + {}^{-}1 =$

4. Rewrite these sentences using symbols, and solve the resulting sums.

a. The sum of seven positives and five negatives.

b. Add −3 and −11.

c. Positive 100 and negative 15 added together.

5. Write a sum for each situation and solve it.

a. Your checking account is overdrawn by $50. (This means your account is negative). Then you deposit $60. What is the balance in your account now?

b. Hannah owed $20 to her mom. Then, she borrowed $15 more from her mom. What is Hannah's "balance" now?

6. Consider the four expressions $2 + 6$, $(-2) + (-6)$, $(-2) + 6$, and $2 + (-6)$. Write these expressions in order from the one with **least** value to the one with **greatest** value.

7. Find the number that is missing from the equations.

a. $-3 +$ _____ $= -7$	**b.** $-3 +$ _____ $= 3$	**c.** $3 +$ _____ $= (-7)$
d. _____ $+ (-15) = -22$	**e.** $2 +$ _____ $= -5$	**f.** _____ $+ (-5) = 0$

Comparing number line jumps and counters

We can think of $-5 + (-3)$ as five negatives and three negatives, totaling 8 negatives or -8. We also know that $-5 - 3$ is like starting at -5 and jumping three steps towards the left on the number line, ending at -8.

Since both have the same answer, the two expressions $-5 + (-3)$ and $-5 - 3$ are equal:

$$-5 + (-3) = -5 - 3$$

It is as if the "+ −" in the middle is changed into a single − sign. This, indeed, is a *shortcut*!

Similarly, $2 + (-7)$ is the same as $2 - 7$. Either (1) think of having 2 positive and 7 negative counters, totaling 5 negatives, (2) or think of being at 2 and taking 7 steps to the left, ending at -5.

When solving integer problems, you can think of number line jumps or of counters, whichever is easier.

8. Compare how $-7 + 4$ is modeled on the number line and with counters.

a. On the number line, $-7 + 4$ is like starting at _____, and moving _____ steps to the _____, ending at _____.

b. With counters, $-7 + 4$ is like _____ negatives and _____ positives added together. We can form _____ negative-positive pairs that cancel, and what is left is ____ negatives.

9. Add.

a. $4 + (-10) =$	**b.** $-8 + (-8) =$	**c.** $-5 + (-7) =$	**d.** $11 + (-2) =$
$-6 + 8 =$	$7 + (-8) =$	$12 + (-5) =$	$-10 + 20 =$

10. **a.** Find the value of the expression $x + (-4)$ for four different values of x. You can choose the values.

b. For which value of x does the expression $x + (-4)$ have the value 0?

11. Solve the problems, and observe the patterns.

a. $3 - 2 =$	**b.** $^{-}7 - 0 =$	**c.** $^{-}5 + 0 =$	**d.** $^{-}6 + 6 =$
$3 - 3 =$	$^{-}7 - 1 =$	$^{-}5 + 1 =$	$^{-}6 + 7 =$
$3 - 4 =$	$^{-}7 - 2 =$	$^{-}5 + 2 =$	$^{-}6 + 8 =$
$3 - 5 =$	$^{-}7 - 3 =$	$^{-}5 + 3 =$	$^{-}6 + 9 =$
$3 - 6 =$	$^{-}7 - 4 =$	$^{-}5 + 4 =$	$^{-}6 + 10 =$

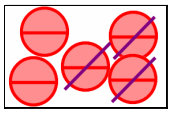
Subtracting a Negative Integer

We have already looked at such subtractions as 3 − 5 or −2 − 8, which you can think of as number line jumps. But what about **subtracting a negative integer?** What is 5 − (−4)? Or (−5) − (−3)?

Let's look at this kind of expression with a "double negative" in several different ways.

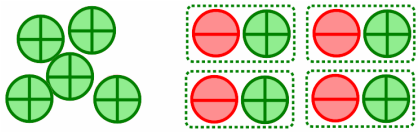
1. Subtraction as "taking away":

We can model subtracting a negative number using counters. (−5) − (−3) means we start with 5 negative counters, and then we *take away* 3 negative counters. That leaves 2 negatives, or −2.



5 − (−4) cannot easily be modeled that way, because it is hard to take away 4 negative counters when we do not have any negative counters to start with. But you *could* do it this way:

Start out with 5 positives. Then *add* four positive-negative pairs, which is just adding zero! Now you can take away four negatives. You are left with nine positives.



Start out with 5. Add four positive-negative pairs, which amount to zero.

Lastly, cross out four negatives. You are left with nine positives.

2. Subtracting a negative number as a number line jump:

5 − (−4) is like standing at 5 on the number line, and getting ready to subtract, or go to the left. But, since there is a minus sign in front of the 4, it "turns you around" to face the positive direction (to the right), and you take 4 steps to the right instead. So, 5 − (−4) = 5 + 4 = 9.

(−5) − (−3) is like standing at −5, ready to go to the left, but the minus sign in front of 3 turns you "about face," and you take 3 steps to the right instead. You end up at −2.

3. Subtraction as a difference/distance:

To find the difference between 76 and 329, you subtract 329 − 76 = 253 (the smaller-valued number from the bigger-valued one). If you subtract the numbers the other way, 76 − 329, the answer is −253.

By the same analogy, we can think of 5 − (−4) as meaning the difference (distance) between 5 and −4. From the number line we can see the distance is **9**.

(−5) − (−3) *could* be the distance between −5 and −3, except it has the larger number, −3, subtracted from the smaller number, −5.

If we turn them around, (−3) − (−5) would give us the distance (difference) between those two numbers, which is 2. Then, (−5) − (−3) would be the opposite of that, or −2.

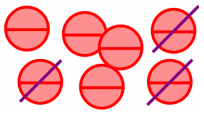
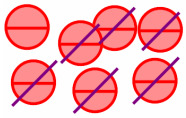
Two negatives make a positive!

You have probably already noticed that, any way you look at it, we can, in effect, replace those two minuses in the middle with a + sign. In other words, 5 − (−4) has the same answer as 5 + 4. And (−5) − (−3) has the same answer as −5 + 3. It may look a bit strange, but it works out really well.

$$5 - (-4)$$
$$5 + 4 = 9$$

$$(-5) - (-3)$$
$$(-5) + 3 = -2$$

1. Write a subtraction sentence to match the pictures.

a. 	**b.** 

2. Write an addition or subtraction sentence to match the number line movements.

a. You are at −2. You jump 6 steps to the left.

b. You are at −2. You get ready to jump 6 steps to the left,
but turn around at the last minute and jump 6 steps to the right instead.

3. Find the distance between the two numbers. Then, write a matching subtraction sentence. To get a positive distance, remember to *subtract the smaller number from the bigger number.*

a. The distance between 3 and −7 is _____. Subtraction: _____ − _____ = _____	**b.** The distance between −3 and −9 is _____. Subtraction: _____ − _____ = _____
c. The distance between −2 and 10 is _____. Subtraction: _____ − _____ = _____	**d.** The distance between −11 and −20 is _____. Subtraction: _____ − _____ = _____

4. Solve. Remember the shortcut: you can change each double minus "− −" into a plus sign.

a. −8 − (−4) =	**b.** −1 − (−5) =	**c.** 12 − (−15) =
8 − (−4) =	1 − (−5) =	−12 + 15 =
−8 + (−4) =	−1 − 5 =	−12 − 15 =
8 + (−4) =	1 − 5 =	12 + (−15) =

5. Connect with a line the expressions that are equal (have the same value).

a.		**b.**	
10 − (−3)	10 − 3	−9 + 2	−9 + (−2)
10 + (−3)	10 + 3	−9 − 2	−9 − (−2)

6. Write an integer addition or subtraction to describe the situations.

a. A roller coaster begins at 90 ft above ground level.
Then it descends 105 feet.

b. Matt has $25. He wants to buy a bicycle from his friend that costs $40.
How much will he owe his friend?

Solve −1 + (−2) − (−3) − 4.

Puzzle Corner

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Chapter 9: Geometry
Introduction

The focus topics of this chapter are:

- the area of triangles
- the area of polygons
- nets and the surface area of prisms and pyramids
- the volume of prisms with sides of fractional length

However, the chapter starts out with some review topics from earlier grades. We review the different types of quadrilaterals and then students do some basic drawing exercises. In these drawing problems, students will need a ruler to measure lengths and a protractor to measure angles.

One focus of the chapter is the area of polygons. To reach this goal, we follow a step-by-step development. First, we study how to find the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Next, we build on the idea that the area of a parallelogram is the same as the area of the related rectangle, and from that we develop the usual formula for the area of a parallelogram as the product of its base times its height. This formula then gives us a way to generalize finding the area of any triangle as *half* of the area of the corresponding parallelogram.

Finally, the area of a polygon can be determined by dividing it into triangles and rectangles, finding the areas of those, and summing them up. Students also practice their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find the lengths of their sides, perimeters, and areas.

Nets and surface area is another major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They also learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with edges of fractional length. (Students have already studied this topic in fifth grade for prisms with edges that are a whole number long.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges ½ or ⅓ unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

There are quite a few videos available to match the lessons in this chapter at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 9

Helpful Resources on the Internet

QUADRILATERALS REVIEW AND DRAWING

Classifying Quadrilaterals Review
Test your knowledge of quadrilaterals with this multiple-choice quiz.
http://www.thegreatmartinicompany.com/geometry/classifying-quadrilaterals-triangles.html

Quadrilateral Types
Identify quadrilaterals based on pictures or attributes in this interactive online quiz.
https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-polygons/e/quadrilateral_types

Quadrilateral Problems in the Co-ordinate Grid
Try these five problems involving quadrilaterals on the co-ordinate plane.
https://www.khanacademy.org/math/6th-engage-ny/engage-6th-module-5/6th-module-5-topic-b/a/rectangles-on-the-coordinate-plane-examples

Drawing Triangles
Drag the vertices of the triangle to construct a new triangle that has the given angle measures.
https://khanacademy.org/math/in-seventh-grade-math/seventh-practical-geometry/new-topic-2015-11-17T01:34:20.426Z/e/constructing-triangles

AREA

Area of a Parallelogram
Explore the area of parallelograms with this activity that goes through step-by-step and shows you how to come up with a formula (in words or algebra) to calculate the area of any parallelogram.
http://www.interactive-maths.com/area-of-a-parallelogram-ggb.html

Area of Parallelograms Quiz
Practice finding the area of parallelograms in this interactive online quiz.
http://www.xpmath.com/forums/arcade.php?do=play&gameid=11

Area of Triangles
This article explains why the area of a triangle is half of base times height. Practice problems are included.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area-triangle/a/area-of-triangle

Triangle Explorer
Practice calculating the area of a triangle using this interactive tool.
http://www.shodor.org/interactivate/activities/TriangleExplorer/

Area of Polygons by Drawing
Explore polygons and their areas with this interactive online tool.
http://www.mathsisfun.com/geometry/area-polygon-drawing.html

Area Tool
Use this tool to determine how the base and the height of a figure can be used to determine its area. Can you find the similarities and differences between the area formulas for trapezoids, parallelograms, and triangles?
http://illuminations.nctm.org/Activity.aspx?id=3567

Area of Shapes on a Grid
Practice finding the areas of triangles and quadrilaterals on grids in this interactive online exercise.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-geometry-topic/cc-6th-area-shapes-grid/e/area-of-triangles-2

Area of Irregular Shapes
Find the area of various irregular shapes in this interactive online activity.
https://www.studyladder.com/games/activity/area-of-irregular-shapes-13136

Free worksheets for the area of triangles, quadrilaterals, and polygons
Generate printable worksheets for finding the area of triangles, parallelograms, trapezoids, or polygons in the coordinate grid. Options include scaling, image size, workspace, border, and more.
http://www.homeschoolmath.net/worksheets/area_triangles_polygons.php

BBC Bitesize - Area
Brief revision (review) "bites," including a few interactive questions, about the area of triangles, parallelograms, and compound shapes.
http://www.bbc.co.uk/bitesize/ks3/maths/measures/area/revision/4/

VOLUME & SURFACE AREA

Solid Shapes and Their Nets
Read about simple solid shapes, click to rotate them, and practice finding their nets.
http://gwydir.demon.co.uk/jo/solid/index.htm

Geometric Solids
Manipulate (rotate) various geometric solids by dragging with the mouse and see their nets.
http://illuminations.nctm.org/Activity.aspx?id=3521

Explore and Play with Prisms
Use the animation at the bottom of the page to explore the properties of four prisms. You can highlight the numbers of faces (F), vertices (V), and edges (E) for each prism.
http://www.learner.org/interactives/geometry/3d_prisms.html

Explore and Play with Pyramids
Use the animation at the bottom of the page to explore the properties of four pyramids. You can highlight the numbers of faces (F), vertices (V), and edges (E) for each prism.
http://www.learner.org/interactives/geometry/3d_pyramids.html

Interactive Solids
Visualize and rotate the following solids: cube, cuboids (a.k.a rectangular prism), tetrahedron, square pyramid, octahedron, triangular prism, and cylinder. Click the button "unfold" to see the net of each solid.
https://www.homeschoolmath.net/interactives/3D_shapes.php

Nets of Polyhedra
Practice matching 2D nets to the 3D shapes that they fold up into in this interactive activity.
https://www.khanacademy.org/math/geometry/hs-geo-foundations/hs-geo-area/e/nets-of-3d-figures

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume

Making Cuboids
An interactive activity to explore the surface area and volume of cuboids, calculate them, or find the volume when the areas of the faces are known.
http://ww.mrbartonmaths.com/resources/keystage3/shape/Volume%20and%20Surface%20Area%20of%20Cuboids.swf

Surface Area of a Rectangular Prism
Practice calculating the surface area of a rectangular prism with this interactive animation.
https://www.learner.org/interactives/geometry/area_surface.html

Surface Area
Practice finding the area of 3D objects in this interactive online activity.
https://www.khanacademy.org/math/in-fifth-grade-math/boxes-sketching/representing-3d-objects-nets/e/surface-areas

Worksheets for the Volume and Surface Area of Rectangular Prisms
Customizable worksheets for volume/surface area of cubes and rectangular prisms. Includes the option of using fractional edge lengths.
http://www.homeschoolmath.net/worksheets/volume_surface_area.php

Volume with Fractions
Practice finding the volume of rectangular prisms that have fractional side lengths in this interactive exercise.
https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-with-fractions/e/volume_with_fractions

Volume Problems with Fractions
Solve volume problems involving objects like fish tanks, truck beds, and refrigerators in this online exercise.
https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-with-fractions/e/volume-word-problems-with-fractions

Exploring Surface Area, Nets, and Volume
This interactive resource allows the user to explore the concepts of surface area, volume, 3D shapes, and nets. Shapes include rectangular and triangular prisms; rectangular and triangular pyramids; cylinders; and cones. The resource includes print activities, solutions, and learning strategies.
http://learnalberta.ca/content/mejhm/?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.SURF&lesson=html/object_interactives/surfaceArea/use_it.html

JUST FOR FUN

Online Kaleidoscope
Create your own kaleidoscope pattern with this interactive tool.
http://www.zefrank.com/dtoy_vs_byokal/

Interactivate! Tessellate
Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!
http://www.shodor.org/interactivate/activities/Tessellate

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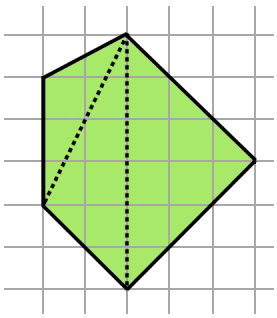
Area of Polygons

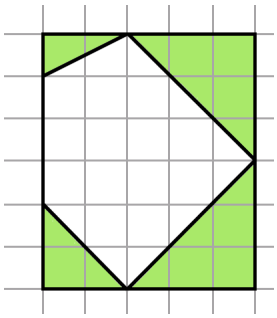
To calculate the **area of a polygon**, all you have to do is divide it into easy shapes, such as rectangles and triangles. Calculate the area of each easy shape separately, and add them to find the total area.

1. This figure is called a _____.



Calculate its area using the three triangles.
For each triangle, use the *vertical* side as the base.

2. Here is another way of calculating the area of the same figure.

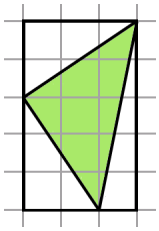


1. Calculate the area of the rectangle that encloses the figure.
2. Calculate the areas of the four shaded triangles.
3. Subtract.

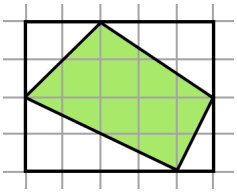
Use this method and verify that you get the same result as above.

3. Find the areas of the shaded figures.

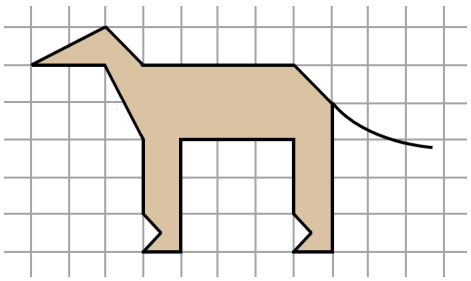
a.



b.



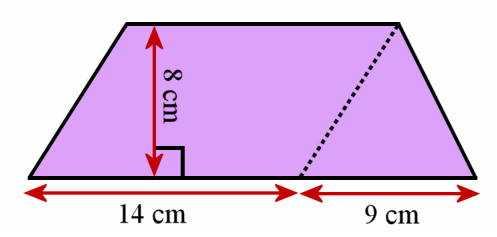
4. **a.** The side of each little square in the drawing on the right is 1 inch. Find the area of the polygon.



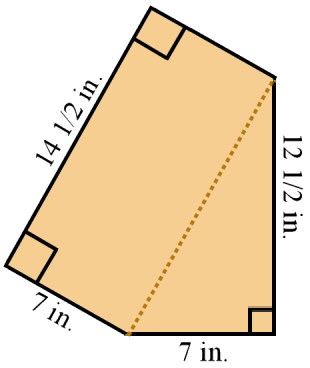
b. Imagine that the side of each little square is 2 inches instead. What is the area now?

5. Calculate the total area of the figures.

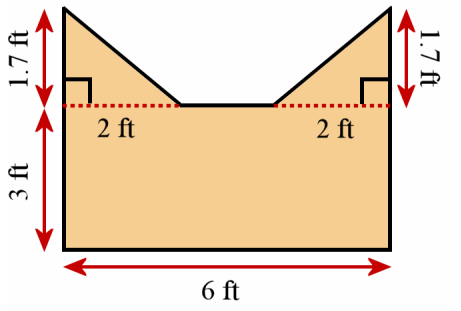
a.



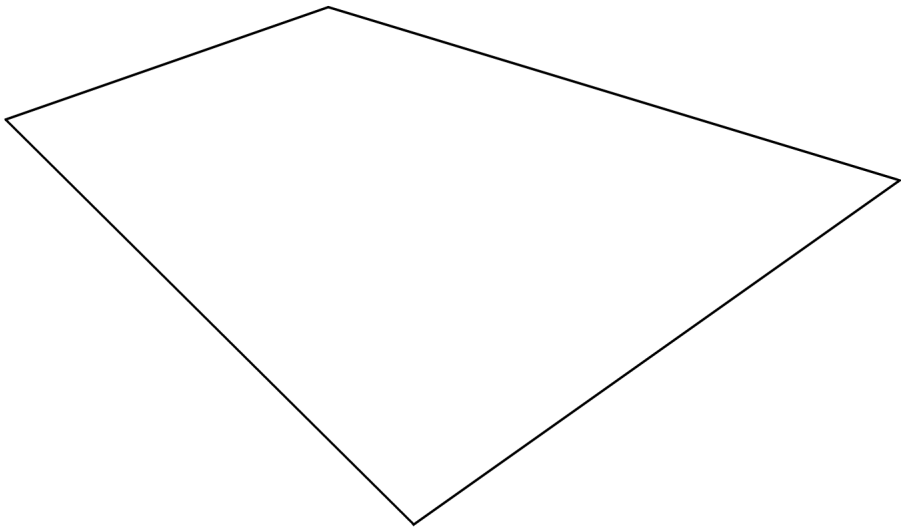
b.



c.

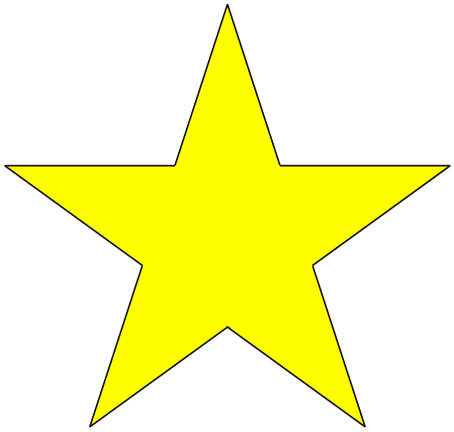


6. Divide this quadrilateral into two triangles, and then find its area in square centimeters. You may use a calculator.



Puzzle Corner

Measure what you need to from this star to find: (**a**) its perimeter in centimeters and (**b**) its area in square centimeters.

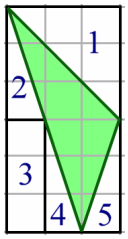


Polygons in the Coordinate Grid

Here is a neat way to **find the area of any polygon whose vertices are points in the grid.**

(1) Draw a rectangle around the polygon.
(2) Divide the area between the polygon and the rectangle into triangles and rectangles.
(3) Calculate those areas.
(4) Subtract the calculated areas from the total area of the large rectangle to find the area of the polygon.

Example. To find the area of the colored triangle, we draw a rectangle around it that is 3 units by 6 units. Then we find the areas marked with 1, 2, 3, 4, and 5:



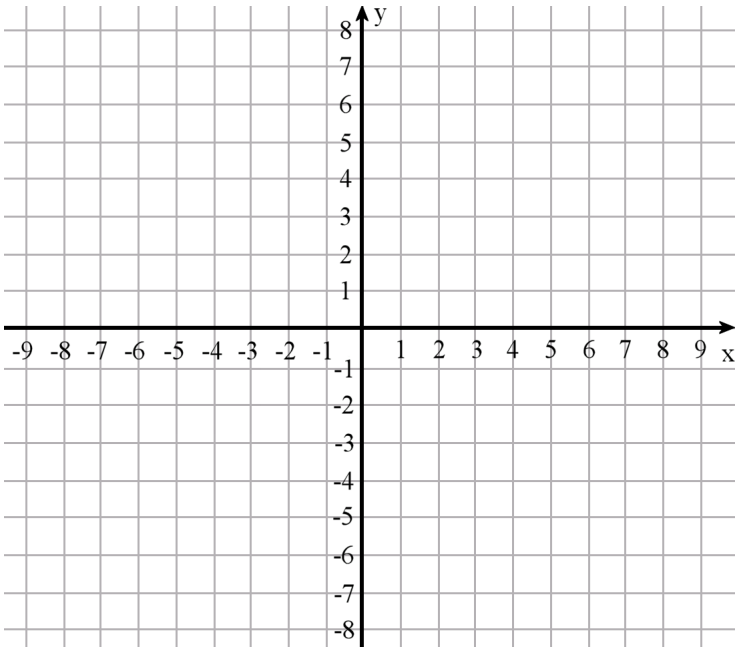
1: a triangle; $3 \times 3 \div 2 = 4.5$ square units
2: a triangle; $1 \times 3 \div 2 = 1.5$ square units
3: a rectangle; $1 \times 3 = 3$ square units
4: a triangle; $1 \times 3 \div 2 = 1.5$ square units
5: a triangle; $1 \times 3 \div 2 = 1.5$ square units
The total for the shapes 1, 2, 3, 4, and 5 is 12 square units.

Therefore, the area of the colored triangle is 18 square units − 12 square units = 6 square units.

1. Use the method above to find the area of these triangles with given vertices.

a. (−8, 7), (−5, 3), and (4, 0).

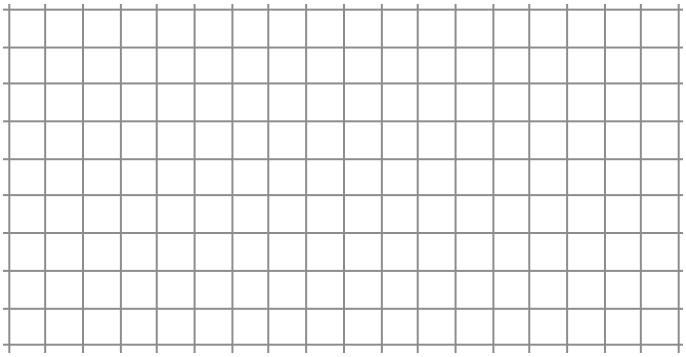
b. (−7, −2), (−2, −1), and (−4, −7).



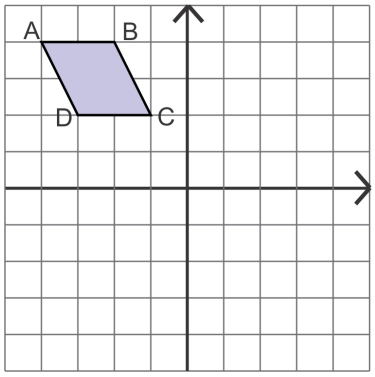
2. Draw a quadrilateral in the grid with vertices (8, 5), (3, 4), (4, −5), (7, −6)

Use the same technique to find its area.

3. Draw any pentagon using grid points as vertices. Then find its area.



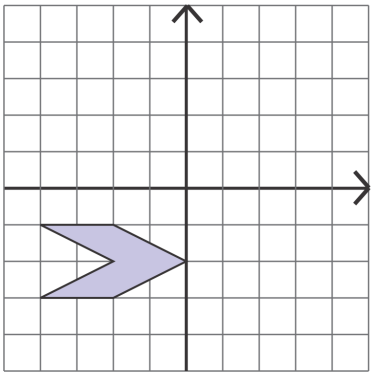
4. Name the polygons, transform them, and find their area.



a. What is this polygon called?

b. Reflect it in the *x*-axis.

c. Find its area.



d. What is this polygon called?

e. Move it 4 units up, and 2 units to the right.

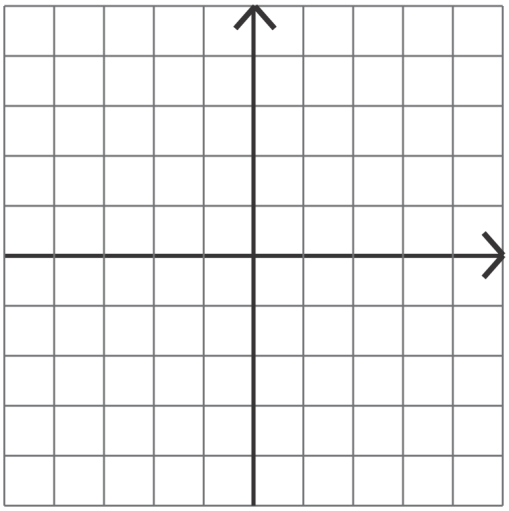
f. Find its area.

5. The points (1, 2.4), (2.4, 1), (2.4, −1), (1, −2.4) are four vertices of a water fountain in the shape of a regular octagon. The other four points are found by reflecting these four in the *y*-axis.

a. Draw the octagon.

b. Find the length of *one* side of the fountain.

c. Find its perimeter.

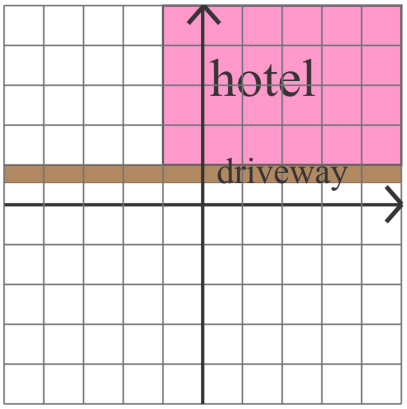
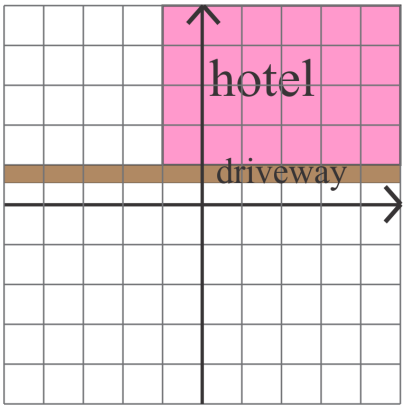
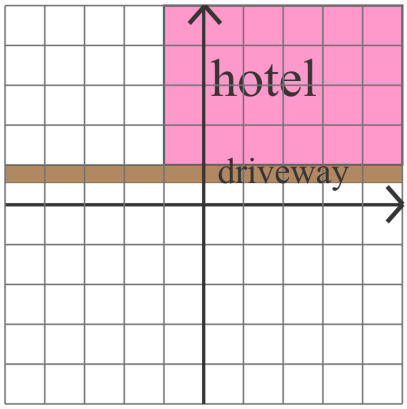


6. A hotel wants to build a swimming pool with a total area between 100 and 140 square meters. One of its sides has to be 12.5 meters.

a. Suggest three different rectangular shapes and draw them in the grids below. Each unit in the grid is 5 m.

b. For each pool, calculate its area.

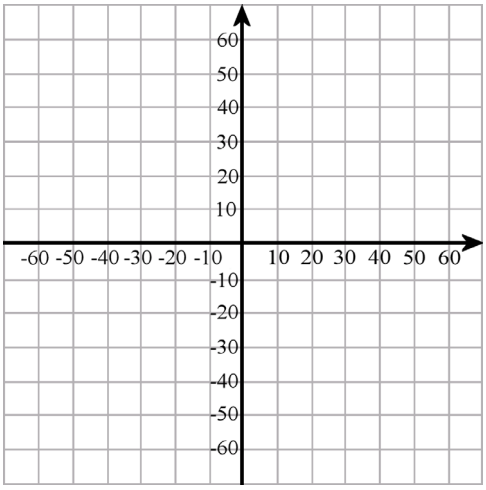
c. For each pool, give the distance from the driveway to the edge of the pool.

Pool Design 1:	**Pool Design 2:**	**Pool Design 3:**
		
Area:	Area:	Area:
Distance from the driveway:	Distance from the driveway:	Distance from the driveway:

Puzzle Corner

Join the following points in order with line segments, and then find the area of the resulting polygon.

(−35, −40), (−35, 40), (−20, 40), (20, −15), (20, 40), (35, 40), (35, −40), (20, −40), (−20, 15), (−20, −40) and (−35, −40)



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Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its center, spread, and overall shape. The shape is read from a graph, such as a dot plot or a bar graph.

Two major concepts when summarizing and analyzing distributions are its center and its variability. First we study the center, in the lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of center to the shape of the data distribution and the type of data.

In the lesson *Measures of Variation* we study range, interquartile range, and mean absolute deviation. The last one takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program, such as Excel.

Then in the next lessons, students learn to make several different kinds of graphs: histograms, boxplots, and stem-and-leaf plots. In those lessons, students continue summarizing distributions by giving their shape, a measure of center, and a measure of variability.

There are some videos available for these topics at https://www.mathmammoth.com/videos/ (choose 6th grade).

The Lessons in Chapter 10

Helpful Resources on the Internet

Statistical Questions
Practice spotting the difference between statistical and non-statistical questions in this interactive online exercise.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6-statistical-questions/e/statistical-questions

Shape of Distributions
Practice explaining the shapes of data distributions in this interactive online activity.
https://www.khanacademy.org/math/probability/data-distributions-a1/displays-of-distributions/e/shape-of-distributions

MEASURES OF CENTRAL TENDENCY

Mean, Median, Mode, and Range
Lesson on how to calculate the mean, median, and mode for a set of data given in different ways.
It also has interactive exercises.
http://www.cimt.org.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

Measures of Center Quiz
Test your knowledge of mean, median, mode, and range with this interactive multiple-choice quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=ana&wcsuffix=8254

Central Measures
This page includes an illustrated lesson about how to find the central values. Scroll down to the bottom and click on the questions to practice the concept.
http://www.mathsisfun.com/data/central-measures.html

Mean and Median from Plots Quiz
Practice finding the mean and median by reading various types of plots in this 10-question online quiz.
https://www.thatquiz.org/tq-5/?-jr0t0-l2-nk-p0

Choosing the Best Measure of Center
Read the lesson and use the practice problems to help you learn how to choose the best measure of center.
https://www.khanacademy.org/math/probability/data-distributions-a1/summarizing-center-distributions/a/choosing-the-best-measure-of-center

How Mean and Median Are Affected When Adding a Data Point
Practice figuring out how the mean and median are affected when a data point is added to, taken from, or shifted within a data set.
https://www.khanacademy.org/math/probability/data-distributions-a1/summarizing-center-distributions/e/effects-of-shifting-adding-removing-data-point

Measures Activity
Enter your own data and the program will calculate mean, median, mode, range and some other statistical measures.
http://www.shodor.org/interactivate/activities/Measures

GCSE Bitesize Mean, Mode and Median Lessons
Tutorials with simple examples.
http://www.bbc.co.uk/schools/gcsebitesize/maths/statistics/measuresofaveragerev1.shtml

Math Goodies Interactive Statistics Lessons
Clear lessons with examples, interactive quiz questions, practice exercises, and challenge exercises over topics that include range, arithmetic mean, non-routine mean, median, and mode.
http://www.mathgoodies.com/lessons/toc_vol8.html

MEASURES OF VARIATION

Interquartile Range (IQR)
Practice finding the interquartile range of small sets of data.
https://www.khanacademy.org/math/probability/data-distributions-a1/summarizing-spread-distributions/e/calculating-the-interquartile-range--iqr-

Mean Deviation
A simple explanation about what the mean absolute deviation is, how to find it, and what it means.
http://www.mathsisfun.com/data/mean-deviation.html

Mean Absolute Deviation
Several videos explaining how to calculate the mean absolute deviation of a data set.
http://www.onlinemathlearning.com/measures-variability-7sp3.html

Working with the Mean Absolute Deviation (MAD)
A tutorial and questions where you are asked to create line plots with a specified mean absolute deviation.
http://www.learner.org/courses/learningmath/data/session5/part_e/working.html

Mean Deviation
This page contains an illustrated lesson on the mean deviation. Scroll down the page to find questions to practice the concept.
http://www.mathsisfun.com/data/mean-deviation.html

Calculate the Mean Absolute Deviation
Practice finding the mean absolute deviation (MAD) of a data set in this interactive online exercise.
https://www.khanacademy.org/math/cc-sixth-grade-math/cc-6th-data-statistics/cc-6-mad/e/calculating-the-mean-absolute-deviation--mad-

GRAPHING AND GRAPHS

Create a Histogram
Explore already-made histograms using given sets of data, or use your own data to make your own. Try changing the interval size (the bin size) to see how it affects the graph.
http://www.shodor.org/interactivate/activities/Histogram/

Make Your Own Histogram
To use this histogram, set your interval (this varies depending on the numerical values of your data), describe the x and y axes, and enter the numerical data. Lastly, click "create graph."
http://mrnussbaum.com/graph/histogram/

Boxplots Quiz
Practice reading boxplots in this 10-question online quiz.
https://www.thatquiz.org/tq-5/?-jo7t0-l3-p0

Understanding Quartiles
Practice understanding the meaning of quartiles of data sets in this interactive online activity.
https://www.khanacademy.org/math/probability/data-distributions-a1/box--whisker-plots-a1/e/interpreting-quartiles-on-box-plots

Make Your Own Boxplot
Enter values from your own data, and this web page creates your box-and-whisker plot for you.
http://www.mrnussbaum.com/graph/bw.htm

Create a Boxplot
You can explore boxplots using the given sets of data, or make your own. Try adding more data to the existing data sets and see how the plot changes.
http://www.shodor.org/interactivate/activities/BoxPlot/

Box-and-Whisker Plots Quiz
Review box-and-whisker plots with this interactive self-check quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=bja&wcsuffix=1203

Measures of Center and Quartiles Quiz from ThatQuiz.org
An online quiz about the measures of center and quartiles in boxplots, stem-and-leaf plots, and dot plots.
http://www.thatquiz.org/tq-5/?-jr0t0-l1-p0

Reading Stem-and-Leaf Plots
Read the stem-and-leaf plot and answer the question correctly.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-stem-leaf/e/reading_stem_and_leaf_plots

Stem-and-Leaf Plots Quiz
Practice the mean and median in this 10-question online quiz involving stem-and-leaf plots.
https://www.thatquiz.org/tq-5/?-ji4t0-l2-p0

Make Your Own Stem-and-Leaf Plot
Enter values from your own data, and this web page creates your stem-and-leaf plot for you.
http://www.mrnussbaum.com/graph/sl.htm

Stem-and-Leaf Plots Quiz
An online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.phschool.com/webcodes10/index.cfm?wcprefix=asa&wcsuffix=0905&area=view

Graphs Quiz from ThatQuiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, boxplot, scattergraph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
http://www.thatquiz.org/tq-5/math/graphs

Create a Graph
Children can create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view and print.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Statistics Interactive Activities from Shodor
A set of interactive tools for exploring and creating different kinds of graphs and plots. You can enter your own data or explore the examples.

http://www.shodor.org/interactivate/activities/BarGraph/

http://www.shodor.org/interactivate/activities/Histogram/

http://www.shodor.org/interactivate/activities/CircleGraph/

http://www.shodor.org/interactivate/activities/MultiBarGraph/

http://www.shodor.org/interactivate/activities/PlopIt/

PlotLy
A comprehensive, collaborative data analysis and graphing tool. Bring data in from anywhere, do the math, graph it with interactive plots (scatter, line, area, bar, histogram, heatmap, box, and more), and export it.
http://plot.ly

Comparing Data Displays
Practice interpreting and comparing dot plots, histograms, and box plots in this interactive online exercise.
https://www.khanacademy.org/math/pre-algebra/pre-algebra-math-reasoning/pre-algebra-frequency-dot-plot/e/comparing-data-displays

Exploring Election Data
This interactive mathematics resource allows students to explore and interpret Alberta provincial election results from 1905 to 2004 using a pictograph, line graph, bar graph, circle graph, or data table. The resource includes print activities, solutions, and learning strategies.
http://learnalberta.ca/content/mejhm/?l=0&ID1=AB.MATH.JR.STAT&ID2=AB.MATH.JR.STAT.DATA&lesson=html/object_interactives/Data_Display/use_it.html

FACTS & FIGURES

GapMinder
Visualizing human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.
http://www.gapminder.org/data/

WorldOdometers
World statistics updated in real time. Useful for general educational purposes - for some stunning facts.
http://www.worldometers.info

UN Data
The United Nations offers the ability to search across its statistical databases, including education, human development, population, trade, and more.
http://data.un.org

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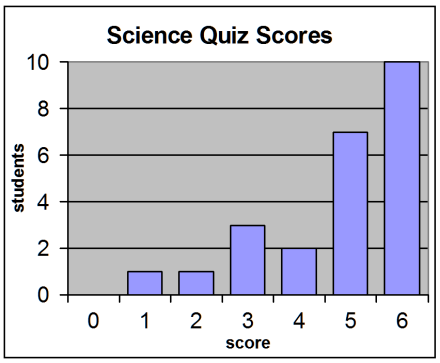
Using Mean, Median, and Mode

Whether you use mean, median, or mode depends both

- on the **type of data** *and*
- on the **shape of distribution.**

Example. This distribution of science quiz scores is heavily skewed (asymmetrical), and its "peak" is at 6. Which of the three measures of center would best describe this distribution?



Let's calculate the mean, median, and mode.

Mode: We can see from the graph that the mode is 6.

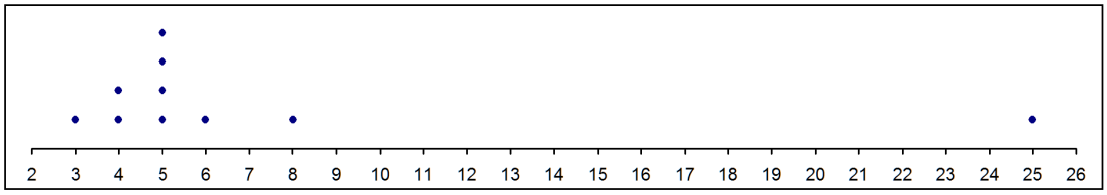
Median: There are 24 students. The students' actual scores are 1, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6.

The median is the average of the 12th and 13th scores, which is 5.

The mean is $\dfrac{1+2+3+3+3+4+4+7\times 5+10\times 6}{24} = 4.79167 \approx 4.79.$

Notice that the mean is less than 5, but the two highest bars on the graph are at 5 and 6. In this case, the mean does *not* describe the peak of the distribution very well because it actually falls outside the peak! Both the median and the mode do describe it well.

1. **a.** Find the mean, median, and mode of this data set: 3, 4, 4, 5, 5, 5, 5, 6, 8, 25.



mean _____ median _____ mode _____

b. Which of the three, mean, median, or mode, best describes the center of this data?

Clearly, either the _____ or the _____ , but *not* the _____!

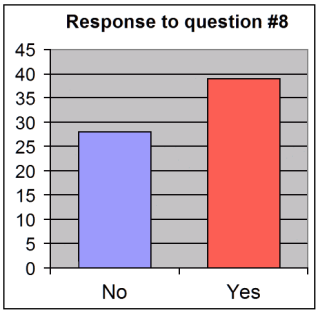
The _____ is off from the central peak of the distribution.

The reason for this is that the data item "25" throws it off. This 25 is very different from the other data items in the set, and could even be a typing error! Such an item is called an **outlier**.

2. The graph shows the response to a certain question in a survey. It was measured as a *yes/no* question. Which of the below are possible to determine? (Mark with an "x").

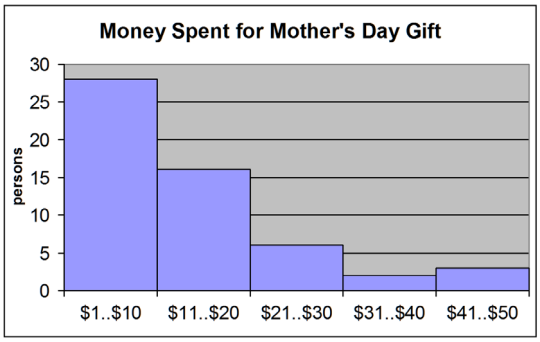
____ mean ____ median ____ mode

Hint: Imagine what the original data that was used to create the graph looks like.



Guidelines for using the mean, median, and mode	Sometimes, the median and the mean do not fall where the peak of the distribution is.
• The *mode* can be used with any type of data. • The *median* can only be used if the data can be put in order. • The *mean* can only be used if the data is numerical.	• The mean works best if the distribution is fairly close to a bell shape and does not have outliers. • If the distribution is very skewed or has outliers, it is better to use median than mean.

3. Judith asked 55 teenagers about how much money they spent to purchase Mother's Day gifts.



a. Which of the numbers \$11 and \$9 is the mean? Which is the median?

b. Would mean or median better describe this data? Why?

c. *Approximately* what percentage of these teenagers spent \$10 or less on a Mother's Day gift?

4. • Name what is being studied (usually the *title* of the graph tells you this).
   • Describe how the data was measured and in what units. For example, the respondents have given numerical answers in dollars. Or perhaps they chose either "yes" or "no."
   • Indicate whether the mean, median, or mode can be calculated. You do not have to find the mean, even when it is possible.

Hint: Think what kind of data was used to create the graph (the original data).

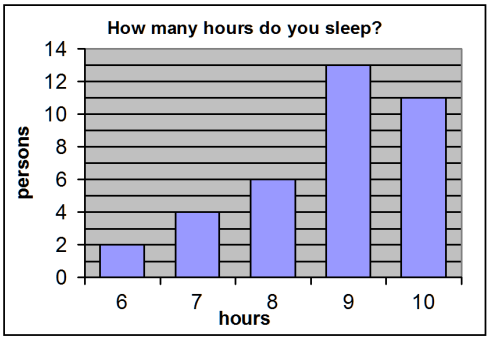
a. What is being measured or studied? _____

How is it measured?

Which are possible? (Mark with an "x").

____ mean ____ median ____ mode

The mode is: _____ The median is: _____



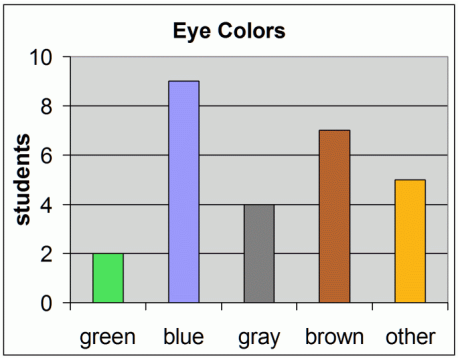
b. What is being measured or studied? _____

How is it measured?

Which are possible? (Mark with an "x").

____ mean ____ median ____ mode

The mode is: _____ The median is: _____



For the following data sets:

- Create a dot plot or a bar graph.
- Name your graph.
- Describe the shape of the distribution.
- Indicate how many observations there are.
- Choose measure(s) of center that describe the peak of the distribution, and calculate them.

5. **a.** The length of words on three pages in a children's storybook:

7 5 6 8 3 6 6 2 4 2 2 3 3 4 4 3 5 5 4
5 4 3 2 5 2 1 4 4 7 5 4 8 3 3 3 3 3 5
5 3 4 2 3 1 6 2 5 4 4 3 4 3 2 8

Here is the same data sorted:

1 1 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 4 4 4
4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 6 6 6 6 7 7 8 8 8

b. A restaurant asked its customers some questions about their food and service. One question was, "How would you rate the meal you ate today?" There were five possible answers: "excellent," "good," "normal," "not so good," and "poor." The customers' responses are listed below:

normal poor excellent good good excellent good
normal not so good excellent good good good
normal normal good excellent good good good
not so good not so good excellent good

Puzzle Corner

Can you find a quick, *mental math* method for calculating the mean for this data set? 102, 94, 99, 105, 96, 107, 101, 104
(the weights of a litter of kittens at birth, in grams)